# An Output Feedback $\mathcal{H}_\infty$ Controller Design for Linear Systems Subject to Sensor Nonlinearities

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***Abstract*—In this paper, the output feedback $\mathcal{H}_\infty$ controller design problem is addressed for linear systems subject to sensor nonlinearity. First, the existence condition of an output feedback controller is derived for systems with sensor sector nonlinearity. A design method for the output feedback $\mathcal{H}_\infty$ controller is proposed using a linear-matrix inequality (LMI) based approach. The result is then applied to the design of a regional output feedback $\mathcal{H}_\infty$ controller for the systems subject to sensor saturation. An LMI optimization based approach is proposed to computing the feedback matrices of the regional output feedback $\mathcal{H}_\infty$ controller. At last a numerical example is presented to show the effectiveness of the results.**

***Index Terms*—Linear systems, sector nonlinearity, sensor saturation, $\mathcal{H}_\infty$ control, $\mathcal{L}_2$ gain.**

## I. Introduction

In FEEDBACK control systems, feedback device nonlinearities, including actuator and sensor saturation, arise frequently. They can severely degrade the closed-loop system performance and sometimes even make an otherwise stable closed-loop system unstable. The issue of closed-loop system stability and performance in the presence of feedback device nonlinearities thus carries a great deal of practical importance. While in practice it is desirable to choose actuators and sensors that are large enough so that they operate in their linear regions, cheaper actuators and sensors can be used if their saturation can be satisfactorily handled.

While actuator nonlinearity has been addressed in much detail, (see, for example, [1], [13], [5] and the references therein), few results are available that deal with the sensor nonlinearity. Among these few results, observability of a linear system subject to sensor saturation was studied in detail in [12]. A discontinuous dead beat controller was constructed for single-input–single-output (SISO) linear systems in the presence of output saturation in [11]. A semiglobal stabilizing linear output feedback controller design method was proposed for the SISO linear systems subject to sensor saturation in [14].

In this paper, we will use the circle criterion theory to address the issues related to analysis and design for linear systems subject to sensor nonlinearity. In particular, we will focus on the output feedback $\mathcal{H}_\infty$ controller design for linear systems with sensor sector nonlinearity. We will present an LMI condition under which the system with sensor sector nonlinearities is $\mathcal{H}_\infty$ stabilizable. A design method for the globally stabilizing output feedback $\mathcal{H}_\infty$ control laws will be proposed by an LMI optimization based approach. These results will then be applied to design the regional $\mathcal{H}_\infty$ controller for linear systems subject to sensor saturation. Our method was motivated by [3], where an explicit controller computing formulation was proposed for linear systems based on the LMI optimization approach.

We note that output feedback control in the context of linear systems subject to actuator saturation has already been addressed by several authors. In particular, Kapila and Haddad [9] considered the fixed-structure controller design problem. Nguyen and Jabbari [15] studied the output feedback disturbance attenuation problem for linear systems subject to actuator amplitude and rate saturation using LMI approach. Despite the impression of its duality with actuator saturation and the similar effects on a given closed-loop system as those of actuator saturation, sensor saturation is fundamentally different from actuator saturation. For example, for a system subject to actuator saturation, the feedback gain is a design variable. Thus, for a given set of initial conditions, there is a possibility of designing the feedback gain such that saturation is completely avoided. For a system subject to sensor saturation, the output matrix is fixed. Hence, if the sensor saturation occurs for some initial conditions of the system, this saturation cannot be avoided by any control design.

This paper is organized as follows. Problem formulation and preliminaries will be given in Section II. Stability and $\mathcal{L}_2$ gain of the output feedback control systems with sensor sector nonlinearities will be analyzed in Section III. The design method for a dynamic output feedback $\mathcal{H}_\infty$ control law will be proposed in Section IV. In Section V, this design approach will be applied to linear systems subject to sensor saturation such that the closed-loop system has a prespecified regional $\mathcal{L}_2$ gain. A numerical example will be given in Section VI to illustrate the design method. The paper will be concluded in Section VII.

*Notations:* The following notation will be used throughout the paper. $\mathcal{R}$ denotes the set of real numbers, $\mathcal{R}^m$ denotes the $m$ dimensional Euclidean space, and $\mathcal{R}^{n\times m}$ denotes the set of all real $n\times m$ matrices. $A^T$ denotes the transpose of $A, A^+$ is the Moore–Penrose inverse of $A, \mathrm{Ker}(A)$ and $\mathrm{Im}(A)$ denote the null space and range space of $A$, respectively, $A^\perp$ denotes a matrix with the following properties: $\mathrm{Ker}(A^\perp)=\mathrm{Im}(A)$ and

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$A^\perp A^{\perp T} > 0$. Note that $A^\perp$ exists if and only if $A$ has linearly dependent rows. Also note that, for a given $A, A^\perp$ is not unique but throughout the paper, any choice is acceptable. In the sequel, if not explicitly stated, matrices are assumed to have compatible dimensions. The notation $M > (\geq, <, \leq) 0$ is used to denote a symmetric positive definite (positive semidefinite, negative definite, negative semidefinite, respectively) matrix. $I$ and $0$ denote the identity matrix and zero matrix of compatible dimensions.

## II. Problem Statement and Preliminaries

Consider a linear system with sensor saturation

$$\dot{x} = Ax + B_1 w + Bu \tag{1}$$
$$z = C_1 x + D_{11} w + D_{12} u \tag{2}$$
$$y = \psi(Cx) + D_{21} w \tag{3}$$

where $x \in \mathcal{R}^n$ is the state vector, $u \in \mathcal{R}^m$ the control input vector, $z \in \mathcal{R}^q$ the controlled output vector, $y \in \mathcal{R}^p$ the measured output vector, $w \in \mathcal{R}^r$ the disturbance input vector; and $w \in \mathcal{L}_2[0, \infty)$, and $A, B_1, B, C_1, C, D_{11}, D_{12}$ and $D_{21}$ are real-valued matrices of appropriate dimensions. Also assume that $(A, B, C)$ is stabilizable and detectable. The function $\psi \in [K_1, K_2]$, for some diagonal matrices $K_1 \geq 0$ and $K_2 \geq 0$ with $K_2 > K_1$, denotes the standard vector-valued sector nonlinearity defined as follows [10].

*Definition 1:* A memoryless nonlinearity $\psi : \mathcal{R}^p \to \mathcal{R}^p$ is said to satisfy a sector condition if

$$(\psi(v) - K_1 v)^T (\psi(v) - K_2 v) \leq 0 \qquad \forall v \in \mathcal{R}^p \tag{4}$$

for some diagonal real matrices $K_1, K_2 \in \mathcal{R}^{p \times p}$, where $K = K_2 - K_1$ is a positive-definite symmetric matrix. In this case, we say that $\psi$ belongs to the sector $[K_1, K_2]$.

The problem considered in this paper can be described as follows.

*Problem 1:* For a given system (1)–(3) and a $\gamma > 0$, find a dynamic output feedback control law of the form

$$\dot{x}_c = A_c x_c + B_c y, x_c(0) = 0 \tag{5}$$
$$u = C_c x_c + D_c y \tag{6}$$

such that, for any $\psi \in [K_1, K_2]$, the closed-loop system is globally asymptotically stable at the origin and the $\mathcal{L}_2$ gain from the disturbance input $w(t)$ to the performance output $z(t)$ is less than or equal to $\gamma$, i.e.,

$$\int_0^T \|z(t)\|^2 \, dt \leq \gamma^2 \int_0^T \|w(t)\|^2 \, dt$$
$$x(0) = 0 \qquad \forall w \in \mathcal{L}_2[0, T) \qquad \forall T \geq 0. \tag{7}$$

Under the feedback law (5)–(6), the closed-loop system can be written as

$$\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}_1 w + \tilde{B}\psi \tag{8}$$
$$z = \tilde{C}_1 \tilde{x} + \tilde{D}_{11} w + \tilde{C}_2 \psi(v) \tag{9}$$
$$v = \tilde{C}\tilde{x} \tag{10}$$

where

$$\tilde{x}(t) = \begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix}$$
$$\tilde{A} = \begin{bmatrix} A & BC_c \\ 0 & A_c \end{bmatrix}$$
$$\tilde{B} = \begin{bmatrix} BD_c \\ B_c \end{bmatrix}$$
$$\tilde{B}_1 = \begin{bmatrix} B_1 + BD_c D_{21} \\ B_c D_{21} \end{bmatrix}$$
$$\tilde{C}_1 = [C_1 \quad D_{12} C_c]$$
$$\tilde{C}_2 = D_{12} D_c$$
$$\tilde{C} = [C \quad 0]$$
$$\tilde{D}_{11} = D_{11} + D_{12} D_c D_{21}.$$

*Lemma 1:* [8] Let matrices $B \in \mathcal{R}^{n \times m}, C \in \mathcal{R}^{p \times n}$, and $Q = Q^T \in \mathcal{R}^{n \times n}$ be given and suppose that $\operatorname{rank}(B) = m \leq n$ and $\operatorname{rank}(C) = p \leq n$. Then, there exists a matrix $G \in \mathcal{R}^{m \times p}$ such that

$$BGC + (BGC)^T + Q < 0$$

if and only if

$$B^\perp Q B^{\perp T} < 0 \quad C^{T\perp} Q C^{T\perp T} < 0.$$

## III. Stability and $\mathcal{L}_2$-Gain Analysis

In this section, we analyze the closed-loop stability and the $\mathcal{L}_2$ gain for the system (8)–(10) by applying the multivariable circle criterion. We first present the following result on global asymptotic stability.

*Lemma 2:* For the system (8)–(10) with $w(t) \equiv 0$, suppose that $p \times p$ diagonal matrices $K_1$ and $K_2$ are given such that $K = K_2 - K_1$ is positive definite and that matrix $\tilde{A} + \tilde{B}K_1\tilde{C}$ is Hurwitz. If there exists a matrix $P > 0$ satisfying

$$\begin{bmatrix} (\tilde{A} + \tilde{B}K_1\tilde{C})^T P + P(\tilde{A} + \tilde{B}K_1\tilde{C}) & (K\tilde{C} + \tilde{B}^T P)^T \\ K\tilde{C} + \tilde{B}^T P & -2I \end{bmatrix} < 0 \tag{11}$$

then, system (8)–(10) in the absence of $w$, is globally asymptotically stable at the origin for any $\psi \in [K_1, K_2]$.

*Proof:* Decompose the nonlinear function $\psi(v)$ into a linear and a nonlinear part as

$$\psi(v) = \psi_s(v) + K_1 v \tag{12}$$

where the nonlinearity $\psi_s$ belongs to the set $\Phi_s$ given by

$$\Phi_s = \{\psi_s : \mathcal{R}^p \to \mathcal{R}^p : (\psi_s(v))^T (\psi_s(v) - Kv) \leq 0\}. \tag{13}$$

Then, (8) with $w \equiv 0$ can be rewritten as

$$\dot{\tilde{x}} = (\tilde{A} + \tilde{B}K_1\tilde{C})\tilde{x} + \tilde{B}\psi_s(v).$$

Select a Lyapunov function as $V(t) = \tilde{x}^T(t) P \tilde{x}(t)$. By (11), we have the first equation shown at the bottom of the next page, which implies that the system (8)–(10) is globally asymptotically stable at the origin for any $\psi \in [K_1, K_2]$. ■

*Lemma 3:* For the system (8)–(10), suppose that $p \times p$ diagonal matrices $K_1$ and $K_2$ are given such that $K = K_2 - K_1$ is positive definite and and that matrix $\tilde{A} + \tilde{B}K_1\tilde{C}$ is Hurwitz. If there exists a matrix $P > 0$ satisfying (14) at the bottom of the page. Then, for any $\psi \in [K_1, K_2]$, system (8)–(10) is globally asymptotically stable at the origin and the $\mathcal{L}_2$ gain from $w$ to $z$ is less than or equal to $\gamma$.

*Proof:* First note that the matrix inequality (14) implies (11). Hence if (14) holds, then the system (8)–(10) with $w = 0$ is globally asymptotically stable at the origin for any $\psi \in [K_1, K_2]$.

Decompose the nonlinear function $\psi(v)$ into a linear and a nonlinear part as in (12), the system (8)–(10) can be rewritten as

$$\begin{aligned}\dot{\tilde{x}} &= \hat{A}_0\tilde{x} + \tilde{B}_1 w + \tilde{B}\psi_s(v)\\ z &= \hat{C}_1\tilde{x} + \tilde{D}_{11} w + \tilde{C}_2\psi_s(v)\\ v &= \tilde{C}\tilde{x}\end{aligned}$$

where $\hat{A}_0 = \tilde{A} + \tilde{B}K_1\tilde{C}$ and $\hat{C}_1 = \tilde{C}_1 + \tilde{C}_2K_1\tilde{C}$. Define a Lyapunov function as $V(x) = \tilde{x}^T P \tilde{x}$. Then, we have the last equation at the bottom of the page, where

$$\xi(t) = \begin{bmatrix} \tilde{x}(t) \\ w(t) \\ \psi_s(v(t)) \end{bmatrix}.$$

By (14) and Schur complement, we further have

$$\|z(t)\|^2 - \gamma^2\|w(t)\|^2 + \dot{V} < 0 \qquad \forall \xi \neq 0.$$

Integrating both sides of the above inequality and noting that $V(0) = 0, V(x(T)) \geq 0$, we have

$$\int_0^T \|z(t)\|^2 dt \leq \gamma^2 \int_0^T \|w(t)\|^2)\, dt \qquad \forall w \in \mathcal{L}_2[0, T] \quad \forall T \geq 0.$$

■

## IV. $\mathcal{H}_\infty$ CONTROL DESIGN

First, we will derive a condition under which an output feedback controller of the form (5)-(6) exists such that the closed-loop system (8)–(10) in the absence of $w$ is globally asymptotically stable at the origin for any $\psi \in [K_1, K_2]$. Then, an $\mathcal{H}_\infty$ controller design method will be proposed.

*Theorem 4:* The system (1)–(3) is globally asymptotically stabilizable by an output feedback control law of the form (5)–(6) if there exist matrices $X > 0$ and $Y > 0$ that satisfy the following matrix inequalities:

$$\begin{bmatrix} B^\perp(AX + XA^T)B^{\perp T} & B^\perp(KCX)^T \\ KCXB^{\perp T} & -2I \end{bmatrix} < 0 \tag{15}$$

$$YA + A^T Y - 2C^T K_1 K_2 C < 0 \tag{16}$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} \geq 0. \tag{17}$$

*Proof:* Feedback controller (5)–(6) such that the closed-loop system (8)–(10) is globally asymptotically stable at the origin for any $\psi \in [K_1, K_2]$ if there exists a matrix $P > 0$ satisfying

$$\begin{bmatrix} (\tilde{A} + \tilde{B}K_1\tilde{C})^T P + P(\tilde{A} + \tilde{B}K_1\tilde{C}) & (K\tilde{C} + \tilde{B}^T P)^T \\ K\tilde{C} + \tilde{B}^T P & -2I \end{bmatrix} < 0. \tag{18}$$

In what follows, we will prove that (18) holds if and only if (15), (16), and (17) hold for some $X > 0$ and $Y > 0$.

We first note that

$$\begin{aligned}\begin{bmatrix} (\tilde{A} + \tilde{B}K_1\tilde{C}) & \tilde{B} \\ K\tilde{C} & -I \end{bmatrix} &= \begin{bmatrix} A + BD_cK_1C & BC_c & BD_c \\ B_cK_1C & A_c & B_c \\ KC & 0 & -I \end{bmatrix}\\ &= \hat{A} + \hat{B}\hat{F}\hat{C}\end{aligned}$$

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$$\begin{aligned}\dot{V} &= \tilde{x}^T((\tilde{A} + \tilde{B}K_1\tilde{C})^T P + P(\tilde{A} + \tilde{B}K_1\tilde{C}))\tilde{x} + 2\tilde{x}^T P\tilde{B}\psi_s(v)\\ &\leq \tilde{x}^T((\tilde{A} + \tilde{B}K_1\tilde{C})^T P + P(\tilde{A} + \tilde{B}K_1\tilde{C}))\tilde{x} + 2\tilde{x}^T P\tilde{B}\psi_s(v) - 2(\psi_s(v))^T(\psi_s(v) - Kv)\\ &= \begin{bmatrix} \tilde{x} \\ \psi_s(v) \end{bmatrix}^T \begin{bmatrix} (\tilde{A} + \tilde{B}K_1\tilde{C})^T P + P(\tilde{A} + \tilde{B}K_1\tilde{C}) & (K\tilde{C} + \tilde{B}^T P)^T \\ K\tilde{C} + \tilde{B}^T P & -2I \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \psi_s(v) \end{bmatrix}\\ &< 0, \quad \forall \tilde{x} \neq 0\end{aligned}$$

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$$\begin{bmatrix} (\tilde{A} + \tilde{B}K_1\tilde{C})^T P + P(\tilde{A} + \tilde{B}K_1\tilde{C}) & P\tilde{B}_1 & (\tilde{C}_1 + \tilde{C}_2K_1\tilde{C})^T & (K\tilde{C} + \tilde{B}^T P)^T \\ \tilde{B}_1^T P & -\gamma^2 I & \tilde{D}_{11}^T & 0 \\ \tilde{C}_1 + \tilde{C}_2K_1\tilde{C} & \tilde{D}_{11} & -I & \tilde{C}_2 \\ K\tilde{C} + \tilde{B}^T P & 0 & \tilde{C}_2^T & -2I \end{bmatrix} < 0 \tag{14}$$

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$$\|z(t)\|^2 - \gamma^2\|w(t)\|^2 + \dot{V} = \xi^T \begin{bmatrix} \hat{A}_0^T P + P\hat{A}_0 + \hat{C}_1^T\hat{C}_1 & P\tilde{B}_1 + \hat{C}_1^T\tilde{D}_{11} & (K\tilde{C} + \tilde{B}^T P + \tilde{C}_2^T\hat{C}_1)^T \\ \tilde{B}_1^T P + \tilde{D}_{11}^T\hat{C}_1 & \tilde{D}_{11}^T\tilde{D}_{11} - \gamma^2 I & \tilde{D}_{11}^T\tilde{C}_2 \\ K\tilde{C} + \tilde{B}^T P + \tilde{C}_2^T\hat{C}_1 & \tilde{C}_2^T\tilde{D}_{11} & -2I + \tilde{C}_2^T\tilde{C}_2 \end{bmatrix} \xi$$

where

$$\hat{A} = \begin{bmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ KC & 0 & -I \end{bmatrix}$$
$$\hat{B} = \begin{bmatrix} B & 0 \\ 0 & I \\ 0 & 0 \end{bmatrix}$$
$$\hat{F} = \begin{bmatrix} D_c & C_c \\ B_c & A_c \end{bmatrix}$$
$$\hat{C} = \begin{bmatrix} K_1 C & 0 & I \\ 0 & I & 0 \end{bmatrix}.$$

Then, the matrix inequality (18) can be rewritten as

$$Q + (\hat{P}\hat{B}\hat{F}\hat{C})^T + \hat{P}\hat{B}\hat{F}\hat{C} < 0 \tag{19}$$

where

$$Q = \hat{A}^T \hat{P} + \hat{P}\hat{A}, \qquad \hat{P} = \begin{bmatrix} P & 0 \\ 0 & I \end{bmatrix}.$$

By Lemma 1, (19) holds if and only if

$$(\hat{P}\hat{B})^{\perp} Q (\hat{P}\hat{B})^{\perp T} < 0 \quad \hat{C}^{T\perp} Q \hat{C}^{T\perp T} < 0. \tag{20}$$

It is easy to see that

$$(\hat{P}\hat{B})^{\perp} = \begin{bmatrix} B^{\perp} & 0 & 0 \\ 0 & 0 & I \end{bmatrix} \hat{P}^{-1}$$
$$\hat{C}^{T\perp} = [I \quad 0 \quad -C^T K_1].$$

Partition $P$ and $P^{-1}$ as

$$P = \begin{bmatrix} Y & N \\ N^T & U \end{bmatrix} \quad P^{-1} = \begin{bmatrix} X & M \\ M^T & V \end{bmatrix} \tag{21}$$

where $0 < X \in \mathcal{R}^{n \times n}$ and $0 < Y \in \mathcal{R}^{n \times n}$. Then, we have

$$Q = \begin{bmatrix} YA + A^T Y & A^T N & (KC)^T \\ N^T A & 0 & 0 \\ KC & 0 & -2I \end{bmatrix}$$
$$\hat{P}^{-1} Q \hat{P}^{-1} = \begin{bmatrix} AX + XA^T & AM & (KCX)^T \\ M^T A^T & 0 & (KCM)^T \\ KCX & KCM & -2I \end{bmatrix}$$
$$(\hat{P}\hat{B})^{\perp} Q (\hat{P}\hat{B})^{\perp T} = \begin{bmatrix} B^{\perp}(AX + XA^T)B^{\perp T} & B^{\perp}(KCX)^T \\ KCXB^{\perp T} & -2I \end{bmatrix}$$
$$\hat{C}^{T\perp} Q \hat{C}^{T\perp T} = YA + A^T Y - 2C^T K_1 K_2 C.$$

This implies that the two matrix inequalities in (20) are equivalent to (15) and (16), respectively. On the other hand, because of the formulation of $X$ and $Y$ in (21), we can prove that $P > 0$ is equivalent to (17) [7], [4]. ■

The above derivation procedure is similar to that of [7], [8], [4]. Similarly, if the solution $(X > 0, Y > 0)$ of the LMIs (15), (16) and (17) satisfies the rank condition

$$n_c = \operatorname{rank}(I - XY) < n$$

then, an $n_c$th-order controller can be constructed. Select $U > 0$ and $N$ such that $NU^{-1}N^T = Y - X^{-1}$ and define

$$\hat{P} = \begin{bmatrix} P & 0 \\ 0 & I \end{bmatrix} \quad P = \begin{bmatrix} Y & N \\ N^T & U \end{bmatrix}.$$

Then, a stabilizing output feedback controller can be constructed by solving the LMI (19) for $\hat{F}$.

We note here that the matrix inequality (15) is equivalent to

$$B^{\perp}(AX + XA^T + XC^T KKCX)B^{\perp T} < 0$$

which is stronger than the stabilizability condition of $(A, B)$ [2].

In what follows, we will present an approach to explicitly computing the $\mathcal{H}_\infty$ control law through the variable linearizing change approach of [3].

*Theorem 5:* Given a system (1)–(3) and a constant $\gamma > 0$, there exists an output feedback controller (5)–(6) such that, for any $\psi \in [K_1, K_2]$, the closed-loop system (8)–(10) is globally asymptotically stable at the origin and the $\mathcal{L}_2$ gain from $w$ to $z$ is less than or equal to $\gamma$ if there exist matrices $X > 0, Y > 0, \bar{A}, \bar{B}, \bar{C}$ and $\bar{D}$ of appropriate dimensions such that the LMIs shown in (22) at the bottom of the page, and as follows, hold:

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} \geq 0 \tag{23}$$

where $*$ represents blocks that are readily inferred by symmetry. Suppose that $(X, Y, \bar{A}, \bar{B}, \bar{C}, \bar{D})$ is a feasible solution of the LMIs (22) and (23). Then, the system matrices of the desired output feedback controller (5)–(6) can be computed by

$$\begin{bmatrix} D_c & C_c \\ B_c & A_c \end{bmatrix} = \begin{bmatrix} I & 0 \\ YB & N \end{bmatrix}^{-1} \begin{bmatrix} \bar{D} & \bar{C} \\ \bar{B} & \bar{A} - YAX \end{bmatrix} \begin{bmatrix} I & K_1 CX \\ 0 & M^T \end{bmatrix}^{-1} \tag{24}$$

where $M \in \mathcal{R}^{n \times n_c}$ and $N \in \mathcal{R}^{n \times n_c}$ are two matrices satisfying

$$MN^T = I - XY. \tag{25}$$

*Proof:* By Lemma 3, the closed-loop system (8)–(10) is globally asymptotically stable at the origin and the $\mathcal{L}_2$ gain is less than or equal to $\gamma$ if there exists a matrix $P > 0$ satisfying (26), shown at the bottom of the next page. Partition $P$ and $P^{-1}$ as

$$P = \begin{bmatrix} Y & N \\ N^T & U \end{bmatrix} \quad P^{-1} = \begin{bmatrix} X & M \\ M^T & V \end{bmatrix}$$

$$\begin{bmatrix} AX + B\bar{C} + (*) & * & * & * & * \\ \bar{A} + A^T + C^T K_1 \bar{D}^T B^T & YA + \bar{B}K_1 C + (*) & * & * & * \\ B_1^T + D_{21}^T \bar{D}^T B^T & B_1^T Y + D_{21}^T \bar{B}^T & -\gamma^2 I & * & * \\ C_1 + D_{12}\bar{C} & C_1 + D_{12}\bar{D}K_1 C & D_{11} + D_{12}\bar{D}D_{21} & -I & * \\ KCX + \bar{D}^T B^T & KC + \bar{B}^T & 0 & \bar{D}^T D_{12}^T & -2I \end{bmatrix} < 0 \tag{22}$$

where $X>0$ and $Y>0$. Then, we have

$$XY+MN^T=I, M^TY+VN^T=0$$
$$XN+MU=0, M^TN+VU=I.$$

Define the linearizing change of the control variables as follows:

$$\begin{bmatrix}\bar{D} & \bar{C}\\ \bar{B} & \bar{A}\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & YAX\end{bmatrix}+\begin{bmatrix}I & 0\\ YB & N\end{bmatrix}\begin{bmatrix}D_c & C_c\\ B_c & A_c\end{bmatrix} \times\begin{bmatrix}I & K_1CX\\ 0 & M^T\end{bmatrix} \quad (27)$$

and

$$Q_1=\begin{bmatrix}X & I\\ M^T & 0\end{bmatrix} \quad Q_2=\begin{bmatrix}I & Y\\ 0 & N^T\end{bmatrix}.$$

It is not difficult to see that $PQ_1=Q_2$ and

$$\bar{P}=Q_1^TPQ_1=Q_2^TP^{-1}Q_2=\begin{bmatrix}X & I\\ I & Y\end{bmatrix} \quad (28)$$

$$Q_2^T(\tilde{A}+\tilde{B}K_1\tilde{C})Q_1=\begin{bmatrix}AX+B\bar{C} & A+B\bar{D}K_1C\\ \bar{A} & YA+\bar{B}K_1C\end{bmatrix} \quad (29)$$

$$Q_2^T\tilde{B}_1=\begin{bmatrix}B_1+BD_cD_{21}\\ YB_1+(YBD_c+NB_c)D_{21}\end{bmatrix} =\begin{bmatrix}B_1+B\bar{D}D_{21}\\ YB_1+\bar{B}D_{21}\end{bmatrix} \quad (30)$$

$$Q_2^T\tilde{B}=\begin{bmatrix}B\bar{D}\\ \bar{B}\end{bmatrix}, \tilde{C}Q_1=[CX \quad C] \quad (31)$$

$$(\tilde{C}_1+\tilde{C}_2K_1\tilde{C})Q_1=[C_1+D_{12}\bar{C} \quad C_1+D_{12}\bar{D}K_1C]. \quad (32)$$

Premultiplying and postmultiplying (26) by $\mathrm{diag}(Q_1^T,I,I,I)$ and $\mathrm{diag}(Q_1,I,I,I)$, respectively, and using (28)–(32), we can find that (26) and $P>0$ hold if and only if (22) and (23) hold, respectively. ■

## V. Regional $\mathcal{H}_\infty$ Design for Sensor Saturation

In this section, we will consider the regional $\mathcal{H}_\infty$ controller design for the systems with sensor saturation. That is, we assume that in system (1)–(3)

$$\psi(Cx)=\sigma(Cx) \quad (33)$$

where $\sigma(\cdot)$ is the standard vector-valued saturation function defined as follows:

$$\sigma(v)=\begin{bmatrix}\sigma_1(v_1)\\ \sigma_2(v_2)\\ \cdots\\ \sigma_p(v_p)\end{bmatrix}$$

with $\sigma_i(v_i)=\mathrm{sign}(v_i)\min\{v_{i,\max},|v_i|\}$. Here, $v_{i,\max}$ denotes the $i$th element of the vector $v_{\max}$, the saturation level.

Let $\tilde{c}_i$ stand for the $i$th row of the matrix $\tilde{C}$. We define the symmetric polyhedron

$$\mathcal{L}(\tilde{C},v_{\max})=\{\tilde{x}\in\mathcal{R}^{n+n_c}:|\tilde{c}_i\tilde{x}|\le v_{i,\max}, \quad i=1,2,\ldots,p\}.$$

If control $v$ does not saturate for all $i=1,2,\ldots,p$, that is $x\in\mathcal{L}(\tilde{C},v_{\max})$, then the nonlinear dynamics (8) admits the following linear representation:

$$\dot{\tilde{x}}=(\tilde{A}+\tilde{B}\tilde{C})\tilde{x}+\tilde{B}_1w. \quad (34)$$

Note that the saturation function $\sigma(v)$ can be written as a sector nonlinearity described by (4) with $K_1=0$ and $K_2=I$. That is, $\sigma\in[0,I]$. Hence, global asymptotic stability of the closed-loop system subject to sensor saturation can be analyzed by the results of Section IV. By Theorem 4, it is easy to see that the existence of the globally stabilizing output feedback controller requires the open-loop system to be asymptotically stable, i.e., $A$ is a Hurwitz matrix. In what follows, we will design a regional stabilizing controller based on the approach developed in the previous section for general systems. In our regional control design, we do not require the saturation function to belong to a sector $[K_1,K_2]$ globally and thus $K_1$ does not have to be 0.

To analyze the regional $\mathcal{L}_2$ gain, we also assume that the disturbance $w\in\mathcal{W}$, where

$$\mathcal{W}=\left\{w\in\mathcal{L}_2[0,\infty):w^TRw\le 1\right\}$$

for some $R>0$.

For the regional analysis of the linear system subject to sensor saturation, we first present the following definitions.

*Definition 2:* Consider the closed-loop system subject to sensor saturation (8)–(10) and (33). A set in $\mathcal{R}^{n+n_c}$ is said to be invariant if all the trajectories $\varphi(t,x_0,w)$ starting from within it will remain in it regardless of $w\in\mathcal{W}$. An ellipsoid $\Omega(P,\rho)=\{\tilde{x}:V(\tilde{x})=\tilde{x}^TP\tilde{x}\le\rho\}$ is invariant if $\dot{V}\le 0$ for all $w\in\mathcal{W}$ and all $\tilde{x}\in\partial\Omega(P,\rho)$, the boundary of $\Omega(P,\rho)$.

*Definition 3:* For a given set $X_\infty\subset\mathcal{R}^{n+n_c}$, the system subject to sensor saturation (8)–(10) and (33) is said to have a regional $\mathcal{L}_2$ gain less than or equal to $\gamma$ in $X_\infty$ for some $\gamma>0$ if $X_\infty$ is invariant, i.e., $x(t)\in X_\infty$ for all $t\in[0,\infty)$, and

$$\int_0^T\|z(t)\|^2dt\le\gamma^2\int_0^T\|w(t)\|^2\,dt \quad \tilde{x}(0)=0 \qquad \forall w\in\mathcal{L}_2[0,T) \qquad \forall T\ge 0. \quad (35)$$

We can then prove the following theorem.

*Theorem 6:* Given a system subject to sensor saturation (1)–(2) and (33), a dynamic feedback control law (5)–(6) and a constant $\gamma>0$, the ellipsoid $\Omega(P,\rho)$ is invariant and the regional $\mathcal{L}_2$ gain from $w$ to $z$ is less than or equal to $\gamma$ if

$$\begin{bmatrix}(\tilde{A}+\tilde{B}K_1\tilde{C})^TP+P(\tilde{A}+\tilde{B}K_1\tilde{C}) & P\tilde{B}_1 & (\tilde{C}_1+\tilde{C}_2K_1\tilde{C})^T & (K\tilde{C}+\tilde{B}^TP)^T\\ \tilde{B}_1^TP & -\gamma^2I & \tilde{D}_{11}^T & 0\\ \tilde{C}_1+\tilde{C}_2K_1\tilde{C} & \tilde{D}_{11} & -I & \tilde{C}_2\\ K\tilde{C}+\tilde{B}^TP & 0 & \tilde{C}_2^T & -2I\end{bmatrix}<0 \quad (26)$$

there exist two positive diagonal matrices $K_1, K_2 \in \mathcal{R}^{p\times p}$ with $K_1 < I, K_2 \geq I$ and a constant $\eta > 0$ such that (36) and (37), shown at the bottom of the page, hold, and $\Omega(P,\rho) \subset \mathcal{L}(K_1\tilde{C}, \; v_{\max})$, i.e., $|k_{1i}\tilde{c}_i\tilde{x}| \leq v_{i,\max}$ for all $\tilde{x} \in \Omega(P,\rho), i = 1,2,\ldots,p,$ where $k_{1i}$ denotes the $i$th diagonal element of $K_1$.

*Proof:* First we establish the set invariance. That is, for $V(x) = \tilde{x}^T P\tilde{x}$, we will show that

$$\dot{V} = 2\tilde{x}^T P(\tilde{A}\tilde{x} + \tilde{B}_1 w + \tilde{B}\sigma(v)) < 0 \qquad \forall \tilde{x} \in \partial\Omega(P,\rho), \qquad w^T Rw \leq 1.$$

Following the procedure in the proof of Lemma 2, we can show that for each $\tilde{x} \in \Omega(P,\rho)$

$$\begin{aligned}\dot{V} &= 2\tilde{x}^T P((\tilde{A} + \tilde{B}K_1\tilde{C})\tilde{x} + \tilde{B}_1 w + \tilde{B}\sigma_s(v)) \\ &\leq \begin{bmatrix}\tilde{x}\\ \sigma_s\end{bmatrix}^T \begin{bmatrix}(\tilde{A}+\tilde{B}K_1\tilde{C})^T P + P(\tilde{A}+\tilde{B}K_1\tilde{C}) & * \\ K\tilde{C} + \tilde{B}^T P & -2I\end{bmatrix} \\ &\quad \times \begin{bmatrix}\tilde{x}\\ \sigma_s\end{bmatrix} + 2\tilde{x}^T P\tilde{B}_1 w\end{aligned}$$

where $\sigma_s(v) = \sigma(v) - K_1 v$. Note that

$$\begin{aligned}2\tilde{x}^T P\tilde{B}_1 w &\leq \tilde{x}^T P\tilde{B}_1 R^{-1}\tilde{B}_1^T P\tilde{x} + w^T Rw \\ &\leq \eta^{-1}\tilde{x}^T P\tilde{B}_1 R^{-1}\tilde{B}_1^T P\tilde{x} + \eta\end{aligned}$$

we obtain the third equation at the bottom of the page. It follows from (36) and Schur complement, that for all $\tilde{x} \in \Omega(P,\rho)\setminus\{0\}$ and $w^T Rw \leq 1$,

$$\dot{V} \leq -\frac{\eta}{\rho}\tilde{x}^T P\tilde{x} + \eta.$$

Observing that on the boundary of $\Omega(P,\rho), \tilde{x}^T P\tilde{x} = \rho$, hence $\dot{V} < 0$. It follows that $\Omega(P,\rho)$ is an invariant set. On the other hand, following the procedure in the proof of Lemma 3, it is easy to see that if (37) holds then the inequality (35) holds. ∎

It is easy to check that under the condition of Theorem 6, $\dot{V} < 0, \forall\tilde{x} \in \Omega(P,\rho)\setminus\{0\}$, in the absence of disturbance. This implies that (8)–(10) with $w = 0$ is asymptotically stable at the origin with $\Omega(P,\rho)$ contained in the domain of attraction.

Note that the constraint

$$\Omega(P,\rho) \subset \mathcal{L}(K_1\tilde{C}, v_{\max})$$

is equivalent to

$$k_{1i}^2\tilde{c}_i(P/\rho)^{-1}\tilde{c}_i^T \leq v_{i,\max}^2 \Leftrightarrow \begin{bmatrix}\frac{1}{\rho}v_{i,\max}^2 & k_{1i}\tilde{c}_i \\ k_{1i}\tilde{c}_i^T & P\end{bmatrix} \geq 0, \qquad i = 1,2,\ldots,p. \quad (38)$$

With the predesigned dynamic control law (5)–(6) and the given $\mathcal{H}_\infty$ performance index $\gamma$, we can then present the following optimization problem to estimate the invariant set $\Omega(P,\rho)$ with a given $\mathcal{L}_2$ gain $\gamma$

$$\max_{P>0,\rho} \Omega(P,\rho)$$
$$\text{s.t. matrix inequalities (36), (37), (38).}$$

Similar to Section IV, with the given $K_1$ and $K_2$, the stabilizing output feedback controller can be directly computed by the variable linearizing change approach.

*Theorem 7:* Given a system subject to sensor saturation (1)–(3) and (33), and a $\gamma > 0$, if, for two given positive diagonal matrices $K_1, K_2 \in \mathcal{R}^{p\times p}$ with $K_1 < I, K_2 \geq I$, there exist constant $\rho > 0, \eta > 0$ and matrices $X > 0, Y > 0, \bar{A}, \bar{B}, \bar{C}$ and $\bar{D}$ of compatible dimensions satisfying (22), (23), and (39) shown at the bottom of the next page, and

$$\begin{bmatrix}\frac{1}{\rho}(v_{i,\max}^2) & k_{1i}c_i X & k_{1i}c_i \\ k_{1i}Xc_i^T & X & I \\ k_{1i}c_i^T & I & Y\end{bmatrix} \geq 0, \qquad i = 1,2,\ldots,p \quad (40)$$

then, there exists a dynamic output feedback controller of the form (5)–(6) such that the regional $\mathcal{L}_2$ gain from $w$ to $z$ is less than or equal to $\gamma$ in the region $\Omega(P,\rho)$, where

$$P = \begin{bmatrix}Y & N \\ N^T & U\end{bmatrix} \quad (41)$$

for any matrices $N$ and $U$ satisfying decomposition

$$NU^{-1}N^T = Y - X^{-1}. \quad (42)$$

$$\begin{bmatrix}(\tilde{A}+\tilde{B}K_1\tilde{C})^T P + P(\tilde{A}+\tilde{B}K_1\tilde{C}) + \frac{\eta}{\rho}P & P\tilde{B}_1 & (K\tilde{C}+\tilde{B}^T P)^T \\ \tilde{B}_1^T P & -\eta R & 0 \\ K\tilde{C}+\tilde{B}^T P & 0 & -2I\end{bmatrix} < 0 \quad (36)$$

$$\begin{bmatrix}(\tilde{A}+\tilde{B}K_1\tilde{C})^T P + P(\tilde{A}+\tilde{B}K_1\tilde{C}) & P\tilde{B}_1 & (\tilde{C}_1+\tilde{C}_2K_1\tilde{C})^T & (K\tilde{C}+\tilde{B}^T P)^T \\ \tilde{B}_1^T P & -\gamma^2 I & \tilde{D}_{11}^T & 0 \\ \tilde{C}_1+\tilde{C}_2K_1\tilde{C} & \tilde{D}_{11} & -I & \tilde{C}_2 \\ K\tilde{C}+\tilde{B}^T P & 0 & \tilde{C}_2^T & -2I\end{bmatrix} < 0 \quad (37)$$

$$\dot{V} \leq \begin{bmatrix}\tilde{x}\\ \sigma_s\end{bmatrix}^T \begin{bmatrix}(\tilde{A}+\tilde{B}K_1\tilde{C})^T P + P(\tilde{A}+\tilde{B}K_1\tilde{C}) + \eta^{-1}P\tilde{B}_1 R^{-1}\tilde{B}_1^T P & * \\ K\tilde{C}+\tilde{B}^T P & -2I\end{bmatrix}\begin{bmatrix}\tilde{x}\\ \sigma_s\end{bmatrix} + \eta$$

Suppose that $(X, Y, \bar{A}, \bar{B}, \bar{C}, \bar{D})$ is a feasible solution of the LMIs (22), (23), (39), and (40). Then, the system matrices of the output feedback controller (5)–(6) can be computed by (24).

*Proof:* By the linearizing change of variables (27), we find that (36) and (37) are equivalent to (39) and (22), respectively. Changing variable as shown in (28)–(32), we have

$$\begin{aligned}\tilde{C}P^{-1}\tilde{C}^T &= \tilde{C}Q_1(Q_1^T P Q_1)^{-1}(\tilde{C}Q_1)^T \\ &= [CX \quad C]\bar{P}^{-1}[CX \quad C]^T.\end{aligned}$$

By (38), we have

$$\rho k_{1i}^2 \tilde{c}_i P^{-1}\tilde{c}_i^T \le v_{i,\max}^2 \Longleftrightarrow \begin{bmatrix} \frac{1}{\rho}\left(v_{i,\max}^2\right) & k_{1i}c_i X & k_{1i}c_i \\ k_{1i}Xc_i^T & X & I \\ k_{1i}c_i^T & I & Y \end{bmatrix} \ge 0, \quad i = 1, 2, \ldots, p. \tag{43}$$

This means the constraint $\Omega(P, \rho) \subset \mathcal{L}(K_1\tilde{C},\ v_{\max})$ can be written as LMI (40). Then, by Theorem 6, we can prove the result. ■

It is easy to see that a different decomposition (42) will lead to a different invariant set $\Omega(P, \rho)$, but its projection on state $x$, i.e., with $x_c = 0$, ellipsoid $\Omega(Y, \rho)$ is the same. A typical solution of (42) is $N = U = Y - X^{-1}$.

## VI. A Numerical Example

The following state equations describe the longitudinal dynamics of the F-8 aircraft:

$$\begin{aligned}\dot{x} &= \begin{bmatrix} -0.8 & -0.006 & -12 & 0 \\ 0 & -0.014 & -16.64 & -32.2 \\ 1 & -0.0001 & -1.5 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} x \\ &\quad - \begin{bmatrix} 19 & 3 \\ 0.66 & 0.5 \\ 0.16 & 0.5 \\ 0 & 0 \end{bmatrix} u + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} w \\ y &= \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} x \\ z &= [0 \quad 0 \quad 0 \quad 1]x + [0 \quad 1]u.\end{aligned}$$

This example is borrowed from [16]. The disturbance $w$ satisfying $\|w\|_2 \le 1$ is added to study the $\mathcal{L}_2$ performance under output feedback control. We assume that the sensors are subject to saturation with $y_{i,\max} = 1$ for $i = 1, 2$. With $\gamma = 0.5, \rho = 1, \eta = 0.1, R = I, K_1 = 0.9I, K_2 = I$, we obtain the following controller by Theorem 7:

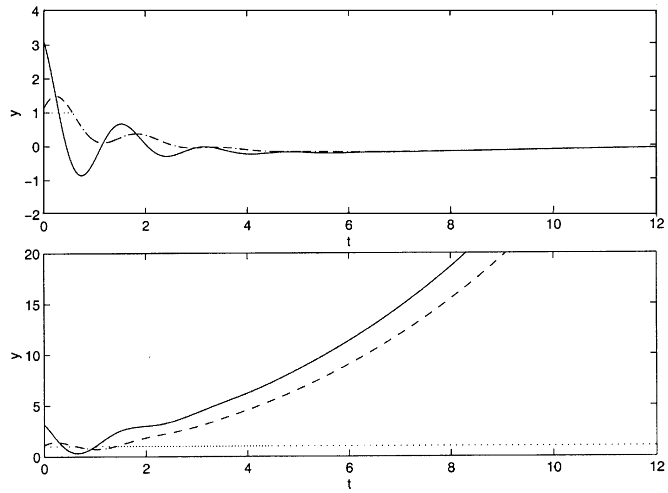


Fig. 1. Output responses with sensor saturation. (a) Proposed controller. (b) Central $H_\infty$ controller.

$$A_c = \begin{bmatrix} 0.0047 & 0.0014 & -1.9213 & -0.8306 \\ -4.6671 & 0.0048 & 6.6572 & -3.7604 \\ 0.0126 & -0.0000 & -0.0364 & 0.0038 \\ 0.0280 & -0.0000 & -0.0405 & 0.0231 \end{bmatrix} \times 10^3$$

$$B_c = \begin{bmatrix} -1.3735 & -0.5297 \\ 701.9195 & 442.7050 \\ -1.4853 & -0.6303 \\ -3.5543 & -2.0714 \end{bmatrix}$$

$$C_c = \begin{bmatrix} 0.8523 & 0.0711 & -101.1930 & -43.0930 \\ 0.0053 & 0.0001 & -0.5264 & 0.4506 \end{bmatrix}$$

$$D_c = 0$$

$$X = \begin{bmatrix} 0.0163 & 0.0010 & 0.0001 & -0.0000 \\ 0.0010 & 1.3081 & 0.0003 & 0.0014 \\ 0.0001 & 0.0003 & 0.0000 & 0.0000 \\ -0.0000 & 0.0014 & 0.0000 & 0.0000 \end{bmatrix} \times 10^5$$

$$Y = \begin{bmatrix} 0.1936 & 0.0004 & -0.5490 & 0.3814 \\ 0.0004 & 0.0000 & -0.0033 & 0.0042 \\ -0.5490 & -0.0033 & 5.4277 & -2.9348 \\ 0.3814 & 0.0042 & -2.9348 & 3.1918 \end{bmatrix}.$$

Fig. 1 shows the output responses with sensor saturation under the controller proposed in this paper and the central $H_\infty$ controller as obtained by the MATLAB command hinfric with the specification of $\gamma = 0.5$. In this simulation,

$$w_1 = \sin(t) \quad w_2 = \cos(t)$$

$$\begin{bmatrix} AX + B\bar{C} + (*) + \frac{\eta}{\rho}X & * & * & * \\ \bar{A} + A^T + C^T K_1 \bar{D}^T B^T + \frac{\eta}{\rho}I & YA + \bar{B}K_1C + (*) + \frac{\eta}{\rho}Y & * & * \\ B_1^T + D_{21}^T \bar{D}^T B^T & B_1^T Y + D_{21}^T \bar{B}^T & -\eta R & * \\ KCX + \bar{D}^T B^T & KC + \bar{B}^T & 0 & -2I \end{bmatrix} < 0 \tag{39}$$

and the initial condition is

$$x_0 = [-4.7991 \quad -3.5246 \quad 2.0517 \quad 3.1592]^T$$

which is a random vector generated by the MATLAB. Fig. 1(b) shows that the closed-loop system under the standard $H_\infty$ controller loses stability in the presence of sensor saturation. This is because the sensor saturation was not taken into account in the control design.

## VII. Conclusion

In this paper, we analyzed the stability and the $\mathcal{L}_2$ gain for linear systems with sensor nonlinearities based on the circle criterion theory. A globally stabilizing output feedback controller design approach was proposed using the LMI based approach. The results were then applied to the systems with sensor saturation nonlinearities. A regional stabilizing output feedback controller design method was proposed such that the closed-loop systems has a given regional $\mathcal{L}_2$ gain.

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