Robust and H_∞-control

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1. Introduction

Robustness has always been a main paradigm in the analysis and design of a control system. In recent times, the main motivation that spurred the research activity was the study of performances over finite and bounded variation of parameters, whereas in classical multi-variable control the designer was only able to ensure robustness at the face of small parameter variations. Indeed, quantitative opposite to qualitative robustness is a watershed between classical and modern design methodologies. A mainstream of research for quantitative robustness is the so-called H_∞ control theory, which dominated the scenario over the last decades, being able to bridge classical frequency-domain and state-space techniques in an elegant unified mathematical framework. This is certainly one of the most interesting historical merits of the H_∞ approach, whose versatile nature permitted to incorporate in the same mathematical framework historically different problems such as filtering, factorization, interpolation and conjugation.

The idea underlying the H_∞ theory is rather simple: minimize a worst-case measure of the input–output map between disturbances and performance variables. When reduced to robustness of stability, this worst-case paradigm leads to the celebrated small-gain theorem (Zames, 1966). A success of H_∞ control theory is in the easy way the various specifications on the closed-loop system that can be incorporated through appropriate shaping functions which reflect the desired dynamic behavior (McFarlane & Glover, 1992). Notably, a posteriori it was seen that, in the state-space context, the theory has some structural similarities with the classical LQG theory, the latter being recovered in the case where a design parameter (the so-called attenuation level) goes to infinity.

It is a fact that the arguments underlying H_∞ control and related problems constitute a solid scientific background for the new researchers entering the field. Nowadays one can count hundreds of papers, many books (Francis, 1987; Helton & Merino, 1988; Maciejowski, 1989; Mustafa & Glover, 1990; Stoorvogel, 1992; Doyle, Francis, & Tannenbaum, 1992; Van Keulen, 1993; Heyde, 1995; Subrahmanyam, 1995; Basar & Bernhard, 1995; Dahleh & Diaz-Bobillo, 1995; Colaneri, Geromel, & Locatelli, 1997; Ball, Gohberg, & Rodman, 1990; Green & Limebeer, 1995; Zhou, Doyle, & Glover, 1995) and regular university courses at all levels in this subject. Interestingly, the success of H_∞ control in a number of important applications, contributes to reduce the historical gap between theory and application, despite the significant inherent mathematical sophistication required to understand the underlying theory.

The theory of H_∞ was first posed in an input–output setting thanks to the pioneering paper by Zames (1981). At that times, two main technical paths were undertaken: the Nevanlinna–Pick interpolation technique (Nevanlinna, 1919; Pick, 1916; Francis & Zames, 1984) and the AAK method (Adamjan, Arov, & Krein, 1978), mainly based on the Nehari extension problem (Glover, 1984) and on the unitary dilation technique in operator theory (Davis, Kahan, & Weinberger, 1982). The interpolation theory was originally part of the circuit theory (Hazony, 1961; Youla, 1961), and only in successive years became object of investigation by control theorists, for the solution of the disturbance reduction problem (Chang & Pearson, 1984) and the robust stabilization problem (Kimura, 1984; Glover, 1986). The state-space counterpart of the interpolation theory was worked-out in (Kimura, 1989) via the notion of J-lossless conjugation, where the role of the Pick matrix was translated in terms of the solution of a Riccati equation. Successive development of the H_∞ control theory were in the framework of the so-called J-spectral factorization approach (Green, Glover, Limebeer, & Doyle, 1990) and chain-scattering representation of the plant (Wieland & Willems, 1989). Also the almost disturbance decoupling problem (Kimura, 1997), which has an extensive literature behind and is an important problem *per se*, was cast in the general H_∞ formulation. In the late 1980s the time became mature for the development of a state-space technique for the solution of the general multi-input multi-output H_∞ control problem (Doyle, 1984). The main drawback was that it was required to compute the solution of a high order Riccati equation. This difficulty was removed later, and this was due to many contributors (Doyle, Glover, Khargonekar, & Francis, 1989). In the recent paper (Liu, Chen, & Lin, 2001), a comprehensive picture is traced and a complete solution is given of the robust and perfect tracking problem.

2. The book

The book consists of 16 chapters. After the introductory Chapter 1, the author presents the main facts of linear system theory in Chapter 2, especially for what concerns the structural concepts of zeros and canonical decomposition in an algebraic setting. In particular, a special coordinate transformation is introduced, which permeates all the technical results presented in the book.

Mappings from continuous to discrete time systems are presented in Chapter 3. This chapter furnishes some useful hints on how the bilinear transformation acts with reference to the structural subspaces of the system zeros. Existence condition for H_∞ suboptimal controllers are reviewed in Chapter 4, whereas in the successive chapter the solution of a general discrete-time algebraic Riccati equation is worked out by using the (bilinear) Caley-Hamilton transformation. Very interesting is the novel contribution in Chapters 6 and 10, where non-iterative procedures to compute exactly the value of the infimum attainable attenuation level γ^* for the full information and the partial information problem in continuous-time and discrete-time, respectively, are provided. The dependence of the solution of the Riccati equation with respect to the design parameter γ is also studied. The computation is also extended to cope with plants with imaginary axis zeros. The continuous-time H_∞ control problem is solved in Chapter 7, in a non-classical way. The solution is given in closed-loop form, explicitly parameterized in terms of γ. In this way this parameter can be tuned to obtain the desired level of attenuation. The derivation hinges on the state-space transformation introduced in Section 2. The same problems, via bilinear transformations, are dealt with in Chapter 11. The almost disturbance decoupling problem is tackled in Section 8 for continuous-time systems and in Chapter 12 for discrete-time systems. This problem is interesting *per se*, even though it can be viewed as a particularization of the general H_∞ control problem. Both the cases of state measurement and output measurement are treated. Chapters 9 and 13 deal with the robust and perfect tracking of continuous-time systems for continuous and discrete-time systems, respectively. Again the derivation hinges on a special coordinate basis illustrated in Chapter 2. Both the state-feedback case and the measurement case are considered. The theories considered so far are illustrated by three real-life applications in Chapters 14–16. In particular, Chapter 14 for voice-coil-motor actuator of computer hard disk drives. The aim is to show that the modern robust and tracking solutions outperform over the classical PID controllers performances. The almost decoupling approach is illustrated through a piezo-electric actuator control system in Chapter 15. Finally the perfect tracking approach is investigated for a gyro- stabilized mirror targeting system. All simulations show that the applications are very satisfactory.

3. Conclusions

The book provides a rather complete picture of the theory of H_∞ control for both continuous and discrete-time systems. The derivation follows non-classical paths, the main effort being to work out closed-loop formulae for the controller, parameterized by the attenuation parameter γ. The technical machinery involves coordinate transformation to allow the derivation of the closed-form solution and bilinear transformation to cast the discrete-time problems into equivalent continuous-time ones. The book surely provides some new insight and new numerical procedures for the solution of the various H_∞ problems. Particularly interesting are the results concerning the almost disturbance decoupling problem and the robust and tracking control problem. This is a way, the book somehow complements other books on similar arguments. Therefore, it is intended to graduate students willing to learn a different and modern technique and to understand the differences between various theoretical approaches to H_∞ control.

Patrizio Colaneri
Dipartimento di Elettronica e Informazione, Politecnico di Milano, Piazza Leonardo da Vinci 32, 20133 Milano, Italy
E-mail address: colaneri@elet.polimi.it

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About the reviewer

Patrizio Colaneri was born in Palmoli, Italy on 12 October 1956. He received the Doctor's degree (Laurea) in Electrical Engineering in 1981 from the Politecnico di Milano, Italy, and the Ph.D. degree (Dottorato di Ricerca) in Automatic Control in 1987 from the Ministero della Pubblica Istruzione of Italy. From 1982 to 1984, he worked in industry on simulation and control of electrical power plants. From 1984 to 1992, he was with the Centro di Teoria dei Sistemi of the Italian National Research Council (CNR). He spent a period of research at the Systems Research Center of the University of Maryland and held a visiting position at the Johannes Kepler University in Linz. He is currently Professor of Automatica at the Faculty of Engineering of the Politecnico di Milano. Dr. Colaneri was a YAP (Young Author Prize) finalist at the 1990 IFAC World Congress, Tallin, USSR. He is a member of the IFAC Technical Committee on Robust Control, the chair the IFAC Technical Committee on Control Design and a senior member of the IEEE. He was a member of the International Program Committee of the 1999 Conference of Decision and Control. Dr. Colaneri serves as Associate Editor of Automatica. His main interests are in the area of periodic systems and control, robust filtering and control, and digital and multi-rate control. On these subjects, he has authored or co-authored about 120 papers and the book "Control Systems Design: an RH-2 and RH-infinity viewpoint", published by Academic Press.