Distributed Optimal Solutions for Multiagent Pursuit-Evasion Games for Capture and Formation Control

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***Abstract*—In this article, distributed optimal solutions are designed for networked multiagent pursuit-evasion (MPE) games for capture and formation control. In the games, the pursuers aim to minimize the distance from their target evaders while the evaders attempt to maximize it, and at the same time, all the players desire to maintain cohesion with their teammates. The goals of agents are obviously reflected in the obtained optimal control strategies, which consist of an attracting term and/or a repelling term. Nash equilibrium is obtained by means of optimal strategies using the solutions of the Hamilton–Jacobi–Isaacs equations. Furthermore, three scenarios are considered in the MPE game: one-pursuer one-evader, multiple-pursuer one-evader, and multiple-pursuer multiple-evader, where sufficient conditions are given for pursuers in achieving exponential capture or formation control with ultimate zero or bounded errors. It is shown that the conditions depend on the structure of the communication graph, the parameters in the controllers, and the expected formation configurations. Finally, both simulations and real flight experiments successfully demonstrate the effectiveness of the proposed strategies.**

***Index Terms*—Differential games, formation control, multiagent systems, pursuit-evasion (PE) games.**

I. Introduction

THE last decade has witnessed wide development of multiagent systems due to their high application values in cooperative transportation, warehouse management, security surveillance, and logistic delivery, to name just a few. Pursuit-evasion (PE) games are one of the most interesting research topics. They are widely used both in military implementations such as missile guidance and aircraft control [1], [2], [3] and in civilian areas such as sport strategies. In nature, animal hunting behaviors are also PE differential games. Therefore, the agents in PE games can be unmanned aerial vehicles, unmanned ground vehicles, autonomous mobile robots, spacecraft, or living organisms.

The study of PE games starts from the simplest case with a single pursuer and a single evader [4], [5]. The PE game in [5] is formulated as a zero-sum game, which is solved using the Hamilton–Jacobi–Isaacs (HJI) equations. The result is extended to the cases of two pursuers versus one evader [6], [7] and multiple pursuers versus one evader [8], [9]. However, it is difficult to solve the HJI equations for nonlinear systems. Instead of solving the HJI equations, the control strategies were derived by differentiation of a particular value function in [10]. Geometrical methods are proposed for PE games in [11], [12], [13].

In recent years, more general multiple-pursuer multiple-evader PE games [14], [15] have gained much attention, owing to the increased interest in multiagent problems. In [16], a distributed hybrid controller is proposed for each pursuer using both local coordination protocols and time-varying potential fields. Conditions for guaranteed capture or guaranteed evasion are analyzed in [17] for multiple nonlinear players. Suboptimal approaches for the multiplayer PE differential games were presented in [18] by decoupled player control strategies. In [19], distributed optimal strategies are obtained for all the players by using a graph-theoretic approach, which depends on the player's teammates and neighbors of the opposite team. The obstacle avoidance PE games are further studied in [20]. The framework of [19] was extended by Qian et al. [21] to search for an adaptive Nash equilibrium solution for the differential games.

In regard to designing distributed solutions for the multiagent pursuit-evasion (MPE) games, a local error variable is defined as the position difference of each agent with respect to both its pursuer neighbors and evader neighbors [19], [20], [21], according to the pursuers' interest of capture of the evader or the evaders' interest in avoiding capture. Then, individual performance index functions are used to find solutions for the games.

In most PE games, the objective of pursuers is to capture the target evader, that is, to achieve position consensus [4], [5], [6],

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[7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21]. However, this kind of perish-together strategy may lead to the ruin of pursuers. Instead, formation control or surrounding control is of more practical [22], [23], [24], [25]. With surrounding the target, the pursuers can jet a mesh to capture the target and then carry it to a safety zone. Specifically, a distributed estimation-and-control hierarchical framework is developed in [22], [23] for, respectively, linear systems and surface vessels. The surrounding formation control can also be achieved by defining an expected displacement, under which the evaders lie in the convex hull formed by the pursuers. Besides, the evaders may also want to maintain some formation configurations to better complete their tasks. Compared with the PE games in [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], the formation control is also considered in this article in designing the distributed optimal solutions for MPE games. We will analyze how the expected displacement and communication graph will influence the result.

In this article, distributed optimal control strategies for MPE games for capture and formation control are designed over complex communication graphs. The MPE games present some interesting technical difficulties in designing *optimal* solutions for all the agents to reach their various goals and in obtaining the conditions so that the game is in Nash equilibrium. Due to the complexity of interactions among the pursuers and evaders, it is nontrivial to formulate their different objectives and analyze their different behaviors. The objective of formation control brings extra difficulties for analysis. Therefore, it requires tools that combine both differential game theory and cooperation control theory to find the optimal control protocols. The contributions of this article are summarized as follows.

1) Unlike [19], [20], [21], considering the players' goals of maintaining cooperation with their teammates and minimizing (maximizing) the distance to the evaders (pursuers), we appropriately defined distinct local error variables and novel performance indices for players in both teams, based on which the obtained distributed optimal solutions consist of an attracting term and/or a repelling term that reflect the goals of agents. More importantly, when group cohesion is ignored, the solutions for the evaders are still valid for them to maximize their distance from the pursuers.
2) Compared with the PE games in [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], formation control is also studied when developing optimal solutions for the MPE games. Besides capture, the formation control can achieve surrounding control of the target and also considers the case that the evaders desire to maintain some formation configurations. The results show that diverse expected formation configurations may result in zero or bounded formation control error.
3) We present conditions for capture and formation control for three scenarios: one-pursuer one-evader, multiple-pursuer one-evader, and multiple-pursuer multiple-evader, which is different from the works focusing on one particular scenario [4], [5], [6], [7], [8], [9], [10], [11], [12], [13]. Under a novel analysis, the results present that both the communication graph and the expected configuration will affect the capture and formation control. Due to the decoupling of the solutions in achieving the goals of agents, the interdependence of subsystems caused by the three communication graphs, and the existence of expected formation configurations, the closed-loop system is complex to analyze. Besides, possible conditions for evaders to avoid being captured are also discussed.

It is common that the multiple-pursuer multiple-evader PE games include the PE games of one-pursuer one-evader and multiple-pursuer one-evader as special cases [14], [15], [16], [17], [18], [19], [20], [21]. In this article, due to the complex communication graphs and formation control, the MPE game is not a simple combination of the PE games of one-pursuer one-evader or multiple-pursuer one-evader.

The rest of this article is organized as follows. Section II provides some preliminaries that include the communication graphs and definitions of local error variables. Problem formulations and control strategies for the MPE games are presented in Section III. In Section IV, conditions for target capture or formation control are analyzed for three cases. Simulation and experiments are shown in Section V to verify our strategies.

II. Preliminaries

Consider a team of N pursuers who have dynamics

$$\dot{x}_i^p = Ax_i^p + Bu_i^p,\ i = 1, \ldots, N \tag{1}$$

where $x_i^p \in \mathbb{R}^n$ and $u_i^p \in \mathbb{R}^m$ are the state and input of the ith pursuer, respectively. Consider also a group of M evaders with dynamics

$$\dot{x}_j^e = Ax_j^e + Bu_j^e,\ j = 1, \ldots, M \tag{2}$$

where $x_j^e \in \mathbb{R}^n$ and $u_j^e \in \mathbb{R}^m$ are the state and input of the jth evader, respectively.

The pursuers (1) and evaders (2) form a group of $N + M$ agents. Define $\mathcal{G}_p = (\mathcal{V}_p, \mathcal{E}_p)$ as the communication graph among the N pursuers, where $\mathcal{V} = \{v_{p1}, \ldots, v_{pN}\}$ and $\mathcal{E}_p = \mathcal{V}_p \times \mathcal{V}_p$. $(v_{pk}, v_{pi}) \in \mathcal{E}_p$ if and only if pursuer i has access to the information of pursuer k, and we say agent k is a neighbor of agent i. Let a_{ik} be the communication weight of the graph $\mathcal{G}_p$, with $a_{ik} = 1$ if $(v_{pk}, v_{pi}) \in \mathcal{E}_p$, otherwise, $a_{ik} = 0$. Let $\mathcal{A}_p = [a_{ik}] \in \mathbb{R}^{N\times N}$ be the weighted adjacency matrix where $a_{ii} = 0$. Denote by $d_i^{pp} = \sum_{k=1}^N a_{ik}$ the in-degree of pursuer i and $\mathcal{D}_{pp} = \mathrm{diag}\{d_i^{pp}\}$ the in-degree matrix of the graph. Then, the Laplacian matrix can be defined as $\mathcal{L}_p = \mathcal{D}_{pp} - \mathcal{A}_p$. Similarly, the interaction topology among evaders is represented by $\mathcal{G}_e = (\mathcal{V}_e, \mathcal{E}_e)$ with the nodes $\mathcal{V}_e = \{v_{e1}, \ldots, v_{eM}\}$. The edge weights are b_{jl} with $b_{jl} = 1$ if $(v_{el}, v_{ej}) \in \mathcal{E}_e$ and $b_{jl} = 0$ otherwise. The in-degree of evader j is $d_j^{ee} = \sum_{l=1}^M b_{jl}$ and the in-degree matrix $\mathcal{D}_{ee} = \mathrm{diag}\{d_j^{ee}\}$. Define the matrices $\mathcal{A}_e = [b_{jl}]$ and $\mathcal{L}_e = \mathcal{D}_{ee} - \mathcal{A}_e$.

Let $\mathcal{G}_{pe} = (\mathcal{V}_{pe}, \mathcal{E}_{pe})$ represent the communication topology among all the agents. Specifically, for $i \in \mathcal{V}_p$ and $j \in \mathcal{V}_e$, the edge weight $c_{ij} = 1$ if pursuer i can obtain the information of evader j; otherwise, $c_{ij} = 0$. Similarly, $e_{ji} = 1$ if evader j knows the information of pursuer i. The in-degree of pursuer i in the graph $\mathcal{G}_{pe}$ is defined as $d_i^{pe} = \sum_{j=1}^{M} c_{ij}$, and the in-degree of evader j is $d_j^{ep} = \sum_{i=1}^{N} e_{ji}$. The graph is undirected if $(i, j) \in \mathcal{E}$ implies $(j, i) \in \mathcal{E}$ with $i \neq j$. In this article, we assume that the graph $\mathcal{G}_{pe}$ is undirected.

Based on the above information, we define two local error variables for each pursuer, with respect to its pursuer neighbors and evader neighbors, respectively,

$$\tilde{x}_i^{pp} = \sum_{k=1}^{N} a_{ik}(x_k^p - x_i^p),\ \tilde{x}_i^{pe} = \sum_{j=1}^{M} c_{ij}\left(x_j^e - x_i^p + \Delta x_{ij}^{pe}\right) \tag{3}$$

where Δx_{ij}^{pe} is the expected displacement between the pursuer i and the evader j, and it can be a zero vector. The justification for the formation is that, in many practical applications, the team of pursuers may want to surround the target evader, instead of achieving state consensus to collide with it.

Similarly, we define another two local errors for each evader, with respect to its evader neighbors and pursuer neighbors, respectively,

$$\tilde{x}_j^{ee} = \sum_{l=1}^{M} b_{jl}\left(x_l^e - x_j^e + \Delta x_{jl}^{ee}\right),\ \tilde{x}_j^{ep} = \sum_{i=1}^{N} e_{ji}\left(x_i^p - x_j^e\right) \tag{4}$$

where Δx_{jl}^{ee} denotes the expected displacement between evaders j and l. In many application scenarios, the evaders desire to move in formation to increase the opportunity to complete the tasks.

Remark 1: It is well known that, for $x_1, x_2 \in \mathbb{R}^n$, $x_2 - x_1$ is a vector pointing from x_1 to x_2. It physically represents an attracting force of agent 2 to agent 1, and also a repelling force of agent 1 to agent 2. Thus, $\tilde{x}_i^{pp}$ and $\tilde{x}_i^{pe}$ denote the attracting forces from the pursuer neighbors and evader neighbors, respectively, to pursuer i. Similarly, $\tilde{x}_j^{ee}$ and $\tilde{x}_j^{ep}$ are the attracting forces from the evader neighbors and repelling forces from the pursuer neighbors to evader j, respectively.

III. Problem Formulation and Solutions for MPE Games

In the MPE game, the objective of pursuers is to minimize the distance from their neighboring evaders to intercept them or achieve the desired formation for the surrounding control. Moreover, the pursuers also intend to stay close to their teammates to keep the group cohesion. Therefore, the control strategy of each pursuer can be divided into two parts. The first part is for remaining close to its teammates, and the second part is for pursuing the evaders, that is, $u_i^p = u_i^{p1} + u_i^{p2}$.

The goals of each pursuer can be formulated as a scalar function $J_{pi}\left(\tilde{x}_i^{pp}, \tilde{x}_i^{pp}, u_i^{p1}, u_i^{p2}\right)$, regarded as the performance index for pursuer i, which is defined as

$$J_{pi} = \int_0^\infty \left[(\tilde{x}_i^{pp})^{\mathrm{T}} Q_i^{pp} \tilde{x}_i^{pp} + \left(u_i^{p1}\right)^{\mathrm{T}} R_i^{pp} u_i^{p1} + (\tilde{x}_i^{pe})^{\mathrm{T}} Q_i^{pe} \tilde{x}_i^{pe} + \left(u_i^{p2}\right)^{\mathrm{T}} R_i^{pe} u_i^{p2} \right] dt \tag{5}$$

where Q_i^{pp}, Q_i^{pe}, R_i^{pp}, and R_i^{pe} are positive-definite matrices with appropriate dimension. Pursuer i is, thus, concerned with minimizing J_{pi}.

Pursuer i is concerned with the minimization of the performance index J_{pi}. The dependence of J_{pi} on the local errors $\tilde{x}_i^{pp}$ and $\tilde{x}_i^{pe}$ can be explained as the goals of pursuer i to stay close to its teammates and minimize the distance from its evaders with the minimum control effort u_i^{p1} and u_i^{p2}.

On the contrary, the goals of the evaders are to maximize the distance from their neighboring pursuers and, at the same time, to stay close to their teammates. Similarly, the control input of evader j consists of two parts, i.e., $u_j^e = u_j^{e1} + u_j^{e2}$. The performance index for evader j can be defined as

$$J_{ej} = \int_0^\infty \left[\left(\tilde{x}_j^{ee}\right)^{\mathrm{T}} Q_j^{ee} \tilde{x}_j^{ee} + \left(u_j^{e1}\right)^{\mathrm{T}} R_j^{ee} u_j^{e1} - \left(\tilde{x}_j^{ep}\right)^{\mathrm{T}} Q_j^{ep} \tilde{x}_j^{ep} + \left(u_j^{e2}\right)^{\mathrm{T}} R_j^{ep} u_j^{e2} \right] dt \tag{6}$$

where matrices Q_j^{ee}, Q_j^{ep}, R_j^{ee}, and R_j^{ep} are positive definite. Notice that minimizing the third term $-(\tilde{x}_j^{ep})^{\mathrm{T}} Q_j^{ep} \tilde{x}_j^{ep}$ equals maximizing the distance from the pursuers, which implies escaping from them.

Based on the above definitions, we define the following MPE differential games on communication graphs $\mathcal{G}_{pe}$.

Definition 1 (MPE game): The MPE game is defined as

$$V_{pi} = \min_{u_i^{p1}, u_i^{p2}} J_{pi}\left(\tilde{x}_i^{pp}, \tilde{x}_i^{pe}, u_i^{p1}, u_i^{p2}\right) \tag{7}$$

$$V_{ej} = \min_{u_j^{e1}, u_j^{e2}} J_{ej}\left(\tilde{x}_j^{ee}, \tilde{x}_j^{ep}, u_j^{e1}, u_j^{e2}\right) \tag{8}$$

where V_{pi} and V_{ej} are the values of the MPE game for pursuer i and evader j, respectively.

Let u_{-i}^p and u_{-i}^e be the control strategies of the pursuer neighbors and evader neighbors of pursuer i, respectively, and u_{-j}^e and u_{-j}^p be the control strategies of the evader neighbors and pursuer neighbors of evader j, respectively. The Nash equilibrium is defined as follows.

Definition 2 (Nash equilibrium): Control strategies u_i^{p1*}, u_i^{p2*}, $i = 1, \ldots, N$, and u_j^{e1*}, u_j^{e2*}, $j = 1, \ldots, M$, form a Nash equilibrium if the inequalities

$$J_{pi}\left(u_i^{p1*}, u_i^{p2*}, u_{-i}^{p*}, u_{-i}^{e*}\right) \le J_{pi}\left(u_i^{p1}, u_i^{p2}, u_{-i}^{p*}, u_{-i}^{e*}\right)$$

$$J_{ej}\left(u_j^{e1*}, u_j^{e2*}, u_{-j}^{e*}, u_{-j}^{p*}\right) \le J_{ej}\left(u_j^{e1}, u_j^{e2}, u_{-j}^{e*}, u_{-j}^{p*}\right)$$

hold for all the agents in the game.

In the following part of this section, we aim to find the Nash equilibrium policies of the MPE game.

The optimal control strategy of pursuer i can be obtained by the Hamiltonian function [8], [26]

$$\begin{aligned}H_i^p &= (\tilde{x}_i^{pp})^{\mathrm{T}} Q_i^{pp} \tilde{x}_i^{pp} + \left(u_i^{p1}\right)^{\mathrm{T}} R_i^{pp} u_i^{p1} \\ &\quad + (\tilde{x}_i^{pe})^{\mathrm{T}} Q_i^{pe} \tilde{x}_i^{pe} + \left(u_i^{p2}\right)^{\mathrm{T}} R_i^{pe} u_i^{p2} + \dot{V}_{pi}(\tilde{x}_i^{pp}, \tilde{x}_i^{pe}) \\ &= (\tilde{x}_i^{pp})^{\mathrm{T}} Q_i^{pp} \tilde{x}_i^{pp} + \left(u_i^{p1}\right)^{\mathrm{T}} R_i^{pp} u_i^{p1} \\ &\quad + (\tilde{x}_i^{pe})^{\mathrm{T}} Q_i^{pe} \tilde{x}_i^{pe} + \left(u_i^{p2}\right)^{\mathrm{T}} R_i^{pe} u_i^{p2} \\ &\quad + \nabla V_{pi}^T(\tilde{x}_i^{pp})\dot{\tilde{x}}_i^{pp} + \nabla V_{pi}^T(\tilde{x}_i^{pe})\dot{\tilde{x}}_i^{pe}\end{aligned}$$

where V_{pi} is the value defined in (7). Following (3), we have

$$\begin{aligned}\dot{\tilde{x}}_i^{pp} &= -\sum_{k=1}^{N} a_{ik}\left(Ax_i^p + Bu_i^{p1} + Bu_i^{p2}\right) + \sum_{k=1}^{N} a_{ik}\dot{x}_k^p \\ \dot{\tilde{x}}_i^{pe} &= -\sum_{j=1}^{M} c_{ij}\left(Ax_i^p + Bu_i^{p1} + Bu_i^{p2}\right) + \sum_{j=1}^{M} c_{ij}\dot{x}_j^e.\end{aligned}$$

Letting the partial derivative of H_i^p

$$\frac{\partial H_i^p}{\partial u_i^{p1}} = 2R_i^{pp}u_i^{p1} - d_i^{pp}B^{\mathrm{T}}\nabla V_{pi}(\tilde{x}_i^{pp}) - d_i^{pe}B^{\mathrm{T}}\nabla V_{pi}(\tilde{x}_i^{pe}) = 0$$

$$\frac{\partial H_i^p}{\partial u_i^{p2}} = 2R_i^{pe}u_i^{p2} - d_i^{pp}B^{\mathrm{T}}\nabla V_{pi}(\tilde{x}_i^{pp}) - d_i^{pe}B^{\mathrm{T}}\nabla V_{pi}(\tilde{x}_i^{pe}) = 0$$

gives

$$u_i^{p1*} = \frac{1}{2}(R_i^{pp})^{-1}\left(d_i^{pp}B^{\mathrm{T}}\nabla V_{pi}(\tilde{x}_i^{pp}) + d_i^{pe}B^{\mathrm{T}}\nabla V_{pi}(\tilde{x}_i^{pe})\right) \quad (9)$$

$$u_i^{p2*} = \frac{1}{2}(R_i^{pe})^{-1}\left(d_i^{pp}B^{\mathrm{T}}\nabla V_{pi}(\tilde{x}_i^{pp}) + d_i^{pe}B^{\mathrm{T}}\nabla V_{pi}(\tilde{x}_i^{pe})\right) \quad (10)$$

which are the optimal control strategies for pursuer i. V_{pi} is the solution of the coupled HJI

$$\begin{aligned}&(\tilde{x}_i^{pp})^{\mathrm{T}} Q_i^{pp}\tilde{x}_i^{pp} + (u_i^{p1*})^{\mathrm{T}} R_i^{pp} u_i^{p1*} + (\tilde{x}_i^{pe})^{\mathrm{T}} Q_i^{pe}\tilde{x}_i^{pe} \\ &+ (u_i^{p2*})^{\mathrm{T}} R_i^{pe} u_i^{p2*} + \nabla V_{pi}^T(\tilde{x}_i^{pp})\left(-\sum_{k=1}^{N} a_{ik}\left(Ax_i^p + Bu_i^{p1*}\right.\right. \\ &\left.\left. + Bu_i^{p2*}\right) + \dot{x}_k^p\right) + \nabla V_{pi}^T(\tilde{x}_i^{pe})\left(-\sum_{j=1}^{M} c_{ij}\left(Ax_i^p + Bu_i^{p1*}\right.\right. \\ &\left.\left. + Bu_i^{p2*}\right) + \dot{x}_j^e\right) = 0. \qquad (11)\end{aligned}$$

Similarly, the optimal control strategies for evader j are given by

$$u_j^{e1*} = \frac{1}{2}(R_j^{ee})^{-1}\left(d_j^{ee}B^{\mathrm{T}}\nabla V_{ej}(\tilde{x}_j^{ee}) + d_j^{ep}B^{\mathrm{T}}\nabla V_{ej}(\tilde{x}_j^{ep})\right) \quad (12)$$

$$u_j^{e2*} = \frac{1}{2}(R_j^{ep})^{-1}\left(d_j^{ee}B^{\mathrm{T}}\nabla V_{ej}(\tilde{x}_j^{ee}) + d_j^{ep}B^{\mathrm{T}}\nabla V_{ej}(\tilde{x}_j^{ep})\right). \quad (13)$$

The following theorem states that the MPE game is in Nash equilibrium under the control policies (9)–(13).

Theorem 1: Considering the pursuers (1) and evaders (2) with local errors (3) and (4). Let (9)–(13) be the control strategies for pursuer i and evader j, where V_{pi} and V_{ej} are the values of the game for pursuer i and evader j, respectively. Then, the MPE game (7), (8) is in Nash equilibrium. Moreover, the values of the game for pursuer i and evader j are given by $V_{pi}(t_0)$ and $V_{ej}(t_0) - V_{ej}(t_\infty)$, respectively.

Proof: The proof process is similar to the case of [19, Th. 1]. We omit it here due to the limit on the number of pages. ■

Remark 2: Different from [19], due to various objectives of pursuers and evaders, we defined distinct local variables $\tilde{x}_i^{pp}$, $\tilde{x}_i^{pe}$, $\tilde{x}_j^{ee}$, and $\tilde{x}_j^{ep}$ and performances indexes (5) and (6), which results in new control strategies (12) and (13) such that the game is in Nash equilibrium. The control strategies (12) and (13) both consist of two terms that reflect objectives of agents with respect to their teammates and the opponents, respectively.

IV. Conditions for Capture and Formation Control in Three Scenarios

In this section, we consider the MPE game in three scenarios: one-purser one-evader, multiple-pursuer one-evader, and multiple-pursuer multiple-evader.

Suppose that for $i = 1, \ldots, N$ and $j = 1, \ldots, M$, the value functions V_{pi} and V_{ej} have the form

$$V_{pi} = \alpha_{i1}(\tilde{x}_i^{pp})^{\mathrm{T}} P_i^{pp}\tilde{x}_i^{pp} + \alpha_{i2}(\tilde{x}_i^{pe})^{\mathrm{T}} P_i^{pe}\tilde{x}_i^{pe} \quad (14)$$

$$V_{ej} = \beta_{j1}(\tilde{x}_j^{ee})^{\mathrm{T}} P_j^{ee}\tilde{x}_j^{ee} - \beta_{j2}(\tilde{x}_j^{ep})^{\mathrm{T}} P_j^{ep}\tilde{x}_j^{ep} \quad (15)$$

where P_i^{pp}, P_i^{pe}, P_j^{ee}, and P_j^{ep} are positive-definite matrices. Taking ∇V_{pi} into (9)–(13) and following the fact that $u_i^p = u_i^{p1} + u_i^{p2}$ and $u_j^e = u_j^{e1} + u_j^{e2}$, we, thus, have

$$\begin{aligned}u_i^p &= \left((R_i^{pp})^{-1} + (R_i^{pe})^{-1}\right) \\ &\quad \cdot \left(\alpha_{i1}d_i^{pp}B^{\mathrm{T}}P_i^{pp}\tilde{x}_i^{pp} + \alpha_{i2}d_i^{pe}B^{\mathrm{T}}P_i^{pe}\tilde{x}_i^{pe}\right) \qquad (16) \\ u_j^e &= \left((R_j^{ee})^{-1} + (R_j^{ep})^{-1}\right) \\ &\quad \cdot \left(\beta_{j1}d_j^{ee}B^{\mathrm{T}}P_j^{ee}\tilde{x}_j^{ee} - \beta_{j2}d_j^{ep}B^{\mathrm{T}}P_j^{ep}\tilde{x}_j^{ep}\right). \qquad (17)\end{aligned}$$

Remark 3: One can note that the control strategy u_i^p for pursuer i reflects the two attracting forces from its neighboring teammates and evaders, which will drive it to stay close to its teammates and, meanwhile, to capture the target. On the contrary, the repelling force $-\tilde{x}_j^{ep}$ in u_j^e for evader j prevents it from being intercepted by its neighboring pursuers. When group cohesion of the evader team is not considered, i.e., $d_j^{ee} = 0$, the repelling force still holds. In [19], when $d_j^{ee} = 0$, it becomes that $u_j^e = \beta_{j2}d_j^{ep}(R_j^{ep})^{-1}B^{\mathrm{T}}P_j^{ep}\tilde{x}_j^{ep}$, which, conversely, represents the attracting forces from its neighboring pursuers.

Remark 4: $\alpha_{i1}, \alpha_{i2}, \beta_{i1}$, and β_{i2} are scalar gains for pursuers and evaders whose values can be selected according to the objectives of the game. α_{i1} and α_{i2} can be seen as the priorities of pursuers to, respectively, stay close to their teammates and capture the target. The larger α_{i2}, the higher priority of capturing the evader. Similarly, β_{i1} and β_{i2} imply the priority of evaders to stay close to their teammates and to escape from the pursuers. The larger β_{i2}, the higher priority of escaping from the pursuers. In [19], there is one scalar gain for the evaders to set priority to stay close to each other.

Without loss of generality, the R matrices in (16) and (17) are selected as identity matrices, and the P matrices are solutions of the Lyapunov equation

$$PA + A^{\mathrm{T}}P - PBB^{\mathrm{T}}P = -I. \tag{18}$$

Note that the equation is solvable if all the eigenvalues of A have nonpositive real parts. The control strategies, thus, become, for $i = 1,\ldots,N$ and $j = 1,\ldots,M$,

$$u_i^p = 2\left(\alpha_{i1}d_i^{pp}B^{\mathrm{T}}P\tilde{x}_i^{pp} + \alpha_{i2}d_i^{pe}B^{\mathrm{T}}P\tilde{x}_i^{pe}\right) \tag{19}$$

$$u_j^e = 2\left(\beta_{j1}d_j^{ee}B^{\mathrm{T}}P\tilde{x}_j^{ee} - \beta_{j2}d_j^{ep}B^{\mathrm{T}}P\tilde{x}_j^{ep}\right). \tag{20}$$

Now, the control policies depend on the coefficients α_{i1}, α_{i2}, β_{j1}, and β_{j2}. In the following, we will analyze how the coefficients affect the PE games in three scenarios.

A. PE Game for the One-Pursuer One-Evader Problem

When the evaders increase their distance with respect to each other to separate the pursuers, each pursuer must select a single evader as its target. Suppose that pursuer i has selected evader i as the target using the target selection algorithm. In such a case, the local error for pursuer i with respect to the evader is defined as $\tilde{x}_i^{pe} = x_i^e - x_i^p$. Similarly, $\tilde{x}_i^{ep} = -\tilde{x}_i^{pe}$. Following the steps in Section III, the control strategies for pursuer i and evader i are, respectively,

$$u_i^p = \alpha_{i2}B^{\mathrm{T}}P\tilde{x}_i^{pe},\ u_i^e = -\beta_{i2}B^{\mathrm{T}}P\tilde{x}_i^{ep} = \beta_{i2}B^{\mathrm{T}}P\tilde{x}_i^{pe}. \tag{21}$$

Theorem 2: Consider the multiagent system with N pursuers and N evaders with dynamics (1) and (2), respectively, and with the control policies (21). Assume that pursuer i selects evader i as its target. Then, if $\alpha_{i2} \geq \frac{1}{2} + \beta_{i2}$, we have $\lim_{t\to\infty}\tilde{x}_i^{pe} = 0$ exponentially for any initial conditions.

Proof: The derivative of $\tilde{x}_i^{pe}$ satisfies

$$\dot{\tilde{x}}_i^{pe} = [A - (\alpha_{i2} - \beta_{i2})BB^{\mathrm{T}}P]\tilde{x}_i^{pe}. \tag{22}$$

We define the Lyapunov function candidate $V_i = (\tilde{x}_i^{pe})^{\mathrm{T}}P\tilde{x}_i^{pe}$ whose derivative along the trajectory of (22) is

$$\begin{aligned}\dot{V}_i &= (\tilde{x}_i^{pe})^{\mathrm{T}}(A^{\mathrm{T}}P + PA - 2(\alpha_{i2} - \beta_{i2})PBB^{\mathrm{T}}P)\tilde{x}_i^{pe}\\ &= -(\tilde{x}_i^{pe})^{\mathrm{T}}\tilde{x}_i^{pe} - [2(\alpha_{i2} - \beta_{i2}) - 1](\tilde{x}_i^{pe})^{\mathrm{T}}PBB^{\mathrm{T}}P\tilde{x}_i^{pe}.\end{aligned}$$

It is obvious that if $\alpha_{i2} \geq \frac{1}{2} + \beta_{i2}$, $\dot{V}_i \leq -(\tilde{x}_i^{pe})^{\mathrm{T}}\tilde{x}_i^{pe}$. Since $\lambda_{\min}(P)\|\tilde{x}_i^{pe}\|^2 \leq V_i(\tilde{x}_i^{pe}) \leq \lambda_{\max}(P)\|\tilde{x}_i^{pe}\|^2$, and $\dot{V}_i \leq -\|\tilde{x}_i^{pe}\|^2$, we have $\lim_{t\to\infty}\tilde{x}_i^{pe} = 0$ exponentially. ■

B. MPE Game for the Multiple-Pursuer One-Evader Problem

When there are multiple pursuers and one evader, the pursuers may want to intercept the target evader or to achieve the surrounding formation control, and the evader aims to maximize the distance from all the pursuers. In such a case, we have $d_i^{pe} = 1$, $R_j^{ee} = 0$, $d_j^{ee} = 0$, and $d_j^{ep} = N$ for the unique evader $j = 1$. For simplicity, we denote $\beta_{j2} = \beta$. We, thus, have

$$u_1^e = -\beta NB^{\mathrm{T}}P\tilde{x}_1^{ep}. \tag{23}$$

Theorem 3: Consider the multiagent system with N pursuers and one evader with dynamics (1) and (2), respectively, and with control policies (19) and (23), respectively. Then, if $\alpha_{i2} \geq (2\beta N^2 + 1)/4$ for all $i = 1,\ldots,N$, and

(i) if $\Delta x_{i1}^{pe} = 0$ for all $i = 1,\ldots,N$, we have $\lim_{t\to\infty}\tilde{x}_i^{pe} = 0$ exponentially for any initial conditions;

(ii) if $\exists i$ such that $\Delta x_{i1}^{pe} \neq 0$ but $\sum_{i=1}^N \Delta x_{i1}^{pe} = 0$, and $\alpha_{i1}d_i^{pp} = 0$ for all $i = 1,\ldots,N$, we have $\lim_{t\to\infty}\tilde{x}_i^{pe} = 0$ exponentially for any initial conditions;

(iii) if $\sum_{i=1}^N \Delta x_{i1}^{pe} \neq 0$, the equilibrium of the closed system is globally exponentially input-to-state stable (ISS) with input Δx^{pe}, i.e.,

$$\lim_{t\to\infty}\sup_{t'\geq t}\|\tilde{x}^{pe}(t')\| \leq \pi_1\left(\|\Delta x^{pe}\|^2\right)$$

for some class $\mathcal{K}$ function π_1, where $\tilde{x}^{pe} = \mathrm{col}(\tilde{x}_1^{pe},\ldots,\tilde{x}_N^{pe})$ and $\Delta x^{pe} = \mathrm{col}(\Delta x_{11}^{pe},\ldots,\Delta x_{N1}^{pe})$.

Proof: This theorem presents sufficient conditions for capture. We, thus, analyze the result from the viewpoint of the pursuers. Let $\hat{x}_i^p = x_i^p - \Delta x_{i1}^{pe}$. It follows from (3) that $\tilde{x}_i^{pe} = x_1^e - \hat{x}_i^p$, whose dynamics satisfies

$$\begin{aligned}\dot{\tilde{x}}_i^{pe} &= A(x_1^e - \hat{x}_i^p) - \beta NBB^{\mathrm{T}}P\tilde{x}_1^{ep} - Bu_i^p\\ &= A\tilde{x}_i^{pe} - \beta NBB^{\mathrm{T}}P\left(-\sum_{i=1}^N\tilde{x}_i^{pe} + \sum_{i=1}^N\Delta x_{i1}^{pe}\right)\\ &\quad - 2B\left(\alpha_{i1}d_i^{pp}B^{\mathrm{T}}P\tilde{x}_i^{pp} + \alpha_{i2}B^{\mathrm{T}}P\tilde{x}_i^{pe}\right)\\ &= \left(A - 2\alpha_{i2}BB^{\mathrm{T}}P\right)\tilde{x}_i^{pe} + \beta NBB^{\mathrm{T}}P\sum_{i=1}^N\tilde{x}_i^{pe}\\ &\quad - 2\alpha_{i1}d_i^{pp}BB^{\mathrm{T}}P\tilde{x}_i^{pp} - \beta NBB^{\mathrm{T}}P\sum_{i=1}^N\Delta x_{i1}^{pe}.\end{aligned} \tag{24}$$

On the one hand, from the definition of $\tilde{x}_i^{pp}$ in (3), we have

$$\begin{aligned}\tilde{x}_i^{pp} &= \sum_{k=1}^N a_{ik}(x_k^p - x_1^e - \Delta x_{k1}^{pe} - x_i^p + x_1^e + \Delta x_{i1}^{pe} + \Delta x_{k1}^{pe} - \Delta x_{i1}^{pe})\\ &= \sum_{k=1}^N a_{ik}(x_i^{pe} - x_k^{pe}) + \sum_{k=1}^N a_{ik}(\Delta x_{k1}^{pe} - \Delta x_{i1}^{pe}).\end{aligned} \tag{25}$$

Denote $\tilde{x}^{pp} = \mathrm{col}(\tilde{x}_1^{pp},\ldots,\tilde{x}_N^{pp})$. Then, the compact form of (25) is

$$\tilde{x}^{pp} = (\mathcal{L}_p \otimes I_n)\tilde{x}^{pe} - (\mathcal{L}_p \otimes I_n)\Delta x^{pe}. \tag{26}$$

Define the Lyapunov function candidate for the closed-loop system $\tilde{x}^{pe}$ as $V = (\tilde{x}^{pe})^{\mathrm{T}}(I_N \otimes P)\tilde{x}^{pe}$. Its derivative along the trajectory of (24) gives

$$\begin{aligned}\dot{V} = \sum_{i=1}^{N} \Bigg[&(\tilde{x}_i^{pe})^{\mathrm{T}} (A^{\mathrm{T}} P + PA - 4\alpha_{i2} PBB^{\mathrm{T}} P) \tilde{x}_i^{pe} \\ &+ 2\beta N (\tilde{x}_i^{pe})^{\mathrm{T}} PBB^{\mathrm{T}} P \sum_{i=1}^{N} \tilde{x}_i^{pe} \\ &- 4\alpha_{i1} d_i^{pp} (\tilde{x}_i^{pe})^{\mathrm{T}} PBB^{\mathrm{T}} P \tilde{x}_i^{pp} \\ &-2\beta N (\tilde{x}_i^{pe})^{\mathrm{T}} PBB^{\mathrm{T}} P \sum_{i=1}^{N} \Delta x_{i1}^{pe} \Bigg]. \end{aligned} \tag{27}$$

Denote $\alpha_2 = \min\{\alpha_{i2}\}$ for $i = 1, \ldots, N$. By (18), it follows that

$$\begin{aligned}\dot{V} \leq &-(\tilde{x}^{pe})^{\mathrm{T}} \tilde{x}^{pe} - (4\alpha_2 - 1)(\tilde{x}^{pe})^{\mathrm{T}} (I_N \otimes PBB^{\mathrm{T}} P) \tilde{x}^{pe} \\ &+ 2\beta N (\tilde{x}^{pe})^{\mathrm{T}} (1_N 1_N^{\mathrm{T}} \otimes PBB^{\mathrm{T}} P) \tilde{x}^{pe} \\ &- (\tilde{x}^{pe})^{\mathrm{T}} \mathrm{diag}\{4\alpha_{i1} d_i^{pp} PBB^{\mathrm{T}} P\} \\ &\cdot ((\mathcal{L}_p \otimes I_n) \tilde{x}^{pe} - (\mathcal{L}_p \otimes I_n) \Delta x^{pe}) \\ &- 2\beta N (\tilde{x}^{pe})^{\mathrm{T}} (1_N 1_N^{\mathrm{T}} \otimes PBB^{\mathrm{T}} P) \Delta x^{pe} \end{aligned} \tag{28}$$

$$\begin{aligned}\leq &-(\tilde{x}^{pe})^{\mathrm{T}} \tilde{x}^{pe} - (4\alpha_2 - 1 - 2\beta N^2)(\tilde{x}^{pe})^{\mathrm{T}} (I_N \otimes PBB^{\mathrm{T}} P) \tilde{x}^{pe} \\ &+ (\tilde{x}^{pe})^{\mathrm{T}} \mathrm{diag}\{4\alpha_{i1} d_i^{pp} PBB^{\mathrm{T}} P\} (\mathcal{L}_p \otimes I_n) \Delta x^{pe} \\ &- 2\beta N (\tilde{x}^{pe})^{\mathrm{T}} (1_N 1_N^{\mathrm{T}} \otimes PBB^{\mathrm{T}} P) \Delta x^{pe} \end{aligned} \tag{29}$$

where the last inequality holds because $\mathcal{L}_p$ and $PBB^{\mathrm{T}}P$ are positive semidefinite.

Note that if $\alpha_{i2} \geq (2\beta N^2 + 1)/4$ for all $i = 1, \ldots, N$, and the first two terms of (29) are negative, then whether or not V decreases to zero depends on the last two terms. It is obvious that $\dot{V} \leq -(\tilde{x}^{pe})^{\mathrm{T}} \tilde{x}^{pe}$ under the conditions in (i) and (ii), which finally results in $\lim_{t\to\infty} \tilde{x}_i^{pe} = 0$. If $\sum_{i=1}^{N} \Delta x_i^{pe} \neq 0$, the last term in (29) is nonzero. Then, we have

$$\begin{aligned}\dot{V} \leq &-\Big(1 - \frac{\kappa_1}{2} \lambda_{\max}^2 \left(\mathrm{diag}\{4\alpha_{i1} d_i^{pp} PBB^{\mathrm{T}} P\} (\mathcal{L}_p \otimes I_n)\right) \\ &-\kappa_2 \lambda_{\max}^2 \left(1_N 1_N^{\mathrm{T}} \otimes PBB^{\mathrm{T}} P\right) \Big) \|\tilde{x}^{pe}\|^2 + \Big(\frac{1}{2\kappa_1} \\ &+ \frac{1}{\kappa_2}\Big) \|\Delta x^{pe}\|^2 \end{aligned}$$

where $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue of a symmetric matrix. Choose κ_1 and κ_2 small enough such that $1 - \frac{\kappa_1}{2} \lambda_{\max}^2 (\mathrm{diag}\{4\alpha_{i1} d_i^{pp} PBB^{\mathrm{T}} P\} (\mathcal{L}_p \otimes I_n)) - \kappa_2 \lambda_{\max}^2 (1_N 1_N^{\mathrm{T}} \otimes PBB^{\mathrm{T}} P) > 0$. By the ISS Lyapunov theorem (see [27, Th. 1] and [28, Lemma 3.2]), the equilibrium of (24) is globally exponentially ISS, i.e.,

$$\lim_{t\to\infty} \sup_{t' \geq t} \|\tilde{x}^{pe}(t')\| \leq \pi_1(\|\Delta x^{pe}\|^2)$$

for some class $\mathcal{K}$ function π_1. ■

Remark 5: The result (i) in Theorem 3 indicates that pursuers can achieve intercept if they put more effort than the evader. The condition $\alpha_{i1} d_i^{pp} = 0$ implies that the pursuers are not influenced by their neighbors but to intercept the evader. The condition $\sum_{i=1}^{N} \Delta x_{i1}^{pe} = 0$ implies a symmetric formation, under which the sum of repelling forces of the pursuers to the evader is zero. The two conditions, thus, contribute to interception. In result (iii), the asymmetric formation leads to asymmetric forces from the pursuers' neighbor, and the forces do not align with the attractive force from the evader, which, thus, leads to a bounded formation error. Moreover, the greater the asymmetry, the larger the formation error.

C. MPE Game for the Multiple-Pursuer Multiple-Evader Problem

In the multiple-pursuer multiple-evader case, each pursuer desires to intercept its target individually or cooperatively with its neighbors. On the contrary, the evaders will try their best to prevent themselves from being intercepted and simultaneously achieve a desired formation.

We assume that the numbers of pursuers and evaders are the same, i.e., $M = N$. If there are more pursuers, the problem can be decoupled into several multiple-pursuer one-evader cases, and the results follow Theorem 3. If there are more evaders, some of them would be able to escape not unexpectedly. In this section, each pursuer aims to capture the target, and it is trivial to form a formation; we, thus, assume that $\Delta x_{ij}^{pe} = 0$ for $i = 1, \ldots, N$ and j denotes the target evader. For simplicity, we also assume that pursuer i selects evader i as its target.

Theorem 4: Consider the multiagent system with N pursuers and N evaders with dynamics (1) and (2), respectively, and with control policies (19) and (20), respectively. Then, for any $\beta_{j1} \geq \frac{1}{4 \min\{d_j^{ee}\} \lambda_{\min}(\mathcal{L}_e)}$ for each evader j, there exists an $\alpha_2^*(\beta_{j2})$, such that if $\alpha_{i2} \geq \alpha_2^*(\beta_{j2})$, and

(i) if $\Delta x_{jl}^{ee} = 0$ for any evaders j and l, we have $\lim_{t\to\infty} \tilde{x}_i^{pe} = 0$ exponentially for any initial conditions;

(ii) if $\exists j, l$ such that $\Delta x_{jl}^{ee} \neq 0$ but $\alpha_{i1} d_i^{pp} = 0$ for all $i = 1, \ldots, N$, we have $\lim_{t\to\infty} \tilde{x}_i^{pe} = 0$ and $\lim_{t\to\infty} \tilde{x}_j^{ee} = 0$ exponentially for any initial conditions;

(iii) if $\exists j, l$ such that $\Delta x_{jl}^{ee} \neq 0$ and $\exists i$ such that $\alpha_{i1} d_i^{pp} \neq 0$, the equilibrium of the closed system is globally exponentially ISS with input Δx_1^{ee}, i.e.,

$$\lim_{t\to\infty} \sup_{t' \geq t} \|\mathrm{col}(\tilde{x}^{pe}(t'), \tilde{x}^{ee}(t'))\| \leq \pi_2(\|\Delta x_1^{ee}\|^2)$$

for some class $\mathcal{K}$ function π_2, where $\tilde{x}^{pe} = \mathrm{col}(\tilde{x}_1^{pe}, \ldots, \tilde{x}_N^{pe})$, $\tilde{x}^{ee} = \mathrm{col}(\tilde{x}_1^{ee}, \ldots, \tilde{x}_N^{ee})$, and $\Delta x_1^{ee} = \mathrm{col}(\sum_{k=1}^{N} a_{1k} \Delta x_{1k}^{ee}, \ldots, \sum_{k=1}^{N} a_{Nk} \Delta x_{Nk}^{ee})$.

Proof: Similar to Theorem 3, we analyze the conditions for capture from the viewpoint of pursuers. By (3), the derivative of the relative position between pursuer i and its target satisfies

$$\begin{aligned}\dot{\tilde{x}}_i^{pe} = A\tilde{x}_i^{pe} + \sum_{j=1}^{N} 2c_{ij} BB^{\mathrm{T}} P (&\beta_{j1} d_j^{ee} \tilde{x}_j^{ee} - \beta_{j2} \tilde{x}_j^{ep} \\ &- \alpha_{i1} d_i^{pp} \tilde{x}_i^{pp} - \alpha_{i2} \tilde{x}_i^{pe})\end{aligned}$$

$$= \left(A - \sum_{j=1}^{N} 2c_{ij}(\alpha_{i2} - \beta_{j2}) BB^{\mathrm{T}} P \right) \tilde{x}_i^{pe} + \sum_{j=1}^{N} 2c_{ij} BB^{\mathrm{T}} P (\beta_{j1} d_j^{ee} \tilde{x}_j^{ee} - \alpha_{i1} d_i^{pp} \tilde{x}_i^{pp}) \quad (30)$$

where the second equality holds because $\tilde{x}_j^{ep} = -\tilde{x}_i^{pe}$, which represent the relative position between a pair of pursuer and evader from the perspective of evader and pursuer, respectively. It is easy to note that the dynamics of $\tilde{x}_i^{pe}$ depends on $\tilde{x}_j^{ee}$ and $\tilde{x}_i^{pp}$. From (3) and (4), we have

$$\dot{\tilde{x}}_i^{pp} = A\tilde{x}_i^{pp} + \sum_{k=1}^{N} 2a_{ik} BB^{\mathrm{T}} P (\alpha_{k1} d_k^{pp} \tilde{x}_k^{pp} - \alpha_{i1} d_i^{pp} \tilde{x}_i^{pp}) + \sum_{k=1}^{N} 2a_{ik} BB^{\mathrm{T}} P (\alpha_{k2} d_k^{pe} \tilde{x}_k^{pe} - \alpha_{i2} d_i^{pe} \tilde{x}_i^{pe}) \quad (31)$$

and

$$\begin{aligned} \dot{\tilde{x}}_j^{ee} &= \sum_{l=1}^{N} b_{jl} (Ax_l^e - Ax_j^e + A\Delta x_{jl}^{ee}) \\ &\quad + \sum_{l=1}^{N} 2b_{jl} BB^{\mathrm{T}} P (\beta_{l1} d_l^{ee} \tilde{x}_l^{ee} - \beta_{l2} \tilde{x}_l^{ep} \\ &\quad - \beta_{j1} d_j^{ee} \tilde{x}_j^{ee} + \beta_{j2} \tilde{x}_j^{ep}) \\ &= A\tilde{x}_j^{ee} + \sum_{l=1}^{N} 2b_{jl} BB^{\mathrm{T}} P (\beta_{j2} \tilde{x}_j^{ep} - \beta_{l2} \tilde{x}_l^{ep}) \\ &\quad + \sum_{l=1}^{N} 2b_{jl} BB^{\mathrm{T}} P (\beta_{l1} d_l^{ee} \tilde{x}_l^{ee} - \beta_{j1} d_j^{ee} \tilde{x}_j^{ee}). \end{aligned} \quad (32)$$

The system composed of (30)–(32) is complex, and the dynamics of each one are affected by the other two ones. It is difficult to analyze them together. However, it is because they influence each other; we can express $\tilde{x}_i^{pp}$ by the other two variables. We, thus, have

$$\begin{aligned} \tilde{x}_i^{pp} &= \sum_{k=1}^{N} a_{ik} (x_k^p - x_k^e + x_k^e - x_i^p + x_i^e - x_i^e) \\ &= \sum_{k=1}^{N} a_{ik} (\tilde{x}_i^{pe} - \tilde{x}_k^{pe}) + \sum_{k=1}^{N} a_{ik} (x_k^e - x_i^e + \Delta x_{ik}^{ee}) \\ &\quad - \sum_{k=1}^{N} a_{ik} \Delta x_{ik}^{ee}. \end{aligned}$$

Let $\tilde{x}^{pp} = \mathrm{col}(\tilde{x}_1^{pp}, \ldots, \tilde{x}_N^{pp})$. For analysis convenience, we assume that all the players have the same values on α_{i1}, α_{i2}, β_{j1}, β_{j2}, and in-degree, and they are redenoted as α_1, α_2, β_1, β_2, d^{pp}, d^{pe}, d^{ee}, and d^{ep}, respectively. Then, the compact form of $\tilde{x}_i^{pp}$ satisfies

$$\tilde{x}^{pp} = (\mathcal{L}_p \otimes I_n)\tilde{x}^{pe} + (d^{ee})^{-1} (\mathcal{L}_p \otimes I_n) \tilde{x}^{ee} - \Delta x_1^{ee}. \quad (33)$$

Taking (33) into the compact of (30) gives

$$\begin{aligned} \dot{\tilde{x}}^{pe} &= [I_N \otimes (A - 2(\alpha_2 - \beta_2) BB^{\mathrm{T}} P)] \tilde{x}^{pe} \\ &\quad + (I_N \otimes 2\beta_1\, d^{ee} BB^{\mathrm{T}} P) \tilde{x}^{ee} - (I_N \otimes 2\alpha_1\, d^{pp} BB^{\mathrm{T}} P) \tilde{x}^{pp} \\ &= [I_N \otimes (A - 2(\alpha_2 - \beta_2) BB^{\mathrm{T}} P)] \tilde{x}^{pe} \\ &\quad + (I_N \otimes 2\beta_1\, d^{ee} BB^{\mathrm{T}} P) \tilde{x}^{ee} - (\mathcal{L}_p \otimes 2\alpha_1\, d^{pp} BB^{\mathrm{T}} P) \tilde{x}^{pe} \\ &\quad - (d^{ee})^{-1} (\mathcal{L}_p \otimes 2\alpha_1\, d^{pp} BB^{\mathrm{T}} P) \tilde{x}^{ee} + \Xi^{ee} \end{aligned} \quad (34)$$

where

$$\Xi^{ee} = (I_N \otimes 2\alpha_1\, d^{pp} BB^{\mathrm{T}} P) \Delta x_1^{ee}. \quad (35)$$

Now, we define the following Lyapunov function candidate for the system (34):

$$V_{pe} = (\tilde{x}^{pe})^{\mathrm{T}} (I_N \otimes P) \tilde{x}^{pe}$$

whose derivative along the trajectory of (34) gives

$$\begin{aligned} \dot{V}_{pe} &= (\tilde{x}^{pe})^{\mathrm{T}} [I_N \otimes (A - 2(\alpha_2 - \beta_2) BB^{\mathrm{T}} P)]^{\mathrm{T}} (I_N \otimes P) \tilde{x}^{pe} \\ &\quad + (\tilde{x}^{pe})^{\mathrm{T}} (I_N \otimes P) [I_N \otimes (A - 2(\alpha_2 - \beta_2) BB^{\mathrm{T}} P)] \tilde{x}^{pe} \\ &\quad - (\tilde{x}^{pe})^{\mathrm{T}} (\mathcal{L}_p \otimes 2\alpha_1\, d^{pp} BB^{\mathrm{T}} P)^{\mathrm{T}} (I_N \otimes P) \tilde{x}^{pe} \\ &\quad - (\tilde{x}^{pe})^{\mathrm{T}} (I_N \otimes P) (\mathcal{L}_p \otimes 2\alpha_1\, d^{pp} BB^{\mathrm{T}} P) \tilde{x}^{pe} \\ &\quad + 4\beta_1\, d^{ee} (\tilde{x}^{pe})^{\mathrm{T}} (I_N \otimes PBB^{\mathrm{T}} P) \tilde{x}^{ee} \\ &\quad - 4\alpha_1\, d^{pp} (d^{ee})^{-1} (\tilde{x}^{pe})^{\mathrm{T}} (\mathcal{L}_p \otimes PBB^{\mathrm{T}} P) \tilde{x}^{ee} \\ &\quad + 2(\tilde{x}^{pe})^{\mathrm{T}} (I_N \otimes P) \Xi^{ee} \\ &\leq -(\tilde{x}^{pe})^{\mathrm{T}} \tilde{x}^{pe} \\ &\quad - (4(\alpha_2 - \beta_2) - 1) (\tilde{x}^{pe})^{\mathrm{T}} (I_N \otimes PBB^{\mathrm{T}} P) \tilde{x}^{pe} \\ &\quad + 4\beta_1\, d^{ee} (\tilde{x}^{pe})^{\mathrm{T}} (I_N \otimes PBB^{\mathrm{T}} P) \tilde{x}^{ee} \\ &\quad - 4\alpha_1\, d^{pp} (d^{ee})^{-1} (\tilde{x}^{pe})^{\mathrm{T}} (\mathcal{L}_p \otimes PBB^{\mathrm{T}} P) \tilde{x}^{ee} \\ &\quad + 4\alpha_1\, d^{pp} (\tilde{x}^{pe})^{\mathrm{T}} (I_N \otimes PBB^{\mathrm{T}} P) \Delta x_1^{ee}. \end{aligned} \quad (36)$$

We note that (36) depends on $\tilde{x}^{ee}$.

By (32), the compact form is

$$\dot{\tilde{x}}^{ee} = [I_N \otimes A - 2\beta_1\, d^{ee} (\mathcal{L}_e \otimes BB^{\mathrm{T}} P)] \tilde{x}^{ee} - 2\beta_2 (\mathcal{L}_e \otimes BB^{\mathrm{T}} P) \tilde{x}^{pe}. \quad (37)$$

Let

$$V_{ee} = (\tilde{x}^{ee})^{\mathrm{T}} (\mathcal{L}_e \otimes P) \tilde{x}^{ee}. \quad (38)$$

Since the graph is undirected, $\mathcal{L}_e$ is symmetric and $\mathcal{L}_e 1_N = 0$. Then, there exists an orthogonal matrix $C = \left[\frac{1_N}{\sqrt{N}}, D\right]$ with $D \in \mathbb{R}^{N\times(N-1)}$, such that $C^{\mathrm{T}} \mathcal{L}_e C = \Lambda = \mathrm{diag}\{0, \lambda_2, \ldots, \lambda_N\}$. Let $(C^{\mathrm{T}} \otimes I_n)\tilde{x}^{ee} = \zeta = \mathrm{col}\{\zeta_1, \ldots, \zeta_N\}$; we then have

$$\zeta_1 = \left(\frac{1_N^{\mathrm{T}}}{\sqrt{N}} \otimes I_n \right) \tilde{x}^{ee} = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} \tilde{x}_j^{ee} = 0.$$

The last equality holds because the graph $\mathcal{L}_e$ is undirected. It implies that ζ_i, $i = 2, \ldots, N$, will not be all zero if $\tilde{x}^{ee} \neq 0$.

Then, V_{ee} can be written as

$$\begin{aligned}V_{ee} &= (\tilde{x}^{ee})^{\mathrm{T}}(C\otimes I_n)(\Lambda\otimes P)(C^{\mathrm{T}}\otimes I_n)\tilde{x}^{ee}\\ &= \zeta^{\mathrm{T}}(\Lambda\otimes P)\zeta = \sum_{i=2}^{N}\lambda_i\zeta_i^{\mathrm{T}}P\zeta_i.\end{aligned}$$

It is obvious that V_{ee} is positive definite. Therefore, V_{ee} can be chosen as a Lyapunov function candidate.

The derivative of V_{ee} along the trajectory of (37) is

$$\begin{aligned}\dot{V}_{ee}=&(\tilde{x}^{ee})^{\mathrm{T}}[I_N\otimes A-2\beta_1\,d^{ee}(\mathcal{L}_e\otimes BB^{\mathrm{T}}P)]^{\mathrm{T}}(\mathcal{L}_e\otimes P)\tilde{x}^{ee}\\ &+(\tilde{x}^{ee})^{\mathrm{T}}(\mathcal{L}_e\otimes P)[I_N\otimes A-2\beta_1\,d^{ee}(\mathcal{L}_e\otimes BB^{\mathrm{T}}P)]\tilde{x}^{ee}\\ &-4\beta_2(\tilde{x}^{ee})^{\mathrm{T}}(\mathcal{L}_e\otimes P)(\mathcal{L}_e\otimes BB^{\mathrm{T}}P)\tilde{x}^{pe}\\ =&-(\tilde{x}^{ee})^{\mathrm{T}}(\mathcal{L}_e\otimes I_n)\tilde{x}^{ee}-4\beta_2(\tilde{x}^{ee})^{\mathrm{T}}(\mathcal{L}_e^2\otimes PBB^{\mathrm{T}}P)\tilde{x}^{pe}\\ &-(\tilde{x}^{ee})^{\mathrm{T}}\left((4\beta_1\,d^{ee}\mathcal{L}_e^2-\mathcal{L}_e)\otimes PBB^{\mathrm{T}}P\right)\tilde{x}^{ee}\end{aligned}\tag{39}$$

since

$$\begin{aligned}\tilde{x}^{ee} &= (C\otimes I_n)\zeta = \left(\left[\frac{1_N}{\sqrt{N}}, D\right]\otimes I_n\right)\begin{bmatrix}0 & \zeta_2^{\mathrm{T}} & \cdots & \zeta_N^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}\\ &= (D\otimes I_n)\zeta_{2N}\end{aligned}\tag{40}$$

where $\zeta_{2N}=\mathrm{col}\{\zeta_2,\ldots,\zeta_N\}$. Since $C^{\mathrm{T}}\mathcal{L}_eC=\Lambda$ and C is an orthogonal matrix, we have $\mathcal{L}_e=C\Lambda C^{\mathrm{T}}$ and $\mathcal{L}_e^2=C\Lambda^2C^{\mathrm{T}}$. Then, taking (40) into (39) gives

$$\begin{aligned}\dot{V}_{ee} =& -\sum_{i=2}^{N}\lambda_i\zeta_i^{\mathrm{T}}\zeta_i-\zeta^{\mathrm{T}}\left((4\beta_1\,d^{ee}\Lambda^2-\Lambda)\otimes PBB^{\mathrm{T}}P\right)\zeta\\ &-4\beta_2\zeta^{\mathrm{T}}(\Lambda^2C^{\mathrm{T}}\otimes PBB^{\mathrm{T}}P)\tilde{x}^{pe}.\end{aligned}$$

Since $\Lambda=\mathrm{diag}\{0,\lambda_2,\ldots,\lambda_N\}$ with $\lambda_i>0$ for $i=2,\ldots,N$, choose

$$\beta_1\geq\frac{1}{4\,d^{ee}\lambda_{\min}\{\lambda_i\}}=\frac{1}{4\,d^{ee}\lambda_{\min}(\mathcal{L}_e)}.\tag{41}$$

Then, the matrix $(4\beta_1\,d^{ee}\Lambda^2-\Lambda)\otimes PBB^{\mathrm{T}}P$ is positive semidefinite. We, thus, have

$$\dot{V}_{ee}\leq-\sum_{i=2}^{N}\lambda_i\zeta_i^{\mathrm{T}}\zeta_i-4\beta_2\zeta^{\mathrm{T}}(\Lambda^2C^{\mathrm{T}}\otimes PBB^{\mathrm{T}}P)\tilde{x}^{pe}.\tag{42}$$

Let $V=V_{pe}+V_{ee}$. By combining (36) and (42), we obtain

$$\begin{aligned}\dot{V}\leq& -(\tilde{x}^{pe})^{\mathrm{T}}\tilde{x}^{pe}-\sum_{i=2}^{N}\lambda_i\zeta_i^{\mathrm{T}}\zeta_i\\ &-(4(\alpha_2-\beta_2)-1)(\tilde{x}^{pe})^{\mathrm{T}}(I_N\otimes PBB^{\mathrm{T}}P)\tilde{x}^{pe}\\ &+2\beta_1\,d^{ee}\left[\frac{1}{\epsilon_1}(\tilde{x}^{pe})^{\mathrm{T}}(I_N\otimes PBB^{\mathrm{T}}P)\tilde{x}^{pe}\right.\\ &\left.+\epsilon_1(\tilde{x}^{ee})^{\mathrm{T}}(I_N\otimes PBB^{\mathrm{T}}P)\tilde{x}^{ee}\right]\\ &+2\alpha_1\,d^{pp}/d^{ee}\left[\frac{1}{\epsilon_2}(\tilde{x}^{pe})^{\mathrm{T}}(\mathcal{L}_p^2\otimes PBB^{\mathrm{T}}P)\tilde{x}^{pe}\right.\\ &\left.+\epsilon_2(\tilde{x}^{ee})^{\mathrm{T}}(I_N\otimes PBB^{\mathrm{T}}P)\tilde{x}^{ee}\right]\\ &+2\beta_2\left[\frac{1}{\epsilon_3}(\tilde{x}^{pe})^{\mathrm{T}}(\Lambda^4\otimes PBB^{\mathrm{T}}P)\tilde{x}^{pe}\right.\\ &\left.+\epsilon_3\zeta^{\mathrm{T}}(C^{\mathrm{T}}C\otimes PBB^{\mathrm{T}}P)\right]\zeta\\ &+2\alpha_1\,d^{pp}\left[\frac{1}{\epsilon_4}(\tilde{x}^{pe})^{\mathrm{T}}(I_N\otimes PBB^{\mathrm{T}}P)\tilde{x}^{pe}\right.\\ &\left.+\epsilon_4\Delta x_1^{ee}(I_N\otimes PBB^{\mathrm{T}}P)\Delta x_1^{ee}\right]\\ \leq& -(\tilde{x}^{pe})^{\mathrm{T}}\tilde{x}^{pe}-\gamma_1(\tilde{x}^{pe})^{\mathrm{T}}(I_N\otimes PBB^{\mathrm{T}}P)\tilde{x}^{pe}\\ &-\gamma_2(\zeta_{2N})^{\mathrm{T}}\zeta_{2N}+\gamma_3\Delta x_1^{ee}\end{aligned}\tag{43}$$

where

$$\begin{aligned}\gamma_1 =& 4(\alpha_2-\beta_2)-1-2\beta_1\,d^{ee}/\epsilon_1-2\alpha_1\,d^{pp}\lambda_{\max}(\mathcal{L}_p^2)/(d^{ee}\epsilon_2)\\ &-2\beta_2\lambda_{\max}(\mathcal{L}_e^4)/\epsilon_3-2\alpha_1\,d^{pp}/\epsilon_4\\ \gamma_2 =& (\lambda_2-(\epsilon_1+\epsilon_2)\|D^{\mathrm{T}}D\|\|PBB^{\mathrm{T}}P\|-\epsilon_3\|PBB^{\mathrm{T}}P\|)\\ \gamma_3 =& \epsilon_4\lambda_{\max}(PBB^{\mathrm{T}}P)\end{aligned}$$

and λ_2 and λ_N are the minimum and maximum nonzero eigenvalues of $\mathcal{L}_e$, respectively. Choose ϵ_1–ϵ_5 small enough such that $\gamma_2\geq0$; then, there exists a $\alpha_2^*(\beta_2)$, when $\alpha_2\geq\alpha_2^*(\beta_2)$, $\gamma_1\geq0$. In such a case, we have

$$\dot{V}\leq-(\tilde{x}^{pe})^{\mathrm{T}}\tilde{x}^{pe}-\gamma_2(\zeta_{2N})^{\mathrm{T}}\zeta_{2N}+\gamma_3\Delta x_1^{ee}.\tag{44}$$

Therefore, when $\Delta x_{jl}^{ee}=0$ for all evaders j and l, which implies that there is no desired formation among evaders, we have $\dot{V}\leq-(\tilde{x}^{pe})^{\mathrm{T}}\tilde{x}^{pe}-\gamma_2(\zeta_{2N})^{\mathrm{T}}\zeta_{2N}$, which will lead to $\lim_{t\to\infty}\tilde{x}_i^{pe}=0$ exponentially. However, when $\Delta x_{jl}^{ee}\neq0$ and $\alpha_1\,d^{pp}\neq0$, the system composed of (34) and (37) is exponentially ISS with input Δx_{jl}^{ee}.

Note that when $\alpha_1\,d^{pp}=0$, the last term in (36) equals zero. Thus, (43) holds with $\epsilon_4=0$. Then, there exists $\alpha_2^*(\beta_2)$, when $\alpha_2\geq\alpha_2^*(\beta_2)$, $\dot{V}\leq-(\tilde{x}^{pe})^{\mathrm{T}}\tilde{x}^{pe}-\gamma_2(\zeta_{2N})^{\mathrm{T}}\zeta_{2N}$, which leads to the fact that $\lim_{t\to\infty}\tilde{x}_i^{pe}=0$ and $\lim_{t\to\infty}\tilde{x}_j^{ee}=0$ exponentially. □

Remark 6: Note that inequality (41) is a sufficient condition for interception. β_1 denotes the attractive forces from its teammates for the evader. Equation (41) means that the attractive force reaches a certain level; otherwise, the evaders may separate from each other. When pursuers pursue the evaders, the separating would cause large attractive forces from other pursuers for each pursuer. It will cause a failure of capture or the closed-loop system would diverge. Note that this fact provides a strategy for evaders to prevent themselves from being interception. Under condition (41), if $\Delta x_{jl}^{ee}=0$ or $\alpha_{i1}d_i^{pp}=0$, the interception is achieved. Otherwise, the attractive force from its teammates for each pursuer under the formation Δx_{jl}^{ee} is actually a resistance for capture, which conversely leads to the formation of evaders with a bounded error.

Remark 7: Different from [19], which considers the PE game for multiple pursuers and multiple evaders, novel analyses are given for three scenarios in this article: one-pursuer one-evader, multiple-pursuer one-evader, and multiple-pursuer

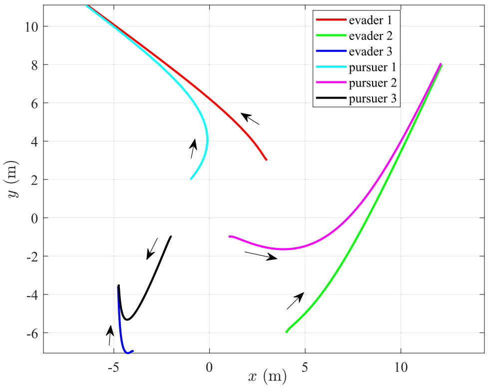

Fig. 1. PE game for the one-pursuer one-evader problem.

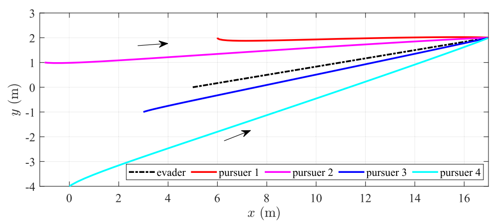

Fig. 2. Capture occurs under the conditions in (i) of Theorem 3.

multiple-evader, and the formation control is also considered. Conditions for capture and formation are different in different scenarios. Compared with the asymptotic convergence of [19], our control strategies contribute to exponential behaviors of agents.

V. Simulation and Experimental Results

In this section, both simulation and experimental results are presented to verify our control strategies. Players are double-integrator systems described by (1) and (2) with $A = \begin{bmatrix} 0_{2\times2} & I_2 \\ 0_{2\times2} & 0_{2\times2} \end{bmatrix}$ and $B = \begin{bmatrix} 0_{2\times2} \\ I_2 \end{bmatrix}$.

Fig. 1 shows capture results that verify the control law (21) designed for the one-pursuer one-evader PE game. In the control law (21), for $i = 1, 2, 3$, we set $\alpha_{i2} = 3$ and $\beta_{i2} = 1$, which obviously satisfies the condition $\alpha_{i2} \geq \frac{1}{2} + \beta_{i2}$.

For the case that multiple pursuers try to capture one evader, the pursuers and evader use control strategies (19) and (23), respectively. In this example, we assume that there are four pursuers; therefore, $N = 4$. To satisfy the sufficient condition $\alpha_{i2} \geq (2\beta N^2 + 1)/4$ for target capture in Theorem 2, we choose $\alpha_{i2} = 9$ and $\beta = 1$. We first consider the case that $\Delta x_{i1}^{pe} = 0$. The value of α_{i1} is chosen randomly. Fig. 2 shows that capture occurs, which verifies the result (i) in Theorem 3. Next, we assume that the pursuers try to achieve surrounding control of the evader, rather than capture it. Let the desired state displacement between the pursuers and the evader be $\begin{bmatrix} \Delta x_{11}^{pe} & \Delta x_{21}^{pe} & \Delta x_{31}^{pe} & \Delta x_{41}^{pe} \end{bmatrix} = \begin{bmatrix} 0 & -2 & 2 & 0 \\ 2 & 0 & 0 & -2 \\ & 0_{2\times4} & & \end{bmatrix}$. Let

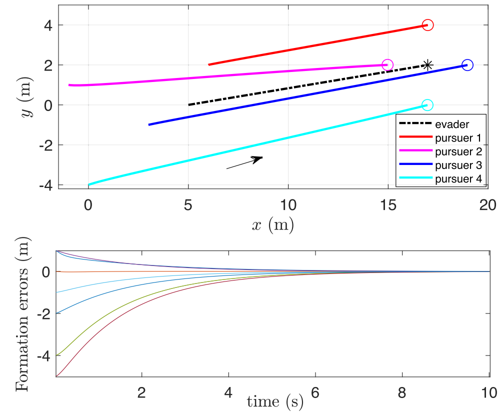

Fig. 3. Trajectories and formation errors for the result (ii) of Theorem 3.

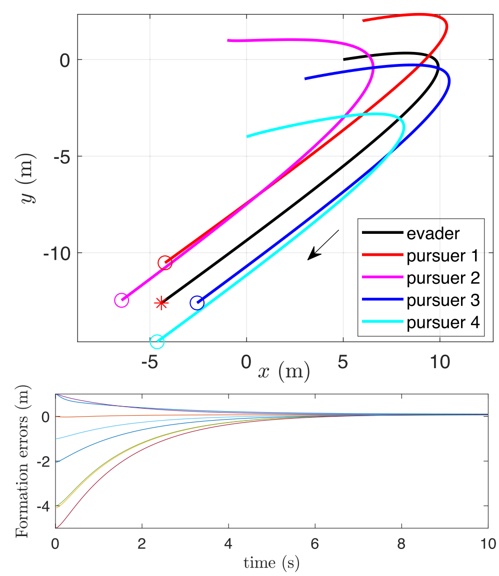

Fig. 4. Trajectories and formation errors for the result (iii) of Theorem 3.

$\alpha_{i1} = 0$ for $i = 1, 2, 3, 4$. It is obvious that the above settings satisfy the condition in (ii) of Theorem 3. The trajectories of players and the formation errors are presented in Fig. 3, which shows that the formation error finally converges to zero. Now, we modify the desired state displacement between the pursuers and the evader as $\begin{bmatrix} \Delta x_{11}^{pe} & \Delta x_{21}^{pe} & \Delta x_{31}^{pe} & \Delta x_{41}^{pe} \end{bmatrix} = \begin{bmatrix} 0 & -1.9 & 2 & 0 \\ 2.01 & 0 & 0 & -1.94 \\ & 0_{2\times4} & & \end{bmatrix}$, which does not satisfy $\sum_{i=1}^{4} \Delta x_{i1}^{pe} \neq 0$. The simulation results in Fig. 4 show the bounded formation error.

Finally, we consider the PE game for multiple pursuers and multiple evaders, who cooperate with their teammates to achieve their objectives. Suppose that there are four pursuers and four evaders. The communication topology is shown in Fig. 5. We, thus, have $d^{ee} = 2$ and $\lambda_{\min}(\mathcal{L}_e) = 1$. A value of $\beta_{j1} = 1, j = 1, 2, 3, 4$, is used to satisfy the condition $\beta_{j1} \geq \frac{1}{4\min\{d_j^{ee}\}\lambda_{\min}(\mathcal{L}_e)}$

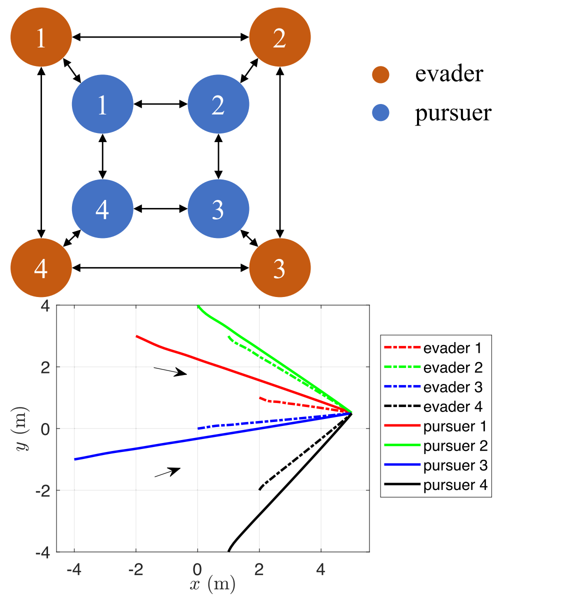

Fig. 5. (Left) Communication topology for the PE game with multiple pursuers and multiple evaders. (Right) Capture occurs under the conditions in (i) of Theorem 4.

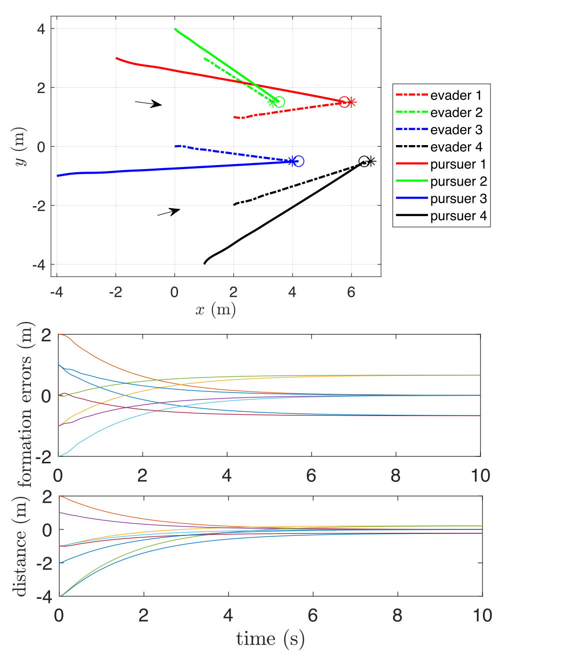

Fig. 6. Trajectories of players, formation errors of evaders, and x and y distances between each pair of pursuer and evader under the conditions (iii) of Theorem 4.

of Theorem 4. The values of other parameters are set as $\beta_{j2} = 6$, $\alpha_{i1} = 1$, and $\alpha_{i2} = 10$ for $i = 1, 2, 3, 4$. First, let $\Delta x_{jl}^{ee} = 0$. Fig. 5 shows that capture occurs. Furthermore, let the desired formation among evaders be $\begin{bmatrix} \Delta x_{12}^{ee} & \Delta x_{23}^{ee} & \Delta x_{34}^{ee} & \Delta x_{41}^{ee} \end{bmatrix} = \begin{bmatrix} 2 & 0 & -2 & 0 \\ 0 & 2 & 0 & -2 \\ & 0_{2\times 4} & & \end{bmatrix}$. Fig. 6 displays the simulation result, where

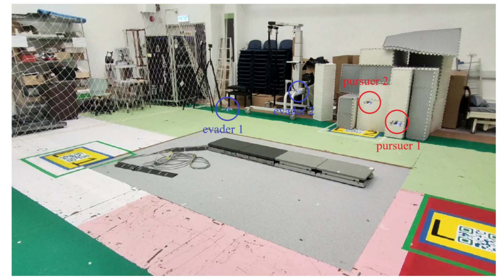

Fig. 7. Experimental environment of our flight test.

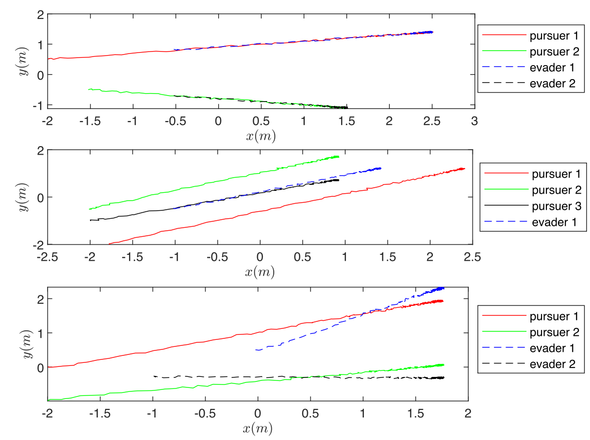

Fig. 8. Real trajectories of quadrotors in our three flight tests.

the distances between each pair of pursuer and evader and formation errors of evaders are bounded. If we further set $\alpha_{i1} = 0$, it will cause capture and zero formation errors.

The *experiment* is done in an indoor environment, as shown in Fig. 7. We use a kind of nano quadrotor helicopter Crazyflie 2.1 as the flying platform, and the localization system we use is VICON. The Crazyflie 2.1 is an open-source platform that weighs only 27 g and is equipped with low-latency/long-range radio. We can control them by combing them with the Crazyradio PA and display data in our computer. A laptop with Intel Core i7CPU is used to run our algorithm.

A group of four Crazyflie is used in our flight test. The actual flight video can be found at https://www.bilibili.com/video/BV1rd4y1x7X3/ and https://youtu.be/uM_U1lQLLLU, and the corresponding real trajectories of quadrotors are shown in Fig. 8.

REFERENCES

[1] P. Wang, X. Zhang, and J. Zhu, “Integrated missile guidance and control: A novel explicit reference governor using a disturbance observer,” *IEEE Trans. Ind. Electron.*, vol. 66, no. 7, pp. 5487–5496, Jul. 2019.

[2] L. Xi, X. Wang, L. Jiao, S. Lai, Z. Peng, and B. M. Chen, “GTO-MPC-based target chasing using a quadrotor in cluttered environments,” *IEEE Trans. Ind. Electron.*, vol. 69, no. 6, pp. 6026–6035, Jun. 2022.

[3] P. Zhou and B. M. Chen, “Formation-containment control of Euler–Lagrange systems of leaders with bounded unknown inputs,” *IEEE Trans. Cybern.*, vol. 52, no. 7, pp. 6342–6353, Jul. 2022.

[4] R. Isaacs, *Differential Games: A Mathematical Theory With Applications to Warfare and Pursuit, Control and Optimization*. New York, NY, USA: Wiley, 1965.

[5] F. L. Lewis, D. Vrabie, and V. L. Syrmos, *Optimal Control*. Hoboken, NJ, USA: Wiley, 2012.
[6] P. Hagedorn and J. Breakwell, “A differential game with two pursuers and one evader,” *J. Optim. Theory Appl.*, vol. 18, no. 1, pp. 15–29, 1976.
[7] M. Foley and W. Schmitendorf, “A class of differential games with two pursuers versus one evader,” *IEEE Trans. Autom. Control*, vol. AC-19, no. 3, pp. 239–243, Jun. 1974.
[8] W. Lin, Z. Qu, and M. A. Simaan, “Nash strategies for pursuit-evasion differential games involving limited observations,” *IEEE Trans. Aerosp. Electron. Syst.*, vol. 51, no. 2, pp. 1347–1356, Apr. 2015.
[9] E. Bakolas and P. Tsiotras, “Relay pursuit of a maneuvering target using dynamic Voronoi diagrams,” *Automatica*, vol. 48, no. 9, pp. 2213–2220, 2012.
[10] J. S. Jang and C. Tomlin, “Control strategies in multi-player pursuit and evasion game,” in *Proc. AIAA Guid., Navigation, Control Conf. Exhib.*, 2005, Art. no. 6239.
[11] K. Mizukami and K. Eguchi, “A geometrical approach to problems of pursuit-evasion games,” *J. Franklin Inst.*, vol. 303, no. 4, pp. 371–384, 1977.
[12] A. Von Moll, D. W. Casbeer, E. Garcia, and D. Milutinović, “Pursuit-evasion of an evader by multiple pursuers,” in *Proc. Int. Conf. Unmanned Aircr. Syst.*, 2018, pp. 133–142.
[13] V. R. Makkapati, W. Sun, and P. Tsiotras, “Optimal evading strategies for two-pursuer/one-evader problems,” *J. Guid., Control, Dyn.*, vol. 41, no. 4, pp. 851–862, 2018.
[14] Y. Xu, H. Yang, B. Jiang, and M. M. Polycarpou, “Multi-player pursuit-evasion differential games with malicious pursuers,” *IEEE Trans. Autom. Control*, vol. 67, no. 9, pp. 4602–4615, Sep. 2022.
[15] F. Yan, J. Jiang, K. Di, Y. Jiang, and Z. Hao, “Multiagent pursuit-evasion problem with the pursuers moving at uncertain speeds,” *J. Intell. Robot. Syst.*, vol. 95, pp. 119–135, 2019.
[16] M. M. Zavlanos and G. J. Pappas, “Distributed hybrid control for multiple-pursuer multiple-evader games,” in *Proc. Int. Workshop Hybrid Syst.: Comput. Control*, 2007, pp. 787–789.
[17] D. M. Stipanovic, A. Melikyan, and N. Hovakimyan, “Guaranteed strategies for nonlinear multi-player pursuit-evasion games,” *Int. Game Theory Rev.*, vol. 12, no. 1, pp. 1–17, 2010.
[18] J. Ge, L. Tang, J. Reimann, and G. Vachtsevanos, “Suboptimal approaches to multiplayer pursuit-evasion differential games,” in *Proc. AIAA Guid., Navigation, Control Conf. Exhib.*, 2006, Art. no. 6786.
[19] V. G. Lopez, F. L. Lewis, Y. Wan, E. N. Sanchez, and L. Fan, “Solutions for multiagent pursuit-evasion games on communication graphs: Finite-time capture and asymptotic behaviors,” *IEEE Trans. Autom. Control*, vol. 65, no. 5, pp. 1911–1923, May 2020.
[20] L. Cheng and Y. Yuan, “Multi-player obstacle avoidance pursuit-evasion games with adaptive parameter estimation,” *IEEE Trans. Ind. Electron.*, vol. 70, no. 5, pp. 5171–5181, May 2023.
[21] Y.-Y. Qian, M. Liu, Y. Wan, F. L. Lewis, and A. Davoudi, “Distributed adaptive Nash equilibrium solution for differential graphical games,” *IEEE Trans. Cybern.*, vol. 53, no. 4, pp. 2275–2287, Apr. 2021.
[22] F. Chen, W. Ren, and Y. Cao, “Surrounding control in cooperative agent networks,” *Syst. Control Lett.*, vol. 59, no. 11, pp. 704–712, 2010.
[23] B.-B. Hu, H.-T. Zhang, B. Liu, H. Meng, and G. Chen, “Distributed surrounding control of multiple unmanned surface vessels with varying interconnection topologies,” *IEEE Trans. Control Syst. Technol.*, vol. 30, no. 1, pp. 400–407, Jan. 2022.
[24] P. Zhou and B. M. Chen, “Semi-global leader-following consensus-based formation flight of unmanned aerial vehicles,” *Chin. J. Aeronaut.*, vol. 35, no. 1, pp. 31–43, 2022.
[25] L. Kou, Y. Huang, Z. Chen, S. He, and J. Xiang, “Cooperative fencing control of multiple second-order vehicles for a moving target with and without velocity measurements,” *Int. J. Robust Nonlinear Control*, vol. 31, no. 10, pp. 4602–4615, 2021.
[26] A. W. Starr and Y.-C. Ho, “Nonzero-sum differential games,” *J. Optim. Theory Appl.*, vol. 3, pp. 184–206, 1969.
[27] E. D. Sontag and Y. Wang, “On characterizations of the input-to-state stability property,” *Syst. Control Lett.*, vol. 24, no. 5, pp. 351–359, 1995.
[28] C. Guiver and H. Logemann, “The exponential input-to-state stability property: Characterisations and feedback connections,” *Math. Control, Signals, Syst.*, vol. 35, pp. 375–398, 2023.

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