A Hard-Disk-Drive Servo System Design Using Composite Nonlinear-Feedback Control With Optimal Nonlinear Gain Tuning Methods

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***Abstract*—This paper investigates the design of composite nonlinear-feedback (CNF) control law for a hard-disk-drive (HDD) servo system. First, a scaled nonlinear function is introduced for the CNF control law, in which a parameter is scaled by the error between the amplitude of the target reference and the initial value of the system controlled output. The closed-loop system under the scaled function has robust transient performance to the variation of the amplitude of the target reference. Then, the parameters of the selected nonlinear function are tuned by optimal tuning methods. More specifically, the parameter-tuning problem is formulated as an optimization problem, which can be solved efficiently via numerical methods. The simulation and experimental results show that the control law designed using the new approach yields excellent performance for both track seeking and track following in the HDD servo system.**

***Index Terms*—Hard disk drive (HDD), nonlinear feedback, servo system, tracking control, transient performance.**

I. Introduction

AS AN IMPORTANT and cheap data-storage medium, hard disk drive (HDD) has been widely used not only in computers but also in consumer electronic devices such as cell phones and digital video camera recorders. To improve the performance of the HDD, various control methods are addressed in the literature, e.g., [2], [3], [8], [19]. In the HDD, data are read from or write to the tracks of rotating disks with a read/write (R/W) head. There are two main functions for the HDD servo system, i.e., track seeking and track following. Track seeking is to move the R/W head from the present track to a destination track in minimum time, whereas track following is to maintain the head as close as possible to the destination-track center while data are read from or written to the disk under disturbances [8]. Currently, a typical HDD servo system has two different controllers for track seeking and track following separately. In such a case, mode switching that switches the track-seeking controller and the track-following controller is necessary (please refer to [21], [23], and [24] for model switching control of HDD servo systems). To avoid mode switching, a nonlinear-feedback technique called composite nonlinear feedback (CNF) is introduced to design the HDD servo system [7], [8], [15], [22]. The CNF control technique was first proposed by Lin *et al.* [16] for a class of second-order linear system with input saturation to improve the transient performance of the closed-loop system. The CNF control law consists of a linear-feedback law and a nonlinear-feedback law. The linear part is designed to yield a closed-loop system with a small damping ratio for a quick response. The nonlinear part is introduced to increase the damping ratio of the closed-loop system while the system output approaches the target reference to reduce the overshoot caused by the linear part. In fact, a CNF controller switches smoothly a linear controller that results in a closed-loop system with low damping ratio to another linear controller that results in a closed-loop system with high damping ratio by a smooth nonlinear function. Thus, it is possible to design an HDD servo system with a single CNF controller for both track seeking and track following.

After the work of Lin *et al.* [16], Turner *et al.* [20] extended the CNF control technique to the multivariable systems. Furthermore, Chen *et al.* [7] developed a CNF control to a more general class of systems with measurement feedback. The results in [7] were extended to multivariable system in [12]. More recently, Lan *et al.* [14] extended the CNF control technique to a class of nonlinear systems. Besides the HDD servo systems, the CNF control technique is successfully applied to the helicopter flight-control system [5], servo position systems [9], [10], etc. However, the tuning of the nonlinear function of the CNF control law is not addressed in the literature. In the design of the CNF control law, we need to construct a linear-feedback part and a nonlinear-feedback part. Once the linear-feedback part is fixed, the performance of CNF control relies on the selection of the nonlinear function in the CNF control law. Different nonlinear function will result in different transient performance of the closed-loop system. There are several versions of the nonlinear function in the literature. However, for some of these functions, the change of the amplitude of the reference target may recede the transient performance obviously. The parameters of the nonlinear functions are required to be tuned and saved for a series of references to preserve the transient performance [8]. To avoid this inconvenience and reduce the

Manuscript received December 1, 2008; revised September 2, 2009. First published September 22, 2009; current version published April 14, 2010. This work was supported in part by the Natural Science Foundation of Fujian Province of China under Grant 2008J0033 and in part by the National Natural Science Foundation of China under Grants 60674037 and 60704042.

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Digital Object Identifier 10.1109/TIE.2009.2032205

sensitivity of the parameters to the change of the reference amplitude, the selection of the nonlinear functions is discussed in [13]. We revise the nonlinear function such that the parameter is scaled to the reference amplitude. Simulation result shows that such revision is efficient.

After fixing the form of the nonlinear function, the designer needs to tune the parameters of the nonlinear function to obtain the desired performance. The number of the parameters depends on the form of the nonlinear function. Currently, the parameters are tuned manually by trial and error, the efficiency of tuning the CNF control law relies on the experience of the designer. Can we develop an autotuning method to simplify the design of CNF control law? Our idea is to convert the parameter-tuning problem into a minimization problem of some performance criteria [13]. Since we are concerned about the transient performance of the closed-loop system, we use the traditional criteria of transient response, such as the integral of absolute value of error (IAE) and the integral of time-multiplied absolute value of error (ITAE) [11]. The value of the parameters is given by the solution of an optimization problem. Although it is difficult to solve the optimization problem analytically, we can easily obtain the solution with numerical methods. In this paper, the optimal nonlinear gain tuning method is applied to design the CNF control law for an HDD servo system. Simulation and experimental results show that the HDD servo system has satisfied performance for both track seeking and track following.

II. CNF Design Procedure

Consider a linear system with input saturation

$$\begin{aligned} \dot{x} &= Ax + B\text{sat}(u), \qquad x(0) = x_0 \\ y &= C_1 x \\ h &= C_2 x \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}$ is the control input, $y \in \mathbb{R}^p$ is the measurement output, and $h \in \mathbb{R}$ is the controlled output. A, B, C_1, and C_2 are appropriate dimensional constant matrices, and sat: $\mathbb{R} \to \mathbb{R}$ represents the actuator saturation defined as

$$\text{sat}(u) = \text{sgn}(u) \min\{u_{\max}, |u|\} \tag{2}$$

where $u_{\max}$ is the saturation level of the input. The objective is to design a feedback control law such that the controlled output can track a step reference input asymptotically with a nice transient performance. To design a state-feedback CNF control law, the following assumptions on the system matrices are required:

A1 (A, B) is stabilizable;
A2 (A, B, C_2) is invertible and has no zeros at $s = 0$;
A3 (A, C_1) is detectable.

A state-feedback CNF control law can be constructed by the following step-by-step design procedure:

Step S.1) Design a linear-feedback law

$$u_L = Fx + Gr \tag{3}$$

where r is a step command input and F is designed such that the following are met: 1) $(A + BF)$ is an asymptotically stable matrix, and 2) the closed-loop system $C_2(sI - A - BF)^{-1}B$ has certain desired properties, e.g., having a small damping ratio. Moreover, G is a scalar and is given by

$$G = -\left[C_2(A + BF)^{-1}B\right]^{-1}.$$

Step S.2) Compute

$$\begin{aligned} H &:= \left[1 - F(A + BF)^{-1}B\right] G \\ x_e &:= -(A + BF)^{-1}BGr. \end{aligned}$$

The nonlinear-feedback control law is given by

$$u_N = \rho(r, h)B^{\mathrm{T}}P(x - x_e) \tag{4}$$

where $\rho(r, h)$ is any nonpositive function locally Lipschitz in h, which is used to change the damping ratio of the closed-loop system as the output approaches the step command input. $P > 0$ is the solution of

$$(A + BF)^{\mathrm{T}}P + P(A + BF) = -W$$

for some given $W > 0$.

Step S.3) The linear- and nonlinear-feedback laws derived in the previous steps are now combined to form a CNF controller

$$u = u_L + u_N = Fx + Gr + \rho(r, h)B^{\mathrm{T}}P(x - x_e). \tag{5}$$

We rephrase the following theorem from [7] to show the properties of the closed-loop system comprising with the given plant in (1) and the CNF control law of (5).

Theorem 2.1: Consider the given system in (1), the linear control law of (3), and the CNF control law of (5). For any $\delta \in (0, 1)$, let $c_\delta > 0$ be the largest positive scalar satisfying the following condition:

$$|Fx| \le u_{\max}(1 - \delta) \qquad \forall x \in \mathbf{X}_\delta := \{x : x^{\mathrm{T}}Px \le c_\delta\}. \tag{6}$$

Then, the linear control law of (3) is capable of driving the system controlled output $h(t)$ to track asymptotically a step command input r, provided that the initial state x_0 and r satisfy

$$\tilde{x} := (x_0 - x_e) \in \mathbf{X}_\delta, \qquad |Hr| \le \delta u_{\max}. \tag{7}$$

Furthermore, for any nonpositive function $\rho(r, h)$, locally Lipschitz in h, the CNF law in (5) is capable of driving the system controlled output $h(t)$ to track asymptotically the step command input of amplitude r, provided that the initial state x_0 and r satisfy (7). ■

Remark 2.1: As shown in [7], the measurement-feedback CNF control law can be designed under assumptions A1, A2, and A3. Specifically, a full-order measurement-feedback control law is given by

$$\begin{aligned} \dot{x}_v &= (A + KC_1)x_v + Ky + B\text{sat}(u) \\ u &= Fx_v + Gr + \rho(r, h)B^{\mathrm{T}}P(x_v - x_e) \end{aligned}$$

where F and K are selected such that $A + BF$ and $A + KC_1$ is asymptotically stable. Moreover, if the system (1) can be written as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \mathrm{sat}(u)$$
$$y = [\, I_p \quad 0\,] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$
$$h = C_2 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

then, a reduced-order measurement-feedback control law is given by

$$\begin{aligned} \dot{x}_v =& (A_{22} + K_R A_{12}) x_v + (B_2 + K_R B_1)\mathrm{sat}(u) \\ &+ [A_{22} + K_R A_{11} - (A_{22} + K_R A_{12}) K_R]\, y \\ u =& F \begin{pmatrix} y \\ x_v - K_R y \end{pmatrix} + Gr \\ &+ \rho(r,h) B^{\mathrm{T}} P \left[\begin{pmatrix} y \\ x_v - K_R y \end{pmatrix} - x_e \right] \end{aligned}$$

where K_R is designed such that $A_{22} + K_R A_{12}$ is asymptotically stable. ■

III. Optimal Nonlinear Gain Tuning Methods

To complete the design of the CNF control law, we need to select an appropriate nonlinear function $\rho(r,h)$ such that the closed-loop system has desired transient performance. For this purpose, we need to answer two questions. First, what kind of nonlinear function should it be, or what is the form of the function? Second, how to tune the parameters of the function after given its form? These two questions are answered in the next two sections. In Section III-A, we specify a form of the nonlinear function $\rho(r,h)$. The function is scaled by the initial tracking error such that the transient performance of the closed-loop system is robust to the variation of tracking target. In Section III-B, the tuning problem is transformed to a minimization problem, and the parameters of the nonlinear function can be tuned automatically by solving the minimization problem.

A. Scaled Nonlinear Function

Consider the linear system with input saturation (1) and the state-feedback CNF control law (5). As required in step S.1), the linear-feedback gain F is designed such that the closed-loop system matrix $A + BF$ is asymptotically stable, and the resulting closed-loop system $C_2(sI - A - BF)^{-1}B$ has a small damping ratio. Such an F can be worked out by using some well-studied methods, for example, LQR, H_2, and H_∞ optimization approaches (see, e.g., [1], [6], [18]). The purpose of introducing the nonlinear part $\rho(r,h)B^{\mathrm{T}}P(x - x_e)$ is to increase the damping ratio of the closed-loop system as the system output approaches the target reference to reduce the overshoot caused by the linear part. The difficulty of designing a CNF control law with a desired transient performance is how to select an appropriate nonlinear function $\rho(r,h)$. To guarantee the stability of the closed-loop system, the only requirement on the nonlinear function is $\rho(r,h) \le 0$ for all r and h. However, to obtain the desired transient performance with quick response and small overshoot, $\rho(r,h)$ needs to be selected carefully. We usually choose $\rho(r,h)$ as a function of the tracking error $h - r$, which in most practical situations is known and available for feedback. As a general guideline, $\rho(r,h)$ needs to be selected such that it has the following two properties.

P1 When the controlled output h is far away from the final set point, i.e., when $|h - r|$ is large, $|\rho(r,h)|$ is small, and thus the effect of the nonlinear part of the CNF control law is very limited.

P2 When the controlled output h approaches the set point, i.e., when $|h - r|$ is small, $|\rho(r,h)|$ becomes larger and larger, and the nonlinear part of the CNF control law will become effective.

Remark 3.1: Assume that the quick-response requirement can be achieved by a linear-feedback control law. By taking this linear-feedback control law as the linear part of the CNF control law, the property P1 guarantees that the closed-loop system under the CNF control law satisfies the quick-response requirement. On the other hand, the second property P2 takes effect when the system output approaches the tracking target to increase the damping ratio of the closed-loop system, thus the overshoot can be also reduced. ■

It is clear that the selection of $\rho(r,h)$ is not unique. In fact, there are various versions of $\rho(r,h)$ in the literature. For example, in [16], $\rho(r,h)$ is of the form

$$\rho(r,h) = -\beta e^{-\alpha|h-r|} \tag{8}$$

where $\beta > 0$ and $\alpha > 0$ are tuning parameters. It is easy to see that this nonlinear function (8) has the properties P1 and P2. In fact, $|\rho(r,h)|$ tends to zero as $|h - r|$ tends to infinity, and $|\rho(r,h)|$ becomes larger and larger when $|h - r|$ tends to zero, and reaches its maximal value β when $h = r$. A modified version of (8) is given in [7]

$$\rho(r,h) = -\frac{\beta}{1 - e^{-1}} \left(e^{-|1-(h-h_0)/(r-h_0)|} - e^{-1} \right) \tag{9}$$

where $h_0 = h(0)$ and $\beta > 0$ is a tuning parameter. The function $|\rho(r,h)|$ changes from 0 to β as the tracking error approaches zero. To avoid division by zero when $r = h_0$, (9) is further revised in [12] to the form of

$$\rho(r,h) = -\beta \left| e^{-\alpha|h-r|} - e^{-\alpha|h_0-r|} \right| \tag{10}$$

where $\alpha > 0$ and $\beta > 0$ are tuning parameters. $|\rho(r,h)|$ starts from zero and increases to $\beta|1 - e^{-\alpha|h_0-r|}|$ as the controlled output h approaches to the target reference r. Because the tuning parameters α and β in (8) and (10) are tuned for a fixed target reference r, the CNF control law cannot hold the performance when the magnitude of the target reference r is changed. For example, an overshoot can be observed when the target reference is changed, which will be shown in Case 1 of Example 3.1.

To adapt the variation of tracking targets, we modify the nonlinear function (8) by introducing a scaling parameter α_0

which is determined by r and h_0. The scaled nonlinear function $\rho(r,h)$ is in the form of

$$\rho(r,h) = -\beta e^{-\alpha\alpha_0|h-r|} \tag{11}$$

where

$$\alpha_0 = \begin{cases} \frac{1}{|h_0-r|}, & h_0 \neq r \\ 1, & h_0 = r. \end{cases} \tag{12}$$

Because α_0 changes with different tracking target r, the closed-loop performance is robust to the variation of tracking targets.

Remark 3.2: Without considering the saturation and letting $\tilde{x} = x - x_e$, the closed-loop system is given by

$$\dot{\tilde{x}} = (A + BF - \beta e^{-\alpha r} BB^{\mathrm{T}} P)\tilde{x}.$$

Assume $h_0 = 0$ and $r \neq h_0$. If the nonlinear function $\rho(r,h)$ is given by the unscaled function (8), the action of the nonlinear function $\rho(r,h)$ is smoothly switching the closed-loop system from the initial system

$$\dot{\tilde{x}} = (A + BF - \beta e^{-\alpha} BB^{\mathrm{T}} P)\tilde{x} \tag{13}$$

to the steady-state system

$$\dot{\tilde{x}} = (A + BF - \beta BB^{\mathrm{T}} P)\tilde{x}.$$

On the other hand, if the nonlinear function $\rho(r,h)$ is given by the scaled function (11), the action of the nonlinear function $\rho(r,h)$ is smoothly switching the closed-loop system from the initial system

$$\dot{\tilde{x}} = (A + BF - \beta e^{-\alpha} BB^{\mathrm{T}} P)\tilde{x} \tag{14}$$

to the steady-state system

$$\dot{\tilde{x}} = (A + BF - \beta BB^{\mathrm{T}} P)\tilde{x}.$$

It is clear that the initial system (13) under the unscaled function (8) relies on the tracking target r, but the initial system (14) under the scaled function (11) does not. Thus, the closed-loop system under the scaled function (11) can have much stronger robustness to the variations of tracking target than the one under the unscaled function (8). Similarly, the closed-loop system under the function (9) also has robust performance to the variation of tracking target. In fact, the nonlinear function $\rho(r,h)$ given by (9) is also a scaled function, because

$$\begin{aligned}\rho(r,h) &= -\frac{\beta}{1-e^{-1}}\left(e^{-|1-(h-h_0)/(r-h_0)|} - e^{-1}\right) \\ &= -\frac{\beta}{1-e^{-1}}\left(e^{-|1-\alpha_0(h-h_0)|} - e^{-1}\right)\end{aligned}$$

where $\alpha_0 = 1/(r - h_0)$. The closed-loop system under the scaled function (9) is switching from the initial system

$$\dot{\tilde{x}} = (A + BF)\tilde{x}$$

to the steady-state system

$$\dot{\tilde{x}} = (A + BF - \beta BB^{\mathrm{T}} P)\tilde{x}.$$

Both the initial system and the steady-state system are not involved in the tracking target r. ■

The following example illustrates the virtue of the scaled nonlinear functions $\rho(r,h)$ given by (9) and (11).

Example 3.1: Consider the system

$$\begin{aligned}\dot{x} &= \begin{bmatrix} -1.5 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \mathrm{sat}(u) \\ h &= [1 \quad 1 \quad 0]\, x\end{aligned} \tag{15}$$

with $u_{\max} = 3$. A CNF control law

$$u = Fx + Gr + \rho(r,h) B^{\mathrm{T}} P(x - x_e) \tag{16}$$

is designed with

$$\begin{aligned}F &= [0 \quad -4.5 \quad -1] \\ G &= 2.55 \\ P &= \begin{bmatrix} 0.4449 & 0.1673 & -0.4844 \\ 0.1673 & 0.7353 & -0.5000 \\ -0.4844 & -0.5000 & 2.6250 \end{bmatrix} \\ x_e &= -(A + BF)^{-1} BGr.\end{aligned}$$

Case 1: Unscaled function (8).

The nonlinear function $\rho(r,h)$ is in the form of (8), and the tuning parameters are tuned for $r = 1$ and $x(0) = 0$. α and β are tuned to minimize the performance index $\int_0^\infty |e| dt$ which gives $\alpha = 5$ and $\beta = 5.125$. The optimal tuning methods will be introduced in the next section.

Case 2: Scaled function (11).

The nonlinear function $\rho(r,h)$ is given by (11) where α and β are the same as Case 1, i.e., $\alpha = 5$ and $\beta = 5.125$.

Case 3: Scaled function (9).

The nonlinear function $\rho(r,h)$ is given by (9) with $\beta = 1.45$.

Fig. 1 shows the output responses of the closed-loop systems for $r = 1$ and $r = 2$ under the same initial condition $x(0) = 0$. When $r = 1$, Case 1 is identical to Case 2, and the output responses under the three different nonlinear functions have satisfactory transient performance, since the parameters of the three functions are all tuned for $r = 1$. However, when r is changed from $r = 1$ to $r = 2$, we can observe an overshoot of 12.56% for Case 1. However, the transient performance is preserved very well for Cases 2 and 3.

From Example 3.1, we can see that, under the unscaled function (8), the closed-loop system cannot preserve the transient performance well. However, under the scaled functions (9) and (11), the performance is robust to the variation of r. Thus, both (9) and (11) can be the qualified candidate of the nonlinear function. However, for the function (9), we can change the magnitude of $|\rho(r,h)|$ only by tuning the parameter β. On the other hand, for the function (11), we not only can change the magnitude by tuning β but also can change the decaying ratio by tuning α. Thus, the nonlinear function (11) is more adapted to various control applications.

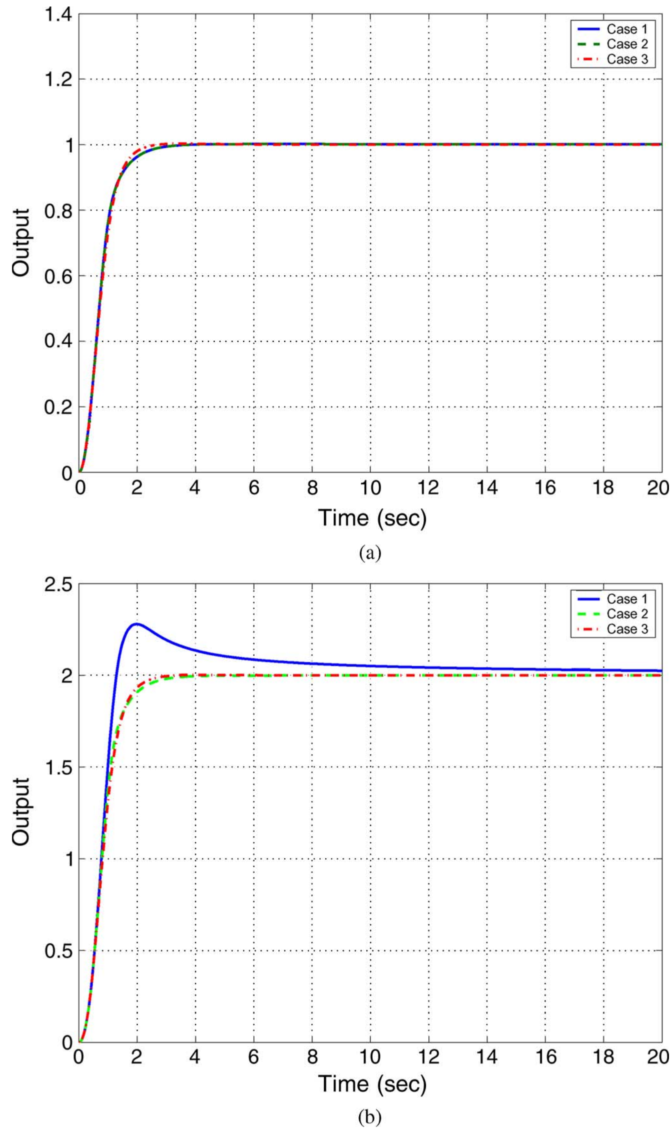

Fig. 1. Performance robustness under different nonlinear functions. (a) $r = 1$. (b) $r = 2$.

B. Optimal Gain Tuning

In this section, we will consider how to determine the parameters α and β for the scaled nonlinear function (11) such that the closed-loop system has a desired transient performance. At present, the parameters are tuned manually by trial and error. Here, we are trying to develop a systematic method to tune the parameters automatically. The idea is to formulate the parameter-tuning problem into a minimization problem so that the parameters can be tuned automatically by solving the minimization problem.

1) Two parameters optimization: Consider the closed-loop system consisting of (1) and (5)

$$\begin{aligned} \dot{x} &= Ax + B\mathrm{sat}(u) \\ u &= Fx + Gr + \rho(r,h)BB^{\mathrm{T}}P(x - x_e) \\ h &= Cx \end{aligned} \tag{17}$$

where

$$\rho(r,h) = -\beta e^{-\alpha\alpha_0|h-r|}.$$

By Theorem 2.1, the closed-loop system is asymptotically stable for any $\alpha > 0$ and $\beta > 0$. Thus, we can tune the parameters α and β by solving the following two parameters optimization problem:

$$\min_{\alpha>0,\beta>0} I(e) \tag{18}$$

where $e = h - r$ is the tracking error of the closed-loop system (17), and $I(e)$ is some performance index. Since we are considering a tracking-control problem, a direct and simple performance index is IAE

$$I(e) = \int_0^\infty |e|dt. \tag{19}$$

To address the overshoot and quick response in the transient performance, we can weight the performance index by time t, i.e., ITAE

$$I(e) = \int_0^\infty t|e|dt. \tag{20}$$

Because the closed-loop system is a nonlinear system, it is not easy to solve the minimization problem (18) analytically. However, since the closed-loop system is asymptotically stable for any $\alpha > 0$ and $\beta > 0$, the performance index $I(e)$ can be calculated for any $\alpha > 0$ and $\beta > 0$. Thus, we can solve the minimization problem (18) numerically by using some direct-search-based algorithms, such as Hooke–Jeeves method [4].

Example 3.2: Consider the same system (15) and the CNF control law (16) given in Example 3.1, where F, G, and P are given by

$$F = [\,2.5 \quad -4.5 \quad -0.5\,]$$
$$G = 1.7$$
$$P = \begin{bmatrix} 0.6094 & 0.4141 & -0.4200 \\ 0.4141 & 1.0412 & -0.5000 \\ -0.4200 & -0.5000 & 3.4000 \end{bmatrix}$$

and $\rho(r,h)$ is in the form of (11), i.e.,

$$\rho(r,h) = -\beta e^{-\alpha\alpha_0|y-r|}.$$

The parameters α and β are obtained by solving the minimization problem (18) with Hooke–Jeeves method. Specifically, we begin the Hooke–Jeeves algorithm with $\alpha = 5$ and $\beta = 4$. For both the IAE index and the ITAE index, the algorithm gives the optimal α and β with

$$\alpha = 8 \quad \beta = 3.625. \tag{21}$$

The simulation results are shown in Fig. 2, where $r = 2$ and $x(0) = 0$. The CNF controller with the parameters given by (21) results in a satisfied transient performance.

2) Single parameter optimization: Using the direct-search-based algorithm, the solution of the two parameters optimization problem (18) may not be unique. As shown in Fig. 3, there are numberless local minimums for both the IAE index and

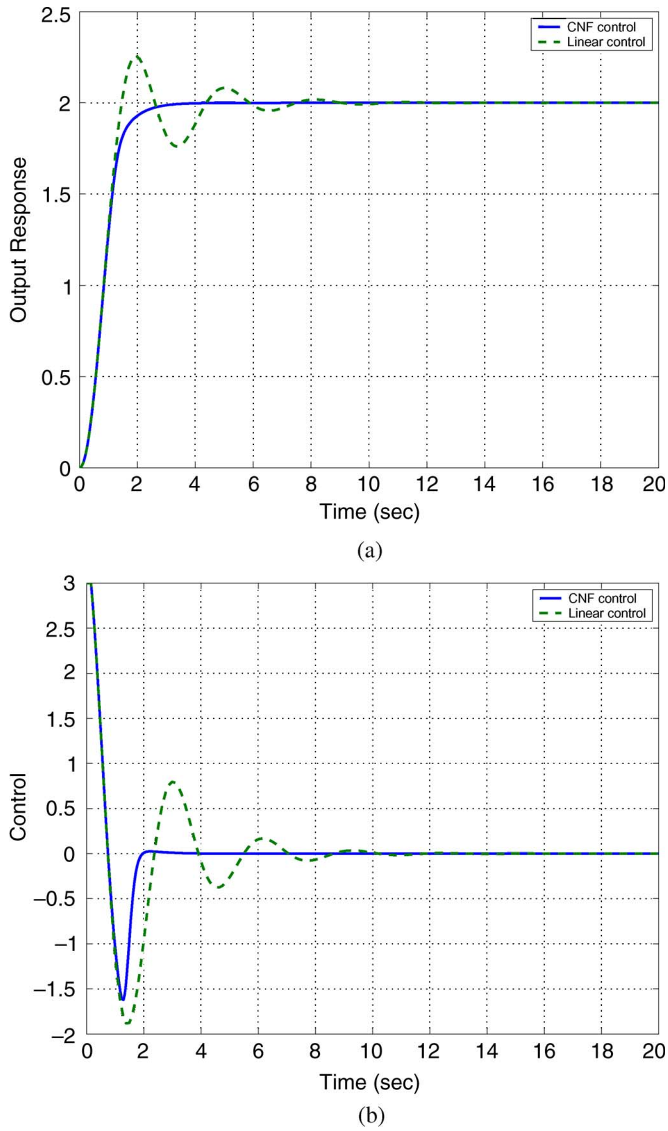

Fig. 2. Simulation result for Example 3.2. (a) Profile of output responses. (b) Profile of control inputs.

the ITAE index in the region of $2 \leq \alpha \leq 10$ and $0 \leq \beta \leq 5$ in Example 3.2. Thus, the search algorithm may result in different optimal α and β for different initial conditions. However, from Fig. 3, we can see that there do exist a global minimum for each fixed β, particularly when the β is large enough. Can we select an appropriate β first and then simplify the two-parameters-optimization problem (18) into a single-parameter-optimization problem? To answer this question, let us consider the steady-state closed-loop system. Typically, the control input will not be saturated at the steady-state period. Thus, the closed-loop steady-state system is given by

$$\dot{x} = Ax + Bu$$
$$u = Fx + Gr + \rho(r,h)B^{\mathrm{T}}P(x - x_e)$$
$$h = Cx$$

where $\rho(r,h)$ is given by (11). Defining

$$\tilde{x} = x - x_e \quad e = h - r$$

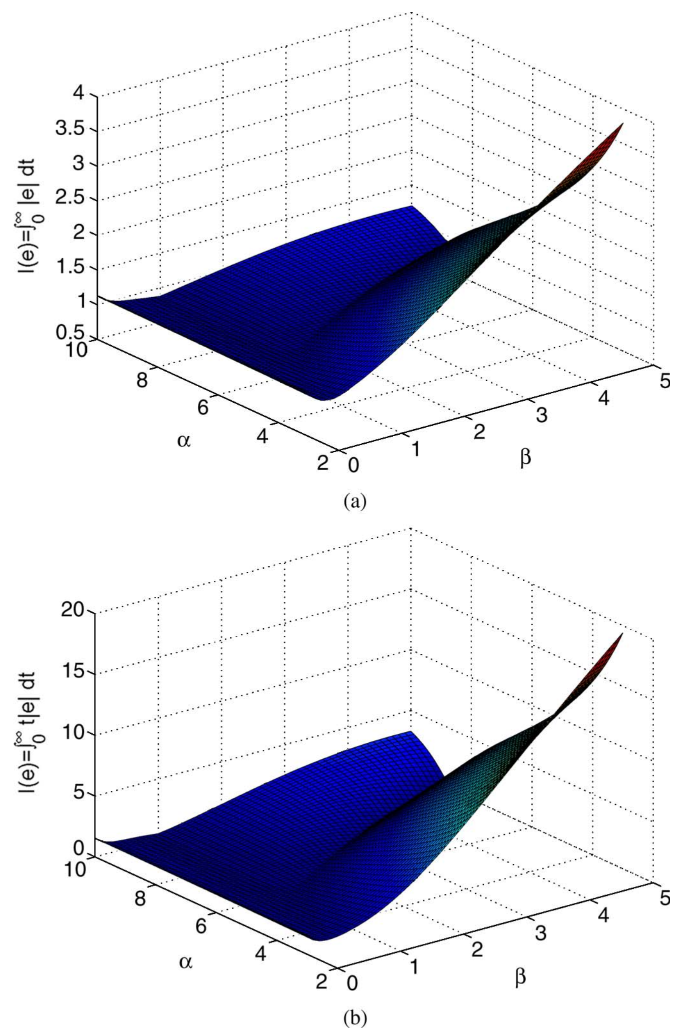

Fig. 3. Profile of performance indexes. (a) $I(e) = \int_0^\infty |e|dt$. (b) $I(e) = \int_0^\infty t|e|dt$.

the closed-loop system is described by

$$\dot{\tilde{x}} = (A + BF + \rho(r,h)BB^{\mathrm{T}}P)\tilde{x} \tag{22}$$
$$e = C\tilde{x}. \tag{23}$$

By Theorem 2.1, $\lim_{t\to\infty} h(t) = r$, which implies

$$\lim_{t\to\infty} \rho\left(r, h(t)\right) = -\beta. \tag{24}$$

Thus, the steady-state system is given by

$$\dot{\tilde{x}} = (A + BF - \beta BB^{\mathrm{T}}P)\tilde{x} \quad e = C\tilde{x}. \tag{25}$$

Now, let us define a design specification ξ_∞ for the closed-loop steady-state system, named steady-state damping ratio, we can work out β by setting the steady-state system (25) as a dominated damping ratio ξ_∞. Once β is fixed, the two-parameters-optimization problem (18) is simplified to a single-parameter-optimization problem

$$\min_{\alpha>0,\beta \text{ fixed}} I(e)$$

where $I(e)$ is some appreciable criterion, such as IAE and ITAE criteria.

In summary, the parameters α and β can be tuned by the following procedure:

Step T1) select a desired steady-state damping ratio ξ_∞;

Step T2) calculate β by letting the steady-state system (25) have a damping ratio of ξ_∞;

Step T3) find an optimal α by solving the minimization problem

$$\min_{\alpha>0,\beta \text{ fixed}} \int_0^\infty |e|\, dt \quad \text{or} \quad \min_{\alpha>0,\beta \text{ fixed}} \int_0^\infty t|e|\, dt$$

where $e = h - r$ is the tracking error of the closed-loop system.

Example 3.3: Consider the asymptotic tracking-control problem of the system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{sat}(u)$$
$$h = [1 \quad 0]\, x$$

with $r = 1$ and $u_{\max} = 1$. A CNF control that beat the time-optimal control (TOC) is designed in [7], [8]

$$u = Fx + Gr + \rho(r,h)B'P(x - x_e) \qquad (26)$$

where

$$F = [-6.5 \quad -1]$$
$$G = 6.5$$
$$B'P = [1.4481 \quad 10.8609]$$
$$x_e = [1 \quad 0]^{\mathrm{T}}.$$

If the settling time is defined to be the total time that it takes for the controlled output h to enter $\pm 1\%$ range of the target reference, the CNF control (26) can beat the TOC in settling time when the nonlinear function $\rho(r,h)$ is given by

$$\rho(r,h) = -\left(e^{-|r-h|} - 0.36788\right). \qquad (27)$$

To illustrate the efficiency of the proposed optimal tuning method, we will tune a new nonlinear gain for the CNF control law (26) with the nonlinear function (11) in which α_0 is defined by (12), α and β are the parameters to be tuned.

By selecting the steady-state damping ratio $\xi_\infty = 5$, it is easy to obtain $\beta = 2.9264$. We use IAE and ITAE as the criteria to tune the parameter α. Fig. 4(a) shows the performance indexes for various values of α when the tracking reference $r = 1$. The optimal α is $\alpha = 5$ for both IAE and ITAE. Thus,

$$\rho(r,h) = -2.9264e^{-5\alpha_0|h-r|}. \qquad (28)$$

If the nonlinear function is given by (27), we call (26) as the CNF control law that beat TOC. Otherwise, if the nonlinear function is given by (28), we call (26) as the autotuned CNF control law. The output responses under the autotuned CNF control and the CNF control that beat TOC are shown in Fig. 4(b). The controlled output under the autotuned CNF control is very close to the output under the carefully designed CNF control that beat TOC. Moreover, there is no overshoot for the resulting output response of the autotuned CNF control. The control inputs are shown in Fig. 4(c). It is clear that the proposed autotuning method is efficient to tune the nonlinear function

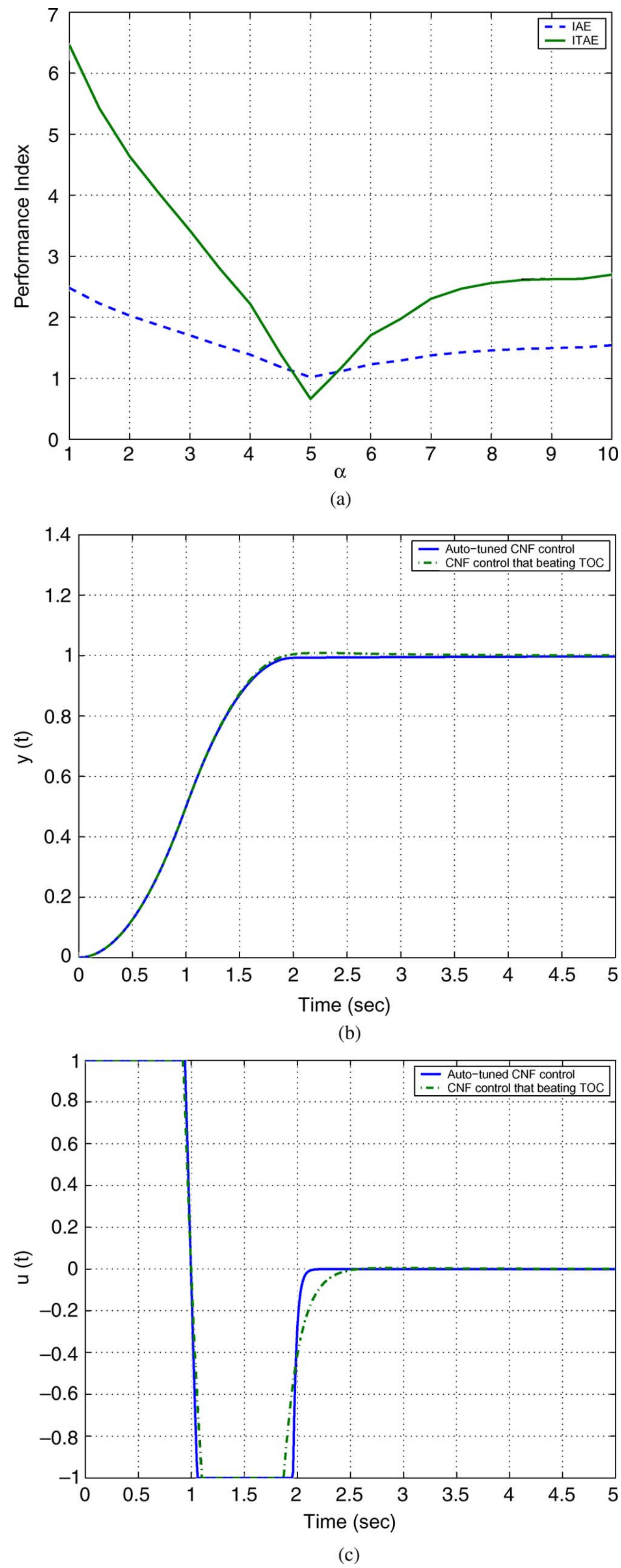

Fig. 4. Tuning of α for Example 3.3. (a) Performance index for various values of α. (b) Profile of output responses. (c) Profile of control inputs.

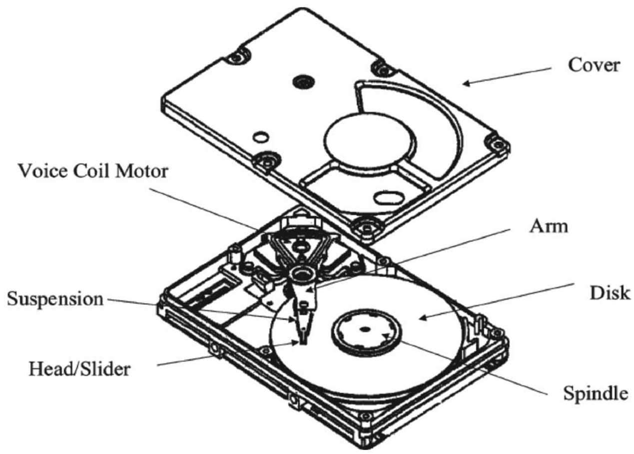

Fig. 5. HDD with a VCM actuator.

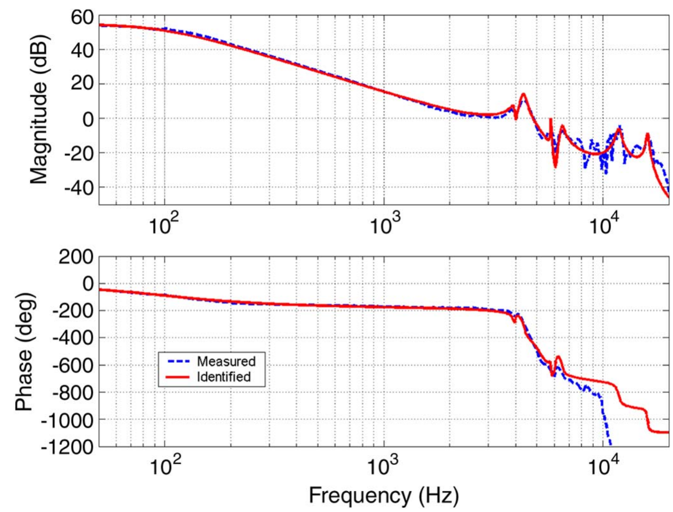

Fig. 6. Frequency responses of the VCM actuator (LDV range 2 μm/V).

of the CNF control law such that the closed-loop system has satisfied transient performance.

IV. HDD Servo System Design

In this section, we proceed to design a CNF control law for an HDD servo system by using CNF control with the proposed optimal nonlinear gain tuning methods.

A. Disk-Drive Modeling

Fig. 5 shows the typical picture of the 3.5-in HDD with a VCM actuator that will be used in our experiment. It consists of a stack of flat rotating disks with position (servo) information embedded on their surfaces. Position error signal calculated from the servo information is used to measure the position of the R/W heads.

The model of the plant to be controlled, i.e., the VCM actuator mounted with the suspension is obtained by system input–output identification. In this paper, during our implementation, instead of using the servo information, the position measurement of the plant is obtained using a Polytec OFV 3001S Scanning Laser Doppler Vibrometer (LDV) in the range of 2 μm/V. A Dynamic Signal Analyzer is used to generate a swept sine signal to excite the actuator so as to capture the frequency response of the dynamics of the actuator in frequency domain. Fig. 6 shows the measured frequency response of the actuator and its nominal model.

The 16th-order nominal plant model of the actuator is obtained using the curve-fitting technique that approximates the measured frequency responses. Its transfer function is given by

$$P(s) = \frac{2.18 \times 10^8}{s^2 + 1005s + 3.948 \times 10^5} P_{\mathrm{pd}}(s) \prod_{i=1}^{6} P_{\mathrm{rm},i}(s) \tag{29}$$

with six main resonance modes given by

$$P_{\mathrm{rm},1}(s) = \frac{0.9702s^2 + 487.7s + 6.13 \times 10^8}{s^2 + 990.2s + 6.13 \times 10^8} \tag{30}$$

$$P_{\mathrm{rm},2}(s) = \frac{0.563s^2 + 820s + 7.47 \times 10^8}{s^2 + 1367s + 7.47 \times 10^8} \tag{31}$$

$$P_{\mathrm{rm},3}(s) = \frac{0.91s^2 + 695.3s + 1.33 \times 10^9}{s^2 + 182.2s + 1.33 \times 10^9} \tag{32}$$

$$P_{\mathrm{rm},4}(s) = \frac{1.51s^2 - 7.95 \times 10^4 s + 5.50 \times 10^9}{s^2 + 3411s + 5.50 \times 10^9} \tag{33}$$

$$P_{\mathrm{rm},5}(s) = \frac{1.17s^2 + 1.74 \times 10^5 s + 1.01 \times 10^{10}}{s^2 + 2011s + 1.01 \times 10^{10}} \tag{34}$$

$$P_{\mathrm{rm},6}(s) = \frac{1.668 \times 10^9}{s^2 + 1634s + 1.668 \times 10^9} \tag{35}$$

and the second-order Padé delay

$$P_{\mathrm{pd}}(s) = \frac{s^2 - 6535s + 1.07 \times 10^9}{s^2 + 6535s + 1.07 \times 10^9}. \tag{36}$$

B. CNF Servo System Design

Next, we proceed to design a servo control system for the HDD with the models given earlier. Prior to any controller design, the plant is precompensated with three second-order notch filters given by

$$F_{\mathrm{nf},1}(s) = \frac{s^2 + 1913s + 7.47 \times 10^8}{s^2 + 2.624 \times 10^4 s + 7.47 \times 10^8} \tag{37}$$

$$F_{\mathrm{nf},2}(s) = \frac{s^2 + 3770s + 5.69 \times 10^9}{s^2 + 3.77 \times 10^4 s + 5.69 \times 10^9} \tag{38}$$

$$F_{\mathrm{nf},3}(s) = \frac{s^2 + 5027s + 1.011 \times 10^{10}}{s^2 + 5.03 \times 10^4 s + 1.011 \times 10^{10}} \tag{39}$$

the plant model $P(s)$ in (29) can be approximated by the second-order model $P_{\mathrm{comp}}(s)$, whose state-space representation is given by

$$\dot{x} = Ax + B\mathrm{sat}(u) \tag{40}$$

$$y = h = Cx \tag{41}$$

with

$$A = \begin{bmatrix} 0 & 1 \\ a_{21} & a_{22} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ b \end{bmatrix} \quad C = [1 \quad 0] \tag{42}$$

where $a_{21} = -3.948 \times 10^5$, $a_{22} = -1005$, and $b = 2.18 \times 10^8$. In (40) and (41), y represents the relative displacement of the R/W head and u stands for the control input, which is limited within $\pm u_{\max}$ with $u_{\max} = 3$ V.

It is not difficult to verify that the system (40) and (41) satisfies all the assumptions A1 to A3. Thus, we can construct the CNF control law for the HDD servo system. Specifically, design a linear-feedback law

$$u_L = Fx + Gr$$

where r is a step command input and F is designed to be

$$F = -\frac{1}{b}\left[w_0^2 + a_{21} \quad 2\xi_0 w_0 + a_{22}\right] \tag{43}$$

with $\xi_0 = 0.328$ and $w_0 = 3925$. It is clear that $(A + BF)$ is an asymptotically stable matrix, and the closed-loop system $C(sI - A - BF)^{-1}B$ has damping ratio 0.328 and natural frequency 3925 rad/s. The feedforward gain G is a scalar and is given by

$$G = -\left[C(A + BF)^{-1}B\right]^{-1} = 0.0707.$$

Next, we choose W of the following Lyapunov equation:

$$(A + BF)'P + P(A + BF) = -W \tag{44}$$

to be a diagonal matrix with diagonal elements being 0.0117 and 2×10^{-10}, respectively. Consequently, solving (44), we obtain

$$P = \begin{bmatrix} 3.8481 \times 10^{-6} & 3.7973 \times 10^{-10} \\ 3.7973 \times 10^{-10} & 1.8632 \times 10^{-13} \end{bmatrix}. \tag{45}$$

Setting $\xi_\infty = 1.8934$ would give $\beta = 3.5$. Since the relative velocity of the R/W head is not measurable, a reduced-order output feedback CNF control law has been designed and is given by

$$\dot{x}_v = -6000x_v + 2.18 \times 10^8 \text{sat}(u) - 3.04 \times 10^7 y$$

$$u = F\begin{bmatrix} y \\ x_v + 4995y \end{bmatrix} + Gr + \rho(r, y)B'P\left[\begin{pmatrix} y \\ x_v + 4995y \end{pmatrix} - x_e\right]$$

with $x_e = -(A + BF)^{-1}BGr$. Let $r = 2$ μm, and solving the minimization problem

$$\min_{\alpha>0,\beta=3.5} \int_0^\infty t|e|\, dt$$

by Hooke–Jeeves algorithm yields $\alpha = 4.21$. Thus, the nonlinear function is given by

$$\rho(r, y) = -3.5e^{-4.21\alpha_0(y-r)}.$$

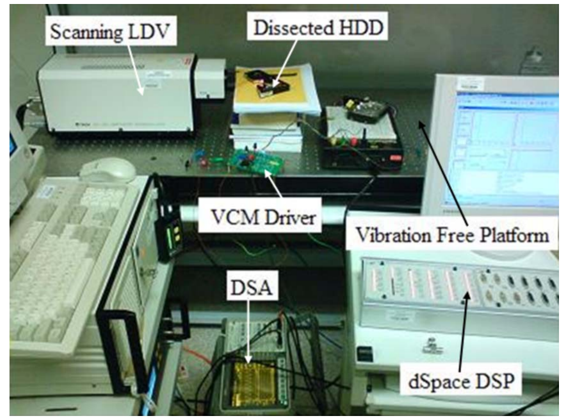

Fig. 7. Experimental setup.

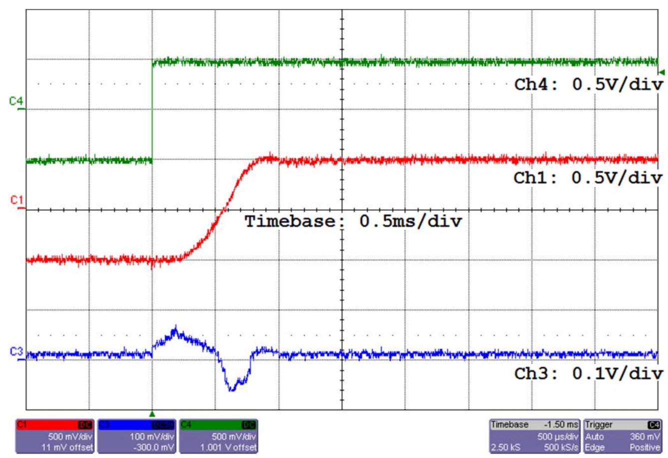

Fig. 8. CNF: 2-μm seek. Ch1: LDV measurement (2 μm/V), $y(t)$. Ch3: VCM-driver input, u. Ch4: Seek Command Input (2 μm/V), r.

C. Implementation Results

The experimental setup is shown in Fig. 7, where a dissected HDD is placed on a vibration-free platform, the displacement of the R/W head is measured with a scanning LDV. The designed CNF control law is discretized using bilinear rule at a sampling frequency of 40 kHz for digital control design. On the other hand, the notch filters are discretized at the same sampling frequency using the matched pole-zero rule instead. All final digital controllers are implemented via a dSpace DSP installed on a desktop PC at this frequency.

Figs. 8 and 9 show the seeking performance with zero initial conditions for a seek span of 2 and 20 μm, respectively. From these figures, it is apparent that a fast and smooth seeking performance with a settling time of 0.8 ms can be achieved for any seek length within ± 20 μm under a single CNF control law.

To show the influence of the scaling parameter α_0, we repeated the experiments by removing the scaling parameter α_0 in the CNF control law. The achieved 2-μm seeking performance is unaffected by removing the scaling parameter α_0, since the parameters of the function are tuned based on a nominal 2-μm seek. The track-following performance is unaffected by the removal of α_0 as well. However, as shown in Fig. 10, the overshoot is increased to 42% and the settling time is delayed

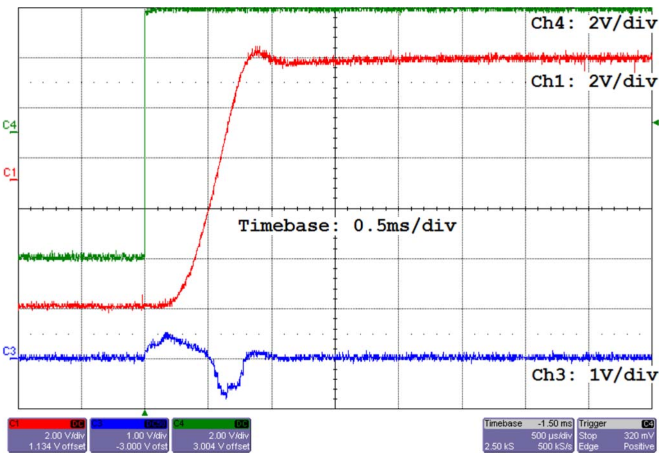

Fig. 9. CNF: 20-μm seek. Ch1: LDV measurement (2 μm/V), $y(t)$. Ch3: VCM-driver input, u. Ch4: Seek Command Input (2 μm/V), r.

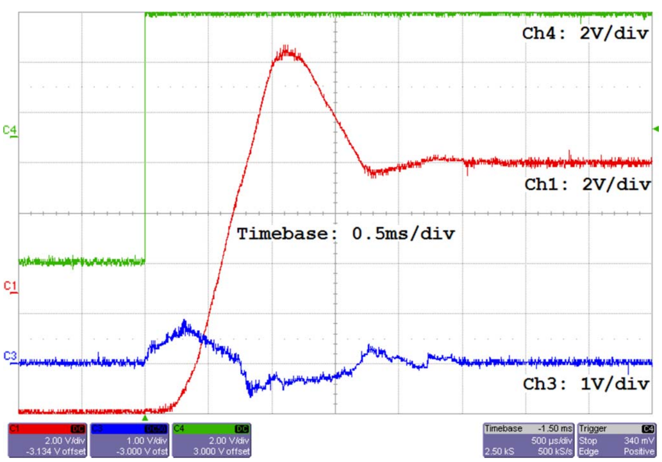

Fig. 10. CNF without scaling: 20-μm seek. Ch1: LDV measurement (2 μm/V), $y(t)$. Ch3: VCM-driver input, u. Ch4: Seek Command Input (2 μm/V), r.

to approximately 2.5 ms for 20-μm seeking when the scaling factor is not present.

The measured open-loop frequency response and the sensitivity function are, respectively, shown in Figs. 11 and 12. As shown in Fig. 11, the proposed CNF controller design achieves an excellent level of closed-loop stability margins with a phase margins of 50° and a minimum gain margin of 7.5 dB, as well as a good servo bandwidth of 1.1 kHz. Fig. 12 shows the measured sensitivity function of the closed-loop system during track following with a sensitivity hump of less 6 dB is achieved with the proposed single CNF control law.

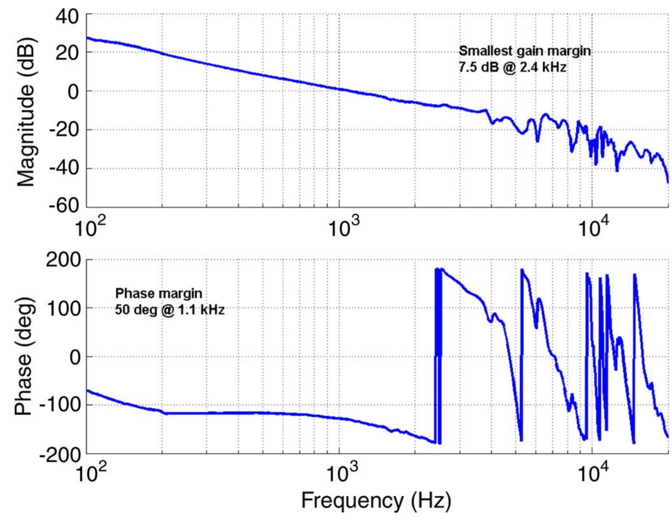

Fig. 11. Measured open-loop frequency response (LDV range 2 μm/V).

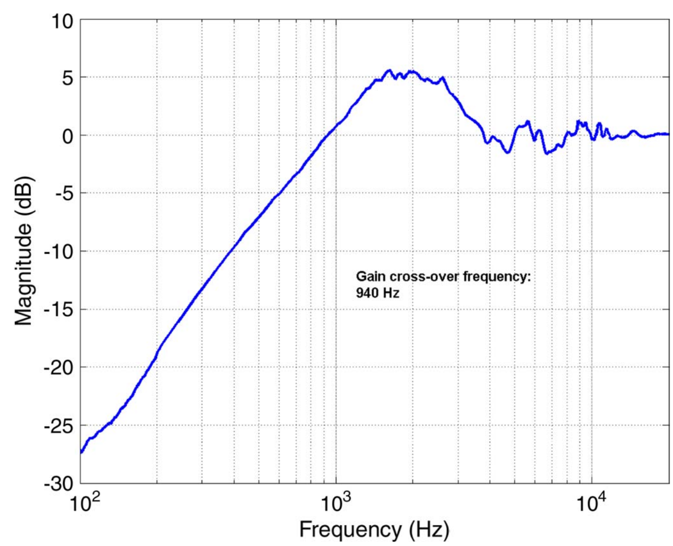

Fig. 12. Measured frequency gain response of the sensitivity function.

V. Conclusion

An autotuning method was proposed to tune the parameters of the nonlinear function $\rho(r, h)$ in the CNF control law. A new nonlinear function is selected to hold the transient performance for different reference amplitudes. There are two parameters α and β that need to be tuned in the new function. β is determined by the steady-state damping ratio ξ_∞. α is tuned by solving a minimization problem to minimize IAE or ITAE. The proposed method is applied to design the CNF control law for a HDD servo system. Track seeking and track following are achieved for any seek length within ± 20 μm with a single CNF control law.

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