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Cooperative target fencing of multiple double-integrator systems with connectivity preservation

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Abstract

In this article, we study the cooperative target-fencing problem of multiple double-integrator systems over a state-dependent communication network. We propose a distributed control law to asymptotically fence a moving target while achieving both collision avoidance and connectivity preservation of the vehicles. In particular, a distributed observer is employed by each vehicle to estimate the position and the velocity of the target. Compared with the existing results, our approach can accommodate a larger class of target's trajectories. Furthermore, our control law is more practical since we do not require all vehicles to know the trajectory of the target. The effectiveness of our approach is illustrated by a numerical example.

KEYWORDS

collision avoidance, connectivity preservation, distributed control, disturbance rejection, target fencing

1 | INTRODUCTION

The cooperative control of multi-agent systems has been extensively studied due to its wide applications in the coordination of multiple aerial vehicles,[1–3] the formation control of multiple wheeled robots,[4–6] and the object transport of multiple robotic systems.[7–9] These practical applications have given rise to various cooperative control problems of multi-agent systems, such as flocking,[10,11] consensus,[12,13] rendezvous,[14–16] formation control,[17–19] containment control,[20,21] and surrounding control.[22] Specifically, a distributed control law was designed in Reference 16 to solve the rendezvous problem of multiple single integrators while considering connectivity preservation and time delays. Furthermore, References 14 and 15 investigated the connectivity-preserving rendezvous problem using a distributed observer for the leader system. In Reference 23, an event-triggered distributed observer was proposed to achieve leader-following attitude consensus for a group of rigid-body systems over jointly connected switching networks. More recently, the cooperative target-fencing problem of multi-agent systems, also known as the target-surrounding problem in some literature,[24,25] has received much attention in the scientific community. The goal of the cooperative target-fencing problem is to design a control law for a group of vehicles such that they can asymptotically fence a given target into the convex hull formed by the vehicles.[26]

Under the assumption that the target is stationary, a cooperative control law was proposed in Reference 26 to fence the target into the convex hull formed by multiple single-integrator systems. A two-level distributed control law was developed in Reference 27 such that a group of unmanned surface vessels can asymptotically fence a stationary target vessel. Some efforts have been made to fence a moving target with a constant velocity. Specifically, two control laws were established in Reference 28 to fence a moving target with an unknown constant velocity by multiple single-integrator systems in a rotation motion and a rigid formation, respectively. Without the availability of velocity measurements, Reference 29 investigated the fencing problem of a target with a constant velocity by a group of vehicles with second-order dynamics.

The target-fencing problem was studied in Reference 25 to fence the target with a predefined distance and an equational angle, where a distributed bearing-only control law was developed for multiple unmanned surface vessels to fence the target with a constant velocity. The result of Reference 25 was further extended to handle a target with uniformly continuous bounded time-varying trajectories in Reference 30. For this reason, the existing results on the cooperative target-fencing problems are still quite limited in terms of the trajectories of the target and the dynamics of the vehicles.

Besides, most of the existing approaches in the literature for cooperative target fencing are not fully distributed. In fact, the results in References 26 and 30 relied on the assumption that all vehicles have access to the trajectory of the target for all time. This requirement is demanding in real-world scenarios due to the limited sensing range of the vehicles. To address this issue, so-called decentralized estimators were proposed in References 28 and 29 to generate the estimates of the trajectory of the target for the vehicles. Under the assumptions that the communication network of the vehicles is connected for all time and there is at least one vehicle that has access to the trajectory of the target, it was shown in References 28 and 29 that these estimates converge to the true trajectory of the target. However, the approaches in References 28 and 29 themselves did not guarantee the connectivity of the state-dependent communication network of the vehicles. Thus, it is desirable to design a distributed control law that can preserve the connectivity of the state-dependent communication network while requiring only a portion of vehicles to know the trajectory of the target.

In this article, we further study the cooperative target-fencing problem of multiple double-integrator systems with connectivity preservation and collision avoidance. The main contributions of this article are summarized as follows:

1. We establish a distributed control law for multiple double-integrator systems to asymptotically fence a moving target over a state-dependent communication network without using the velocity measurements of the vehicles.
2. Our approach achieves both the connectivity preservation of the communication network and the collision avoidance among the vehicles.
3. Our approach can handle a larger class of target's trajectories compared with the existing results on the cooperative target-fencing problem.
4. Our control law is distributed in the sense that only a portion of the vehicles need to know the trajectory of the target.

Compared with the existing results on the cooperative target-fencing problem in the literature, our result has the following distinct features. (i) Unlike the results obtained in References 26,30,31 that assumed all vehicles know the trajectory of the target, our control law is fully distributed in the sense that it only requires at least one of the vehicles know the trajectory of the target. (ii) Although, the decentralized estimators in References 28 and 29 can handle the case where only a portion of vehicles know the position of the target, the approaches of References 28 and 29 cannot guarantee the connectivity of the communication network. By contrast, our approach can guarantee the connectivity of the communication graph. (iii) Compared with the approaches in References 26,28–31, our approach can handle a larger class of trajectories of the target which contains arbitrary polynomials and sinusoidal functions as special cases. (iv) While none of the existing results considered external disturbances, our approach can handle a large class of disturbances generated by a general linear exosystem. (v) While the results in References 26,28–31 were restricted to two-dimensions systems, our result applies to n-dimensional systems.

In addition, the cooperative target-fencing problem studied in this article is different from the consensus problem studied in Reference 15 and the formation tracking problem studied in Reference 19 in the following aspects. First, the control objective is different. The objective of the cooperative target-fencing problem is to fence the target into the convex hull formed by the positions of vehicles. In contrast, the objective of the leader-following consensus problem investigated in Reference 15 is to drive a group of followers to achieve both position and velocity consensus with the leader asymptotically. Second, unlike the formation tracking problem considered in Reference 19 that required predefined offsets for defining the desired formation configuration with respect to the leader, we do not need to specify predefined offsets in the cooperative target-fencing problem. Third, we further consider inter-vehicle collision avoidance, while References 15 and 19 did not consider collision avoidance.

The rest of this article is organized as follows. In Section 2, we will summarize some preliminaries and formulate our problem. In Section 3, we will present our main result. An example is provided in Section 4 to illustrate our approach. The article is closed in Section 5 with some concluding remarks.

Notation: $\otimes$ denotes the Kronecker product of matrices. $\|x\|$ denotes the Euclidean norm of a vector x. For column vectors a_i, $i = 1, \dots, s$, $\text{col}(a_1, \dots, a_s) = [a_1^\top \dots a_s^\top]^\top$. For a real symmetric matrix A, $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denote its minimum and maximum eigenvalues, respectively.

2 | PRELIMINARIES AND PROBLEM FORMULATION

Consider a group of N vehicles whose dynamics are described by the following double-integrator systems:

$$\ddot{q}_i = u_i + d_i, \ i = 1, \dots, N, \tag{1}$$

where, for the ith vehicle, $i = 1, \dots, N$, $q_i \in \mathbb{R}^n$ is the position; $u_i \in \mathbb{R}^n$ is the input; and $d_i \in \mathbb{R}^n$ is external disturbance. The external disturbance is assumed to be generated by the following linear exosystem:

$$\dot{\omega}_i = S_i\omega_i, \quad d_i = D_i\omega_i, \ i = 1, \dots, N, \tag{2}$$

where, for $i = 1, \dots, N$, $\omega_i \in \mathbb{R}^{s_i}$, and $S_i \in \mathbb{R}^{s_i \times s_i}$ and $D_i \in \mathbb{R}^{n \times s_i}$ are constant matrices. It can be assumed without loss of generality that the pair (D_i, S_i) is detectable.[32]

Remark 1. Since (2) is a general linear system, the external disturbance $d_i(t)$ generated by (2) can model a large class of disturbance signals, including arbitrary polynomials of time t, arbitrary sinusoidal functions of time t, arbitrary exponential functions of time t, and their finite combinations.

The dynamics of the target is described as follows:

$$\ddot{q}_0 = S_{01}q_0 + S_{02}\dot{q}_0, \tag{3}$$

where $q_0 \in \mathbb{R}^n$ is the position of the target, and $S_{01}, S_{02} \in \mathbb{R}^{n\times n}$ are constant matrices.

Remark 2. The trajectory generated by (3) can be arbitrary polynomial and sinusoidal functions of time t. Hence, the target (3) contains the stationary target in Reference 26, the target with constant velocity in References 29 and 29, and the target with time-varying velocity in Reference 31 as special cases. Therefore, our approach can accommodate a larger class of target's trajectories.

To describe our problem, we introduce the following design parameters:

1. $r \in \mathbb{R}$ is the maximum sensing range of each vehicle.
2. $\epsilon \in (0, r)$ is to introduce the effect of hysteresis.
3. $\underline{r} \in \mathbb{R}$ is the minimum distance between two vehicles for inter-vehicle collision avoidance.

The system composed of (1) and (3) can be viewed as a multi-agent system of $N + 1$ agents with (3) as the target and N subsystems of (1) as the N vehicles. Given the multi-agent system composed of (1) and (3), we can define a digraph $\overline{\mathcal{G}}(t) = (\overline{\mathcal{V}}, \overline{\mathcal{E}}(t))$, where $\overline{\mathcal{V}} = \{0, 1, \dots, N\}$ is the node set with node 0 associated with the target and node i, $i = 1, \dots, N$, associated with the ith vehicle of (1), and $\overline{\mathcal{E}}(t) \subseteq \overline{\mathcal{V}} \times \overline{\mathcal{V}}$ is the edge set. Let $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$ be a subgraph of $\overline{\mathcal{G}}(t)$, where $\mathcal{V} = \{1, \dots, N\}$ and $\mathcal{E}(t) \subseteq \mathcal{V} \times \mathcal{V}$. For $i = 1, \dots, N$, let $\overline{\mathcal{N}}_i(t)$ denote the neighbor set of the ith vehicle at time t. The neighbor set of node i with respect to $\mathcal{V}$ is defined as $\mathcal{N}_i(t) = \overline{\mathcal{N}}_i(t) \bigcap \mathcal{V}$. Let $\overline{\mathcal{A}}(t) = [a_{ij}(t)]_{i,j=0}^N \in \mathbb{R}^{(N+1)\times(N+1)}$ be the adjacency matrix of $\overline{\mathcal{G}}(t)$. For $i = 1, \dots, N, j = 0, 1, \dots, N$, and $i \neq j$, we let $a_{ij}(t) = 1$ whenever $(j, i) \in \overline{\mathcal{E}}(t)$ and $a_{ij}(t) = 0$ otherwise.

Given any $r > 0$ and $\epsilon \in (0, r)$, for any $t \geq 0$, the edge set $\mathcal{E}(t)$ of the state-dependent communication graph $\mathcal{G}(t)$ is defined as follows:

1. $\mathcal{E}(0) = \{(i,j) | \|q_i(0) - q_j(0)\| < (r - \epsilon), i, j = 1, \dots, N\}$;
2. If $\|q_i(t) - q_j(t)\| \geq r$, then $(i,j) \notin \mathcal{E}(t)$;
3. For $i, j = 1, \dots, N$, if $(i,j) \notin \mathcal{E}(t^-)$ and $\|q_i(t) - q_j(t)\| < (r - \epsilon)$, then $(i,j) \in \mathcal{E}(t)$;
4. For $i, j = 1, \dots, N$, if $(i,j) \in \mathcal{E}(t^-)$ and $\|q_i(t) - q_j(t)\| < r$, then $(i,j) \in \mathcal{E}(t)$.

For any $t \geq 0$, the edge set $\overline{\mathcal{E}}(t)$ of $\overline{\mathcal{G}}(t)$ satisfies the following:

1. $\mathcal{E}(t) \subseteq \overline{\mathcal{E}}(t)$;
2. For $i = 1, \dots, N$, $(0, i) \in \overline{\mathcal{E}}(t)$ if and only the ith vehicle can access the trajectory of the target;
3. For $i = 1, \dots, N$, $(i, 0) \notin \overline{\mathcal{E}}(t)$.

It is noted that the state-dependent graph $\mathcal{G}(t)$ is undirected. Since the target (3) does not have an input, there is no edge from a vehicle to the target for all $t \geq 0$. That is, for all $t \geq 0$, the edge set $\overline{\mathcal{E}}(t)$ does not contain edges of the form $(i, 0)$, $i = 1, \ldots, N$. The edge set $\mathcal{E}(t)$ of $\mathcal{G}(t)$ is obtained from the edge set $\overline{\mathcal{E}}(t)$ of $\overline{\mathcal{G}}(t)$ by removing all edges between node 0 and node i, $i = 1, \ldots, N$.

As in Reference 26, for any $t \geq 0$, the convex hull formed by the N vehicles is defined by

$$\mathrm{co}(q(t)) = \left\{ \sum_{i=1}^{N} \lambda_i q_i(t) | \lambda_i \geq 0, i \in \mathcal{V}, \text{ and } \sum_{i=1}^{N} \lambda_i = 1 \right\}, \tag{4}$$

where $q = \mathrm{col}(q_1, \ldots, q_N)$. For any $t \geq 0$, the distance from the point $q_0(t) \in \mathbb{R}^n$ to the convex hull $\mathrm{co}(q(t))$ is

$$P_{q_0(t)}(q(t)) = \min_{s \in \mathrm{co}(q(t))} \|q_0(t) - s\|. \tag{5}$$

Thus, $q_0(t) \in \mathrm{co}(q(t))$ if and only if $P_{q_0(t)}(q(t)) = 0$.

We consider a control law of the following abstract form:

$$\begin{aligned} u_i &= l_i\Big(q_i, q_j, \zeta_i, \zeta_j, j \in \overline{\mathcal{N}}_i(t)\Big), \\ \dot{\zeta}_i &= g_i\Big(q_i, q_j, \zeta_i, \zeta_j, j \in \overline{\mathcal{N}}_i(t)\Big), \ i = 1, \ldots, N, \end{aligned} \tag{6}$$

where $\zeta_0 = \mathrm{col}(q_0, \dot{q}_0)$, and, for $i = 1, \ldots, N$, $l_i(\cdot)$ and $g_i(\cdot)$ are some sufficiently smooth functions, $\zeta_i \in \mathbb{R}^{(2n+s_i+2n)}$ is the estimate of $\mathrm{col}(q_i, \dot{q}_i, \omega_i, q_0, \dot{q}_0)$.

Remark 3. Since the ith vehicle only makes use of the position information of its neighbors and itself for feedback, the control law (6) is called a distributed position feedback control law.

We describe the cooperative target-fencing problem with connectivity preservation as follows.

Problem 1. Consider the multi-agent system composed of (1), (2), and (3). Given any $r > 0$, $\epsilon \in (0, r)$, and $\underline{r} \in (0, r - \epsilon)$, design a distributed control law of the form (6) such that, for any initial conditions $q_0(0)$, $\dot{q}_0(0)$, $\omega_i(0)$, $q_i(0)$, $\dot{q}_i(0)$, $\zeta_i(0)$, $i = 1, \ldots, N$, that make $\mathcal{G}(0)$ connected, the solution of the closed-loop system satisfies the following properties:

1. $\mathcal{G}(t)$ is connected for all $t \geq 0$.
2. $\|q_i - q_j\| > \underline{r}$ for all $i \neq j$, $i, j = 1, \ldots, N$, and $t \geq 0$.
3. $\lim_{t\to\infty}(\dot{q}_i(t) - \dot{q}_0(t)) = 0$, $i = 1, \ldots, N$.
4. $\lim_{t\to\infty} P_{q_0(t)}(q(t)) = 0$.

Remark 4. Unlike References 28 and 29 that assumed $\mathcal{G}(t)$ is connected for all $t \geq 0$, Problem 1 is more challenging since we only assume $\mathcal{G}(0)$ is connected and we need to preserve the connectivity of $\mathcal{G}(t)$ for all $t \geq 0$ in addition to achieving target fencing.

We need the following assumption to guarantee the solvability of the problem.

Assumption 1. For all $t \geq 0$, the trajectory $x_0(t)$ of the target can be accessed by at least one vehicle.

3 | MAIN RESULT

As in Reference 15, we employ the output regulation theory to deal with external disturbances. For this purpose, let us rewrite the system (1) as follows:

$$\dot{x}_i = Ax_i + Bu_i + E_i\omega_i, \tag{7a}$$

$$y_i = Cx_i, \tag{7b}$$

$$e_i = x_i - x_0, \ i = 1, \ldots, N, \tag{7c}$$

where $x_i = \mathrm{col}(q_i, p_i)$ with $p_i = \dot{q}_i$, $y_i \in \mathbb{R}^n$, and $e_i \in \mathbb{R}^{2n}$ being the state, the measurement output, and the regulated output of the ith vehicle, respectively; $x_0 = \mathrm{col}(q_0, p_0)$ with $p_0 = \dot{q}_0$; $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \otimes I_n$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes I_n$, $E_i = \begin{bmatrix} 0_{n\times s_i} \\ D_i \end{bmatrix}$, and $C = \begin{bmatrix} 1 & 0 \end{bmatrix} \otimes I_n$.

Also, we rewrite system (3) as follows:

$$\dot{x}_0 = S_0 x_0, \tag{8}$$

where $S_0 = \begin{bmatrix} 0_{n\times n} & I_n \\ S_{01} & S_{02} \end{bmatrix}$.

Remark 5. Let $\hat{A}_i = \begin{bmatrix} A & E_i \\ 0_{s_i\times 2n} & S_i \end{bmatrix}$ and $\hat{C}_i = \begin{bmatrix} C & 0_{n\times s_i} \end{bmatrix}$. Without loss of generality, we can assume that the pair $(\hat{C}_i, \hat{A}_i)$ is detectable since the pair (C, A) is observable.[32] Therefore, there exists a gain matrix $L_i = \mathrm{col}(L_{i1}, L_{i2})$ with $L_{i1} \in \mathbb{R}^{2n\times n}$ and $L_{i2} \in \mathbb{R}^{s_i\times n}$ such that $\hat{A}_i + L_i\hat{C}_i$ is Hurwitz. Furthermore, there exists a symmetric and positive definite matrix $\overline{P}_i$ such that $\left(\hat{A}_i + L_i\hat{C}_i\right)^{\top}\overline{P}_i + \overline{P}_i\left(\hat{A}_i + L_i\hat{C}_i\right) = -I_{2n+s_i}$.

Let

$$U_i = \begin{bmatrix} S_{01} & S_{02} & -D_i \end{bmatrix}. \tag{9}$$

Perform on system (7) the following coordinate transformations:

$$\bar{x}_i = \begin{bmatrix} \overline{q}_i \\ \overline{p}_i \end{bmatrix} = x_i - x_0, \tag{10a}$$

$$\bar{u}_i = u_i - U_i v_i, \; i = 1, \ldots, N, \tag{10b}$$

where $v_i = \mathrm{col}(x_0, \omega_i)$, $i = 1, \ldots, N$. Then, system (7) is transformed into the following double-integrator system without external disturbance:

$$\dot{\overline{q}}_i = \overline{p}_i, \quad \dot{\overline{p}}_i = \bar{u}_i, \; i = 1, \ldots, N. \tag{11}$$

Define potential function $\psi(s) : [0, r) \to [\frac{1}{2r^2}, \infty)$ for connectivity preservation and the potential function $\rho(s) : (\underline{r}, r) \to (0, \infty)$ for collision avoidance in following forms:

$$\psi(s) = \frac{1}{2\left(r^2 - s^2\right)}, \; 0 \le s < r, \tag{12}$$

$$\rho(s) = \frac{1}{2\left(s^2 - \underline{r}^2\right)^2}, \; \underline{r} < s < r. \tag{13}$$

The function $\psi(s)$ is nonnegative over $[0, r)$, and its derivative is

$$\frac{d\psi(s)}{ds} = \frac{s}{\left(r^2 - s^2\right)^2} \tag{14}$$

which is positive for all $s \in (0, r)$. The function $\rho(s)$ is nonnegative over $(\underline{r}, r)$, and its derivative is

$$\frac{d\rho(s)}{ds} = -\frac{2s}{\left(s^2 - \underline{r}^2\right)^3} \tag{15}$$

which is negative and bounded over $(\underline{r}, r)$.

Motivated by Reference 15, we propose the following distributed feedback control law:

$$u_i = -(q_i - \eta_{1i}) - \alpha \sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)\nabla_{\overline{q}_i}\psi\left(\left\|\overline{q}_i - \overline{q}_j\right\|\right) - \sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)\nabla_{\overline{q}_i}\rho\left(\left\|\overline{q}_i - \overline{q}_j\right\|\right) - \sum_{j\in\overline{\mathcal{N}}_i(t)} a_{ij}(t)\left(\xi_{2i} - \xi_{2j}\right) + U_i\mathrm{col}(\eta_i, \hat{w}_i) \tag{16a}$$

$$\dot{\xi}_i = A\xi_i + Bu_i + E_i\hat{w}_i + L_{i1}(C\xi_i - y_i) \tag{16b}$$

$$\dot{\hat{w}}_i = S_i\hat{w}_i + L_{i2}(C\xi_i - y_i) \tag{16c}$$

$$\dot{\eta}_i = S_0\eta_i + \gamma \sum_{j\in\overline{\mathcal{N}}_i(t)} a_{ij}(t)\left(\eta_j - \eta_i\right),\ i = 1, \dots, N, \tag{16d}$$

where, for $i = 1, \dots, N$, $\eta_i = \mathrm{col}(\eta_{1i}, \eta_{2i})$ with $\eta_{1i} \in \mathbb{R}^n$ and $\eta_{2i} \in \mathbb{R}^n$, $\hat{w}_i \in \mathbb{R}^{s_i}$, $\xi_i = \mathrm{col}(\xi_{1i}, \xi_{2i})$ with $\xi_{1i} \in \mathbb{R}^n$ and $\xi_{2i} \in \mathbb{R}^n$; $\eta_0 = x_0$ and $\xi_{20} = p_0$; α is an arbitrary positive real number; γ is a positive real number to be specified; and L_i is as defined in Remark 5. It can be verified that (16) is in the form of (6) with $\zeta_i = \mathrm{col}(\xi_i, \hat{\omega}_i, \eta_i)$, $i = 1, \dots, N$.

In the distributed position feedback control law (16), Equation (16a) is the control input of the ith vehicle; Equations (16b) and (16c) are linear observers for the states of the vehicle and the external disturbance, respectively; and Equation (16d) is the distributed observer for the target.

Let $\overline{\xi}_i = \xi_i - x_i$, $\overline{\omega}_i = \hat{\omega}_i - \omega_i$, $i = 1, \dots, N$, and $\overline{\eta}_i = \eta_i - x_0$, $i = 0, 1, \dots, N$. Then, the closed-loop system composed of (11) and (16) is

$$\dot{\overline{q}}_i = \overline{p}_i \tag{17a}$$

$$\dot{\overline{p}}_i = -\left(\overline{q}_i - \overline{\eta}_{1i}\right) - \alpha \sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)\nabla_{\overline{q}_i}\psi\left(\left\|\overline{q}_i - \overline{q}_j\right\|\right) - \sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)\nabla_{\overline{q}_i}\rho\left(\left\|\overline{q}_i - \overline{q}_j\right\|\right) - D_i\overline{w}_i - \sum_{j\in\overline{\mathcal{N}}_i(t)} a_{ij}(t)\left(\overline{p}_i - \overline{p}_j\right) - \left(\sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)\left(\overline{\xi}_{2i} - \overline{\xi}_{2j}\right) + a_{i0}(t)\overline{\xi}_{2i}\right) + \left[S_{01} S_{02}\right]\overline{\eta}_i \tag{17b}$$

$$\begin{bmatrix} \dot{\overline{\xi}}_i \\ \dot{\overline{w}}_i \end{bmatrix} = \left(\hat{A}_i + L_i\hat{C}_i\right)\begin{bmatrix} \overline{\xi}_i \\ \overline{w}_i \end{bmatrix} \tag{17c}$$

$$\dot{\overline{\eta}}_i = S_0\overline{\eta}_i + \gamma \sum_{j\in\overline{\mathcal{N}}_i(t)} a_{ij}(t)\left(\overline{\eta}_j - \overline{\eta}_i\right),\ i = 1, \dots, N. \tag{17d}$$

Remark 6. To preserve the connectivity of the communication network and avoid inter-vehicle collisions, the distributed control law (16) makes use of the gradients of the potential functions (12) and (13). Hence, the closed-loop system (17) turns out to be a nonlinear time-varying system.

We define some matrices associated with the graph $\overline{\mathcal{G}}(t)$. For $t \geq 0$, let the Laplacian matrix of the graph $\mathcal{G}(t)$ be

$$\mathcal{L}(t) = \begin{bmatrix} \bar{a}_1(t) & -a_{12}(t) & \cdots & -a_{1N}(t) \\ -a_{21}(t) & \bar{a}_2(t) & \cdots & -a_{2N}(t) \\ \vdots & \vdots & \ddots & \vdots \\ -a_{N1}(t) & -a_{N2}(t) & \cdots & \bar{a}_N(t) \end{bmatrix}, \tag{18}$$

where, for $i = 1, \dots, N$, $\bar{a}_i(t) = \sum_{j=1, j\neq i}^{N} a_{ij}(t)$. Let $H(t) = \mathcal{L}(t) + \Delta(t)$ where $\Delta(t) = \mathrm{diag}(a_{10}(t), \dots, a_{N0}(t))$. Let

$$P_0(t) = \begin{bmatrix} H(t) \otimes I_n & \frac{\Lambda(t)}{2} \\ \frac{\Lambda^{\top}(t)}{2} & \theta I_\iota \end{bmatrix}, \tag{19}$$

where $\Lambda(t) = \begin{bmatrix} 0_{Nn\times Nn} & H(t)\otimes I_n & D \end{bmatrix}$ with $D = \text{block diag}(D_1, \dots, D_N)$, $\iota = 2Nn + s_1 + \cdots + s_N$, and θ is a positive real number such that

$$\theta > \lambda_{\max}\left(\frac{\Lambda^\top(t)}{2}\left(H^{-1}(t)\otimes I_n\right)\frac{\Lambda(t)}{2}\right),\ \forall t \geq 0. \tag{20}$$

Let

$$P(t) = \begin{bmatrix} P_0(t) & Z \\ Z^\top & Y(t) \end{bmatrix}, \tag{21}$$

where $Z = \begin{bmatrix} -\frac{1}{2}I_N \otimes \begin{bmatrix} S_{01}+I_n & S_{02} \end{bmatrix} \\ 0_{\iota\times 2Nn} \end{bmatrix}$ and $Y(t) = \gamma H(t)\otimes I_{2n} - I_N \otimes \frac{1}{2}(S_0 + S_0^\top)$ with γ being a positive real number such that

$$\gamma > \frac{\lambda_{\max}\left(I_N \otimes \frac{S_0+S_0^\top}{2} + Z^\top P_0^{-1}(t) Z\right)}{\lambda_{\min}(H(t))},\ \forall t \geq 0. \tag{22}$$

Since $H(t)$ is uniquely determined by $\overline{\mathcal{G}}(t)$ and there are only a finite number of connected graphs with $N+1$ nodes, such θ satisfying (20) and γ satisfying (22) always exist.

The solvability of Problem 1 is summarized in the following theorem.

Theorem 1. *Under Assumption 1, Problem 1 is solvable by the distributed feedback control law (16) with γ being a positive real number satisfying (22) for all $t \geq 0$.*

Proof. The proof consists of the following five parts.

Part I: In this part, we determine the parameters for the control law (16).

Let $\overline{\eta} = \text{col}(\overline{\eta}_1, \dots, \overline{\eta}_N)$, $\overline{q} = \text{col}(\overline{q}_1, \dots, \overline{q}_N)$, $\overline{p} = \text{col}(\overline{p}_1, \dots, \overline{p}_N)$, $\overline{\xi} = \text{col}\left(\overline{\xi}_1, \dots, \overline{\xi}_N\right)$, $\mu_i = \text{col}\left(\overline{\xi}_i, \overline{\omega}_i\right)$, $i = 1, \dots, N$, and $\mu = \text{col}(\mu_1, \dots, \mu_N)$. Let $\overline{\mu} = \text{col}\left(\overline{\xi}_{11}, \dots, \overline{\xi}_{1N}, \overline{\xi}_{21}, \dots, \overline{\xi}_{2N}, \overline{w}_1, \dots, \overline{w}_N\right) = T\mu$ with

$$T = \begin{bmatrix} I_n & 0_{n\times n} & 0_{n\times s_1} & \cdots & 0_{n\times n} & 0_{n\times n} & 0_{n\times s_N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0_{n\times n} & 0_{n\times n} & 0_{n\times s_1} & \cdots & I_n & 0_{n\times n} & 0_{n\times s_N} \\ 0_{n\times n} & I_n & 0_{n\times s_1} & \cdots & 0_{n\times n} & 0_{n\times n} & 0_{n\times s_N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0_{n\times n} & 0_{n\times n} & 0_{n\times s_1} & \cdots & 0_{n\times n} & I_n & 0_{n\times s_N} \\ 0_{s_1\times n} & 0_{s_1\times n} & I_{s_1} & \cdots & 0_{s_1\times n} & 0_{s_1\times n} & 0_{s_1\times s_N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0_{s_N\times n} & 0_{s_N\times n} & 0_{s_N\times s_1} & \cdots & 0_{s_N\times n} & 0_{s_N\times n} & I_{s_N} \end{bmatrix}. \tag{23}$$

Note that $(T^{-1})^\top T^{-1} = I_\iota$. Therefore, T^{-1} is orthogonal.

Given $r > 0$, $\epsilon \in (0, r)$, the control law (16) is determined by the design parameter γ. Similar to the proof of Lemma 3.1 of Reference 15, it can be shown that, if θ satisfies (20) and γ satisfies (22) for all $t \geq 0$, then $P(t)$ is positive definite for all possible connected $\overline{\mathcal{G}}(t)$ with $N+1$ nodes and all $t \geq 0$. Fix such θ and γ.

Part II: Next, we show that under Assumption 1, the graph $\overline{\mathcal{G}}(t)$ is connected for all $t \geq 0$.

For this purpose, we introduce the following energy function:

$$\begin{aligned} &V(\overline{q}, \overline{p}, \mu, \overline{\eta}, t) \\ &= \frac{1}{2}\sum_{i=1}^{N}\left(\alpha \sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)\psi\left(\left\|\overline{q}_i - \overline{q}_j\right\|\right) + \sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)\rho\left(\left\|\overline{q}_i - \overline{q}_j\right\|\right) + \overline{q}_i^\top\overline{q}_i + \overline{p}_i^\top\overline{p}_i \right. \\ &\quad \left. + \overline{\eta}_i^\top\overline{\eta}_i + 2\theta\mu_i^\top \overline{P}_i \mu_i\right), \end{aligned} \tag{24}$$

where $\overline{P}_i$, $i = 1, \dots, N$, are as defined in Remark 5.

The time derivative of (24) along the trajectories of the closed-loop system (17) is

$$\begin{aligned}
\dot{V} &= \frac{1}{2}\sum_{i=1}^{N}\left(\alpha \sum_{j\in\mathcal{N}_i(t)} \dot{\psi}\left(\left\|\overline{q}_i - \overline{q}_j\right\|\right) + \sum_{j\in\mathcal{N}_i(t)} \dot{\rho}\left(\left\|\overline{q}_i - \overline{q}_j\right\|\right)\right. \\
&\quad \left. + 2\dot{\overline{q}}_i^\top \overline{q}_i + 2\dot{\overline{p}}_i^\top \overline{p}_i + 2\theta\left(\dot{\mu}_i^\top \overline{P}_i \mu_i + \mu_i^\top \overline{P}_i \dot{\mu}_i\right)\right) \\
&= -\sum_{i=1}^{N}\overline{p}_i^\top \sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)(\overline{p}_i - \overline{p}_j) - \theta\sum_{i=1}^{N}\mu_i^\top \mu_i + \sum_{i=1}^{N}\overline{p}_i^\top \left[\, S_{01} + I_n \;\; S_{02} \,\right]\overline{\eta}_i \\
&\quad - \sum_{i=1}^{N}\overline{p}_i^\top D_i \overline{w}_i - \sum_{i=1}^{N}\overline{p}_i^\top\left(\sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)\left(\overline{\xi}_{2i} - \overline{\xi}_{2j}\right) + a_{i0}(t)\overline{\xi}_{2i}\right) \\
&\quad + \sum_{i=1}^{N}\overline{\eta}_i^\top\left(S_0\overline{\eta}_i + \gamma\sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)(\overline{\eta}_j - \overline{\eta}_i)\right).
\end{aligned} \tag{25}$$

Using the notation of Kronecker product, (25) can be rewritten in the following compact form:

$$\begin{aligned}
\dot{V} &= -\overline{p}^\top (H(t)\otimes I_n)\overline{p} + \overline{p}^\top\left(I_N \otimes \left[\, S_{01} + I_n \;\; S_{02}\,\right]\right)\overline{\eta} \\
&\quad - \overline{p}^\top \Lambda(t)\overline{\mu} - \theta\overline{\mu}^\top (T^{-1})^\top T^{-1}\overline{\mu} + \overline{\eta}^\top\left(I_N \otimes \frac{S_0 + S_0^\top}{2} - \gamma H(t)\otimes I_{2n}\right)\overline{\eta} \\
&= -\begin{bmatrix}\overline{p}\\ \overline{\mu}\\ \overline{\eta}\end{bmatrix}^\top
\begin{bmatrix}
H(t)\otimes I_n & \frac{\Lambda(t)}{2} & -\frac{1}{2}I_N \otimes \left[S_{01} + I_n \quad S_{02}\right] \\
\frac{\Lambda^\top(t)}{2} & \theta I_\iota & 0_{\iota\times 2Nn} \\
-\frac{1}{2}I_N \otimes \left[S_{01} + I_n \quad S_{02}\right]^\top & 0_{2Nn\times\iota} & Y(t)\otimes I_{2n}
\end{bmatrix}
\begin{bmatrix}\overline{p}\\ \overline{\mu}\\ \overline{\eta}\end{bmatrix} \\
&= -\begin{bmatrix}\overline{p}\\ \overline{\mu}\\ \overline{\eta}\end{bmatrix}^\top P(t)\begin{bmatrix}\overline{p}\\ \overline{\mu}\\ \overline{\eta}\end{bmatrix}.
\end{aligned} \tag{26}$$

In what follows, we first show that under the control law (16), the graph $\mathcal{G}(t)$ is connected for all $t \geq 0$. Let $V(t) = V(\overline{q}(t), \overline{p}(t), \mu(t), \overline{\eta}(t), t)$. By the continuity of the solution of the closed-loop system (17), there exists a $0 < t_1 \leq \infty$ such that $\mathcal{G}(t) = \mathcal{G}(t_1)$ for all $t \in [0, t_1)$.

If $t_1 = \infty$, then $\mathcal{G}(t) = \mathcal{G}(0)$ for all $t \geq 0$, which shows that $\mathcal{G}(t)$ is connected for all $t \geq 0$ since $\mathcal{G}(0)$ is. Therefore, $P(t) = P(0)$ is positive definite for all $t \geq 0$ with the γ and θ we choose. Then,

$$V(t) \leq V(0), \;\; \forall t \geq 0. \tag{27}$$

If $t_1 < \infty$, then $\mathcal{G}(t) = \mathcal{G}(0)$ does not hold for all $t \geq 0$. In this case, without loss of generality, we can assume that t_1 is such that

$$\begin{aligned}
\mathcal{G}(t) &= \mathcal{G}(0), \;\; t \in [0, t_1), \\
\mathcal{G}(t_1) &\neq \mathcal{G}(0).
\end{aligned} \tag{28}$$

It is noted that, for a connected $\mathcal{G}(0)$, the γ and θ we choose guarantees that $P(t) = P(0)$ is positive definite for all $t \in [0, t_1)$. Then, from (26), there exists a positive real number W_1 with $W_1 < \infty$ such that

$$V(t) \leq V(0) \leq W_1, \;\; \forall t \in [0, t_1). \tag{29}$$

We claim that $\mathcal{G}(t_1) \supset \mathcal{G}(0)$. Otherwise, there exists some edge (i,j) such that $(i,j) \in \mathcal{E}(0)$ and $(i,j) \notin \mathcal{E}(t_1)$. Then, $\lim_{t \to t_1^-} \|q_i(t) - q_j(t)\| = r$, which implies that $\lim_{t \to t_1^-} V(t) = \infty$. From (29), we have $V(t) \leq W_1$ for all $t \in [0, t_1)$, which leads to a contradiction. Thus, we have $\mathcal{G}(t_1) \supset \mathcal{G}(0)$, which implies that $\mathcal{G}(t_1)$ is also connected.

If there exists a $t_2 > t_1$ such that

$$\begin{aligned} &\mathcal{G}(t) = \mathcal{G}(t_1), \ t \in [t_1, t_2), \\ &\mathcal{G}(t_2) \neq \mathcal{G}(t_1), \end{aligned} \tag{30}$$

then we claim that $\mathcal{G}(t_2) \supset \mathcal{G}(t_1)$.

We have already shown that $\mathcal{G}(t_1)$ is connected. Moreover, $P(t)$ is positive definite for all $t \in [t_1, t_2)$ with the γ and θ we choose.

A direct calculation gives

$$\psi(r - \epsilon) = \frac{1}{2\left(r^2 - (r - \epsilon)^2\right)} < \infty. \tag{31}$$

Since a t_1 satisfying (28) exists and $\mathcal{G}(t_1) \supset \mathcal{G}(0)$, there exists at least one edge (i,j) such that $(i,j) \in \mathcal{E}(t_1)$ and $(i,j) \notin \mathcal{E}(0)$. Without loss of generality, we assume that there exists τ edges $(i_1, j_1), \dots, (i_\tau, j_\tau)$ such that $(i_k, j_k) \in \mathcal{E}(t_1)$ and $(i_k, j_k) \notin \mathcal{E}(0)$ for $k \in \{1, \dots, \tau\}$, where τ is a positive integer. By (29) and (31), there exists a positive real number W_2 with $W_2 < \infty$ such that

$$V(t) \leq V(t_1) \leq V(0) + \alpha\tau\psi(r - \epsilon) \leq W_2, \ \forall t \in [t_1, t_2). \tag{32}$$

Again, we show that $\mathcal{G}(t_2) \supset \mathcal{G}(t_1)$ by contradiction. To this end, suppose $\mathcal{G}(t_2) \not\supset \mathcal{G}(t_1)$, that is, there exists some edge (i,j) such that $(i,j) \in \mathcal{E}(t_1)$ and $(i,j) \notin \mathcal{E}(t_2)$. Then, we have $\lim_{t \to t_2^-} \|q_i(t) - q_j(t)\| = r$, which implies that $\lim_{t \to t_2^-} V(t) = \infty$. From (32), $V(t) \leq W_2$ for all $t \in [t_1, t_2)$, which leads to a contradiction. Thus, the graph $\mathcal{G}(t)$ will not lose edges at time t_2, that is, $\mathcal{G}(t_2) \supset \mathcal{G}(t_1)$.

It is noted that $\mathcal{G}(t)$ can only have a finite number of edges. Thus, by repeating the above arguments, we can conclude that there exists a finite integer $k > 0$ such that

$$\begin{aligned} &\mathcal{G}(t) = \mathcal{G}(0), \ t \in [0, t_1), \\ &\mathcal{G}(t) = \mathcal{G}(t_i) \supset \mathcal{G}(t_{i-1}), \ t \in [t_i, t_{i+1}), \ i = 1, \dots, k-1, \\ &\mathcal{G}(t) = \mathcal{G}(t_k) \supset \mathcal{G}(t_{k-1}), \ t \in [t_k, \infty). \end{aligned} \tag{33}$$

Hence, the graph $\mathcal{G}(t)$ remains connected for all $t \geq 0$ under the control law (16). Under Assumption 1, we further conclude that the graph $\overline{\mathcal{G}}(t)$ is connected for all $t \geq 0$.

Part III: Next, we show that $\|q_i(t) - q_j(t)\| > \underline{r}$, $i \neq j$, $i,j = 1, \dots, N$, for all $t \geq 0$.

We have shown that $\dot{V}(t) \leq 0$ for all $t \geq t_k$. Then, from (33),

$$V(t) \leq V(t_k), \ \forall t \geq t_k. \tag{34}$$

Note that k is a finite integer, thus,

$$V(t) \leq \max_{i=1,\dots,k} V(t_i), \ \forall t \geq 0. \tag{35}$$

We claim that $\|q_i(t) - q_j(t)\| > \underline{r}$, $i \neq j$, $i,j = 1, \dots, N$, for all $t \geq 0$. We prove it by contradiction. Suppose there exists a $t_c \in (0, \infty)$ such that $\|q_i(t_c) - q_j(t_c)\| = \underline{r}$ for some i and j, $i,j = 1, \dots, N$, $i \neq j$. Then, we have $\lim_{t \to t_c^-} \rho(\|q_i(t) - q_j(t)\|) = \infty$. Since $\|q_i(t_c) - q_j(t_c)\| < r - \epsilon$, we can see that $(i,j) \in \mathcal{E}(t_c)$ according to the definition of the graph. Thus, $\lim_{t \to t_c^-} V(t) = \infty$, which contradicts (35). Hence, such a t_c does not exist, and $\|q_i(t) - q_j(t)\| > \underline{r}$, $i \neq j$, $i,j = 1, \dots, N$, for all $t \geq 0$.

Part IV: In what follows, we show that $\lim_{t \to \infty}(\dot{q}_i(t) - \dot{q}_0(t)) = 0$ for $i = 1, \dots, N$.

Recall that $V(t) \geq 0$ and it is nonincreasing for all $t \geq t_k$. Thus, $\lim_{t\to\infty} V(t)$ exists, and, for $i = 1, \ldots, N$, $\overline{p}_i(t)$, $\overline{\eta}_i(t)$, $\overline{q}_i(t)$, $\mu_i(t)$, and $\overline{q}_i(t) - \overline{q}_j(t)$ with $j \in \mathcal{N}_i(t)$ are all bounded. Thus, from (17), we can further conclude that $\dot{\overline{p}}_i(t)$, $\dot{\mu}_i(t)$, and $\dot{\overline{\eta}}_i(t)$, $i = 1, \ldots, N$, are bounded. Then, for all $t \geq t_k$, $\ddot{V}(t)$ is bounded and $\dot{V}(t)$ is uniformly continuous. Thus, by Barbalat's lemma, we have $\lim_{t\to\infty} \dot{V}(t) = 0$ which together with (26) imply that $\lim_{t\to\infty} \overline{p}_i(t) = \lim_{t\to\infty} \left(\dot{q}_i(t) - \dot{q}_0(t)\right) = 0, i = 1, \ldots, N$.

Part V: Finally, we show that $\lim_{t\to\infty} P_{q_0(t)}(q(t)) = 0$.

To this end, for $t \geq 0$, define the center of the N vehicles as

$$q_c(t) = \frac{1}{N}\sum_{i=1}^{N} q_i(t) \tag{36}$$

and the velocity of the center as

$$p_c(t) = \dot{q}_c(t) = \frac{1}{N}\sum_{i=1}^{N} p_i(t). \tag{37}$$

Furthermore, let

$$\overline{q}_c(t) = q_c(t) - q_0(t) = \frac{1}{N}\sum_{i=1}^{N}\overline{q}_i(t) = \frac{1}{N}\sum_{i=1}^{N}(q_i(t) - q_0(t)), \tag{38}$$

$$\overline{p}_c(t) = \dot{\overline{q}}_c(t) = \frac{1}{N}\sum_{i=1}^{N}\overline{p}_i(t). \tag{39}$$

Then, we show that $\lim_{t\to\infty} \overline{q}_c(t) = 0, i = 1, \ldots, N$.

For $i = 1, \ldots, N$, by a direct calculation, $\ddot{\overline{p}}_i(t)$ is bounded over $[t_k, \infty)$. Thus, $\dot{\overline{p}}(t)$ is uniformly continuous for all $t \geq t_k$. Since $\lim_{t\to\infty} \overline{p}_i(t) = 0$, by Barbalat's lemma,

$$\begin{aligned}
&\lim_{t\to\infty} \dot{\overline{p}}_i(t) \\
&= -\overline{q}_i(t) - \alpha \sum_{j\in\mathcal{N}_i(t)} \frac{\overline{q}_i(t) - \overline{q}_j(t)}{\left(r^2 - \left\|\overline{q}_i(t) - \overline{q}_j(t)\right\|^2\right)^2} + \sum_{j\in\mathcal{N}_i(t)} \frac{2\left(\overline{q}_i(t) - \overline{q}_j(t)\right)}{\left(\left\|\overline{q}_i(t) - \overline{q}_j(t)\right\|^2 - \underline{r}^2\right)^3} \\
&= 0,\ i = 1, \ldots, N.
\end{aligned} \tag{40}$$

Thus, noting (38), we have

$$\begin{aligned}
&\lim_{t\to\infty} \dot{\overline{p}}_c(t) \\
&= \lim_{t\to\infty} \frac{1}{N}\sum_{i=1}^{N}\dot{\overline{p}}_i(t) \\
&= \lim_{t\to\infty}\left(-\left(\frac{1}{N}\sum_{i=1}^{N}(q_i(t) - q_0(t))\right) - \alpha\frac{1}{N}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(t)} \frac{\overline{q}_i(t) - \overline{q}_j(t)}{\left(r^2 - \left\|\overline{q}_i(t) - \overline{q}_j(t)\right\|^2\right)^2}\right. \\
&\quad \left. + \frac{1}{N}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(t)} \frac{2\left(\overline{q}_i(t) - \overline{q}_j(t)\right)}{\left(\left\|\overline{q}_i(t) - \overline{q}_j(t)\right\|^2 - \underline{r}^2\right)^3}\right) \\
&= 0.
\end{aligned} \tag{41}$$

Since $\mathcal{G}(t)$ is undirected, for $i, j = 1, \ldots, N$ and $i \neq j, j \in \mathcal{N}_i(t) \Leftrightarrow i \in \mathcal{N}_j(t)$ for all $t \geq 0$. Thus, for all $t \geq 0$,

$$\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(t)}\frac{\overline{q}_i(t)-\overline{q}_j(t)}{\left(r^2-\left\|\overline{q}_i(t)-\overline{q}_j(t)\right\|^2\right)^2}=0 \tag{42}$$

and

$$\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(t)}\frac{2\left(\overline{q}_i(t)-\overline{q}_j(t)\right)}{\left(\left\|\overline{q}_i(t)-\overline{q}_j(t)\right\|^2-\underline{r}^2\right)^3}=0. \tag{43}$$

Then we further conclude that

$$\lim_{t\to\infty}\dot{\overline{p}}_c(t)=\lim_{t\to\infty}-\left(\frac{1}{N}\sum_{i=1}^{N}(q_i(t)-q_0(t))\right)=0. \tag{44}$$

Therefore,

$$\lim_{t\to\infty}(q_c(t)-q_0(t))=\lim_{t\to\infty}\overline{q}_c(t)=\lim_{t\to\infty}\frac{1}{N}\sum_{i=1}^{N}(q_i(t)-q_0(t))=0. \tag{45}$$

From (4), $q_c(t)\in \mathrm{co}(q(t))$ for all $t\geq 0$. Hence, $\lim_{t\to\infty}P_{q_0(t)}(q(t))=0$. The proof is thus complete. ∎

Remark 7. Unlike the control laws of References 26,30,31 that assumed all vehicles can make use of the trajectory of the target, our control law only requires at least one vehicle to know the trajectory of the target. This is possible because we employ the distributed observer (16d) to generate the estimated leader's trajectory for each vehicle. For this reason, our approach is less demanding than the approaches in References 26,30,31 in terms of communication requirements.

4 | EXAMPLE

Consider a group of four double-integrator systems of the form (1) with $n=3$ as follows:

$$\ddot{q}_i=u_i+d_i,\ \ i=1,2,3,4. \tag{46}$$

For $i=1,2,3,4$, the external disturbance $d_i\in\mathbb{R}^3$ is assumed to be generated by

$$\dot{\omega}_i=S_i\omega_i,\quad d_i=D_i\omega_i \tag{47}$$

with

$$S_1=\begin{bmatrix}0&1\\0&0\end{bmatrix},\quad S_2=1,\quad S_3=\begin{bmatrix}0&1\\-1&0\end{bmatrix},\quad S_4=\begin{bmatrix}0&1\\1&0\end{bmatrix}, \tag{48}$$

$$D_1=\begin{bmatrix}1&0\\0&1\\1&1\end{bmatrix},\quad D_2=\begin{bmatrix}0.2\\0.1\\-0.1\end{bmatrix},\quad D_3=\begin{bmatrix}1&0\\-1&1\\1&1\end{bmatrix},\quad D_4=\begin{bmatrix}1&0\\0&0\\1&1\end{bmatrix}. \tag{49}$$

By solving (47), the external disturbances can be explicitly expressed as follows:

$$d_1(t)=\begin{bmatrix}1-2t\\-2\\-2t-1\end{bmatrix},\quad d_2(t)=\begin{bmatrix}\frac{1}{5}e^t\\\frac{1}{10}e^t\\-\frac{1}{10}e^t\end{bmatrix},\quad d_3(t)=\begin{bmatrix}-\cos(t)-2\sin(t)\\-3\cos(t)-\sin(t)\\\cos(t)-3\sin(t)\end{bmatrix},\quad d_4(t)=\begin{bmatrix}e^{-t}\\-e^{-t}\\0\end{bmatrix}.$$

For this reason, the disturbances considered in this example contain linear functions, sinusoidal functions, and exponential functions of time t.

Consider a target as follows:

$$\ddot{q}_0 = S_{01} q_0 + S_{02} \dot{q}_0, \tag{50}$$

where

$$S_{01} = \begin{bmatrix} -1 & -0.09 & -0.2 \\ 0.1 & -0.08 & 0.3 \\ 0.3 & 0.1 & -0.5 \end{bmatrix}, \quad S_{02} = \begin{bmatrix} -0.13 & -0.1 & 0.1 \\ 0.2 & -0.12 & 0.3 \\ 0 & -0.2 & -0.2 \end{bmatrix}. \tag{51}$$

Let $r = 13$ be the maximum sensing range and $\epsilon = 4.5$. For inter-vehicle collision avoidance, let $\underline{r} = 1$ be the minimum distance between any two vehicles. By Theorem 1, we can design a distributed position feedback control law of the form (16) with $\alpha = 200$, $\gamma = 28561$, and L_i, $i = 1, 2, 3, 4$, such that the spectrum of $\hat{A}_1 + L_1\hat{C}_1$ is $\{-9, -7, -6, -4, -8, -6, -8, -2\}$; the spectrum of $\hat{A}_2 + L_2\hat{C}_2$ is $\{-1, -7, -5, -9, -8, -3, -6\}$; the spectrum of $\hat{A}_3 + L_3\hat{C}_3$ is $\{-2, -8, -6, -8, -9, -6, -5, -4\}$; and the spectrum of $\hat{A}_4 + L_4\hat{C}_4$ is $\{-8, -6, -5, -9, -7, -1, -3, -2\}$.

In the simulation, we let the initial conditions be

$$\begin{aligned}
x_0(0) &= \text{col}(0, 5, 2, 3, -5, 1), \\
x_1(0) &= \text{col}(-1, -1, -1, 2, 1, 2), \\
x_2(0) &= \text{col}(-2, -7, 5, 3, 3, -2), \\
x_3(0) &= \text{col}(3, -7.5, 0, -1, 2, 1), \\
x_4(0) &= \text{col}(5, 4, -3, -1, 3, -2), \\
\omega_1(0) &= \text{col}(1, -2), \\
\omega_2(0) &= 1, \\
\omega_3(0) &= \text{col}(2, -1), \\
\omega_4(0) &= \text{col}(1, -1),
\end{aligned}$$

and the components of $\eta_i(0)$, $\xi_i(0)$, $\hat{\omega}_i(0)$, $i = 1, 2, 3, 4$, are randomly generated within $[-10, 10]$. It is assumed that the 1st vehicle can access the trajectory of the target for all $t \geq 0$, that is, $(0, 1) \in \overline{\mathcal{E}}(t)$ and $a_{10}(t) = 1$ for all $t \geq 0$. Then, it can be verified that a connected graph $\overline{\mathcal{G}}(0)$ is formed under the above initial conditions with edge set $\overline{\mathcal{E}}(0) = \{(0, 1), (1, 3), (3, 1), (1, 4), (4, 1), (2, 3), (3, 2)\}$. For $t > 0$, the state-dependent graph $\overline{\mathcal{G}}(t)$ is updated according to the positions of the vehicles and the target at time t.

Under the above conditions, the simulation results for the closed-loop system composed of the plant (46) and the control law (16) are shown in Figures 1–9. Figures 1 and 2 show the trajectories of the control inputs of the vehicles. Figure 3 shows that the center of the four vehicles asymptotically converges to the position of the target. Figure 4 shows that the velocities of the vehicles asymptotically converge to the velocity of the target. Figure 5 shows the distances between any two vehicles. It can be seen from Figure 5 that the distances between any two initially connected vehicles remain smaller than the maximum sensing range $r = 13$, which implies that the connectivity of the state-dependent graph $\mathcal{G}(t)$ is preserved. Furthermore, the minimum distance between any two distinct vehicles is 1.0438, which is greater than $\underline{r} = 1$. Hence, inter-vehicle collisions are avoided. Figures 6–8 show the estimation errors of the states of the vehicles, the trajectory of the target, and the external disturbances, respectively. It can be seen that the estimation errors all converge to zero as desired. Figure 9 shows the time profiles of the positions of the target and the vehicles. Desired performance is observed as the target is captured by the convex hull of the vehicles.

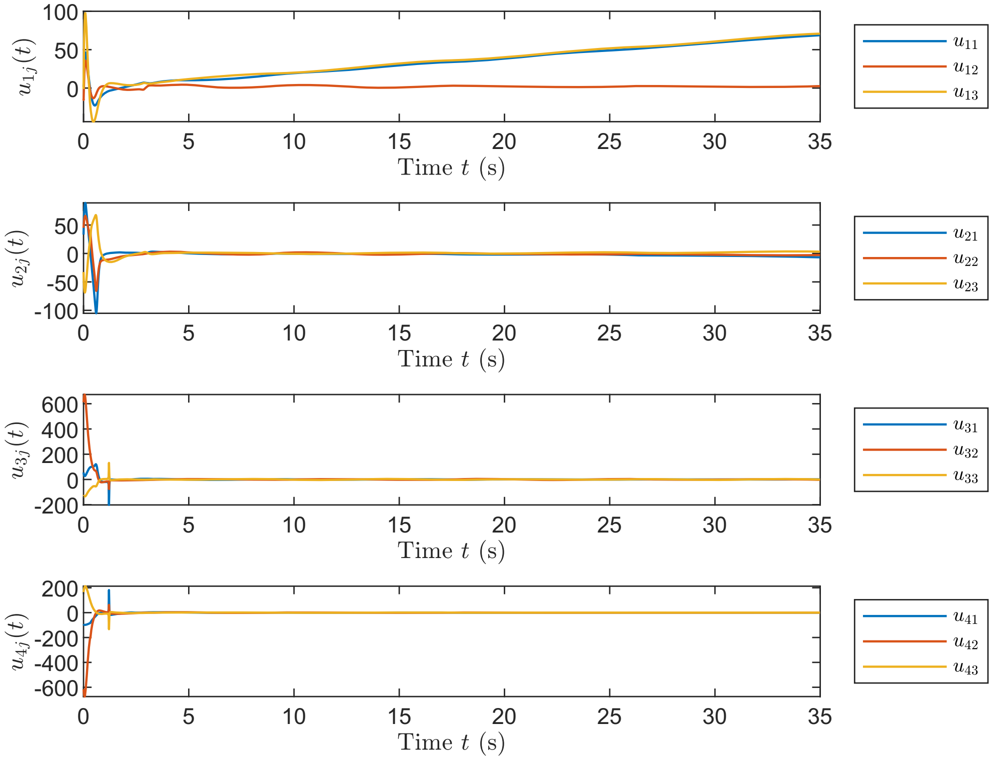

FIGURE 1 Control inputs of the vehicles.

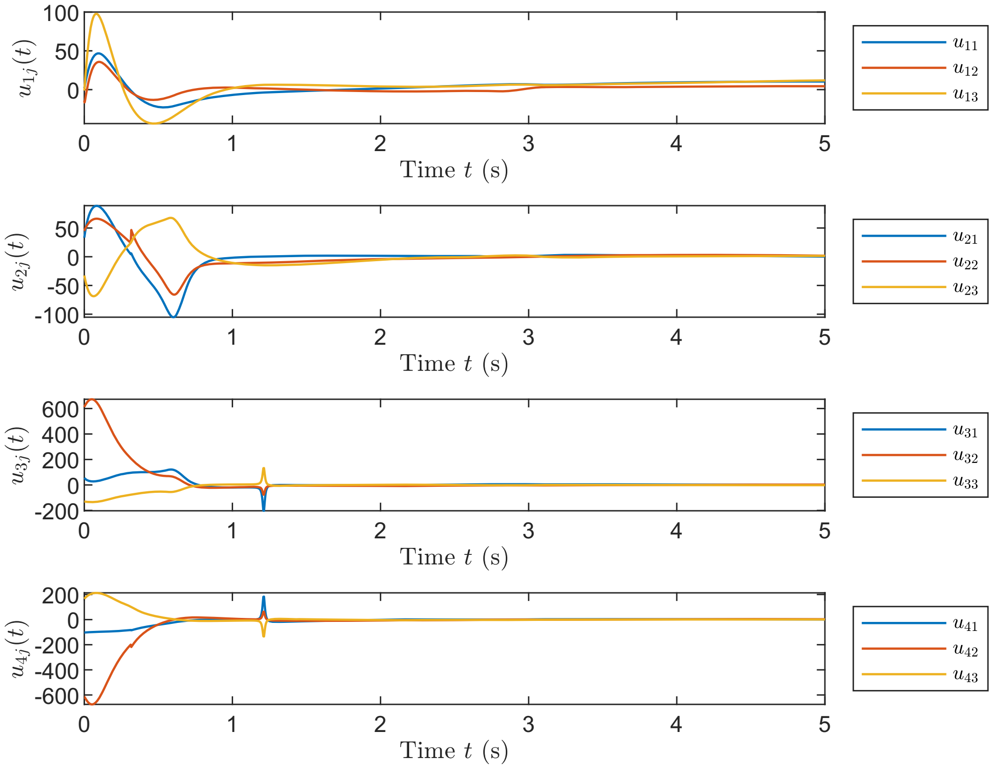

FIGURE 2 Control inputs of the vehicles during $t \in [0, 5]$.

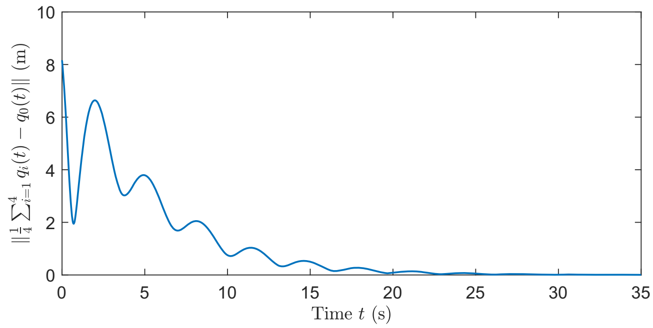

FIGURE 3 Distance between the center of the vehicles and the target.

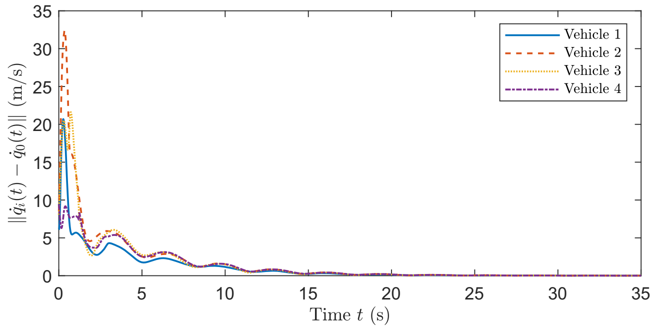

FIGURE 4 Velocity tracking errors of the vehicles.

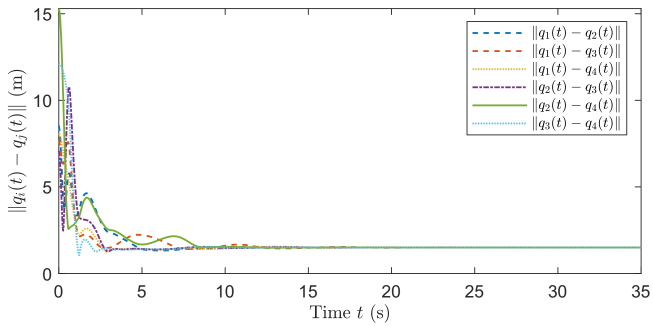

FIGURE 5 Distances between any two vehicles.

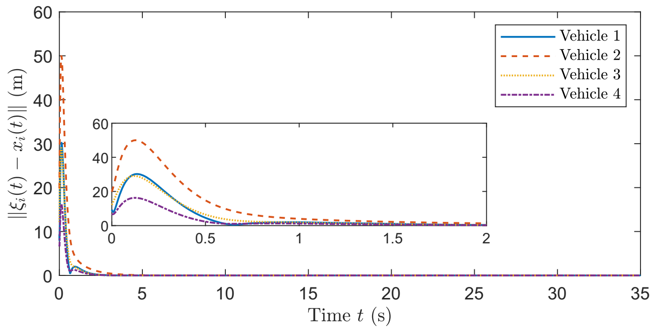

FIGURE 6 Estimation errors of the states of the vehicles.

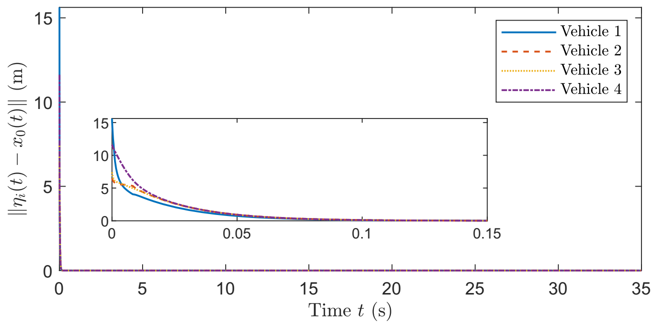

FIGURE 7 Estimation errors of the trajectory of the target.

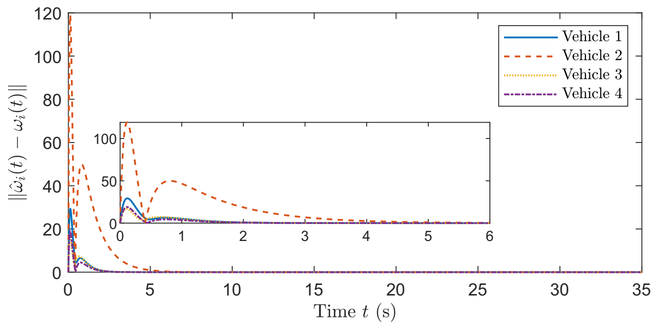

FIGURE 8 Estimation errors of the external disturbances.

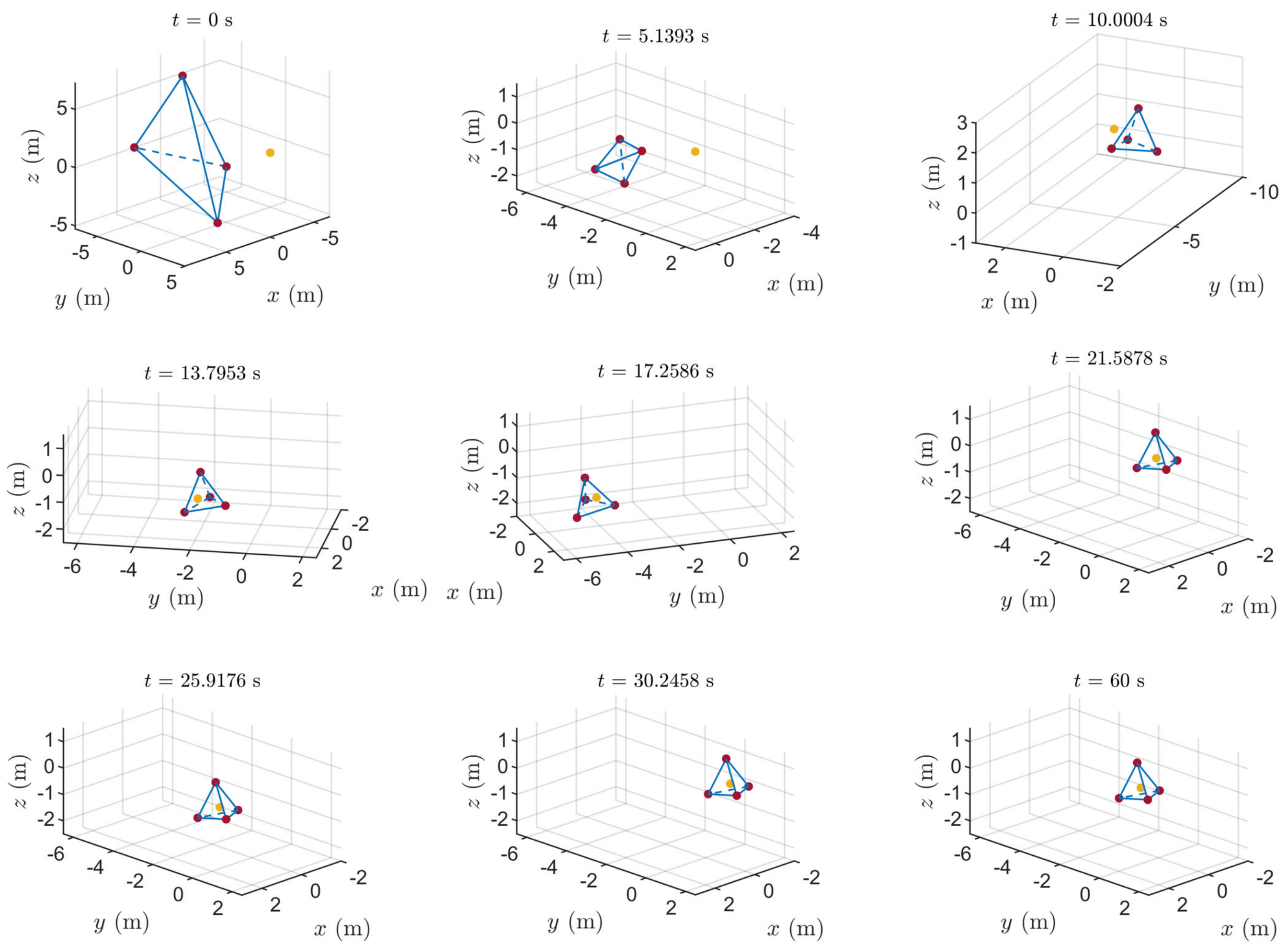

FIGURE 9 Positions of the target and the vehicles.

5 | CONCLUSION

In this article, we have studied the cooperative target-fencing problem of multiple double-integrator systems over a state-dependent communication network. By employing a distributed observer for the target, we have synthesized a distributed position feedback control law to solve the cooperative target-fencing problem. In particular, our approach does not require all vehicles to know the trajectory of the target and is capable of preserving the connectivity of the communication network while avoiding inter-vehicle collisions.

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CONFLICT OF INTEREST STATEMENT
The authors declare that they have no conflict of interest.

DATA AVAILABILITY STATEMENT
Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

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