SCIENCE CHINA
Information Sciences

• RESEARCH PAPER •

July 2017, Vol. 60 072203:1–072203:13
doi: 10.1007/s11432-016-9025-9

Online schedule for autonomy of multiple unmanned aerial vehicles

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Received August 29, 2016; accepted November 29, 2016; published online May 19, 2017

Abstract An online rectangle based scheduling algorithm (RSA) is developed to improve autonomy of multiple unmanned aerial vehicles (UAVs) to search a field of forest together. The purposes of RSA are to online decide the number of the UAVs to be assigned and to schedule the path for the assigned UAVs to search the missed areas resulted from the previous search. The main ideas of RSA are to cover each separated zone of the missed areas with a rectangle and then to schedule the path to search the rectangles. Thus, RSA is robust against the unknown shapes and sizes of the missed areas. The forest search is applied to verify the online RSA in simulation. The simulation results demonstrate that the online RSA is successful to decide the number of the UAVs to be assigned and to schedule the path for the assigned UAVs to search the missed areas.

Keywords online schedule, mission planning, autonomy of UAVs, multiple UAVs, algorithms

Citation Peng K M, Lin F, Chen B M. Online schedule for autonomy of multiple unmanned aerial vehicles. Sci China Inf Sci, 2017, 60(7): 072203, doi: 10.1007/s11432-016-9025-9

1 Introduction

The autonomy of the unmanned aerial vehicles (UAVs) has been one of the attracting topics in the unmanned aerial systems as it improves the capabilities of the UAVs to complete a designated mission independently. There are a lot of publications to improve the capabilities of the UAVs. The 3D path planning was explored for a fixed-wing UAV in [1]. The cooperative path planning was applied for the multiple UAVs in [2–5]. The adaptive consensus strategy was used to implement the formation flight of the multiple spacecraft in [6,7]. The behavioral control methodology was applied to realize the formation control in [8]. The decentralized control strategy was used for the vehicle formation in [9, 10]. The virtual strategy was used to implement the networks of the satellites in [11]. The conflict resolution was considered in the path planning in [12]. Such autonomy results in that the UAVs can complete the assigned missions/ tasks autonomously, they do not rely on the ground station to be controlled by the operators and are not limited by the radio range to communicate each other or with the ground station.

In order to achieve the autonomy, the online schedule is essential. The pro-schedule can improve the autonomy of the UAVs. But it is not robust against the unknown factors of the obstacles and collision. The online schedule is activated once an accident or an event happens and thus it can take the new

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detected factors into account to reschedule to complete the designated mission. The dynamic factors were considered to schedule in [13, 14]. The augmenting path planning was considered with constraints in [15]. The policies and rules were considered in the online scheduling in [16–18]. The dynamic mission planning was considered in [19]. The time-hierarchical scheduling was discussed in [20]. The autonomous mission execution was discussed in [21]. The learning path planning was considered in [22]. Since such new detected factors are considered in the online scheduling, the autonomy of the UAVs can be improved potentially.

Many researchers dedicated their efforts to the online schedule and pre-schedule. The hierarchical path planning approach was developed for the virtual reality in [23]. The path was planned for the robots in [24, 25]. The delegation was used for the high-level mission planning in [26]. The jobs were scheduled for the machines with constraints in [27–30]. The cognitive system architecture was discussed for the task scheduling in [31, 32]. The bi-objective scheduling was explored in [33]. Among those algorithms, the heuristic method is one of the typical methods that are popularly used in the schedule for optimization. However, the core problems are still left to study. One problem is how to formulate the schedule in presence of the unknown factors. That is to improve the robustness against the unknown factors. Another problem is to schedule online with minimal computation. This has been one of the challenges in the online schedule [34, 35].

Our objective is to develop an online scheduling algorithm to decide the number of the UAVs to be assigned and to schedule the path for the assigned UAVs to search the missed areas resulted from the previous search. The missed areas are presented in the 2D grid maps. We do not know how many zones of the missed areas there are. The shapes and sizes of the missed areas are unknown. This is our scenario. We need to take measures to handle the unknown factors to schedule with minimal computation. This motivated us to develop an online rectangle based scheduling algorithm (RSA) for the multiple UAVs together to search a field of forest autonomously.

To develop the online RSA, our ideas are as follows. At first, the number of separated zones of the missed areas is detected. Each separated zone of the missed areas is covered with a minimal-size rectangle. A set of the path is scheduled to search each of the rectangles. Those sets of the path are connected based on the nearest neighbor. Based on the allocated flight distance of the UAVs, the number of the UAVs to be assigned is decided. Finally, schedule the take-off and landing path to connect the searching path of the assigned UAVs. An online schedule is generated eventually. The outline of the paper is as follows. The online RSA is developed step by step in the next section. The developed online RSA is verified in simulation in Section 3. The simulation results will demonstrate that the online RSA is successful to decide the number of the UAVs to be assigned and to schedule the path for the assigned UAVs to search the missed areas. Finally, the concluding remarks are drawn out.

2 Online RSA

We consider a 2D grid map as follows:

$$m_{i,j} = \begin{cases} 1, \text{ detected}, & i \in N_x := \{1, \ldots, n_x\}, \\ 0, \text{ missed}, & j \in N_y := \{1, \ldots, n_y\}, \end{cases} \tag{1}$$

where n_x and n_y denote the size of the map. $m_{i,j}$ denotes the value in the i-th and j-th grid of the map. The 2D grid map denotes the previous search results in which 1 in the grids means detected and 0 in the grids means missed. The indices of the grids mean the coordinates of the grids. The missed areas are shown in Figure 1. The map frame is defined in Figure 1. Note that the indices of the grid at the left and bottom corner are $(1, 1)$.

In order to develop RSA, the assumptions are necessary as follows:

(1) The nominal distance between the two neighbor channels is known;

(2) The maximal flight distance of the UAVs is known;

(3) One side of the forest field is accessible;

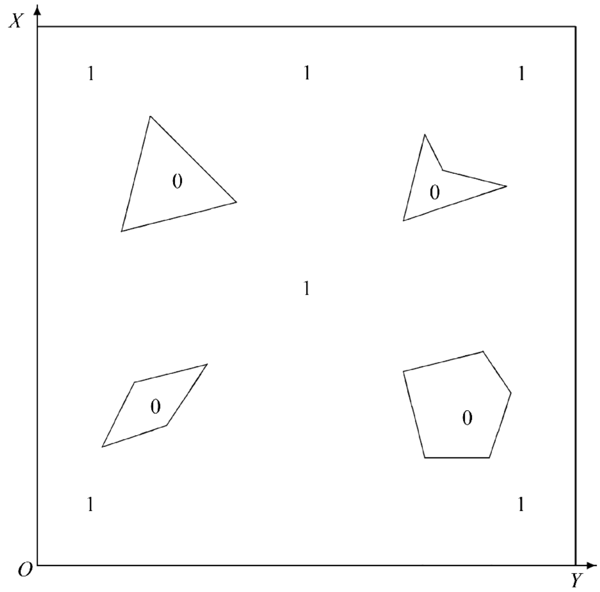

Figure 1 Missed areas in a 2D grid map.

(4) The length of the channels to be scheduled to search a separated zone of the missed areas is not more than the allocated flight distance of the UAVs.

Note that the considered UAVs are the rotary-wing aircraft that fly in low speeds. Such typical UAVs are the quadrotors. The online RSA will be developed in four steps such as

(1) detect the separated zones of the missed areas;

(2) cover the separated zones of the missed areas with rectangles;

(3) schedule the path to search the rectangles and decide the number of the UAVs to be assigned; and

(4) schedule the path for taking off and landing.

Subsequently, we proceed to develop the online RSA step by step.

2.1 Step 1: detect the separated zones of the missed areas

Once the value of a grid in the map is detected 0, it denotes a grid of the missed areas. Then if one of its neighbor grids have been labeled as a known zone of the missed areas, the grid is not a new zone of the missed areas and thus is labeled as its neighbor grids. Otherwise, the grid is a new zone of the missed areas and thus is labeled with a new label. The process can be presented as follows.

Suppose that $n_{\rm ma}$ separated zones of the missed areas have been detected out and each detected separated zone of the missed areas has been labeled with a negative integer such as $-1,\ldots,-n_{\rm ma}$. Once $m_{i,j}=0, i\in N_x, j\in N_y$ is detected out, then

$$m_{i,j}=\begin{cases}\mu, & m_{k,l}=\mu<0,\\ -(n_{\rm ma}+1), & \forall m_{k,l}\geqslant 0,\end{cases} \tag{2}$$

where $(k,l)\in N_{\rm bh}:=\{(i-1,j-1),(i,j-1),(i+1,j-1),(i-1,j),(i+1,j),(i-1,j+1),(i,j+1),(i+1,j+1)\}$. Note that the elements of $N_{\rm bh}$ are subject to their existence. Otherwise, the elements disappear in $N_{\rm bh}$.

The operation is shown in Figure 2. A separated zone of the missed areas may be labeled with more than one negative integers. Thus, the detected outcome will be checked again to correct the mistakes by detecting whether there is any different label in each separated zone of the missed areas. This detection on the map provides us the number of separated zones of the missed areas in the map.

2.2 Step 2: cover the separated zones of the missed areas with rectangles

Each detected separated zone of the missed areas will be covered with a rectangle. By the coverage, we just need to search the rectangles and do not need to know the sizes and shapes of the missed areas.

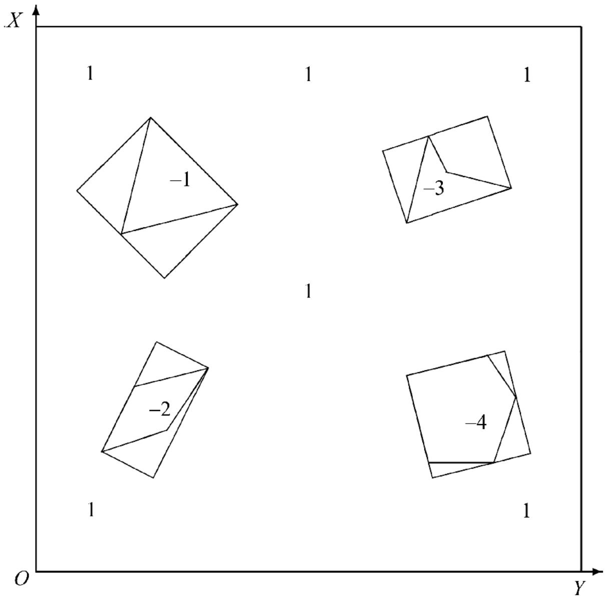

Figure 2 Covered missed areas in a map.

A minimal-size rectangle can be found to cover a separated zone of the missed areas by rotating the frame in an angle of θ to find one of the optimal solutions or a reasonable solution. Note that $\theta \in [0, \pi/2)$ because of the symmetry of rectangles.

Suppose $Z_{\rm ma}$ denotes a separated zone of the missed areas and the frame is rotated in an angle of θ. Then a rectangle can be found to cover $Z_{\rm ma}$. The rectangle, $R_{\rm ct}$, is presented in the coordinates of its four vertices as follows:

$$
\begin{aligned}
R_{\rm ct}(\theta) &:= \left\{ \begin{pmatrix} i_{\min} \\ j_{\min} \end{pmatrix}, \begin{pmatrix} i_{\min} \\ j_{\max} \end{pmatrix}, \begin{pmatrix} i_{\max} \\ j_{\max} \end{pmatrix}, \begin{pmatrix} i_{\max} \\ j_{\min} \end{pmatrix} \right\}, \\
i_{\max} &= \max_{z_{i,j} \in Z_{\rm ma}} i_\theta, \ i_{\min} = \min_{z_{i,j} \in Z_{\rm ma}} i_\theta, \\
j_{\max} &= \max_{z_{i,j} \in Z_{\rm ma}} j_\theta, \ j_{\min} = \min_{z_{i,j} \in Z_{\rm ma}} j_\theta, \\
\begin{pmatrix} i_\theta \\ j_\theta \end{pmatrix} &= B(\theta) \begin{pmatrix} i \\ j \end{pmatrix}, \\
B(\theta) &= \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}.
\end{aligned} \tag{3}
$$

The minimal-size rectangle can be determined as follows:

$$
\begin{aligned}
R_{\rm ct,min} &= \min_{\theta \in [0, \pi/2)} \left[S_{\rm rct}(\theta) / S_{\rm zma} \right], \\
S_{\rm rct}(\theta) &= (i_{\max} - i_{\min})(j_{\max} - j_{\min}),
\end{aligned} \tag{4}
$$

where $S_{\rm zma}$ denotes the size of $Z_{\rm ma}$. Suppose that the optimal solution is found at $\theta = \theta^\star$. Then, the vertex coordinates of $R_{\rm ct}(\theta^\star)$ need to be expressed in the original frame with the conversion as follows:

$$
\begin{pmatrix} i \\ j \end{pmatrix} = \mathrm{round} \left[B'(\theta^\star) \begin{pmatrix} i_\delta \\ j_\rho \end{pmatrix} \right], \quad \delta, \rho \in \{\max, \min\}, \tag{5}
$$

where the indices are rounded to the nearest integers. If two rectangles join together, the corresponding two zones of the missed areas can be combined together to cover with one rectangle. The covered missed areas are shown in Figure 2.

2.3 Step 3: schedule the path to search the rectangles and decide the number of the UAVs to be assigned

Based on the nominal distance between the two neighbor channels and the provided vertex coordinates of the rectangles, the path can be scheduled to search the rectangles.

Suppose the vertex coordinates of the k-th rectangle, $R_{\mathrm{ct},k}$, are $\{x_1, x_2, x_3, x_4\}, x_i \in R^2, i = 1, 2, 3, 4$ and the nominal distance between the two neighbor channels is d_{nbh}. Then the length and width of $R_{\mathrm{ct},k}$ are $l = \|x_1 - x_4\|$ and $w = \|x_1 - x_2\|$ respectively and

$$n_{\mathrm{l}} = \mathrm{round}[l/d_{\mathrm{nbh}}], \quad n_{\mathrm{w}} = \mathrm{round}[w/d_{\mathrm{nbh}}]. \tag{6}$$

If $n_{\mathrm{l}} = 0$, n_{l} is set to 1 and if $n_{\mathrm{w}} = 0$, n_{w} is set to 1. Then, the path is scheduled in six cases as follows.

(1) $n_{\mathrm{l}} = n_{\mathrm{w}} = 1$. Theoretically, the UAV can fly over the zone in any direction. We mainly consider two directions for simplicity. The two sets of the path are presented in a line segment as follows:

$$R_{\mathrm{tg},ik} = s_{\mathrm{gt},1i}, \quad i = 1, 2, \tag{7}$$

where $R_{\mathrm{tg},ik}$ denotes the i-th set of the scheduled path to search $R_{\mathrm{ct},k}$ and

$$s_{\mathrm{gt},11} = \begin{pmatrix} (x_1 + x_2)/2 \\ (x_3 + x_4)/2 \end{pmatrix}, \quad s_{\mathrm{gt},12} = \begin{pmatrix} (x_1 + x_4)/2 \\ (x_2 + x_3)/2 \end{pmatrix}.$$

Note that a line segment of the path is presented in

$$\begin{pmatrix} \text{starting point} \\ \text{end point} \end{pmatrix}.$$

(2) $n_{\mathrm{l}} > n_{\mathrm{w}} = 1$. The UAV can fly over the zone in a direction reasonably. A set of the path is presented as follows:

$$R_{\mathrm{tg},1k} = s_{\mathrm{gt},11}. \tag{8}$$

(3) $n_{\mathrm{w}} > n_{\mathrm{l}} = 1$. A set of the path is presented as follows:

$$R_{\mathrm{tg},1k} = s_{\mathrm{gt},12}. \tag{9}$$

(4) $n_{\mathrm{l}} = n_{\mathrm{w}} > 1$. The UAV can enter the zone in four directions reasonably. The four sets of the path are presented as follows:

$$R_{\mathrm{tg},ik} = \begin{pmatrix} s_{\mathrm{gt},1i} & \cdots & s_{\mathrm{gt},n_i i} \end{pmatrix}, \quad i = 1, 2, 3, 4, \tag{10}$$

where there are n_i of segments in the scheduled path. $n_i = 2n_{\mathrm{w}} - 1$ for $i = 1, 2$ or $n_i = 2n_{\mathrm{l}} - 1$ for $i = 3, 4$. Let $\hat{j} = 2j - 1$ and $\bar{j} = 2j$. Then

$$\begin{aligned}
s_{\mathrm{gt},\hat{j}1} &= \begin{pmatrix} y_{j,12} \\ z_{j,43} \end{pmatrix}, \quad s_{\mathrm{gt},\bar{j}1} = \begin{pmatrix} z_{j,43} \\ z_{j+1,43} \end{pmatrix}, \quad j = 1, \\
s_{\mathrm{gt},\hat{j}1} &= \begin{pmatrix} z_{j,43} \\ z_{j,12} \end{pmatrix}, \quad s_{\mathrm{gt},\bar{j}1} = \begin{pmatrix} z_{j,12} \\ z_{j+1,12} \end{pmatrix}, \quad j = 2, 4, 6, \ldots, \\
s_{\mathrm{gt},\hat{j}1} &= \begin{pmatrix} z_{j,12} \\ z_{j,43} \end{pmatrix}, \quad s_{\mathrm{gt},\bar{j}1} = \begin{pmatrix} z_{j,43} \\ z_{j+1,43} \end{pmatrix}, \quad j = 3, 5, 7, \ldots, \\
s_{\mathrm{gt},\hat{j}1} &= \begin{pmatrix} z_{j,43} \\ y_{j,12} \end{pmatrix}, \quad j = n_{\mathrm{w}} \text{ is an even}, \\
s_{\mathrm{gt},\hat{j}1} &= \begin{pmatrix} z_{j,12} \\ y_{j,43} \end{pmatrix}, \quad j = n_{\mathrm{w}} \text{ is an odd},
\end{aligned}$$

$$z_{j,12} = y_{j,12} + r_{\rm b}(y_{j,43} - y_{j,12}), \quad z_{j,43} = y_{j,43} + r_{\rm b}(y_{j,12} - y_{j,43}),$$
$$r_{\rm b} = d_{\rm nbh}/(2l), \quad y_{j,12} = x_1 + (j - 1/2)(x_2 - x_1)/n_{\rm w}, \quad y_{j,43} = x_4 + (j - 1/2)(x_3 - x_4)/n_{\rm w},$$

$$\begin{aligned}
&s_{{\rm gt},\hat{j}2} = \begin{pmatrix} y_{j,43} \\ z_{j,12} \end{pmatrix}, \quad s_{{\rm gt},\bar{j}2} = \begin{pmatrix} z_{j,12} \\ z_{j+1,12} \end{pmatrix}, \quad j = 1,\\
&s_{{\rm gt},\hat{j}2} = \begin{pmatrix} z_{j,12} \\ z_{j,43} \end{pmatrix}, \quad s_{{\rm gt},\bar{j}2} = \begin{pmatrix} z_{j,43} \\ z_{j+1,43} \end{pmatrix}, \quad j = 2, 4, 6, \ldots,\\
&s_{{\rm gt},\hat{j}2} = \begin{pmatrix} z_{j,43} \\ z_{j,12} \end{pmatrix}, \quad s_{{\rm gt},\bar{j}2} = \begin{pmatrix} z_{j,12} \\ z_{j+1,12} \end{pmatrix}, \quad j = 3, 5, 7, \ldots,\\
&s_{{\rm gt},\hat{j}2} = \begin{pmatrix} z_{j,12} \\ y_{j,43} \end{pmatrix}, \quad j = n_{\rm w} \text{ is an even},\\
&s_{{\rm gt},\hat{j}2} = \begin{pmatrix} z_{j,43} \\ y_{j,12} \end{pmatrix}, \quad j = n_{\rm w} \text{ is an odd},
\end{aligned}$$

$$\begin{aligned}
&s_{{\rm gt},\hat{j}3} = \begin{pmatrix} y_{j,14} \\ z_{j,23} \end{pmatrix}, \quad s_{{\rm gt},\bar{j}3} = \begin{pmatrix} z_{j,23} \\ z_{j+1,23} \end{pmatrix}, \quad j = 1,\\
&s_{{\rm gt},\hat{j}3} = \begin{pmatrix} z_{j,23} \\ z_{j,14} \end{pmatrix}, \quad s_{{\rm gt},\bar{j}3} = \begin{pmatrix} z_{j,14} \\ z_{j+1,14} \end{pmatrix}, \quad j = 2, 4, 6, \ldots,\\
&s_{{\rm gt},\hat{j}3} = \begin{pmatrix} z_{j,14} \\ z_{j,23} \end{pmatrix}, \quad s_{{\rm gt},\bar{j}3} = \begin{pmatrix} z_{j,23} \\ z_{j+1,23} \end{pmatrix}, \quad j = 3, 5, 7, \ldots,\\
&s_{{\rm gt},\hat{j}3} = \begin{pmatrix} z_{j,23} \\ y_{j,14} \end{pmatrix}, \quad j = n_{\rm l} \text{ is an even},\\
&s_{{\rm gt},\hat{j}3} = \begin{pmatrix} z_{j,14} \\ y_{j,23} \end{pmatrix}, \quad j = n_{\rm l} \text{ is an odd},
\end{aligned}$$

$$z_{j,14} = y_{j,14} + r_{\rm b}(y_{j,23} - y_{j,14}), \quad z_{j,23} = y_{j,23} + r_{\rm b}(y_{j,14} - y_{j,23}),$$
$$r_{\rm b} = d_{\rm nbh}/(2w), \quad y_{j,14} = x_1 + (j - 1/2)(x_4 - x_1)/n_{\rm l}, \quad y_{j,23} = x_2 + (j - 1/2)(x_3 - x_2)/n_{\rm l},$$

$$\begin{aligned}
&s_{{\rm gt},\hat{j}4} = \begin{pmatrix} y_{j,23} \\ z_{j,14} \end{pmatrix}, \quad s_{{\rm gt},\bar{j}4} = \begin{pmatrix} z_{j,14} \\ z_{j+1,14} \end{pmatrix}, \quad j = 1,\\
&s_{{\rm gt},\hat{j}4} = \begin{pmatrix} z_{j,14} \\ z_{j,23} \end{pmatrix}, \quad s_{{\rm gt},\bar{j}4} = \begin{pmatrix} z_{j,23} \\ z_{j+1,23} \end{pmatrix}, \quad j = 2, 4, 6, \cdots,\\
&s_{{\rm gt},\hat{j}4} = \begin{pmatrix} z_{j,23} \\ z_{j,14} \end{pmatrix}, \quad s_{{\rm gt},\bar{j}4} = \begin{pmatrix} z_{j,14} \\ z_{j+1,14} \end{pmatrix}, \quad j = 3, 5, 7, \cdots,\\
&s_{{\rm gt},\hat{j}4} = \begin{pmatrix} z_{j,14} \\ y_{j,23} \end{pmatrix}, \quad j = n_{\rm l} \text{ is an even},\\
&s_{{\rm gt},\hat{j}4} = \begin{pmatrix} z_{j,23} \\ y_{j,14} \end{pmatrix}, \quad j = n_{\rm l} \text{ is an odd}.
\end{aligned}$$

(5) $n_{\rm l} > n_{\rm w} > 1$. The UAV can enter the zone in two directions reasonably. The two sets of the path are presented as follows:

$$R_{{\rm tg},ik} = \begin{pmatrix} s_{{\rm gt},1i} & \cdots & s_{{\rm gt},n_i i} \end{pmatrix}, \quad i = 1, 2. \tag{11}$$

(6) $n_{\rm w} > n_{\rm l} > 1$. The two sets of the path are presented as follows:

$$R_{{\rm tg},ik} = \begin{pmatrix} s_{{\rm gt},1\bar{i}} & \cdots & s_{{\rm gt},n_{\bar{i}}\bar{i}} \end{pmatrix}, \quad \bar{i} = i+2, \quad i = 1,2. \tag{12}$$

Note that the UAVs can fly along a set of the path from the starting point to the end point or from the end point to the starting point as request. Thus, an index, h, is introduced as $R_{{\rm tg},hik}, h \in \{1,2\}$. $h = 1$ means that the UAVs fly along $R_{{\rm tg},ik}$ from the starting point to the end point. $h = 2$ means reverse.

Those sets of the scheduled path, $R_{{\rm tg},hik}, h \in N_h := \{1,2\}, i \in N_i := \{1,\dots,n_{\rm set}\}, k \in N_k := \{1,\dots,n_k\}$ are connected based on the nearest neighbor. n_k denotes the number of the rectangles and $n_{\rm set}$ denotes the number of sets of the scheduled path to search the k-th rectangle. Based on the allocated flight distance, the number of the UAVs to be assigned can be decided.

Let

$$l_{{\rm th},hik},\ s_{{\rm p},hik} = \begin{pmatrix} s_{{\rm p}x,hik} \\ s_{{\rm p}y,hik} \end{pmatrix},\ e_{{\rm p},hik} = \begin{pmatrix} e_{{\rm p}x,hik} \\ e_{{\rm p}y,hik} \end{pmatrix}$$

be the length, starting point and end point of $R_{{\rm tg},hik}$. The Y axis is regarded as the accessible side of the forest field. Then,

$$|s_{{\rm p}x,hik}| + l_{{\rm th},hik} + |e_{{\rm p}x,hik}| \leqslant d_{\rm flt},$$

where $d_{\rm flt}$ denotes the allocated flight distance. The inequality is true because of the assumptions.

Suppose the path for the $(k_{\rm v}-1)$-th UAV has been scheduled as $p_{{\rm th},k_{\rm v}-1}$. The path for the $k_{\rm v}$-th UAV is scheduled as follows.

(1) The first set of the path is found out based on the minimal distance to the Y axis as follows:

$$d_{1,k_{\rm v}} = \min_{h,i,k} |s_{{\rm p}x,hik}|, \quad h \in N_h,\ \ i \in N_i,\ \ k \in N_k. \tag{13}$$

Note that the connected sets of the path are not included in the optimization. If there are more than one optimal solutions, the first found is regarded as the optimal solution. Let h_1, i_1, k_1 be the optimal solution. Then

$$\begin{aligned} &e_{1,k_{\rm v}} + |e_{{\rm p}x,h_1 i_1 k_1}| \leqslant d_{\rm flt}, \\ &e_{1,k_{\rm v}} = d_{1,k_{\rm v}} + l_{{\rm th},h_1 i_1 k_1}. \end{aligned} \tag{14}$$

Note that the inequality is true because of the assumption.

(2) The s-th set of the path is supposed to have been found out. Let h_s, i_s, k_s be the optimal solution and

$$\begin{aligned} &e_{s,k_{\rm v}} + |e_{{\rm p}x,h_s i_s k_s}| \leqslant d_{\rm flt}, \\ &e_{s,k_{\rm v}} = e_{s-1,k_{\rm v}} + d_{s,k_{\rm v}} + l_{{\rm th},h_s i_s k_s}, \\ &e_{0,k_{\rm v}} = 0, \ \ s \geqslant 1. \end{aligned} \tag{15}$$

If there is no set of the path to be connected, the schedule for the $k_{\rm v}$-th UAV is completed. Otherwise, proceed to find the t-th $(t := s+1)$ set of the path.

(3) The t-th set of the path is found out based on the nearest neighbor as follows:

$$d_{t,k_{\rm v}} = \min_{h,i,k} \|e_{{\rm p},h_s i_s k_s} - s_{{\rm p},hik}\|, \quad h \in N_h,\ \ i \in N_i,\ \ k \in N_k. \tag{16}$$

Note that the connected and skipped sets of the path are not included in the optimization. Let h_t, i_t, k_t be the optimal solution and check the conditions as follows:

$$\begin{aligned} &e_{s,k_{\rm v}} + d_{t,k_{\rm v}} > d_{\rm flt}, \\ &e_{t,k_{\rm v}} + |e_{{\rm p}x,h_t i_t k_t}| \leqslant d_{\rm flt}, \\ &e_{t,k_{\rm v}} = e_{s,k_{\rm v}} + d_{t,k_{\rm v}} + l_{{\rm th},h_t i_t k_t}, \ \ t \geqslant 2. \end{aligned} \tag{17}$$

(1) If the 1st inequality of (17) is true, the schedule for the $k_{\rm v}$-th UAV is completed.

(2) If the 2nd inequality of (17) is true, proceed to find the $(t+1)$-th set of the path.

(3) If the 2nd inequality of (17) is false, skip this set of the path and proceed to find the subsequent nearer neighbor.

(4) If the 2nd inequality of (17) is false and there is no set of the path to be connected, the schedule for the $k_{\rm v}$-th UAV is completed.

When the schedule for the $k_{\rm v}$-th UAV is completed, if there are still remaining sets of the path to be connected, proceed to schedule the path for the $(k_{\rm v}+1)$-th UAV. Otherwise, the whole schedule is over.

Suppose that there are s sets of the path scheduled for the $k_{\rm v}$-th UAV. Then, $p_{{\rm th},k_{\rm v}}$ is given as follows:

$$\begin{aligned} p_{{\rm th},k_{\rm v}} &= \begin{pmatrix} R_{{\rm tg},h_1 i_1 k_1} & \cdots & R_{{\rm tg},h_r i_r k_r} & s_{r{\rm tg},rr+1} & R_{{\rm tg},h_{r+1} i_{r+1} k_{r+1}} & \cdots & R_{{\rm tg},h_s i_s k_s} \end{pmatrix}, \\ s_{r{\rm tg},rr+1} &= \begin{pmatrix} e_{{\rm p},h_r i_r k_r} \\ s_{{\rm p},h_{r+1} i_{r+1} k_{r+1}} \end{pmatrix}. \end{aligned} \tag{18}$$

The scheduled path to search the rectangles is shown in Figure 3.

2.4 Step 4: schedule the path for taking off and landing

The scheduled path needs to be connected with the take-off and landing locations for the assigned UAVs. The objective of the connection is to avoid collision. There are two ideas. One is that there is no joint between any two channels and one UAV takes one channel. The other is that more than one UAVs share one channel and there is no meeting chance for the UAVs in the channel. The first idea needs sufficient space to allocate the channels for the UAVs. The second idea needs time allocation for the UAVs to share a channel. Here the first idea is shown as follows.

The main concern is the distances between any two of the UAVs to enter and exit the forest field. If there is any distance less than the safety distance, it needs to broaden the distance for safety. Let

$$p_{{\rm s},k} := \begin{pmatrix} p_{{\rm s}x,k} \\ p_{{\rm s}y,k} \end{pmatrix}, \quad p_{{\rm e},k} := \begin{pmatrix} p_{{\rm e}x,k} \\ p_{{\rm e}y,k} \end{pmatrix}$$

be the starting point and end point of the path for the k-th UAV, $p_{{\rm th},k}, k \in N_{\rm v} =: \{1,\ldots,n_{\rm v}\}$, respectively and $d_{\rm sfty}$ be the safety distance between the two flying UAVs, where $n_{\rm v}$ denotes the number of the UAVs to be assigned together. Then, if the distances between any two of the entrance segments and the distances between any two of the exit segments of $p_{{\rm th},k}$, $k \in N_{\rm v}$ are not less than $d_{\rm sfty}$ respectively, the complete path for the k-th UAV, $k \in N_{\rm v}$, is scheduled as follows:

$$\begin{aligned} p_{{\rm ath},k} &= \begin{pmatrix} p_{{\rm tof},k} & p_{{\rm th},k} & p_{{\rm lnd},k} \end{pmatrix}, \quad k \in N_{\rm v}, \\ p_{{\rm tof},k} &= \begin{pmatrix} t_{{\rm of},1k} & \cdots & t_{{\rm of},4k} \end{pmatrix}, \\ p_{{\rm lnd},k} &= \begin{pmatrix} l_{{\rm nd},4k} & \cdots & l_{{\rm nd},1k} \end{pmatrix}, \end{aligned} \tag{19}$$

$$\begin{aligned} &t_{{\rm of},1k} = \begin{pmatrix} t_{0,k} \\ t_{1,k} \end{pmatrix}, \ t_{{\rm of},2k} = \begin{pmatrix} t_{1,k} \\ t_{2,k} \end{pmatrix}, \ t_{{\rm of},3k} = \begin{pmatrix} t_{2,k} \\ t_{3,k} \end{pmatrix}, \ t_{{\rm of},4k} = \begin{pmatrix} t_{3,k} \\ p_{{\rm s},k} \end{pmatrix}, \\ &l_{{\rm nd},1k} = \begin{pmatrix} l_{1,k} \\ l_{0,k} \end{pmatrix}, \ l_{{\rm nd},2k} = \begin{pmatrix} l_{2,k} \\ l_{1,k} \end{pmatrix}, \ l_{{\rm nd},3k} = \begin{pmatrix} l_{3,k} \\ l_{2,k} \end{pmatrix}, \ l_{{\rm nd},4k} = \begin{pmatrix} p_{{\rm e},k} \\ l_{3,k} \end{pmatrix}, \\ &t_{2,k} = \begin{pmatrix} t_{2x,k} \\ t_{2y,k} \end{pmatrix}, \quad t_{3,k} = \begin{pmatrix} t_{2x,k} \\ p_{{\rm s}y,k} \end{pmatrix}, \quad l_{2,k} = \begin{pmatrix} l_{2x,k} \\ l_{2y,k} \end{pmatrix}, \quad l_{3,k} = \begin{pmatrix} l_{2x,k} \\ p_{{\rm e}y,k} \end{pmatrix}, \end{aligned}$$

$t_{0,k}, t_{1,k}$ and $t_{2,k}$ denote the three point for the k-th UAV to take off from the point on the ground to the vertical point in the sky and to another horizontal point. $t_{2,k}$ is chosen so that there is no joint between any two take-off channels for the k-th UAV and the other UAVs. $l_{2,k}, l_{1,k}$ and $l_{0,k}$ denote the three points for the k-th UAV to land from a horizontal point to the vertical point in the sky and to the point on the ground. $l_{2,k}$ is chosen so that there is no joint between any two landing channels for the k-th UAV and the other UAVs.

Assume that $n_{\rm v} \geqslant 3$ for presentation. If the distance between the entrance segments of $p_{{\rm th},k}$, $k \in \{k_1, k_2\} \subset N_{\rm v}$ is less than $d_{\rm sfty}$, $|p_{{\rm s}y,k_1} - p_{{\rm s}y,k_2}| < d_{\rm sfty}$, and the distances between any other two entrance segments of $p_{{\rm th},k}$, $k \in N_{\rm v}$ are not less than $d_{\rm sfty}$, The distance between the entrance segments of $p_{{\rm th},k_1}$ and $p_{{\rm th},k_2}$ has to be broadened to $d_{\rm sfty}$. If the space is sufficient for the broadening not to affect the other entrance segments of $p_{{\rm th},k}$, $k \in N_{\rm v}$ exclusive k_1 and k_2, an additional segment is added to $p_{{\rm th},k_1}$ and $p_{{\rm th},k_2}$ respectively as follows:

$$\begin{aligned} s_{{\rm g}i,k_1} &= \begin{pmatrix} p_{{\rm s}x,k_1} & c_i - d_{\rm sfty}/2 & p_{{\rm s}x,k_1} & p_{{\rm s}y,k_1} \end{pmatrix}', \\ s_{{\rm g}i,k_2} &= \begin{pmatrix} p_{{\rm s}x,k_2} & c_i + d_{\rm sfty}/2 & p_{{\rm s}x,k_2} & p_{{\rm s}y,k_2} \end{pmatrix}', \\ c_i &\in [p_{{\rm s}y,k_2} - d_{\rm sfty}/2, p_{{\rm s}y,k_1} + d_{\rm sfty}/2], \end{aligned} \tag{20}$$

by assuming $p_{{\rm s}y,k_1} \leqslant p_{{\rm s}y,k_2}$ so that c_i exists. If the starting point and end point of an added segment are same, the added segment disappears.

If the distances between two exit segments of $p_{{\rm th},k}$, $k \in \{k_1, k_2, k_3\} \subseteq N_{\rm v}$ are less than $d_{\rm sfty}$, $|p_{{\rm e}y,k_1} - p_{{\rm e}y,k_2}| < d_{\rm sfty}$, $|p_{{\rm e}y,k_2} - p_{{\rm e}y,k_3}| < d_{\rm sfty}$, and the distances between any other two exit segments of $p_{{\rm th},k}$, $k \in N_{\rm v}$ are not less than $d_{\rm sfty}$, The distances between two exit segments of $p_{{\rm th},k}$, $k \in \{k_1, k_2, k_3\}$ have to be broadened to $d_{\rm sfty}$. If the space is sufficient for the broadening not to affect the other exit segments of $p_{{\rm th},k}$, $k \in N_{\rm v}$ exclusive k_1, k_2 and k_3, an additional segment is added to $p_{{\rm th},k}$, $k \in \{k_1, k_2, k_3\}$ respectively as follows:

$$\begin{aligned} s_{{\rm go},k_1} &= \begin{pmatrix} p_{{\rm e}x,k_1} & c_{\rm o} - d_{\rm sfty} & p_{{\rm e}x,k_1} & p_{{\rm e}y,k_1} \end{pmatrix}', \\ s_{{\rm go},k_2} &= \begin{pmatrix} p_{{\rm e}x,k_2} & c_{\rm o} & p_{{\rm e}x,k_2} & p_{{\rm e}y,k_2} \end{pmatrix}', \\ s_{{\rm go},k_3} &= \begin{pmatrix} p_{{\rm e}x,k_3} & c_{\rm o} + d_{\rm sfty} & p_{{\rm e}x,k_3} & p_{{\rm e}y,k_3} \end{pmatrix}', \\ c_{\rm o} &\in [p_{{\rm e}y,k_3} - d_{\rm sfty}, p_{{\rm e}y,k_1} + d_{\rm sfty}], \end{aligned} \tag{21}$$

by assuming $p_{{\rm e}y,k_1} \leqslant p_{{\rm e}y,k_2} \leqslant p_{{\rm s}y,k_3}$ so that $c_{\rm o}$ exists. If the starting point and end point of an added segment are same, the added segment disappears.

The complete path for the three UAVs is scheduled as follows:

$$\begin{aligned} p_{{\rm ath},k} &= \begin{pmatrix} p_{{\rm tof},k} & p_{{\rm th},k} & p_{{\rm lnd},k} \end{pmatrix}, \quad k \in \{k_1, k_2, k_3\}, \\ p_{{\rm tof},k} &= \begin{pmatrix} t_{{\rm of},1k} & \cdots & t_{{\rm of},4k} & s_{{\rm gi},k} \end{pmatrix}, \quad k \in \{k_1, k_2\}, \\ p_{{\rm tof},k_3} &= \begin{pmatrix} t_{{\rm of},1k_3} & \cdots & t_{{\rm of},4k_3} \end{pmatrix}, \\ p_{{\rm lnd},k} &= \begin{pmatrix} s_{{\rm go},k} & l_{{\rm nd},4k} & \cdots & l_{{\rm nd},1k} \end{pmatrix}, \quad k \in \{k_1, k_2, k_3\}, \end{aligned} \tag{22}$$

where as the additional segments are added, the related segments are revised as follows:

$$t_{{\rm of},3k} = \begin{pmatrix} t_{2,k} \\ \bar{t}_{3,k} \end{pmatrix}, t_{{\rm of},4k} = \begin{pmatrix} \bar{t}_{3,k} \\ \bar{t}_{4,k} \end{pmatrix}, \quad k \in \{k_1, k_2\},$$

$$\bar{t}_{3,k_1} = \begin{pmatrix} t_{2x,k_1} \\ c_i - d_{\rm sfty}/2 \end{pmatrix}, \bar{t}_{4,k_1} = \begin{pmatrix} p_{{\rm s}x,k_1} \\ c_i - d_{\rm sfty}/2 \end{pmatrix}, \bar{t}_{3,k_2} = \begin{pmatrix} t_{2x,k_2} \\ c_i + d_{\rm sfty}/2 \end{pmatrix}, \bar{t}_{4,k_2} = \begin{pmatrix} p_{{\rm s}x,k_2} \\ c_i + d_{\rm sfty}/2 \end{pmatrix},$$

$$\begin{aligned} l_{{\rm nd},3k} &= \begin{pmatrix} \bar{l}_{3,k} \\ l_{2,k} \end{pmatrix}, l_{{\rm nd},4k} = \begin{pmatrix} \bar{l}_{4,k} \\ \bar{l}_{3,k} \end{pmatrix}, \quad k \in \{k_1, k_2, k_3\}, \\ \bar{l}_{4,k_1} &= \begin{pmatrix} p_{{\rm e}x,k_1} \\ c_{\rm o} - d_{\rm sfty} \end{pmatrix}, \bar{l}_{3,k_1} = \begin{pmatrix} l_{2x,k_1} \\ c_{\rm o} - d_{\rm sfty} \end{pmatrix}, \\ \bar{l}_{4,k_2} &= \begin{pmatrix} p_{{\rm e}x,k_2} \\ c_{\rm o} \end{pmatrix}, \bar{l}_{3,k_2} = \begin{pmatrix} l_{2x,k_2} \\ c_{\rm o} \end{pmatrix}, \\ \bar{l}_{4,k_3} &= \begin{pmatrix} p_{{\rm e}x,k_3} \\ c_{\rm o} + d_{\rm sfty} \end{pmatrix}, \bar{l}_{3,k_3} = \begin{pmatrix} l_{2x,k_3} \\ c_{\rm o} + d_{\rm sfty} \end{pmatrix}. \end{aligned}$$

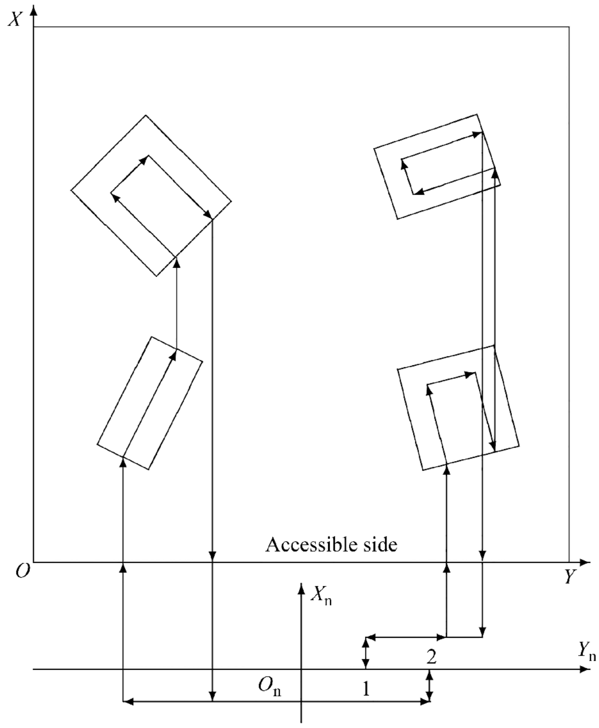

Figure 3 Scheduled path.

The complete scheduled path is shown in Figure 3. $X_nO_nY_n$ denotes the North-East-Down (NED) frame. 1 and 2 denote the two take-off and landing locations in which the local region is enlarged for display. This completes the development of the online RSA.

3 Verification of online RSA

A 2D grid map is drawn with the missed areas to verify the developed online RSA. The results of the four steps are shown in Figure 4. The computed results show that the developed online RSA is successful to decide the number of the UAVs to be assigned and to schedule the complete path for the assigned UAVs to take off to search the missed areas and fly back to land.

The developed online RSA is applied for the forest search with multiple UAVs (quadrotors) together. Nine UAVs are assigned in three batches and three UAVs are assigned in a batch. Each UAV in the first two batches is allocated a round channel to search the field of forest. Each UAV builds a 2D grid map to record its detecting area. Those maps are merged for the assessment. If there is any zone of the missed areas detected on the merged map, RSA is applied to schedule online for the third batch based on the merged 2D grid map.

In the simulation, one UAV is assumed lost in the 1st batch and another UAV is lost in the 2nd batch. In such scenario, RSA is applied to online decide the number of the UAVs to be assigned in the 3rd batch and to schedule the path for the assigned UAVs to search the missed areas. The online scheduled results and the simulation results for the third batch are shown in Figure 5. The simulation results demonstrate that RSA is successful to decide the number of the UAVs to be assigned and to schedule the path for the assigned UAVs so that the UAVs in the 3rd batch successfully search the missed areas.

4 Concluding remarks

RSA has been developed and applied for the forest search with multiple UAVs. RSA is successful to online decide the number of the UAVs to be assigned and to schedule the path for the assigned UAVs to take off

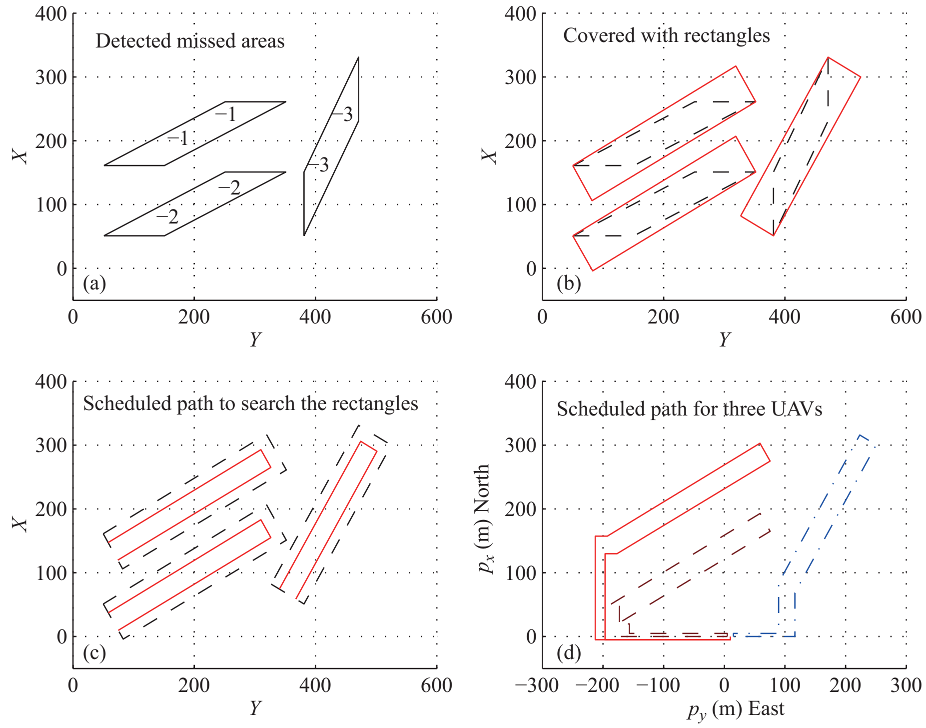

Figure 4 (Color online) Results of an example.

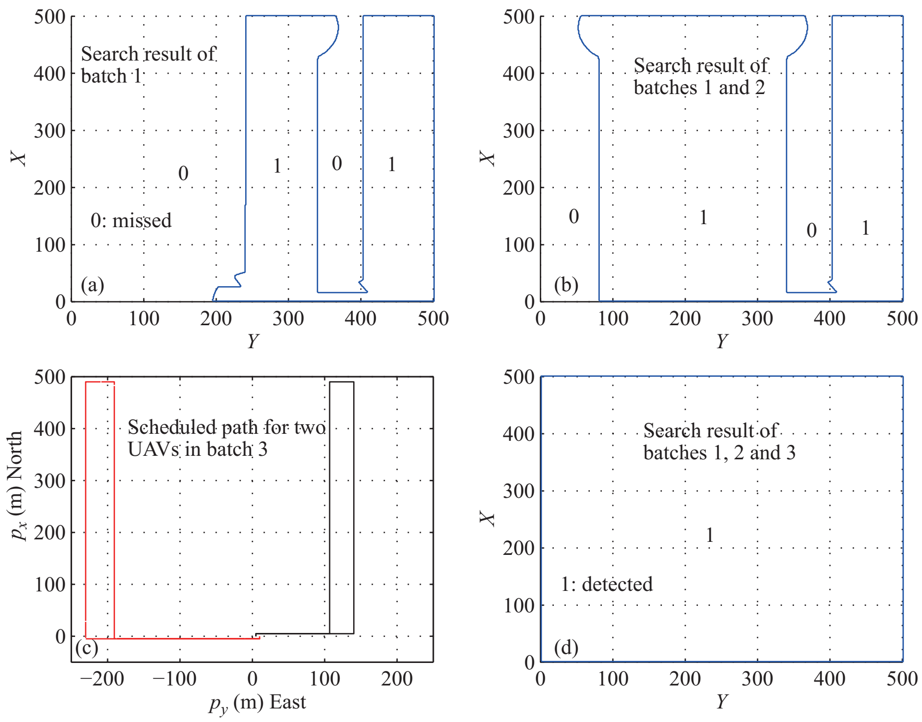

Figure 5 (Color online) Results of the forest search.

to search the missed areas and fly back to land. RSA is applicable to the similar scenarios. Nonetheless, there is still much left to study. The coverage efficiency is to be improved and the assumption on the length of the path to be scheduled to search a zone of the missed areas is to be relaxed.

Conflict of interest The authors declare that they have no conflict of interest.

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