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D. L. Elliott
Associate Editor—Book Reviews
Institute for Systems Research
University of Maryland
A.V. Williams Blvd.
College Park, MD 20742 USA

H_2 Optimal Control— A. Saberi, P. Sannuti, and B. M. Chen (Prentice Hall International Series in Systems and Control Engineering, London, U.K., 1995.) *Reviewed by Siep Weiland.*

I. INTRODUCTION

The theory of H_2 optimal control and linear quadratic Gaussian (LQG) control are cornerstones of modern control theory. Witness the vast amount of literature on this topic with seminal contributions in, e.g., [1]–[4]; one is easily led to wonder whether such a mature research area has still relevant open research questions and whether there is something new to be said on this topic. The appearance of a book on H_2 optimal control may therefore easily lead to some skepticism at first sight.

In its general abstract formulation the H_2 optimal control problem amounts to finding a feedback controller for a given linear time invariant system such that the interconnection of system and controller is internally stable and such that the L_2 norm of the impulse response of the closed-loop system is minimal. In a suitable stochastic setting this problem corresponds to the well known LQG problem. Similarly, the linear quadratic regulator problem, Kalman filtering problems and certain optimization problems for sampled data systems, suitably interpreted, fit in the setting of an H_2 optimal control problem. The relevance of this problem, both for theoretical and practical considerations, is therefore unquestionable.

II. THE BOOK

While other books may well be suitable to address various aspects of the H_2 optimal control problem, this book covers a complete survey of this control problem at a high level of mathematical abstraction. The book is meant for practicing control engineers, graduate students, and researchers in control engineering. The book has no exercises and is less suitable as a textbook. It treats the H_2 control problem in its many facets, including:

- continuous- and discrete-time systems;
- regular and singular plants;
- proper and strictly proper controllers;
- perfect state measurements and imperfect state measurements.

The solution concept that is presented in the book is inspired by a method which transforms the to-be-controlled plant to an auxiliary plant in which the exogenous disturbances need to be completely decoupled from the to-be-controlled outputs. This transformation method dictates the main line of reasoning and analysis in the book. The main feature of this transformation is that it replaces the question to find H_2 optimal controllers by the equivalent question to solve a suitable version of a disturbance decoupling problem for the auxiliary system. Similarly, questions on existence and uniqueness of optimal controllers are reformulated in terms of solvability conditions of disturbance decoupling problems for the auxiliary system. The disturbance decoupling problem in its many variations is therefore extensively treated in the book. This treatment is very much based on concepts from geometric control theory and includes exact and almost disturbance decoupling problems with different information patterns for the feedback control law. It is a distinct feature of this book that it provides explicit algebraic algorithms for the construction, characterization, and parameterization of optimal controllers, closed-loop pole, and zero locations and optimal achievable performance levels. For the constructive aspects, a special coordinate basis of the state space, the input space and the output space has been devised that is used as a main tool throughout the book.

The amount of material that is covered in the book is voluminous. The book consists of eleven chapters. A detailed description of the contents of each chapter is given below.

Chapter 1 introduces the main topic area and contains an extensive section explaining the notation and terminology which is used throughout the book.

Chapter 2 formalizes the general H_2 optimal control problem for continuous- and discrete-time linear time-invariant systems. The relation of this problem to the classical linear quadratic Gaussian (LQG) problem and the deterministic linear quadratic regulator problem is pointed out. The H_2 optimal control problem for sampled data systems is reduced to the discrete-time H_2 optimization problem.

In Chapter 3 the authors introduce a special coordinate basis for state-space representations of linear time-invariant plants. A suitable partitioning of the input, state, and output space is provided which explicitly displays the finite and infinite zero structure and the invertibility properties of the plant. The connection between this coordinate basis and objects from the theory of geometric control [5] is emphasized.

Chapter 4 contains a wealth of results on algebraic Riccati equations, linear matrix inequalities, and quadratic matrix inequalities. Although

The reviewer is with the Department of Electrical Engineering, Eindhoven University of Technology, 5600 MB Eindhoven, The Netherlands (e-mail: s.weiland@ele.tue.nl).

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most results in this chapter are standard, there are few books that cover this material in so much detail.

The next two chapters (5 and 6) deal with the calculation of the optimal achievable H_2 performance levels, and existence and uniqueness conditions of the H_2 optimal controllers for, respectively, continuous-time and discrete-time systems. As discussed above, the authors employ a clever transformation on the given state space parameters of the plant so that H_2 optimal behavior for the given plant coincides with disturbance decoupled behavior for the transformed plant. This transformation then leads to algebraic and geometric characterizations for the existence and uniqueness of H_2 optimal controllers in terms of the auxiliary system.

Chapter 7 and Chapter 8 treat, respectively, the continuous-time and discrete-time state feedback case. A complete parameterization and synthesis is given of the set of all dynamic and static H_2 optimal state feedback controllers. It is shown that a number of closed-loop pole and zero locations are invariant for all H_2 optimal controlled systems and an algorithm is presented for the calculation of these locations. The problem of H_2 optimal control with simultaneous pole placement is solved and the important multi-objective H_2 optimal control problem with an H_∞ norm constraint on a closed-loop transfer function is considered and solved in a special case.

The next two chapters (9 and 10) cover the problem of measurement feedback control for continuous- and discrete-time systems, respectively. An algorithm is provided for the complete parameterization of all H_2 optimal measurement feedback controllers. A thorough treatment is given for the analysis of observer (or estimator) -based architectures of optimal measurement feedback controllers. These chapters contain an interesting analysis of the implementation of H_2 optimal controllers by means of a series interconnection of full or reduced order Luenberger observers (estimators) and state feedback control laws. It is shown that the folk belief of a "separation principle" for H_2 optimal controllers is, in fact, a very delicate issue.

Chapter 11 provides a brief treatment on the suboptimal H_2 control problem. This problem involves the construction of a controller such that the H_2 performance of the controlled systems is arbitrarily close to the optimal performance. A (theoretical) solution to this problem is provided by means of a perturbation argument.

III. Conclusions

Results on (LQG control, the linear quadratic regulator and H_2 optimal control got pretty much scattered in the literature. Looking for specific results in this research area may cost quite some research time in libraries and search efforts on the web. It is for this reason that there is certainly a need for a book that captures, solves, and discusses all relevant problems and tools for the design of H_2 optimal control systems. This book serves as such.

The authors managed to write an excellent and thorough survey of this area which serves as an extensive research reference covering the algebraic state space techniques to solve the general H_2 optimal control problem in its many variations. The book is highly mathematics-oriented with emphasis on a rigorous treatment of the theory rather than on applications. It is for this reason that this work will prove beneficial to both researchers and control engineers involved in the synthesis and analysis of almost all aspects of H_2 optimal control.

As indicated in Section I, the book is more than a collection of existing results. It contributes with quite some new material. The fact that the well-known separation principle for H_2 optimal control design (i.e., an optimal H_2 controller consists of a series interconnection of an optimal state observer and an optimal static state feedback) does *not* hold comes as an interesting outcome of the analysis in Chapter 9. Furthermore, the treatment of the important multi-objective H_2/H_∞ control scheme, the complete coverage of the discrete-time case of the general H_2 control problem, and the results on H_2 optimal control with simultaneous pole placement are interesting and welcome additions to results from the literature.

The book has very much the flavor of an encyclopedic treatment of the subject. Its writing is very compact which occasionally is at the cost of the tutorial exposition of the material. Indeed, the authors promise a self-contained treatment of the research area, but sometimes little guidance and background is given to uninitiated readers looking for a profound system theoretic understanding of the delicate mathematical intricacies. For example, the treatment of the complex special coordinate basis in Chapter 3 leaves too much the impression of a magic tool to solve problems. The encyclopedic style of the book implies that the development of the results is slow, but rigorous. For example, the first controller synthesis results appear more than 150 pages after the formulation of the general H_2 optimal control problem. Less patient readers should therefore definitely consult the table of contents.

The emphasis of the book is on the theoretical and constructive aspects of H_2 optimal controllers. Practical design aspects such as robustness properties, frequency response characteristics, and bandwidth requirements are viewed as "secondary considerations" and are, unfortunately, hardly addressed. It may be true that H_∞ theory is better suited for these issues, but I believe that practicing engineers will miss a treatment of these issues in the context of H_2 optimal designs.

Throughout this book, filtering problems with an H_2-type performance criterion are solved by considering the corresponding dual control problems. From a mathematical point of view there may be quite some elegance in such a treatment, but it provides little system theoretic understanding of the intricate structure of filtering problems. In view of the theoretical and practical importance of the Kalman filter it is somewhat regretful that a book on this topic does not provide a full fledged treatment of the Kalman filter. It seems that this book would have been the proper place for doing so.

Is this book the last work on H_2 optimal control? Certainly not. The importance of this problem in control theory and the maturity of this subject area for linear time invariant systems will be a continuous and inspiring source for generalizations, extensions and new paradigms. To paraphrase Charles Dickens: there indeed will be authors who believe this writing, in the reading, more than the authors believed it in the writing.

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