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# Cooperative Target Fencing of Multiple Vehicles for a General Target with Connectivity Preservation and Collision Avoidance*

**PAN Zini · CHEN Ben M.**

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**Abstract** In this paper, the authors study the cooperative target-fencing problem for $n$-dimensional systems and a target with a general trajectory. Without using the velocity of the vehicles, a position feedback control law is proposed to fence the general target into the convex hull formed by the vehicles. Specifically, the dynamics of each vehicle is described by a double-integrator system. Two potential functions are designed to guarantee connectivity preservation of the communication network and collision avoidance among the vehicles. The proposed approach can deal with a target whose trajectory is any twice continuously differentiable function of time. The effectiveness of the result is verified by a numerical example.

**Keywords** Multi-agent systems, networked systems, target fencing.

## 1 Introduction

In recent years, a variety of cooperative control problems for multiple vehicles have been investigated due to their potential in practical applications, such as rescue[1, 2], transportation[3], surveillance, and reconnaissance[4, 5]. To make a group of vehicles move with a circular motion specified by a given radius, the circular formation problem was investigated in [6]. The circumnavigation problem[7, 8], which is also referred to as the target-capturing problem[9] and the target circular problem[10], was studied to drive multiple vehicles to form a circular motion centering at a target. Recently, the cooperative target-fencing problem[11] was studied to make a group of vehicles asymptotically fence a target into the convex hull formed by their positions. Unlike the circular formation and the circumnavigation problems, which require predefined radii

PAN Zini (Corresponding author) · CHEN Ben M.
*Department of Mechanical and Automation Engineering, The Chinese University of Hong Kong, Hong Kong, China.* Email: znpan@mae.cuhk.edu.hk; bmchen@cuhk.edu.hk.
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in their control designs, the cooperative target-fencing problem does not require such a predefined parameter. Given the variety and complexity of both the dynamics of the robots and their operating environments, designing appropriate radii can itself be challenging. In this sense, the cooperative target-fencing problem is more practical than the circular formation problem and the circumnavigation problem. Furthermore, in the circumnavigation problem, the target may not be in the convex hull formed by the vehicles, even in the desired configuration. These limitations give rise to the investigation of the cooperative target-fencing problem.

In the past few years, there have been some studies on the cooperative target-fencing problem. In [11], the problem was investigated for a static target and a group of single integrators. Later, the cooperative target-fencing problem for a target moving with an unknown velocity and multiple single integrators was studied in [12]. In [13], a control law was proposed to fence a target with a constant velocity by multiple double integrators without using the velocity measurements of the vehicles. A label-free control law was proposed in [14] for a group of double-integrator systems to fence a target with a constant acceleration. To handle a more general target, [15] further studied the cooperative target-fencing problem for a target whose acceleration can be any uniformly continuous bounded function with respect to time. However, it was assumed in [15] that all the vehicles can access the position of the target, and the vehicles have exponentially convergent estimates of both the velocity and the acceleration of the target. It is noted that the existing results on the cooperative target-fencing problem in the aforementioned literature only considered two-dimensional systems and cannot handle external disturbances. Since most practical robotic systems are high-dimensional and are subject to external disturbances, it is desirable to extend the solvability of the cooperative target-fencing problem to accommodate $n$-dimensional systems subject to external disturbances.

In this paper, we will further investigate the cooperative target-fencing problem for $n$-dimensional systems subject to a class of external disturbances. Our approach can handle a general target whose trajectory can be any twice continuously differentiable function of time. Furthermore, it guarantees connectivity preservation and collision avoidance simultaneously. The main contributions of this paper are summarized as follows:

1) We establish a control law for a group of double integrators to asymptotically fence a general target whose trajectory can be any twice continuously differentiable function.

2) Our result does not rely on the velocity measurements of the vehicles.

3) Our approach guarantees the connectivity preservation of the state-dependent communication network and the inter-vehicle collision avoidance, simultaneously.

4) Our control law applies to $n$-dimensional systems and can fully reject a large class of external disturbances.

The rest of the paper is organized as follows: In Section 2, we give some preliminaries and describe the cooperative target-fencing problem. In Section 3, we present our main result. An example is provided in Section 4 to illustrate our approach. The paper is wrapped up with some remarks in Section 5. A preliminary version of this paper was reported in [16].

## 2 Problem Formulation

Let us consider that a general target whose position trajectory $q_0(t) : [0, \infty) \to \mathbb{R}^n$ is any twice continuously differentiable function of time $t$.

The dynamics of the $N$ vehicles is given by

$$\ddot{q}_i = u_i + d_i, \quad i = 1, \cdots, N, \tag{1}$$

where $q_i, u_i, d_i \in \mathbb{R}^n$ denote the position, input, and external disturbance for the $i$-th vehicle, respectively.

It is assumed that the external disturbance $d_i$, $i = 1, \cdots, N$, is generated by the linear exosystem as follows:

$$\dot{\omega}_i = S_i \omega_i, \quad d_i = D_i \omega_i, \quad i = 1, \cdots, N, \tag{2}$$

where $\omega_i \in \mathbb{R}^{s_i}$, $S_i \in \mathbb{R}^{s_i \times s_i}$, and $D_i \in \mathbb{R}^{n \times s_i}$ are constant matrices. Without loss of generality, we assume that $(D_i, S_i)$ is detectable[17].

The system composed of (1) and the target can be viewed as a multi-agent system of $N + 1$ agents with the target as the virtual leader and the $N$ subsystems of (1) as the $N$ followers. Given the multi-agent system composed of the target and (1), we define a digraph $\overline{\mathcal{G}}(t) = (\overline{\mathcal{V}}, \overline{\mathcal{E}}(t))$, where $\overline{\mathcal{V}} = \{0, 1, \cdots, N\}$ is the node set with node 0 associated with the target and node $i$, $i = 1, \cdots, N$, associated with the $i$-th vehicle of (1), and $\overline{\mathcal{E}}(t) \subseteq \overline{\mathcal{V}} \times \overline{\mathcal{V}}$ is the edge set. Let $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$ be a subgraph of $\overline{\mathcal{G}}(t)$, where $\mathcal{V} = \{1, \cdots, N\}$ and $\mathcal{E}(t) \subseteq \mathcal{V} \times \mathcal{V}$. For $i = 1, \cdots, N$, let $\overline{\mathcal{N}}_i(t)$ denote the neighbor set of the $i$-th vehicle at time $t$. The neighbor set of node $i$ with respect to $\mathcal{V}$ is defined as $\mathcal{N}_i(t) = \overline{\mathcal{N}}_i(t) \bigcap \mathcal{V}$.

Taking into account the limited communication range of the onboard sensors, the communication link between a pair of agents exists if the distance between them is less than the maximum sensing range $r \in (0, +\infty)$. To incorporate the effect of hysteresis, we further introduce a parameter $\varepsilon \in (0, r)$. On the other hand, to ensure safety and prevent collisions among the vehicles, we define another parameter $\underline{r} \in (0, r - \varepsilon)$, which is the minimum distance between two vehicles. These parameters are illustrated in Figure 1.

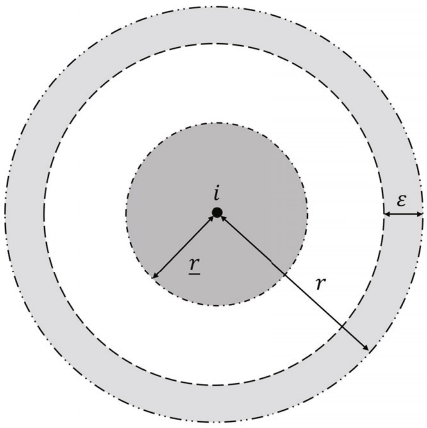


**Figure 1** Illustration of design parameters of the $i$-th vehicle

Then, given any $r > 0$ and $\varepsilon \in (0, r)$, for any $t \geq 0$, the edge set $\mathcal{E}(t)$ of the state-dependent communication graph $\mathcal{G}(t)$ is defined as follows:

1) $\mathcal{E}(0) = \{(i,j) \mid \|q_i(0) - q_j(0)\| < (r - \varepsilon), i, j = 1, \cdots, N\}$;

2) If $\|q_i(t) - q_j(t)\| \geq r$, then $(i,j) \notin \mathcal{E}(t)$;

3) For $i, j = 1, \cdots, N$, if $(i,j) \notin \mathcal{E}(t^-)$ and $\|q_i(t) - q_j(t)\| < (r - \varepsilon)$, then $(i,j) \in \mathcal{E}(t)$;

4) For $i, j = 1, \cdots, N$, if $(i,j) \in \mathcal{E}(t^-)$ and $\|q_i(t) - q_j(t)\| < r$, then $(i,j) \in \mathcal{E}(t)$.

For any $t \geq 0$, the edge set $\overline{\mathcal{E}}(t)$ of $\overline{\mathcal{G}}(t)$ is such that

1) $\mathcal{E}(t) \subseteq \overline{\mathcal{E}}(t)$;

2) For $i = 1, \cdots, N$, $(0, i) \in \overline{\mathcal{E}}(t)$ if and only if the $i$-th vehicle can access the velocity of the target;

3) For $i = 1, \cdots, N$, $(i, 0) \notin \overline{\mathcal{E}}(t)$.

For any $t \geq 0$, the convex hull formed by the $N$ vehicles can be defined as in [11] as follows:

$$\mathrm{co}(q(t)) = \left\{ \sum_{i=1}^{N} \lambda_i q_i(t) \mid \lambda_i \geq 0, i \in \mathcal{V}, \text{ and } \sum_{i-1}^{N} \lambda_i = 1 \right\}, \tag{3}$$

where $q(t) = \mathrm{col}\,(q_1(t), \cdots, q_N(t))$.

For any $t \geq 0$, the distance from the target $q_0(t)$ to the convex hull $\mathrm{co}(q(t))$ is defined as

$$P_{q_0(t)}(q(t)) = \min_{s \in \mathrm{co}(q(t))} \|q_0(t) - s\|. \tag{4}$$

**Remark 1** It is noted that $q_0(t) \in \mathrm{co}(q(t))$ if and only if $P_{q_0(t)}(q(t)) = 0$

We consider a control law of the following abstract form:

$$\begin{aligned} u_i &= l_i \left( \ddot{q}_0, q_i - q_0, q_i - q_j, \zeta_i, \zeta_j, j \in \overline{\mathcal{N}}_i(t) \right), \\ \dot{\zeta}_i &= g_i \left( q_i, \zeta_i, \zeta_j, j \in \overline{\mathcal{N}}_i(t) \right), \quad i = 1, \cdots, N, \end{aligned} \tag{5}$$

where $\zeta_0 = \dot{q}_0$, and, for $i = 1, \cdots, N$, $l_i(\cdot)$ and $g_i(\cdot)$ are some sufficiently smooth functions, $\zeta_i \in \mathbb{R}^{(2n+s_i+2n)}$ is the estimate of $\mathrm{col}(q_i, \dot{q}_i, \omega_i)$.

**Remark 2** The control law of the $i$-th vehicle only makes use of the position information of its neighboring vehicles and itself for feedback. As a result, the control law (5) does not rely on the velocity measurements of the vehicles.

The cooperative general target-fencing problem with connectivity preservation and collision avoidance can be described as follows:

**Problem 1** Consider the multi-agent system composed of (1), (2), and a given target $q_0(t)$. Given any $r > 0$, $\varepsilon \in (0, r)$, and $\underline{r} \in (0, r - \varepsilon)$, design a control law of the form (5) such that, for any initial conditions $\omega_i(0)$, $q_i(0)$, $\dot{q}_i(0)$, $\zeta_i(0)$, $i = 1, \cdots, N$, that make $\overline{\mathcal{G}}(0)$ connected, the solution of the closed-loop system satisfies the following properties:

1) $\overline{\mathcal{G}}(t)$ is connected for all $t \geq 0$;

2) $\|q_i - q_j\| > \underline{r}$ for all $i \neq j$, $i, j = 1, \cdots, N$, and $t \geq 0$;

3) $\lim_{t \to \infty} (\dot{q}_i(t) - \dot{q}_0(t)) = 0$, $i = 1, \cdots, N$;

4) $\lim_{t \to \infty} P_{q_0(t)}(q(t)) = 0$.

We use the following assumptions to guarantee the solvability of Problem 1.

**Assumption 1** The velocity $\dot{q}_0(t)$ of the target can be accessed by at least one of the vehicles for all $t \geq 0$.

**Assumption 2** For all $t \geq 0$, the $i$-th vehicle can access the relative position between the target and itself, which is $q_i(t) - q_0(t)$. Moreover, all the vehicles can access the acceleration $\ddot{q}_0(t)$ of the target.

**Remark 3** Problem 1 considers both connectivity preservation and collision avoidance in cooperative target fencing. These two requirements have been considered in some other cooperative control problems, such as the formation control problem[18, 19] and the flocking control problem[20]. However, they have not been addressed simultaneously in the cooperative target-fencing problem[11–15].

**Remark 4** Problem 1 can be viewed as a generalization of the cooperative target-fencing problem investigated previously in [11–15] from two-dimensional vehicles to $n$-dimensional vehicles.

## 3 Solvability of the Problem

As in [21], we utilize the output regulation theory to deal with external disturbances. To this end, we rewrite the system (1) into the following form:

$$\dot{x}_i = Ax_i + Bu_i + E_i\omega_i, \tag{6a}$$

$$y_i = Cx_i, \quad i = 1, \cdots, N, \tag{6b}$$

where for the $i$-th vehicle, $i = 1, \cdots, N$, the state is denoted as $x_i = \operatorname{col}(q_i, p_i)$ with $p_i = \dot{q}_i$, and $y_i \in \mathbb{R}^n$ is the measurement output, and

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \otimes I_n, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes I_n, \quad E_i = \begin{bmatrix} 0_{n\times s_i} \\ D_i \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \otimes I_n.$$

**Remark 5** Define $\widehat{A}_i = \begin{bmatrix} A & E_i \\ 0_{s_i\times 2n} & S_i \end{bmatrix}$ and $\widehat{C}_i = \begin{bmatrix} C & 0_{n\times s_i} \end{bmatrix}$. It can be assumed, without loss of generality, that the pair $(\widehat{C}_i, \widehat{A}_i)$ is detectable since the pair $(C, A)$ is observable[17]. Therefore, a gain matrix $L_i = \operatorname{col}(L_{i1}, L_{i2})$ exists, where $L_{i1} \in \mathbb{R}^{2n\times n}$ and $L_{i2} \in \mathbb{R}^{s_i\times n}$, such that $\widehat{A}_i + L_i\widehat{C}_i$ is Hurwitz. Additionally, there exists a symmetric and positive definite matrix $\overline{P}_i$ that satisfies $(\widehat{A}_i + L_i\widehat{C}_i)^{\mathrm{T}}\overline{P}_i + \overline{P}_i(\widehat{A}_i + L_i\widehat{C}_i) = -I_{2n+s_i}$.

We consider the following coordinate transformations:

$$\overline{x}_i = \begin{bmatrix} \overline{q}_i \\ \overline{p}_i \end{bmatrix} = x_i - x_0, \tag{7a}$$

$$\overline{u}_i = u_i - \ddot{q}_0 + D_i\omega_i, \quad i = 1, \cdots, N, \tag{7b}$$

where $x_0 = \mathrm{col}\,(q_0, p_0)$ with $p_0 = \dot{q}_0$. Then, the system (6) is transformed into the following form:

$$\dot{\overline{q}}_i = \overline{p}_i, \quad \dot{\overline{p}}_i = \overline{u}_i, \quad i = 1, \cdots, N. \tag{8}$$

**Remark 6** With the coordinate transformations (7), the original system (6) is transformed into a double-integrator system without disturbance, as given in (8).

To guarantee the connectivity of the communication graph, we define the potential function $\psi(s) : [0, r) \to [\frac{1}{2r^2}, \infty)$ in the following form:

$$\psi(s) = \frac{1}{2\left(r^2 - s^2\right)}, \quad 0 \leq s < r. \tag{9}$$

We note that function $\psi(s)$ is nonnegative over $[0, r)$, and its derivative is given as

$$\frac{d\psi(s)}{ds} = \frac{s}{\left(r^2 - s^2\right)^2}, \tag{10}$$

which is positive over $(0, r)$.

Additionally, let us define the potential function $\rho(s) : (\underline{r}, r) \to (\frac{1}{2(r^2-\underline{r}^2)^2}, \infty)$ for collision avoidance as follows:

$$\rho(s) = \frac{1}{2\left(s^2 - \underline{r}^2\right)^2}, \quad \underline{r} < s < r. \tag{11}$$

Here, the function $\rho(s)$ is nonnegative over $(\underline{r}, r)$ with a derivative given as

$$\frac{d\rho(s)}{ds} = -\frac{2s}{\left(s^2 - \underline{r}^2\right)^3}, \tag{12}$$

which is negative for all $s \in (\underline{r}, r)$.

The position feedback control law is given as follows:

$$\begin{aligned} u_i = & -(q_i - q_0) - \alpha \sum_{j \in \mathcal{N}_i(t)} a_{ij}(t) \nabla_{\overline{q}_i} \psi\left(\left\|\overline{q}_i - \overline{q}_j\right\|\right) - \sum_{j \in \mathcal{N}_i(t)} a_{ij}(t) \nabla_{\overline{q}_i} \rho\left(\left\|\overline{q}_i - \overline{q}_j\right\|\right) \\ & - \sum_{j \in \overline{\mathcal{N}}_i(t)} a_{ij}(t)\left(\xi_{2i} - \xi_{2j}\right) + \ddot{q}_0 - D_i \widehat{\omega}_i, \end{aligned} \tag{13a}$$

$$\dot{\xi}_i = A\xi_i + Bu_i + E_i \widehat{w}_i + L_{i1}\left(C\xi_i - y_i\right), \tag{13b}$$

$$\dot{\widehat{w}}_i = S_i \widehat{w}_i + L_{i2}\left(C\xi_i - y_i\right), \quad i = 1, \cdots, N, \tag{13c}$$

where, for $i = 1, \cdots, N$, $\widehat{w}_i \in \mathbb{R}^{s_i}$, $\xi_i = \mathrm{col}\,(\xi_{1i}, \xi_{2i})$ with $\xi_{1i} \in \mathbb{R}^n$ and $\xi_{2i} \in \mathbb{R}^n$; $\xi_{20} = p_0$; $L_i$ is as defined in Remark 5.

**Remark 7** It can be verified that (13) is in the form of (5) with $\zeta_i = \mathrm{col}(\xi_i, \widehat{\omega}_i)$, $i = 1, \cdots, N$.

**Remark 8** The cooperative target-fencing problem was studied under the assumption that the acceleration of the target is uniformly continuous and bounded in [15]. In contrast, we relax this assumption by only requiring the trajectory of the target to be twice continuously differentiable.

Let $\overline{\xi}_i = \xi_i - x_i$, $\overline{w}_i = \widehat{\omega}_i - \omega_i$, $i = 1, \cdots, N$. Then, the closed-loop system composed of (8) and (13) is given as

$$\dot{\overline{q}}_i = \overline{p}_i, \tag{14a}$$

$$\begin{aligned}\dot{\overline{p}}_i = & -\overline{q}_i - \alpha \sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)\nabla_{\overline{q}_i}\psi\left(\left\|\overline{q}_i - \overline{q}_j\right\|\right) - \sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)\nabla_{\overline{q}_i}\rho\left(\left\|\overline{q}_i - \overline{q}_j\right\|\right) - D_i\overline{w}_i \\ & - \sum_{j\in\overline{\mathcal{N}}_i(t)} a_{ij}(t)\left(\overline{p}_i - \overline{p}_j\right) - \left(\sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)\left(\overline{\xi}_{2i} - \overline{\xi}_{2j}\right) + a_{i0}(t)\overline{\xi}_{2i}\right),\end{aligned} \tag{14b}$$

$$\begin{bmatrix}\dot{\overline{\xi}}_i \\ \dot{\overline{w}}_i\end{bmatrix} = \left(\widehat{A}_i + L_i\widehat{C}_i\right)\begin{bmatrix}\overline{\xi}_i \\ \overline{w}_i\end{bmatrix}, \quad i = 1, \cdots, N. \tag{14c}$$

Next, we define some matrices associated with $\overline{\mathcal{G}}(t)$. For $t \geq 0$, we let the Laplacian matrix of the graph $\mathcal{G}(t)$ be as follows:

$$\mathcal{L}(t) = \begin{bmatrix} \overline{a}_1(t) & -a_{12}(t) & \cdots & -a_{1N}(t) \\ -a_{21}(t) & \overline{a}_2(t) & \cdots & -a_{2N}(t) \\ \vdots & \vdots & \ddots & \vdots \\ -a_{N1}(t) & -a_{N2}(t) & \cdots & \overline{a}_N(t) \end{bmatrix}, \tag{15}$$

where $\overline{a}_i(t) = \sum_{j=1,j\neq i}^{N} a_{ij}(t)$ for $i = 1, \cdots, N$. Let $H(t) = \mathcal{L}(t) + \Delta(t)$ where $\Delta(t) = \operatorname{diag}(a_{10}(t), \cdots, a_{N0}(t))$, and define

$$P(t) = \begin{bmatrix} H(t)\otimes I_n & \frac{\Lambda(t)}{2} \\ \frac{\Lambda^{\mathrm{T}}(t)}{2} & \theta I_\iota \end{bmatrix}, \tag{16}$$

where $\Lambda(t) = \begin{bmatrix} 0_{Nn\times Nn} & H(t)\otimes I_n & D \end{bmatrix}$ with $D = \text{block diag}(D_1, \cdots, D_N)$ and $\iota = 2Nn + s_1 + \cdots + s_N$, and $\theta$ is a positive real number such that

$$\theta > \lambda_{\max}\left(\frac{\Lambda^{\mathrm{T}}(t)}{2}\left(H^{-1}(t)\otimes I_n\right)\frac{\Lambda(t)}{2}\right), \quad \forall t \geq 0. \tag{17}$$

**Remark 9** By Lemma 1 of [22], for all $t \geq 0$, $H(t)$ is positive definite if $\overline{\mathcal{G}}(t)$ is connected. Given that $H(t)$ is uniquely determined by $\overline{\mathcal{G}}(t)$, and considering that there are only a finite number of connected graphs with $N + 1$ nodes, such a $\theta$ that satisfies (17) always exists if $\overline{\mathcal{G}}(t)$ remains connected for all $t \geq 0$.

We can now summarize our main result in the following theorem.

**Theorem 1** *Under Assumptions* 1 *and* 2*, Problem* 1 *is solvable by the position feedback control law* (13).

*Proof* The proof consists of the following five parts.

**Part I** In this part, let us determine the parameters for the control law (13).

For $i=1,\cdots,N$, let $\overline{\eta}=\operatorname{col}(\overline{\eta}_1,\cdots,\overline{\eta}_N)$, $\overline{q}=\operatorname{col}(\overline{q}_1,\cdots,\overline{q}_N)$, $\overline{p}=\operatorname{col}(\overline{p}_1,\cdots,\overline{p}_N)$, $\overline{\xi}=\operatorname{col}\left(\overline{\xi}_1,\cdots,\overline{\xi}_N\right)$, and $\mu_i=\operatorname{col}\left(\overline{\xi}_i,\overline{\omega}_i\right)$. We also define $\mu=\operatorname{col}(\mu_1,\cdots,\mu_N)$ and $\overline{\mu}=\operatorname{col}\left(\overline{\xi}_{11},\cdots,\overline{\xi}_{1N},\overline{\xi}_{21},\cdots,\overline{\xi}_{2N},\overline{w}_1,\cdots,\overline{w}_N\right)=T\mu$ with

$$T=\begin{bmatrix} I_n & 0_{n\times n} & 0_{n\times s_1} & \cdots & 0_{n\times n} & 0_{n\times n} & 0_{n\times s_N}\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots\\ 0_{n\times n} & 0_{n\times n} & 0_{n\times s_1} & \cdots & I_n & 0_{n\times n} & 0_{n\times s_N}\\ 0_{n\times n} & I_n & 0_{n\times s_1} & \cdots & 0_{n\times n} & 0_{n\times n} & 0_{n\times s_N}\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots\\ 0_{n\times n} & 0_{n\times n} & 0_{n\times s_1} & \cdots & 0_{n\times n} & I_n & 0_{n\times s_N}\\ 0_{s_1\times n} & 0_{s_1\times n} & I_{s_1} & \cdots & 0_{s_1\times n} & 0_{s_1\times n} & 0_{s_1\times s_N}\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots\\ 0_{s_N\times n} & 0_{s_N\times n} & 0_{s_N\times s_1} & \cdots & 0_{s_N\times n} & 0_{s_N\times n} & I_{s_N} \end{bmatrix}.$$

Here, $T^{-1}$ is orthogonal with $(T^{-1})^{\mathrm{T}}T^{-1}=I_\iota$.

Given $r>0$, $\varepsilon\in(0,r)$, by the proof of Lemma 3.1 in [21], it can be shown that, if $\theta$ satisfies (17) for all $t\geq 0$, then $P(t)$ is positive definite for all possible connected $\overline{\mathcal{G}}(t)$ with $N+1$ nodes and all $t\geq 0$. We fix such a $\theta$.

**Part II** Next, we show that under Assumption 1, the graph $\overline{\mathcal{G}}(t)$ is connected for all $t\geq 0$.

Considering the energy function as follows:

$$\begin{aligned} V(\overline{q},\overline{p},\mu,\overline{\eta},t)=&\frac{1}{2}\sum_{i=1}^{N}\Bigg(\alpha\sum_{j\in\mathcal{N}_i(t)}a_{ij}(t)\psi\left(\left\|\overline{q}_i-\overline{q}_j\right\|\right)\\ &+\sum_{j\in\mathcal{N}_i(t)}a_{ij}(t)\rho\left(\left\|\overline{q}_i-\overline{q}_j\right\|\right)+\overline{q}_i^{\mathrm{T}}\overline{q}_i+\overline{p}_i^{\mathrm{T}}\overline{p}_i\overline{\eta}_i^{\mathrm{T}}\overline{\eta}_i+2\theta\mu_i^{\mathrm{T}}\overline{P}_i\mu_i\Bigg), \end{aligned}\tag{18}$$

where $\overline{P}_i$, $i=1,\cdots,N$, are as defined in Remark 5.

The time derivative of (18) along the trajectories of the closed-loop system (14) is as follows:

$$\begin{aligned} \dot{V}=&\frac{1}{2}\sum_{i=1}^{N}\Bigg(\alpha\sum_{j\in\mathcal{N}_i(t)}\dot{\psi}\left(\left\|\overline{q}_i-\overline{q}_j\right\|\right)+\sum_{j\in\mathcal{N}_i(t)}\dot{\rho}\left(\left\|\overline{q}_i-\overline{q}_j\right\|\right)\\ &+2\dot{\overline{q}}_i^{\mathrm{T}}\overline{q}_i+2\dot{\overline{p}}_i^{\mathrm{T}}\overline{p}_i+2\theta\left(\dot{\mu}_i^{\mathrm{T}}\overline{P}_i\mu_i+\mu_i^{\mathrm{T}}\overline{P}_i\dot{\mu}_i\right)\Bigg)\\ =&-\sum_{i=1}^{N}\overline{p}_i^{\mathrm{T}}\Bigg(\sum_{j\in\mathcal{N}_i(t)}a_{ij}(t)\left(\overline{\xi}_{2i}-\overline{\xi}_{2j}\right)+a_{i0}(t)\overline{\xi}_{2i}\Bigg)\\ &-\sum_{i=1}^{N}\overline{p}_i^{\mathrm{T}}\Bigg(\sum_{j\in\overline{\mathcal{N}}_i(t)}a_{ij}(t)\left(\overline{p}_i-\overline{p}_j\right)+D_i\overline{w}_i\Bigg)-\theta\sum_{i=1}^{N}\mu_i^{\mathrm{T}}\mu_i. \end{aligned}\tag{19}$$

We rewrite (19) into the compact form with the notation of Kronecker product as follows:

$$\begin{aligned}\dot{V} &= -\overline{p}^{\mathrm{T}}\left(H(t)\otimes I_n\right)\overline{p} - \overline{p}^{\mathrm{T}}\Lambda(t)\overline{\mu} - \theta\overline{\mu}^{\mathrm{T}}(T^{-1})^{\mathrm{T}}T^{-1}\overline{\mu} \\ &= -\begin{bmatrix}\overline{p}\\ \overline{\mu}\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}H(t)\otimes I_n & \frac{\Lambda(t)}{2}\\ \frac{\Lambda^{\mathrm{T}}(t)}{2} & \theta I_\iota\end{bmatrix}\begin{bmatrix}\overline{p}\\ \overline{\mu}\end{bmatrix} \\ &= -\begin{bmatrix}\overline{p}\\ \overline{\mu}\end{bmatrix}^{\mathrm{T}} P(t)\begin{bmatrix}\overline{p}\\ \overline{\mu}\end{bmatrix}.\end{aligned}\tag{20}$$

In the following discussion, we show that under the control law (13), the graph $\mathcal{G}(t)$ is connected for all $t \geq 0$.

For simplicity, we first let $V(t) = V(\overline{q}(t), \overline{p}(t), \mu(t), t)$. It is obvious that there exists a $0 < t_1 \leq \infty$ such that $\mathcal{G}(t) = \mathcal{G}(t_1)$ for all $t \in [0, t_1)$ by the continuity of the solution of (14).

First we consider the case that $t_1 = \infty$, that is, $\mathcal{G}(t) = \mathcal{G}(0)$ for all $t \geq 0$. Since the graph is initially connected, $\mathcal{G}(t)$ remains connected for all $t \geq 0$. Additionally, $\overline{\mathcal{G}}(t)$ is also connected for all $t \geq 0$ under Assumption 1. Hence, given the $\theta$ we have chosen previously, for all $t \geq 0$, $P(t) = P(0)$ is positive definite. Therefore,

$$V(t) \leq V(0), \quad \forall t \geq 0. \tag{21}$$

Next, we consider the case that $t_1 < \infty$, that is, $\mathcal{G}(t) = \mathcal{G}(0)$ does not hold for all $t \geq 0$. Without loss of generality, we assume $t_1$ is such that

$$\begin{aligned}&\mathcal{G}(t) = \mathcal{G}(0), \quad t \in [0, t_1),\\ &\mathcal{G}(t_1) \neq \mathcal{G}(0).\end{aligned}\tag{22}$$

Under Assumption 1, $\overline{\mathcal{G}}(0)$ is also connected with a connected $\mathcal{G}(0)$, which means $P(t) = P(0)$ is positive definite for all $t \in [0, t_1)$ with the $\theta$ we chose. According to (20), there exists a $W_1 \in (0, \infty)$ which satisfies

$$V(t) \leq V(0) \leq W_1, \quad \forall t \in [0, t_1). \tag{23}$$

In what follows, we prove that $\mathcal{G}(t_1) \supset \mathcal{G}(0)$. Let us assume to the contrary that there exists some edge $(i, j)$ such that $(i, j) \in \mathcal{E}(0)$ and $(i, j) \notin \mathcal{E}(t_1)$. In this case, $\lim_{t\to t_1^-}\|q_i(t) - q_j(t)\| = r$. Therefore, $\lim_{t\to t_1^-} V(t) = \infty$, which contradicts (23). Thus, $\mathcal{G}(t_1) \supset \mathcal{G}(0)$ holds, which further implies that $\mathcal{G}(t_1)$ is also connected.

If there exists a $t_2 > t_1$ such that

$$\begin{aligned}&\mathcal{G}(t) = \mathcal{G}(t_1), \quad t \in [t_1, t_2),\\ &\mathcal{G}(t_2) \neq \mathcal{G}(t_1),\end{aligned}\tag{24}$$

then we claim $\mathcal{G}(t_2) \supset \mathcal{G}(t_1)$.

We note that for all $t_1 \leq t < t_2$ and the $\theta$ we have chosen, $P(t) = P(t_1)$ is positive definite. Note that

$$\psi(r-\varepsilon) = \frac{1}{2\left(r^2 - (r-\varepsilon)^2\right)} < \infty. \tag{25}$$

Given the existence of a $t_1$ satisfies (22) and the fact that $\mathcal{G}(t_1) \supset \mathcal{G}(0)$, there exists at least one edge $(i,j)$ such that $(i,j) \in \mathcal{E}(t_1)$ and $(i,j) \notin \mathcal{E}(0)$. We assume without loss of generality that there exists $\tau$ edges $(i_1, j_1), \cdots, (i_\tau, j_\tau)$ such that for any $k \in \{1, \cdots, \tau\}$, it holds that $(i_k, j_k) \in \mathcal{E}(t_1)$ and $(i_k, j_k) \notin \mathcal{E}(0)$. Here, $\tau$ is a positive integer. Then, according to (23) and (25), there exists a $W_2 \in (0, \infty)$ such that

$$V(t) \leq V(t_1) \leq V(0) + \alpha\tau\psi\left(r-\varepsilon\right) \leq W_2, \quad \forall t \in [t_1, t_2). \tag{26}$$

Then, the claim that $\mathcal{G}(t_2) \supset \mathcal{G}(t_1)$ can be proved similarly by contradiction as the proof of $\mathcal{G}(t_1) \supset \mathcal{G}(0)$.

Since $\mathcal{G}(t)$ has a finite number of edges, by repeating the above arguments, there exists a finite integer $k > 0$ such that

$$\begin{aligned}
&\mathcal{G}(t) = \mathcal{G}(0), \quad t \in [0, t_1), \\
&\mathcal{G}(t) = \mathcal{G}(t_i) \supset \mathcal{G}(t_{i-1}), \quad t \in [t_i, t_{i+1}),\ i = 1, \cdots, k-1, \\
&\mathcal{G}(t) = \mathcal{G}(t_k) \supset \mathcal{G}(t_{k-1}), \quad t \in [t_k, \infty).
\end{aligned} \tag{27}$$

Therefore, we have shown that under the control law (13), the graph $\mathcal{G}(t)$ is connected for all $t \geq 0$. Hence, the graph $\overline{\mathcal{G}}(t)$ is also connected for all $t \geq 0$ under Assumption 1.

**Part III** Next, let us show that $\|q_i(t) - q_j(t)\| > \underline{r}$, $i \neq j$, $i,j = 1, \cdots, N$, for all $t \geq 0$.

According to (27), we have $\dot{V}(t) \leq 0$ for all $t \geq t_k$. Therefore,

$$V(t) \leq V(t_k), \quad \forall t \geq t_k. \tag{28}$$

For a finite integer $k$, it holds that

$$V(t) \leq \max_{i=1,\cdots,k} V(t_i), \quad \forall t \geq 0. \tag{29}$$

We claim that $\|q_i(t) - q_j(t)\| > \underline{r}$, $i \neq j$, $i,j = 1, \cdots, N$, for all $t \geq 0$. We prove the claim by contradiction. Let us assume that for some $i$ and $j$, $i,j = 1, \cdots, N$, $i \neq j$, there exists a $t_c \in (0, \infty)$ such that $\|q_i(t_c) - q_j(t_c)\| = \underline{r}$. We note that such an $(i,j)$ belongs to $\mathcal{E}(t_c)$. However, $\lim_{t \to t_c^-} \rho(\|q_i(t) - q_j(t)\|) = \infty$, which implies that $\lim_{t \to t_c^-} V(t) = \infty$. It contradicts the boundness of $V(t)$ in (29). Therefore, $\|q_i(t) - q_j(t)\| > \underline{r}$, $i \neq j$, $i,j = 1, \cdots, N$, for all $t \geq 0$.

**Part IV** In what follows, we show that $\lim_{t \to \infty}(\dot{q}_i(t) - \dot{q}_0(t)) = 0$ for $i = 1, \cdots, N$.

We first have $\lim_{t \to \infty} V(t)$ exists due to the fact that $V(t) \geq 0$ is nonincreasing for all $t \geq t_k$. Additionally, we easily find that $\ddot{V}(t)$ is bounded and $\dot{V}(t)$ is uniformly continuous. Therefore, it can be concluded that $\lim_{t \to \infty} \dot{V}(t) = 0$ by Barbalat's lemma. Recall (20), $\lim_{t \to \infty} \overline{p}_i(t) = \lim_{t \to \infty}\left(\dot{q}_i(t) - \dot{q}_0(t)\right) = 0$, $i = 1, \cdots, N$.

**Part V** Finally, we show that $\lim_{t\to\infty} P_{q_0(t)}(q(t)) = 0$.

To this end, for $t \geq 0$, define the center of the $N$ vehicles as

$$q_c(t) = \frac{1}{N}\sum_{i=1}^{N} q_i(t). \tag{30}$$

Furthermore, let

$$\begin{aligned}\overline{q}_c(t) &= q_c(t) - q_0(t)\\ &= \frac{1}{N}\sum_{i=1}^{N} \overline{q}_i(t)\\ &= \frac{1}{N}\sum_{i=1}^{N} \left(q_i(t) - q_0(t)\right).\end{aligned} \tag{31}$$

Then, we show that $\lim_{t\to\infty} \overline{q}_c(t) = 0$, $i = 1, \cdots, N$.

For $i = 1, \cdots, N$, by a direct calculation, $\ddot{\overline{p}}_i(t)$ is bounded over $[t_k, \infty)$. Thus, $\dot{\overline{p}}(t)$ is uniformly continuous for all $t \geq t_k$. Since $\lim_{t\to\infty} \overline{p}_i(t) = 0$, by Barbalat's lemma,

$$\begin{aligned}\lim_{t\to\infty} \dot{\overline{p}}_i(t) &= -\overline{q}_i(t) - \alpha \sum_{j\in\mathcal{N}_i(t)} \frac{\overline{q}_i(t) - \overline{q}_j(t)}{\left(r^2 - \left\|\overline{q}_i(t) - \overline{q}_j(t)\right\|^2\right)^2} + \sum_{j\in\mathcal{N}_i(t)} \frac{2\left(\overline{q}_i(t) - \overline{q}_j(t)\right)}{\left(\left\|\overline{q}_i(t) - \overline{q}_j(t)\right\|^2 - \underline{r}^2\right)^3}\\ &= 0, \quad i = 1, \cdots, N.\end{aligned} \tag{32}$$

Thus, noting (31), we have

$$\begin{aligned}\lim_{t\to\infty} \ddot{\overline{q}}_c(t) &= \lim_{t\to\infty} \frac{1}{N}\sum_{i=1}^{N} \dot{\overline{p}}_i(t)\\ &= \lim_{t\to\infty} \left( -\left(\frac{1}{N}\sum_{i=1}^{N} \left(q_i(t) - q_0(t)\right)\right) - \alpha\frac{1}{N}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(t)} \frac{\overline{q}_i(t) - \overline{q}_j(t)}{\left(r^2 - \left\|\overline{q}_i(t) - \overline{q}_j(t)\right\|^2\right)^2}\right.\\ &\quad \left. + \frac{1}{N}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(t)} \frac{2\left(\overline{q}_i(t) - \overline{q}_j(t)\right)}{\left(\left\|\overline{q}_i(t) - \overline{q}_j(t)\right\|^2 - \underline{r}^2\right)^3}\right)\\ &= 0.\end{aligned} \tag{33}$$

By the definition of the state-dependent communication graph, $\mathcal{G}(t)$ is undirected. Therefore, for $i, j = 1, \cdots, N$ and $i \neq j$, $j \in \mathcal{N}_i(t) \Leftrightarrow i \in \mathcal{N}_j(t)$ for all $t \geq 0$. As a result, for all $t \geq 0$, we have

$$\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(t)} \frac{\overline{q}_i(t) - \overline{q}_j(t)}{\left(r^2 - \left\|\overline{q}_i(t) - \overline{q}_j(t)\right\|^2\right)^2} = 0 \tag{34}$$

and

$$\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(t)}\frac{2\left(\overline{q}_i(t)-\overline{q}_j(t)\right)}{\left(\left\|\overline{q}_i(t)-\overline{q}_j(t)\right\|^2-\underline{r}^2\right)^3}=0. \tag{35}$$

Then we further conclude that

$$\lim_{t\to\infty}\dot{\overline{p}}_c(t)=\lim_{t\to\infty}-\left(\frac{1}{N}\sum_{i=1}^{N}(q_i(t)-q_0(t))\right)=0. \tag{36}$$

Therefore,

$$\lim_{t\to\infty}(q_c(t)-q_0(t))=\lim_{t\to\infty}\overline{q}_c(t)=\lim_{t\to\infty}\frac{1}{N}\sum_{i=1}^{N}(q_i(t)-q_0(t))=0. \tag{37}$$

From (3), $q_c(t)\in\mathrm{co}(q(t))$ for all $t\geq 0$. Hence, $\lim_{t\to\infty}P_{q_0(t)}(q(t))=0$. ∎

## 4 An Example

In this section, we consider an example where four quadrotors need to catch a moving target by fencing the target into the convex hull formed by their positions. The dynamics of each quadrotor is described by a double integrator of the form (1) with $n=3$. The external disturbances $d_i\in\mathbb{R}^3$, $i=1,2,3,4$, are of (2) with

$$S_1=\begin{bmatrix}0&1\\0&0\end{bmatrix},\quad S_2=0.05,\quad S_3=\begin{bmatrix}0&1\\-0.5&0\end{bmatrix},\quad S_4=\begin{bmatrix}0&1\\-1&0\end{bmatrix},$$
$$D_1=\begin{bmatrix}0.9&0\\0&0.5\\0.5&0.1\end{bmatrix},\quad D_2=\begin{bmatrix}0.5\\-0.2\\0.1\end{bmatrix},\quad D_3=\begin{bmatrix}1&0\\1&-1\\0.2&0.5\end{bmatrix},\quad D_4=\begin{bmatrix}0.3&0\\0&0.3\\0&-0.2\end{bmatrix}.$$

The trajectory of the target to be fenced is given as follows:

$$q_0(t)=\begin{bmatrix}0.5\mathrm{e}^{-0.1t+0.2}-0.3\\(0.05t-0.9)^2-0.3\\\log(t+25)-3.5\end{bmatrix}.$$

It is noted that the acceleration of the given target is unbounded and all agents we considered are three-dimensional. Therefore, this example cannot be handled by the approaches in the existing literature.

Throughout the entire fencing process, all vehicles are required to maintain the connectivity of the communication network while avoiding collisions. Therefore, we define $r=13$, $\varepsilon=4.5$, and $\underline{r}=1$. The initial positions of the agents are given by $q_0=[-2.5\quad -4\quad 0]^{\mathrm{T}}$, $q_1=[-1\quad 1\quad 3]^{\mathrm{T}}$, $q_2=[-4\quad -3\quad 2]^{\mathrm{T}}$, $q_3=[2\quad -7\quad 1]^{\mathrm{T}}$, $q_4=[4\quad 5\quad 0]^{\mathrm{T}}$. We assume that the 1st vehicle has access to the velocity of the target for all $t \geq 0$, that is, $(0,1) \in \overline{\mathcal{E}}(t)$ and $a_{10}(t) = 1$ for all $t \geq 0$. Then, we have

$$\overline{\mathcal{E}}(0) = \{(0,1)\,,(1,2)\,,(2,1)\,,(2,3)\,,(3,2)\,,(1,4)\,,(4,1)\}\,, \tag{38}$$

which shows that the graph $\overline{\mathcal{G}}(0)$ is connected.

By Theorem 1, we can design a position feedback control law of the form (13) with $\alpha =$ 28,561. The simulation results are shown in Figures 2–6. Figure 2 shows the distance between the target and the center of the vehicles. Figure 3 shows the velocity tracking errors of the vehicles. The connectivity preservation of the communication network and the inter-vehicle collision avoidance are verified in Figure 4. Figure 5 shows the trajectories of the agents. As shown in Figure 6, which shows the time profiles of the positions of the agents, the target is fenced into the convex hull form by the vehicles.

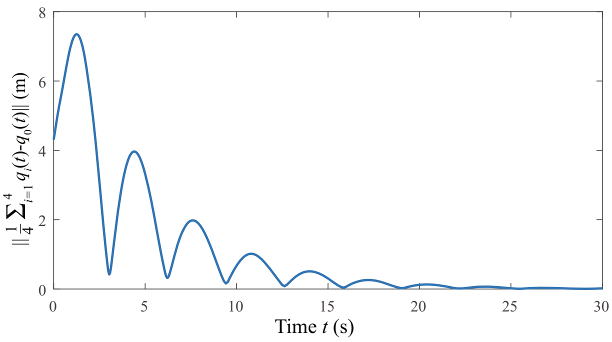


**Figure 2** Distance between the center of vehicles and the target

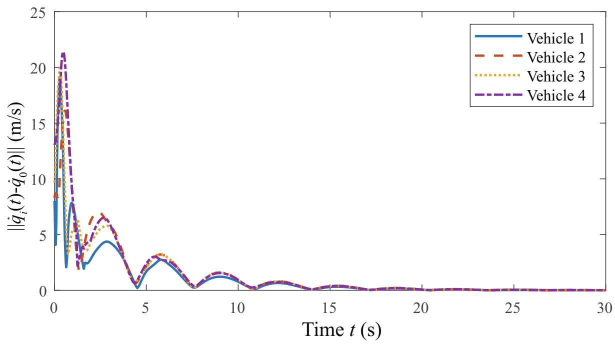


**Figure 3** Velocity tracking errors of the vehicles

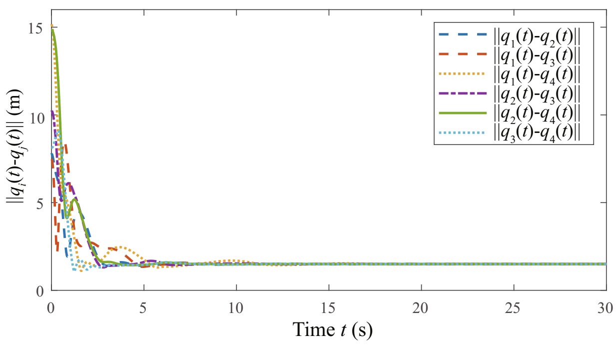


**Figure 4** Distances between any two vehicles

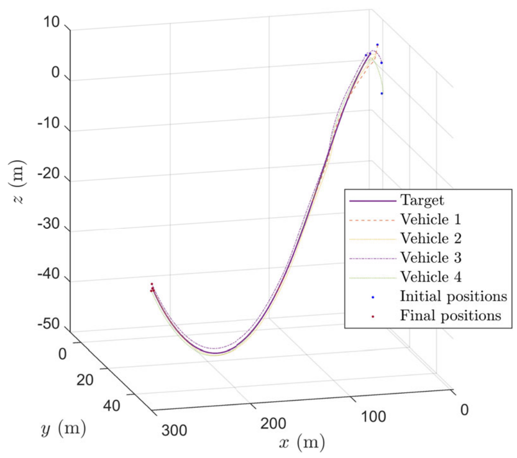


**Figure 5** Trajectories of the target and the vehicles

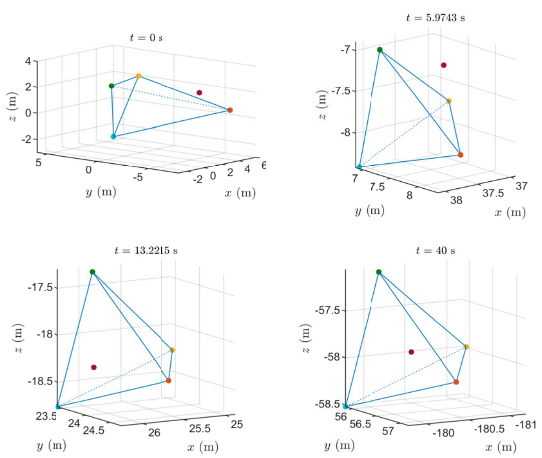


**Figure 6** Positions of the target and the vehicles

## 5 Conclusion

The cooperative target-fencing problem has been addressed in this paper for multiple $n$-dimensional double-integrator systems and a target with any twice continuously differentiable trajectory. Our approach can preserve the connectivity of the state-dependent communication graph and guarantee collision avoidance among the vehicles, simultaneously. In addition, our approach can reject a large class of disturbances and does not rely on the velocity measurements of the vehicles.

## Conflict of Interest

The authors declare no conflict of interest.

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