Market turning points forecasting using wavelet analysis

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H I G H L I G H T S

- A model is proposed to forecast major turning points of stock markets.
- Our model employs the system adaptation framework and wavelet analysis.
- This model is successfully applied to the US, UK and China stock markets.
- This model could be extended to other economic time series or financial markets.

A R T I C L E   I N F O

A B S T R A C T

Based on the system adaptation framework we previously proposed, a frequency domain based model is developed in this paper to forecast the major turning points of stock markets. This system adaptation framework has its internal model and adaptive filter to capture the slow and fast dynamics of the market, respectively. The residue of the internal model is found to contain rich information about the market cycles. In order to extract and restore its informative frequency components, we use wavelet multi-resolution analysis with time-varying parameters to decompose this internal residue. An empirical index is then proposed based on the recovered signals to forecast the market turning points. This index is successfully applied to US, UK and China markets, where all major turning points are well forecasted.

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1. Introduction

Market cycles are the patterns that the price level of market repeats its upward and downward movements over some specific time scales. In financial markets, financial time series always show cycle patterns at all-time scales [1,2], varying from long term cycles to high frequency fluctuations. However, cycle does not imply any regularity in timing or durations. According to duration, market cycles can be classified into three categories: primary cycles, intermediate cycles and short-term cycles [3]. The average length of primary cycles is three to seven years, which are driven by both economic environment and the sentiment of investors [4–6]. Intermediate cycles typically last three to eighteen months, while short-term cycles last six to twelve weeks, which are usually driven by unpredicted news or random events. This kind of short-term fluctuation is inevitable in every financial market. This study mainly focuses on the identification of primary cycles and the forecasting of their turning points.

In terms of the market cycle structure, cycles generally include three phases: uptrend, downtrend, and sideways [7]. Successfully identifying the transition period between two phases is extremely critical in the investment. There are mainly

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two groups of market forecasting methods: fundamental analysis and technical analysis. Fundamental analysis aims to determine the intrinsic value of securities by studying all factors related to the company, e.g. the balance sheet, financial statement and operating environment [8]. When applied to the market index analysis, related macroeconomic variables are studied to understand the main market situations [9,10]. Technical analysis is to study the market data, such as price and volume, to forecast the future price movements [11]. It is believed the past data patterns will repeat themselves, and their statistical characteristics would imply the future activity.

The large quantities of noises is a big challenge to the technical analysts when they study the primary cycles. There are intermediate and short-term cycles, and high-frequency volatilities within the primary cycle ranges. All of these cause interference to the primary cycle recognition and forecasting. The moving average (MA) method is commonly used to filter such noises and highlight the long-term trends. Based on the MA, a moving average convergence/divergence (MACD) indicator is developed to forecast the market turning points. These methods are widely used in the financial industry. However, the MA introduces lags to the data analysis. Instead of identifying the cycle patterns, another technical analysis method, the momentum index, tries to find turning signals from the change rate of price [3,12]. It calculates the price differences between the current market price and the price of some days ago. Once the momentum crosses zero it generates a trading signal. The momentum method is mostly used for the short-term trend analysis. Benefiting from the momentum concept, a group of indicators, e.g. stochastic oscillators and relative strength indices, are developed for the forecasting of price movements [13].

In engineering, there are many advanced methodologies for signal processing and pattern recognition. Known as a "mathematical microscope", the wavelet method is a powerful time–frequency analysis tool in this field. By using wavelet multi-resolution analysis (MRA), a signal can be split into multiple time scales, including large-scale approximation and finer-scale details. It allows us to focus on specific time scales where cycle patterns are critically concerned, and it does not introduce any lags. The development of this method has attracted extensive attentions from economic researchers [14,15]. By using wavelet to investigate the high-frequency data of the Nikkei stock index, Capobianco [16] revealed the hidden periodic components. Yamada and Honda [17] applied the MRA of the discrete wavelet transform (DWT) to Japanese stock prices to retrieve the middle-frequency signals, which were found to contain predictive information of Japanese business turning points.

The maximal overlap discrete wavelet transform (MODWT) is a non-decimated form of the DWT, which applies high and low pass filters to decompose a signal [18]. One of its main advantages is the translation invariance, meaning that a shift in the signal does not change the wavelet and scaling coefficients. Therefore, it is not sensitive to the starting point of a signal. Xue et al. [19] applied the MODWT to extract the multi-frequency components from the intraday equity prices, in which the jump dynamics of equity prices were found to be sensitive to the data sampling frequency. Their results revealed that the high frequency bands contain more jump points than that in the low frequency bands. Based on a MRA of the MODWT, Gençay et al. [20] proposed a method to extract the intraday seasonality which was simple to calculate and free of model selection parameters. Similarly, the MODWT is employed in analyzing the business cycle and growth cycle, see Refs. [21,22]. The multi-scaling extraction of wavelet has also been applied to the volatility analysis, risk hedging and portfolio allocation [23–25]. In recent years, although wavelet methods have been widely used in financial time series analysis, the literature in forecasting market turning points still lacks.

In this paper, we propose a model to forecast the market turning points using wavelet analysis. The model is based on our previous system adaptation framework [26,27], in which the financial market is considered to be a complex dynamic system, see Fig. 1. One of the advantages of this system adaptation framework is its structure, within which not only quantitative relationships between the market and external influences can be modeled, but also the market dynamics and behaviors are well captured. Our study found that the residue of the internal model contains predictive signals on the market cycles [27]. This paper first applied the internal model to the stock prices to generate a signal-rich residue series. The wavelet MRA with time-varying parameters is then applied to decompose the internal residue and retrieve concerned signals, based on which an empirical index for forecasting market turning points is proposed.

The following of this paper is organized as below. Section 2 briefly reviews our system adaptation framework and the internal model used in this paper. Section 3 gives our turning points forecasting methods with an introduction of wavelet analysis. Section 4 presents the empirical results from US, UK, and China stock markets. Section 5 concludes the paper.

Throughout this paper, the notation $\mathbb{R}$ and $\mathbb{Z}$ denote the set of real numbers and integers, respectively. $\mathcal{L}^2(\mathbb{R})$ denotes the vector space of measurable, square-integrable one-dimensional function $f(x)$.

2. Modeling the market: a system adaptation framework

Modeling financial markets in our system adaptation framework is considered as a system identification problem. The real financial market is treated as an unknown plant S whose dynamic behavior is mathematically described by the identification model $\hat{S}$, see Fig. 1. The market in our models is considered to have slow and fast dynamics, which are respectively captured by the internal model I and adaptive filter A. The input r is consisted of external influences and the output $\hat{p}$ is the estimated security price. The actual security price p is the output of the real financial market S.

In this paper, we focus on the internal model, and for the readers who are interested in the detail of our system adaptation models or the design of adaptive filter, please refer to our previous publications [26,27]. To capture the slow dynamic properties, the internal model works as a price trend generator, which makes the estimated prices have the same trends as
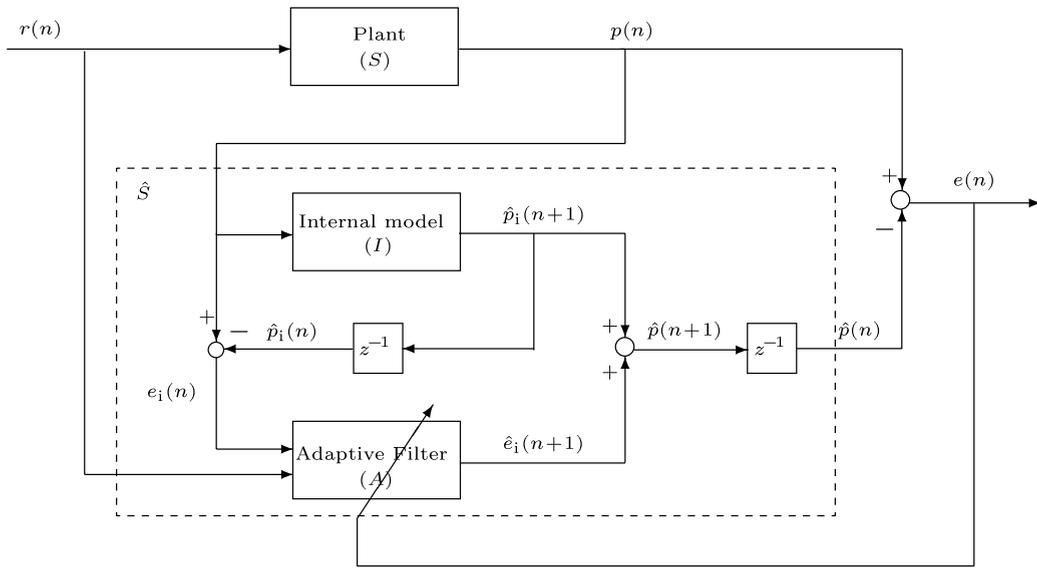
Fig. 1. Block diagram of the system adaptation framework.
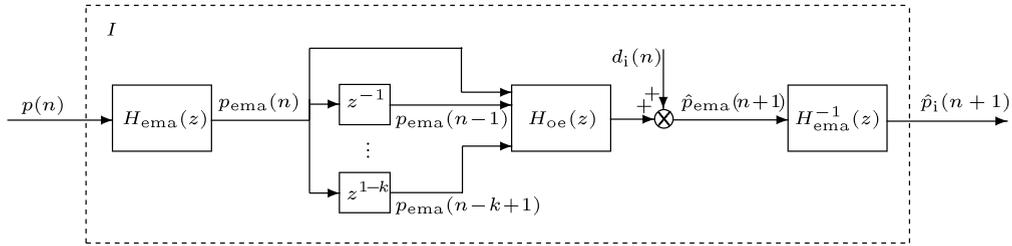
Fig. 2. The internal model of the system adaptation framework.

the actual prices. The differences between the actual price $p(n)$ and estimated price $\hat{p}_i(n)$ is defined as internal residue $e_i(n)$

$$e_i(n) = p(n) - \hat{p}_i(n). \tag{1}$$

This internal residue is analyzed by wavelet MRA methods in the next section.

The following is a brief introduction of the internal model design, see Fig. 2 for its structure. It is approximated by a time-invariant system, where an output-error (OE) model is employed. First, the historical prices are smoothed by exponential moving average (EMA). We use classical 12 days EMA in this paper. The second part is an OE model with multi-inputs and single-output (MISO). Its input $u_{oe}(n)$ includes both current and $k-1$ previous samples of the EMA prices, which is denoted by

$$u_{oe}(n) = \begin{bmatrix} u_{oe,1}(n) \\ u_{oe,2}(n) \\ \vdots \\ u_{oe,k}(n) \end{bmatrix} = \begin{bmatrix} p_{ema}(n) \\ p_{ema}(n-1) \\ \vdots \\ p_{ema}(n-k+1) \end{bmatrix}. \tag{2}$$

Hence, the transfer function of this MISO OE model is

$$H_{oe}(z) = \begin{bmatrix} H_{oe,1}(z) & H_{oe,2}(z) & \cdots H_{oe,k}(z) \end{bmatrix}, \tag{3}$$

where for $j = 1, 2, \ldots, k$, $H_{oe,j}(z)$ is the transfer function for the jth channel of the OE model. The function of $H_{oe,j}(z)$ is defined by

$$H_{oe,j}(z) = \frac{B_j(z)}{F_j(z)}, \tag{4}$$

where

$$B_j(z) = b_{j,1} + b_{j,2}z^{-1} + \cdots + b_{j,n_b}z^{-n_b+1}, \tag{5}$$

and

$$F_j(z) = 1 + f_{j,1}z^{-1} + \cdots + f_{j,n_f}z^{-n_f}. \tag{6}$$

This system is considered to have a disturbance, $d_i(n)$, which is assumed to be white noise. The third part is to transform the estimated EMA price $\hat{p}_{ema}(n+1)$ back to $\hat{p}_i(n+1)$.

3. Turning point forecasting: frequency domain approach

In engineering, frequency domain approaches are frequently used in signal analysis to find out significant features that cannot be presented in the time domain. The signals in the time domain show how signals evolve over time, while in frequency domain it shows the power spectrum at each frequency band. One advantage of analyzing time series in the frequency domain is that it allows us to filter some noisy signals at special frequencies and recombine the remaining components in order to recover the original signals. In this study, we need to extract the middle-frequency components of the internal residue to forecast turning points.

3.1. Wavelet analysis

Fourier transform is a typical method to covert signals from time domain to frequency domain. There are some previous works using Fourier methods to study the market turning periods [27,28]. However, Fourier transform assumes the signal is periodic. It might not be applicable to some non-stationary signals, e.g., the financial time series. Rather than the trigonometric functions in Fourier, wavelets define a finite domain which makes it well localized with respect to both time and frequency. This characteristics allows it to be well used in the study of non-stationary signals. The MRA of the DWT splits a signal into a coarse approximation (large time scale) and a group of finer details (small time scales) [29]. The coarse approximation indicates the trend information of signal, and its finer scales show details of all the other information.

The DWT has two basic types of functions: $\Phi(t)$ and $\Psi(t)$, also known as father wavelet and mother wavelet, respectively. The parents functions can be dilated and translated to get a set of wavelets. In this paper, we use the common dyadic DWT, according to which, the scaling function $\Phi_{j,n}(t)$ and wavelet function $\Psi_{j,n}(t)$ can be obtained by

$$\Phi_{j,n}(t) = \frac{1}{\sqrt{2^j}}\Phi\left(\frac{t-2^j n}{2^j}\right), \tag{7}$$

and

$$\Psi_{j,n}(t) = \frac{1}{\sqrt{2^j}}\Psi\left(\frac{t-2^j n}{2^j}\right), \tag{8}$$

where $j, n \in \mathbb{Z}, j$ is the dilation parameter and n is the translation parameter. $\Phi_{j,n}$ represents the signal approximation or low frequencies of the data, while $\Psi_{j,n}$ captures the other high frequencies. Hence, for a signal with finite energy $f(t) \in \mathcal{L}^2(\mathbb{R})$, its DWT is

$$f(t) = \sum_{n=-\infty}^{\infty} a_{J,n}(t)\Phi_{J,n}(t) + \sum_{j=-\infty}^{J}\sum_{n=-\infty}^{\infty} d_{j,n}(t)\Psi_{j,n}(t), \tag{9}$$

where J is the maximum decomposition level; $a_{J,n}(t) = \langle\Phi_{J,n}(t), f(t)\rangle$ and $d_{j,n}(t) = \langle\Psi_{j,n}(t), f(t)\rangle$, which can be computed by Mallat's pyramid algorithm [29]. In order to capture the fast changing dynamics of signal, parameters in our MRA are set to be time dependent. Let

$$A_J(t) = \sum_{n=-\infty}^{\infty} a_{J,n}(t)\Phi_{J,n}(t), \tag{10}$$

and

$$D_j(t) = \sum_{n=-\infty}^{\infty} d_{j,n}(t)\Psi_{j,n}(t), \tag{11}$$

where the sequence of $A_J(t)$ represents the J-th level wavelet smooth and $D_j(t)$ represents the jth level wavelet details, see Fig. 3 for its mechanism. Since we use the daily data, A_J theoretically captures the nonlinear trend with periodicity greater than 2^{J+1} days and D_j captures the signal details with periodicity between 2^j and 2^{j+1} days.

As introduced above, this paper focuses on market cycles with average periodicity around three to seven years, which theoretically corresponds to the frequencies between D_{10} (2.8 years) and D_{12} (11.2 years). It has been proved that using higher frequency data would better capture signal volatility. Our empirical studies found that the informative frequencies lie in the bands between D_7 (0.35 years) and D_{12}, which are referred to as middle-frequency components. Moreover, each market has its own dynamic features, so that the specific frequency bands for different markets should be selected respectively.
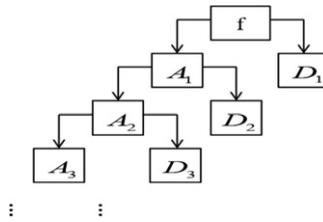
Fig. 3. Wavelet decomposition mechanism.
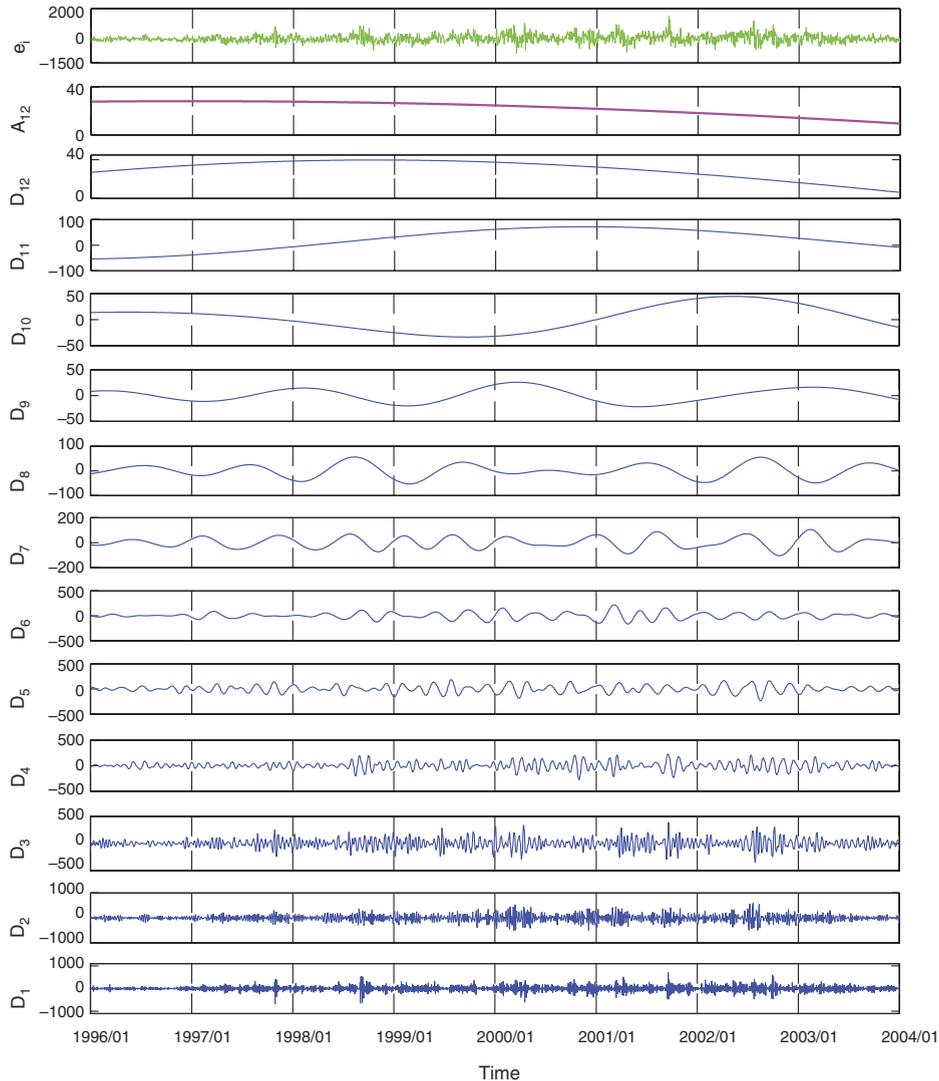
Fig. 4. Decomposition of internal residue of the DJIA with the DWT.

There are various discrete wavelets available for the MRA, e.g., the wavelets family of Daubechies, Harr, coiflets and symlets. The selection of wavelets depends on the signal properties and the problem nature. With the advantage of compact support and orthogonality, the Daubechies wavelets are widely used in the analysis of problems with local high gradient [30]. Considering that the internal residue has nonstationary and drastic fluctuations during some periods, the Daubechies wavelets are employed in this study.

Fig. 4 shows an example of multi-resolution decomposition of the internal residue. The internal residue of the Dow Jones Industrial Index Average (DJIA) is decomposed by wavelet of Daubechies 12 (db12) at level $J = 12$. In this figure, the trend term A_{12} and all the other frequencies from D_{12} to D_1 are precisely decomposed. The middle-frequency signals, $m(n)$, are
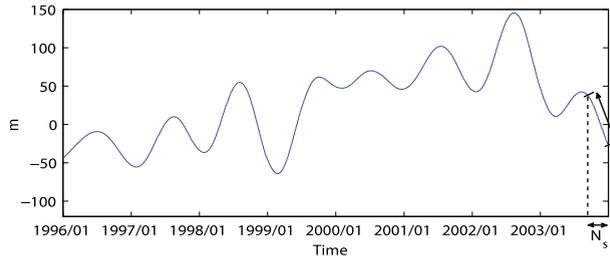
Fig. 5. One snapshot of the retrieved middle-frequency signal *m* with the DWT.
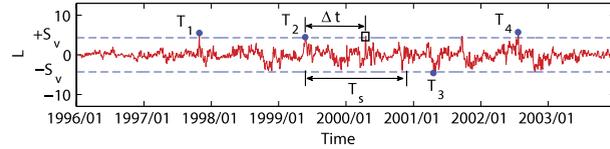
(a) *L* value.
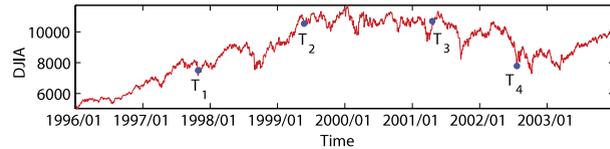
(b) Original DJIA price.

Fig. 6. Turning point identification rules. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

retrieved by

$$m(n) = D_{11}(n) + D_{10}(n) + D_9(n) + D_8(n), \tag{12}$$

see Fig. 5.

The MODWT is a non-orthogonal variant of the DWT. Compared with the DWT, the MODWT is a highly redundant and nonorthogonal transform [18]. It retains downsampled values at each level of the decomposition rather than decimating the coefficients as the DWT. Therefore, the number of wavelet and scaling coefficients at each level remains to be the original sample size. For this reason, the MODWT is also called time-invariant DWT. The MODWT is also employed in this work to have a comparison with the DWT in terms of predicting turning signals.

3.2. Market turning index

To find out the turning information from the retrieved signals, an index which is capable of capturing the dynamical changes in the signals is needed. Our empirical testing found that when the market steps into the turning period between two primary trends, the internal residues usually show some patterns in the middle-frequency components. The slope *L* of retrieved signals in the past N_s days, see Fig. 5, working well as a measurement to capture such kinds of oscillations. The intense fluctuation of *L* indicates a turning point for primary market cycles, i.e., once *L* is large enough to exceed some threshold, the corresponding time is identified as a market turning point.

Based on the index *L*, two rules are proposed to identify the major turning points, which are illustrated in Fig. 6. The forecasted turning points are denoted by T_k, $k = 0, 1, 2, \ldots, n$.

Rule I. A threshold value, S_v, is defined to identify a new turning point. If the slope $L > +S_v$ or $L < -S_v$ it is marked as a candidate of the next turning point, $\hat{T}_{k+1}, k = 0, 1, 2, \ldots, n$.

Rule II. A time slot threshold, T_s, measuring in days, is defined to filter the redundant turning points after a confirmed one. Since our interested market cycles are around three to seven years, once a new turning point is found, the next turning point is not likely to appear in the near future. The time differences between a candidate turning point and the last confirmed turning point is defined as Δt:

$$\Delta t = \hat{T}_{k+1} - T_k. \tag{13}$$

If $\Delta t > T_s$, the candidate $\hat{T}_{k+1}$ is confirmed as a new turning point T_{k+1}, otherwise it is removed as a redundant one.

For the initial condition, we set the starting date of the testing period to be a default turning point T_0. One example for these two rules is shown in Fig. 6, in which the DWT is used for MRA. The parameters are set as: $N_s = 10$, $S_v = 4.3$ and

$T_s = 360$. Fig. 6(a) is the internal residue and Fig. 6(b) shows the original DJIA price with the forecasted turning points correspondingly marked by blue points. In this paper, the initial point T_0 is not presented in the results unless otherwise specified. As demonstrated in Fig. 6(a), the point in the rectangular box satisfies the condition of *Rule* I ($L < -S_v$) and it is marked as a turning point candidate, but it is obvious that $\Delta t < T_s$ which does not satisfy *Rule* II. Thus it is considered as a redundant point.

4. Results

We select three stock market index to do empirical testing, including DJIA, Financial Times Stock Exchange (FTSE) 100, and Shanghai Stock Exchange Composite Index (SSE). Among the three index, two are from developed markets, which correspondingly represent US and Europe market, and one from China that represents emerging market.

4.1. US market

For the US market, we focus on the DJIA and the testing period is from year 1996 to 2013. The daily closing prices from year 1991 to 1995 are used to train the OE model through MATLAB System Identification Toolbox. The identified OE model is

$$H(z) = \begin{bmatrix} \dfrac{2.614z^{-1} - 9.925z^{-2} - 2.945z^{-3} + 8.961z^{-4}}{1 + 0.9776z^{-1} + 0.00428z^{-2}} \\[2mm] \dfrac{10.81z^{-1} - 5.121z^{-2} + 2.79z^{-3} - 2.534z^{-4}}{1 + 0.1485z^{-1} + 0.3395z^{-2}} \\[2mm] \dfrac{-2.859z^{-1} + 0.2158z^{-2} + 1.733z^{-3} - 0.3547z^{-4}}{1 + 0.06269z^{-1} - 0.5227z^{-2}} \end{bmatrix}^{\mathrm{T}}. \tag{14}$$

Fig. 7(b) shows the internal residue. The DWT is used to extract the middle frequency components, where Daubechies 12 (db12) wavelet is selected to decompose the internal residue at level $J = 12$. The middle-frequency bands are selected between D_8 and D_{11}:

$$m(n) = D_{11}(n) + D_{10}(n) + D_9(n) + D_8(n). \tag{15}$$

The other parameters are set as

$$N_s = 10, \qquad S_v = 4.3, \qquad T_s = 360. \tag{16}$$

Fig. 7 presents the forecasting results with the DWT: Fig. 7(a) shows the original index prices, in which the forecasted turning points, $T_k, k = 1, 2, \ldots, n$, are labeled by blue markers respectively; Fig. 7(b) is the internal residue; Fig. 7(c) shows a snapshot of the retrieved middle-frequency signals at the end of the testing period; Fig. 7(d) presents the value of slope L in each step, in which the blue points are the forecasted turning points.

From the results, we can find that nine turning points are forecasted during this period. The first forecasted turning point is in October, 1997, when the market was in a short tranquil period. Two months later, the market began to rise sharply due to the dot-com boom. Therefore, point T_1 gives excellent forecasting for this rapid growth. The second forecasted turning point, T_2, alarms that the market is going to end the rising trend. As expected, after T_2, the market switched to an one-year's tranquil period. Before the end of this tranquil period, our model gives another turning signal at T_3. It correctly signifies the starting of a bear market, which lasted for one year because of the burst of the dot-com bubble. After that, our model provides a successful forecasting for the bottom of this bear market at T_4. It is obvious that, after T_4 the market went through a short fluctuation period, and then entered into a rally period.

The US stock market was heavily hit by the latest sub-prime financial crisis. The market reached its peak in October 2007, and then started to crash quickly. Our model gives an alarm signal T_5 in August 2007 that was two months before this crash. The next forecasted point T_6 suggests the ending of the crash, which has been proved to be accurate. Stimulated by the Federal Reserve's quantitative easing programs, the market began to rebound after a short period of fluctuation. During the recovering period, our model gives several turning alarms from T_7 to T_9. Although they are not the major turning points, there are still significant fluctuations around these points.

The MODWT is also used to study the dynamics of the proposed index. The internal residue is also decomposed at level $J = 12$ by db12. The empirical results find that the MODWT needs higher frequency bands to capture the turning signals than the DWT. The middle frequency bands are thus selected from D_5 to D_7:

$$m(n) = D_7(n) + D_6(n) + D_5(n). \tag{17}$$

The other parameters are set as

$$N_s = 12, \qquad S_v = 20, \qquad T_s = 380. \tag{18}$$

Fig. 8 shows the results forecasted by the MODWT. From Fig. 8(d), we can find that the change of the slope is continuous and smooth. However, compared with the DWT, the MODWT does not provide additional information in this case. The MODWT forecasts one less turning points than the DWT, and the forecasted turning points of T_1 to T_5 are not as precise as the DWT. The MODWT also generates noisy points at T_6, T_7 and T_8.

(a) Original DJIA price.

(b) Internal residue.

(c) Snapshot of retrieved middle frequency signal m.

(d) L value.
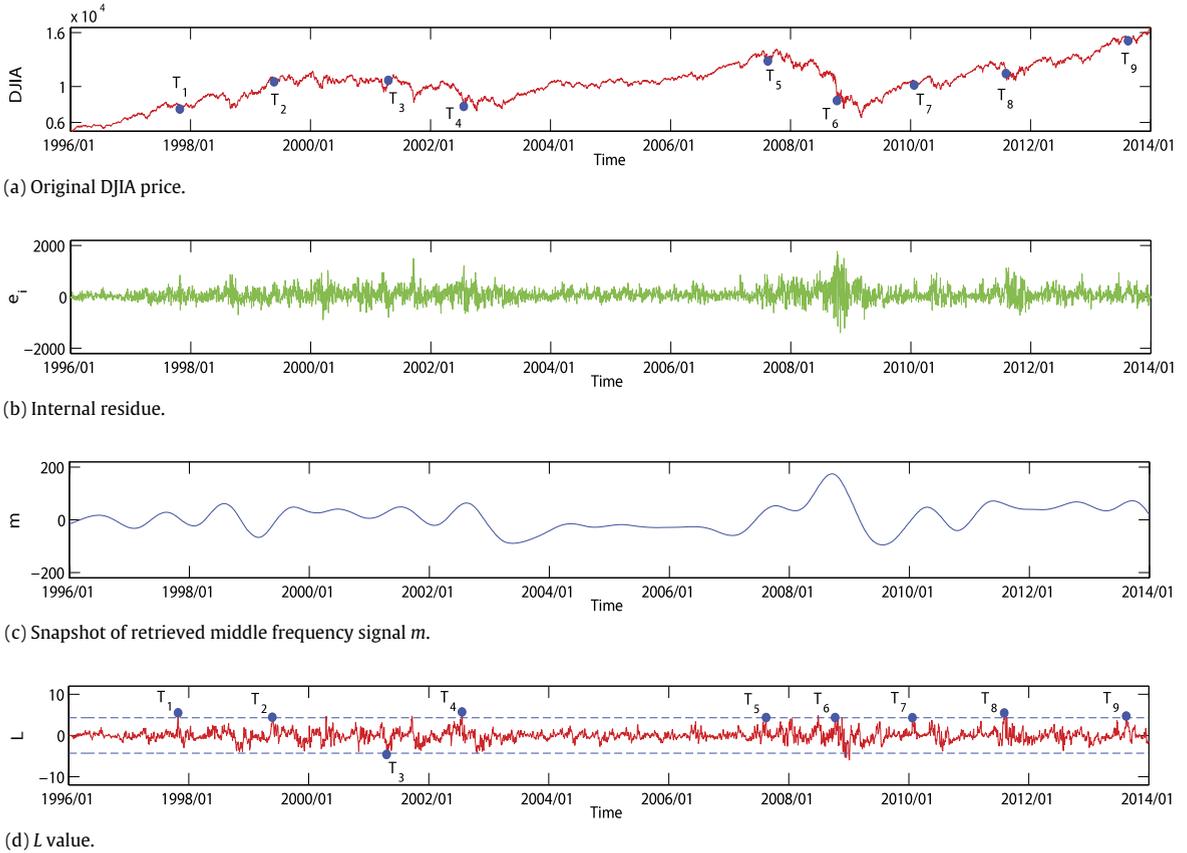
Fig. 7. Turning points forecasting of the DJIA with the DWT. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

4.2. UK market

London stock exchange (LSE) is the only stock exchange in the UK and also the largest one in Europe. In LSE, the most widely used index is FTSE 100, which is a blue-chip index of 100 largest companies on its list. In the past decades, it experienced several primary cycles. In this section, we use FTSE 100 to test our model's performance.

Daily closing prices are used in this model. The training period is selected as from January 1991 to January 2001. The forecasting period is from February 2001 to December 2013, and the corresponding OE model is obtained as Eq. (19):

$$H(z) = \begin{bmatrix} \dfrac{-1.906z^{-1} - 2.196z^{-2} + 0.1489z^{-3} + 0.5238z^{-4}}{1 + 0.8047z^{-1}} \\[2ex] \dfrac{-0.1284z^{-1} + 3.468z^{-2} + 1.158z^{-3} - 0.7299z^{-4}}{1 + 0.1306z^{-1}} \\[2ex] \dfrac{2.605z^{-1} - 4.287z^{-2} + 0.4033z^{-3} + 1.269z^{-4}}{1 - 0.9788z^{-1}} \end{bmatrix}^{\mathrm{T}} . \tag{19}$$

The wavelet we use is Daubechies 8 (db8) wavelet at level $J = 12$. The middle-frequency components are retrieved as

$$m(n) = D_{11}(n) + D_{10}(n) + D_9(n) + D_8(n). \tag{20}$$

The other parameters are selected as

$$N_s = 10, \qquad S_v = 1.81, \qquad T_s = 680. \tag{21}$$

Fig. 9 shows the forecasting results with the DWT. It is interesting to find that since 2000, FTSE 100 has experienced two primary cycles with three major turning periods. The first one is in March, 2003, when FTSE 100 hit its low-point of 3287. Our model gives an alarm signal T_1 for this turning point two month ago. The external factors that triggered the bear market of 2002–2003 mainly come from the economic depression of US and EU, which are major trading partners of UK. During this period, the burst of the dot-com bubble and 9/11 attacks significantly hit their economy, which reduced the foreign
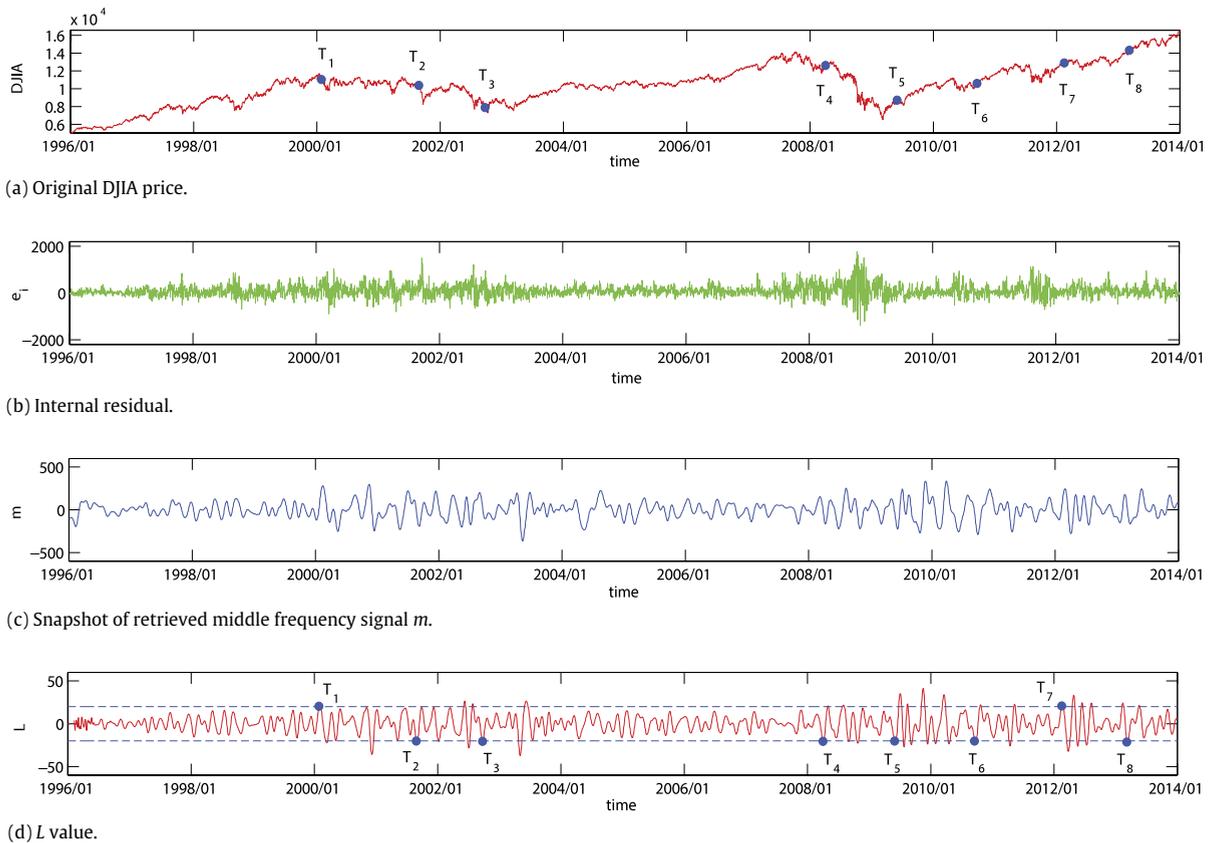
Fig. 8. Turning points forecasting of the DJIA with the MODWT.

investment in the UK. In addition, the fear of terrorism threat and the intense emotion brought by Iraq war severely reduced investors' confidence in the future, which results in the slumped investment and economic growth. There was an overall decline from real estate, manufacturing and service industry until the market reached its record low of this century.

The latest financial crisis of 2007–2008 significantly hits London stock market. When the market reached its peaks in July 2007 it began to sharply fluctuate and then suddenly crashed. This turning point is forecasted by our model at T_2, March 2007. After this turning period, the market declined more than one year until reached its six-year low in March 03, 2009. Our model successfully forecasts this turning point, see T_3 in Fig. 9. Compared to the peak of 2007, FTSE 100 lost almost half of its value. The US subprime mortgage crisis and shrink of foreign investments are the major external factors that account for this economic downturn. Financial sectors, which accounted for 9.4% of UK GDP in 2006, sustained huge losses in the crisis. Another dominant industry, the real estate, also experienced large decline of price and sales volume that significantly influenced the economy. After hitting its bottom, the market had its rally until now. During this rebound, there are some intense fluctuations where our model also gives some corresponding signals. In early August, 2011, the market experienced a sharp and continuous decline, lasting two weeks. Detecting this signal, our system gives a turning point alarm at T_4, but it was proved this fluctuation did not change the primary trend.

The MODWT is also applied to the FTSE 100. The db8 wavelet is used to decompose the internal residue at level $J = 11$. The middle-frequency components are selected as

$$m(n) = D_7(n) + D_6(n) + D_5(n). \tag{22}$$

The other parameters are selected as

$$N_s = 10, \qquad S_v = 10, \qquad T_s = 520. \tag{23}$$

Fig. 10 presents the forecasting results of the MODWT. There are six turning points forecasted during the testing period. The timing of T_1 and T_3 is similar to the results of the DWT. The turning signal T_2 is later than the corresponding one from the DWT, but it still locates in the turning period. After 2009, the MODWT generates more noisy alarm signals than the DWT.

4.3. China market

The stock trading in the emerging markets is very active in recent years. The emerging markets have some unique features distinguishing from developed markets. Their volatilities are much higher than that in the developed markets [31], which

(a) Original FTSE 100 price.

(b) Internal residual.

(c) Snapshot of retrieved middle frequency signal m.

(d) L value.
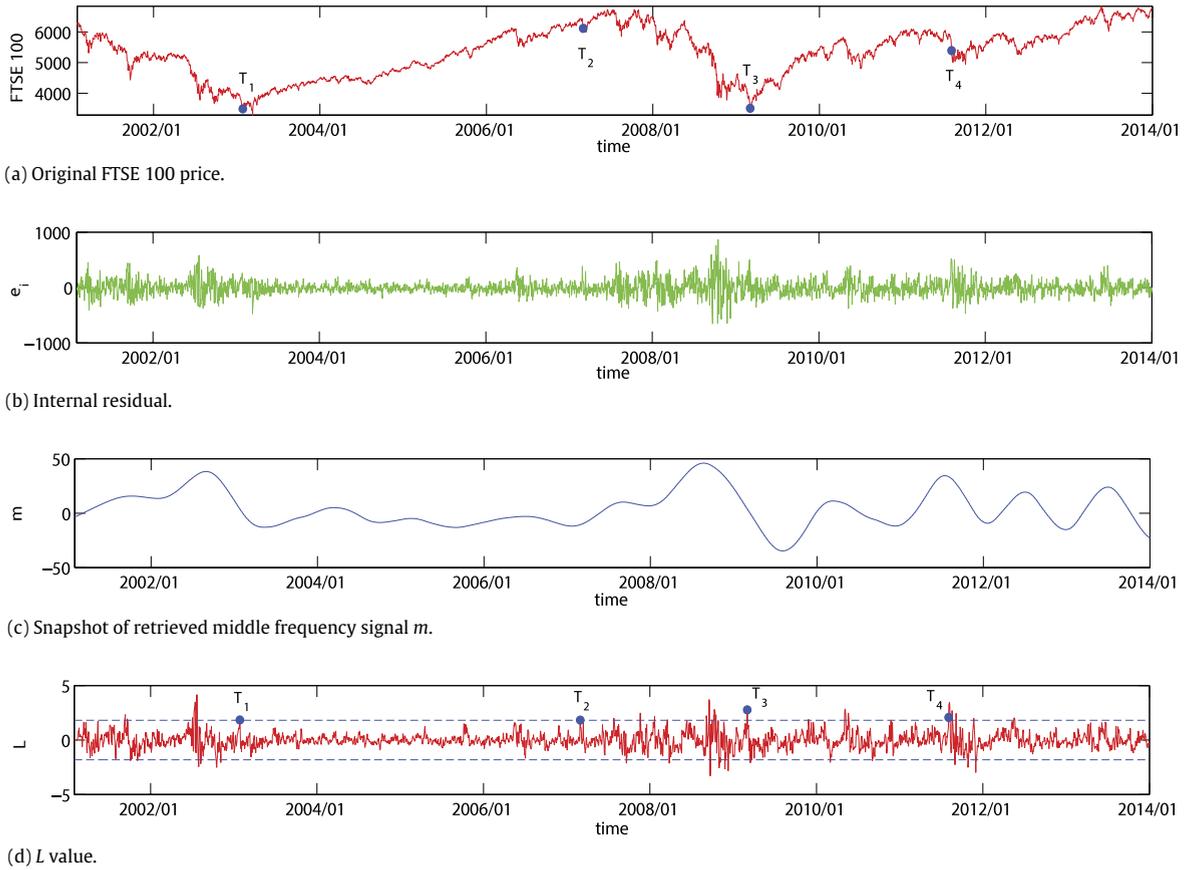
Fig. 9. Turning points forecasting of the FTSE 100 with the DWT.

are characterized by high risk and high return. One possible reason is that these markets are very sensitive to political events, and they always overreact to some new policies. It makes the cycle forecasting in such emerging markets more important but more challenging [32].

As a typical emerging market, the China market is studied in this session. The data we use is the daily closing prices of the SSE. The training period is selected as from year 1999 to 2004. The forecasting period is from year 2005 to 2013, and the corresponding OE model is as below:

$$H(z) = \begin{bmatrix} \dfrac{2.504z^{-1} - 1.086z^{-2} - 0.03676z^{-3}}{1 - 0.1794z^{-1} + 0.002452z^{-2}} \\[2mm] \dfrac{-0.8056z^{-1} + 1.445z^{-2} - 0.8015z^{-3}}{1 - 1.793z^{-1} + 0.995z^{-2}} \\[2mm] \dfrac{0.08992z^{-1} - 0.1419z^{-2} + 0.05945z^{-3}}{1 - 1.681z^{-1} + 0.7428z^{-2}} \end{bmatrix}^{\mathrm{T}} . \tag{24}$$

The DWT is used to forecast the turning points. The wavelet we use is Daubechies 9 (db9) wavelet at level $J = 12$. The middle-frequency components are restored as

$$m(n) = D_{10}(n) + D_9(n) + D_8(n) + D_7(n). \tag{25}$$

The other parameters are selected as

$$N_{\mathrm{s}} = 10, \qquad S_{\mathrm{v}} = 2.3, \qquad T_{\mathrm{s}} = 300. \tag{26}$$

The results are presented in Fig. 11. T_1 forecasted the beginning of a rising market. Although this is not the optimal starting point, it is a good forecasting for the long time economic activities, henceforwards the market stepped into a rapidly growing period. During this period, the China market adjusted its policy to be more open to the international investors, i.e., the non-tradable share reform in 2006. This rising market reached its peak in October 2007. This turning point is precisely forecasted by our model at T_2. After this peak, the China market began to crash. In the following one year it quickly declined until the

(a) Original FTSE 100 price.

(b) Internal residual.

(c) Snapshot of retrieved middle frequency signal m.

(d) S evolution.
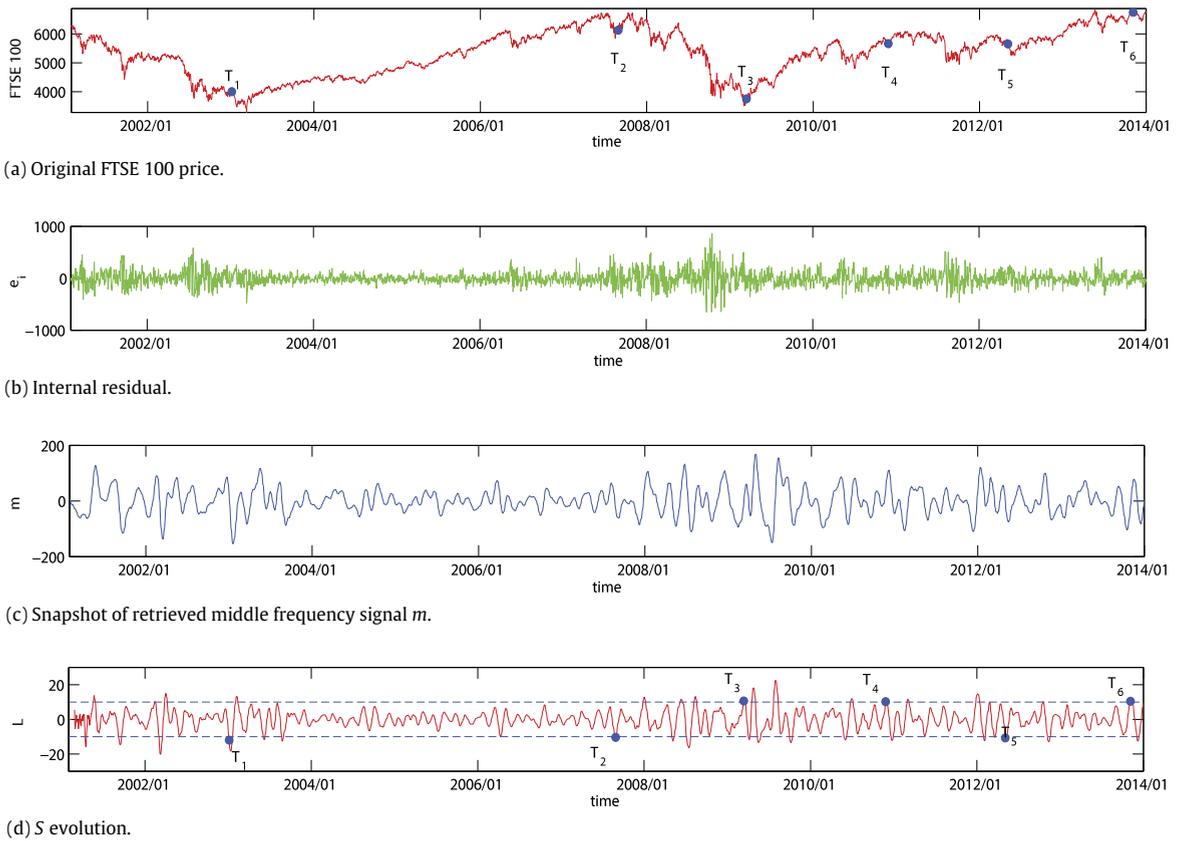
Fig. 10. Turning points forecasting of the FTSE 100 with the MODWT.

end of 2008. It is clear that T_3 very successfully forecasted the bottom of this declined trend. This crash was caused by many reasons, including reform of exchange rate, US financial crises and recession in the global economy. The next forecasted point, T_4, signifies the peak of the following recovery market very well. After this peak, the market entered into a tranquil period until present. T_5 and T_7 capture two stepwise decline in 2010 and 2013, which are results of the continuous recession of global economy.

The MODWT is also applied to the China market. The wavelet is selected as Daubechies 4 (db4) wavelet at level $J = 11$. The middle-frequency signals are reconstructed as

$$m(n) = D_7(n) + D_6(n) + D_5(n). \tag{27}$$

The other parameters are used as

$$N_s = 12, \qquad S_v = 13, \qquad T_s = 300. \tag{28}$$

Fig. 12 presents the forecasting results by using the MODWT. Only three turning points are forecasted in the whole testing period. T_1 gives an alarm of the coming downturn caused by the 2007–2008 financial crisis. The point T_2 indicates an alarm for the bottom of the declining trend, which is around one month earlier than the result of the DWT. T_3 indicates the end of the following rebounding trend, which is a little later than the timing of the DWT. Compared with the results from the DWT, the MODWT results miss the starting point of the bull market between 2006 and 2007, and do not give any alarm after 2010.

4.4. Results analysis

The results with the DWT for the US, UK and China markets demonstrate that our model is capable of capturing all the major turning points during the testing periods. As each market has its specific dynamics, the model parameters should be specified accordingly. However, the same feature is that the middle-frequency components of internal residues in all markets can capture their primary market cycles. This model performs best for the FTSE 100 among the three markets. The UK economy is highly influenced by external environments, making its market dynamics highly consistent with US and EU markets. When its external environments critically change, the market responses to it by giving some oscillating signals that are precisely captured by our model.

(a) Original SSE price.

(b) Internal residue.

(c) Snapshot of retrieved middle frequency signal m.

(d) L value.
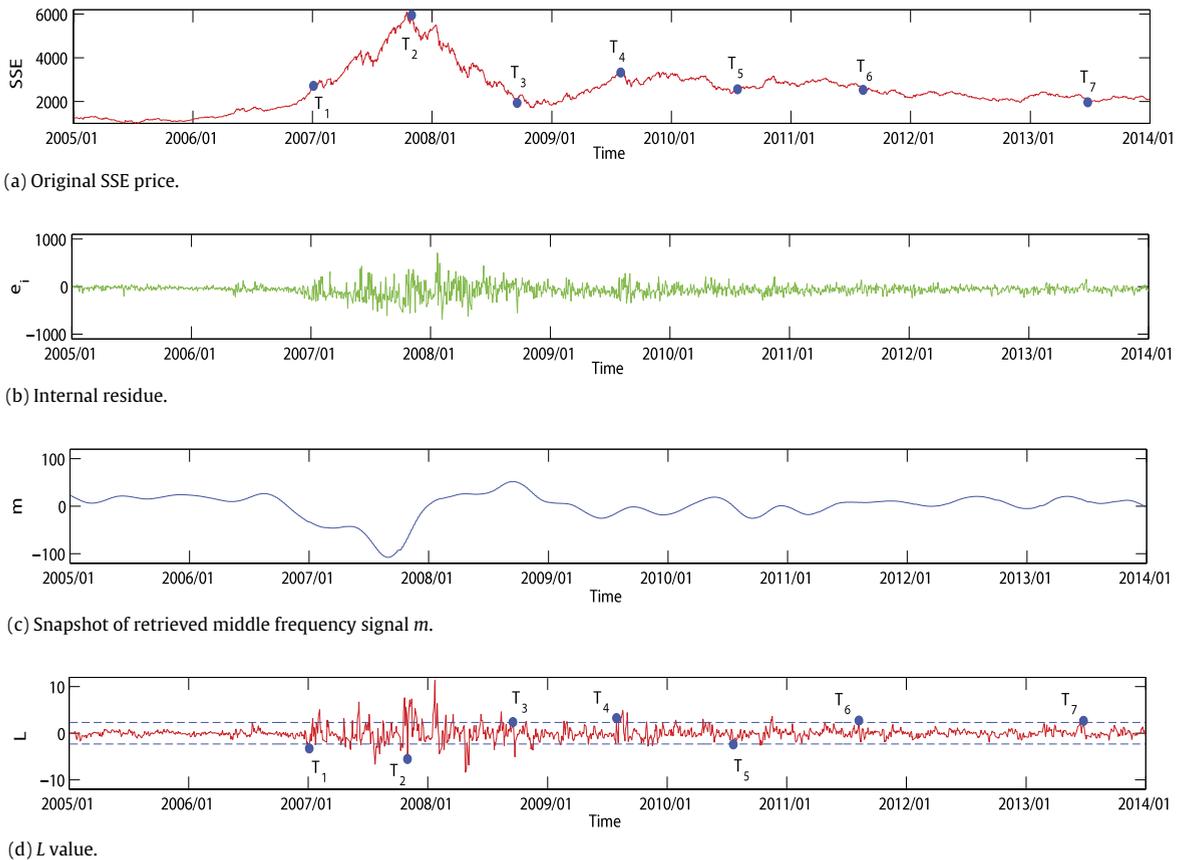
Fig. 11. Turning points forecasting of the SSE with the DWT.

Considering the time-invariant feature of the MODWT, it is a powerful tool in analyzing financial time series. However, it misses some turning points in the China and US markets and generates more noisy alarms in the UK market. In addition, in the US market, most of the forecasted turning points by the MODWT are later than those forecasted by the DWT. The MODWT has its advantages over the DWT in signal decomposition, but it does not provide additional information to the cases in this study. The DWT performs better in our framework in terms of capturing the oscillation signal during the turning period.

Comparing the US with China markets from 2005 to 2013 with the DWT, it is found that more turning points are forecasted in the China market than the US market. The reason may lie in the differences between the dynamical properties of the two markets in nature, e.g., essential differences in market size, structure and functionality, which make the fluctuation of the China stock market more dramatic than the US market.

US has a typical market-based financial system with large size of direct financing and well-developed capital markets, while the financial system in China is bank-based with underdeveloped capital markets as well as relatively isolated and small stock market. Currently, the stock market capitalization in China is still less than one quarter of that in US. Therefore, the China stock market is easily affected by external environments.

In a mature stock market like US, institutional investors usually dominate the trading activities. However, in the China stock market, individual investors account for more than 85% of all trading volume. The majority of this group of investors is lack of basic knowledge about financial investment, portfolio management and risk control, which makes them prone to speculative short-term trading. This kind of trading behavior inevitably results in dramatic fluctuations. In terms of market functionality, unlike the US market, not many listed companies in China stock exchange have significant influence and value. In this way, it cannot maintain a stable and efficient stock market. Additionally, due to the defective regulations in the option trading, short-selling mechanism and exit mechanism, the China stock market is relatively easy to be manipulated. All of these factors intensify the fluctuation in the China stock market.

5. Conclusion

Based on the system adaptation framework we previously proposed [26,27], its internal model is used in this paper to capture the dynamical properties of stock markets and generate a signal-rich residue for turning points forecasting. The

(a) Original SSE price.

(b) Internal residue.

(c) Snapshot of retrieved middle frequency signal m.

(d) L value.
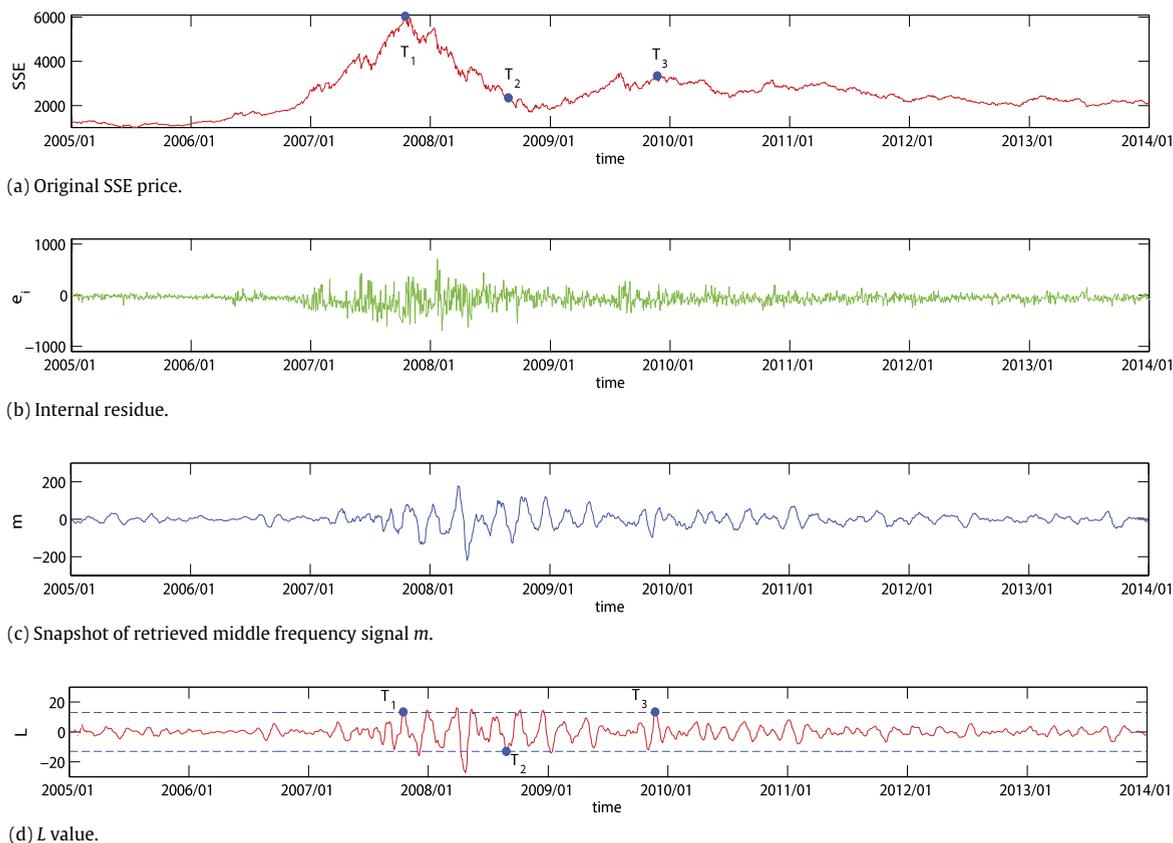
Fig. 12. Turning points forecasting of the SSE with the MODWT.

MRA of the DWT and MODWT are used to decompose the internal residue and further extract its middle-frequency signals. By analyzing the slope of retrieved signals, a turning points forecasting index is proposed.

Compared the results of the DWT with the MODWT, it is found that the DWT works better for this indicator. The testing results of US, UK and China markets demonstrate that nearly all the major turning points in the testing periods can be well forecasted by our index with the DWT, even including some smooth transition timings. In some other early works, the emerging markets are always considered to be more volatile and hard to forecast [32], e.g., China market which is highly driven by policy. Our model finds that the middle-frequency signals can also give remarkable forecasting information at such emerging markets. One reason might be that, in these markets the high-frequency data are more noisy than that in the mature markets. However, such kinds of noises can be effectively filtered by the wavelet methods.

We should notice that this method is not limited to stock markets. It can be widely used to study other economic time series or other financial markets, e.g., markets of future, commodity and other derivative instruments. Based on this framework, related studies could be extended further, such as constructing other forecasting index and doing the forecasting in different frequency bands. In addition, the MODWT still deserves more study in detecting the oscillation of financial time series, which may shed some light in market turning points forecasting.

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