Semiglobal Leader-Following Output Consensus of Discrete-Time Heterogeneous Linear Systems Subject to Actuator Position and Rate Saturation

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***Abstract*—Motivated by the fact that physical systems are usually subject to actuator position and rate saturation, we consider in this article the semiglobal leader-following output consensus problem of a group of discrete-time heterogeneous linear systems with position and rate-limited actuators by output feedback control law. Distributed observers are designed for followers to estimate the state of the leader and the follower itself, based on which a novel distributed control law is proposed. We show that, via the low gain feedback design technique and output regulation theory, the semiglobal leader-following output consensus of discrete-time heterogeneous linear systems can be achieved by the consensus protocol if each follower is reachable from the leader in a directed communication topology.**

***Index Terms*—Actuator position and rate saturation, low gain feedback, multiagent systems, output regulation.**

I. Introduction

Consensus or leader-following consensus, a fundamental coordination control problem for a group of multiagent systems, has gained much attention in the academic community. Many efforts have been made to tackle such a problem for the groups of linear systems [1], [2], [3], Euler–Lagrange systems [4], and other nonlinear systems [5], [6]. Moreover, consensus-based methods can be extended to solve more practical problems, such as formation control [7], [8] and economic dispatch control [9]. The majority of this literature, however, does not take into consideration of actuator limitations.

In the case of position-limited actuators for multiagent systems, a few results on the global consensus were obtained, see, for example, [10] for continuous-time systems and [11] for discrete-time systems. In both results, state feedback and output feedback nonlinear control laws were established for general linear systems, whereas Meng et al. [12] established linear local feedback laws for neutrally stable systems and double integrator systems. Several global consensus control laws were constructed in [13] and [14] by event-triggered approach. In the work of Zhou and Chen [15] where formation-containment control of Euler–Lagrange systems was studied, the leaders are subject to position-limited actuators, while the followers are not. Zhu et al. [16] achieved consensus tracking of multiple second-order systems by bounded nonlinear controllers. A crucial issue related to the global consensus of general systems with position-limited actuators is that, in general, linear feedback control laws cannot achieve global convergence. In view of this situation, a semiglobal consensus was achieved by the low-gain feedback design technique (see Lin [17]) for both continuous-time and discrete-time linear systems subject to actuator position saturation. The low-gain feedback design technique was originally constructed to solve the semiglobal asymptotically stabilization problem for a linear system, see, for example, [18] and [19]. In recent years, the technique has been applied to the coordination control of a group of multiagent systems (see [20] and [21]). The low gain feedback design technique is of great significance in guaranteeing the control input to remain unsaturated by tuning the low gain parameter small enough, given any arbitrarily large and bounded set of initial conditions.

On the other hand, rate saturation is an inevitable part of actuators. For instance, rate saturation has been identified as a contributing factor to the mishaps of YF-22 [22] and Gripen [23]. Further discussions of destabilizing effects of actuator rate saturation were given in [24]. It can be seen that rate-limited actuators may lead to more severe cases when they are also subject to position saturation. The position and rate-limited case were first studied by Lin [25] to solve the semiglobal stabilization problem of a linear system with the open-loop system being stabilizable and all its poles located at the closed left-half complex plane. In [26], the discrete-time counterpart of [25], semiglobal stabilization was achieved by a linear feedback control law if a discrete-time linear system is asymptotically null controllable. Recently, the results were extended to solve the semiglobal leader-following consensus [27] and containment problem [28] of linear systems in the continuous-time domain. However, the systems in [27] and [28] are homogeneous multiagent systems in the continuous-time domain, whereas in this article, we consider heterogeneous systems in the discrete-time setting.

Inspired by our recent work [29] for the continuous-time systems, we investigate in this article its discrete-time counterpart, that is, discrete-time semiglobal leader-following output consensus problem of a class of heterogeneous linear systems in the presence of actuator position and rate saturation. We should note that the control laws in [29] are constructed based on singular perturbation theory, in which a low gain feedback law of the dynamics induces a slow subsystem, and a high gain feedback law with the actuator state and the leader's state induces a fast subsystem. These laws cannot be directly extended to discrete-time systems as the largest gain one can push in the discrete-time setting is that of deadbeat control. We focus in this article to construct an output feedback consensus protocol, in which a set of distributed observers are designed for each follower to estimate the state of the leader and that of the follower itself. It allows the actuator rate to converge to the value it should be when the consensus problem is achieved. Via the low gain approach and output regulation theory, the problem is solved if each follower is reachable from the leader in a directed topology.

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The rest of this article is organized as follows. Section II gives the designed distributed observers and the definition of semiglobal output feedback-based leader-following output consensus problem for discrete-time heterogeneous linear systems with position and rate-limited actuators. The main result is shown in Section III. An illustrative example is given in Section IV. Finally, Section V concludes this article with some remarks.

Notations: For a time constant $T \geq 0$ and a signal $x(t): \mathbb{R}_+ \to \mathbb{R}^s$, $x(t) = [x_1(t), x_2(t), \ldots, x_s(t)]^{\mathrm{T}}$ is a column vector, $|x(t)|$ denotes the Euclidean norm at time t, $\|x(t)\|_\infty = \max_i |x_i(t)|$, and $\|x(t)\|_{T,\infty} = \sup_{t \geq T} |x(t)|$. Throughout this article, we denote $x(t)$, $t \in \mathbb{Z}^+$, by a shorthand notation x when no confusion will occur. For integers M and N, $I[M, N] = \{M, M+1, M+2, \ldots, N\}$. $\lambda_{\max}(X)$ represents the maximum eigenvalue of the symmetric matrix X. $\mathrm{Re}(\lambda)$ is the real parts of the constant λ. $\mathbf{1}_N \in \mathbb{R}^N$ denotes a vector with all elements being 1. $I_N \in \mathbb{R}^{N \times N}$ is the identity matrix. Kronecker product is denoted by $\otimes$. X^{T} stands for the transpose of the vector or matrix X. $\mathbf{0}$ represents a vector or matrix of zero with appropriate dimension.

II. Preliminaries and Problem Formulation

Consider a group of $N+1$ discrete-time heterogeneous systems consisting of a leader and N followers. The leader, labeled as 0, is described as

$$\begin{cases} w(t+1) = Sw \\ y_0(t) = -Qw \end{cases} \tag{1}$$

where $w \in \mathbb{R}^s$ and $y_0 \in \mathbb{R}^m$ are the state and output, respectively. Similar to the system in [26], the dynamics of the ith follower, $i \in I[1, N]$, subject to actuator position and rate saturation, is described by the following equation:

$$\begin{cases} x_i(t+1) = A_i x_i + B_i \sigma_p(v_i) + W_i w \\ v_i(t+1) = v_i + \sigma_r\left((\alpha_i - 1)v_i + u_i\right), \ |\alpha_i| < 1 \\ y_i(t) = C_i x_i \\ e_i(t) = C_i x_i + Qw \end{cases} \tag{2}$$

where $x_i \in \mathbb{R}^{n_i}$, $y_i \in \mathbb{R}^m$, and $u_i \in \mathbb{R}^{q_i}$ are, respectively, the plant state, output, and control input of the ith follower. The second equation denotes the actuator dynamics with state $v_i \in \mathbb{R}^{q_i}$. α_i denotes the time constant of the actuator. $e_i \in \mathbb{R}^m$ denotes output tracking error between the ith follower and the leader. The leader generates both the output to be tracked y_0 and the disturbances to be rejected $W_i w$. Here, $\sigma_p(\cdot), \sigma_r(\cdot) : \mathbb{R}^{q_i} \to \mathbb{R}^{q_i}$ represent vector valued saturation functions. For $v_i = [v_{i,1}, v_{i,2}, \ldots, v_{i,q_i}]^{\mathrm{T}}$, $\sigma_p(v_i) = [\sigma_p(v_{i,1}), \sigma_p(v_{i,2}), \ldots, \sigma_p(v_{i,q_i})]^{\mathrm{T}}$. For each $j = 1, 2, \ldots, q_i$, $\sigma_p(v_{i,j}) = \mathrm{sgn}(v_{i,j}) \min\{|v_{i,j}|, p\}$ is the standard saturation function.

We define a graph (see [30] for a summary of digraph) $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with $\mathcal{V} = \{0, 1, \ldots, N\}$ and $\mathcal{E} = \mathcal{V} \times \mathcal{V}$. For $i, j \in \mathcal{V}$, $(j, i) \in \mathcal{E}$ if and only if node i has access to the information of node j. We use $\mathcal{N}_i := \{j : (j, i) \in \mathcal{E}\}$ to represent the set of neighbors of node i. As in [20], depending on whether or not the followers have access to the information of the leader, the followers are divided into two classes. The informed ones (the first l followers) can obtain the information of the leader, whereas the uninformed ones (the left $N-l$ followers) cannot. The Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{(N+1)\times(N+1)}$ is defined as $l_{ij} = -a_{ij}$ if $i \neq j$, and $l_{ii} = \sum_{j=0}^{N} a_{ij}$. According to the classification of the leader, the informed followers and the uninformed followers, $\mathcal{L}$ can be partitioned as

$$\mathcal{L} = \begin{bmatrix} 0 & \mathbf{0} \\ \mathcal{L}_{fl} & \mathcal{L}_{ff} \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{0} & \mathbf{0} \\ \mathcal{L}_1 & \mathcal{L}_2 & \mathcal{L}_3 \\ \mathbf{0} & \mathcal{L}_4 & \mathcal{L}_5 \end{bmatrix} \tag{3}$$

where $\mathcal{L}_{fl} \in \mathbb{R}^{N\times 1}$, $\mathcal{L}_{ff} \in \mathbb{R}^{N\times N}$, $\mathcal{L}_1 \in \mathbb{R}^{l\times 1}$, $\mathcal{L}_2 \in \mathbb{R}^{l\times l}$, $\mathcal{L}_3 \in \mathbb{R}^{l\times(N-l)}$, $\mathcal{L}_4 \in \mathbb{R}^{(N-l)\times l}$, and $\mathcal{L}_5 \in \mathbb{R}^{(N-l)\times(N-l)}$.

Assumption 1: The graph $\mathcal{G}$ contains a spanning tree with node 0 as the root.

Assumption 2: All eigenvalues of S are located inside or on the unit circle, and those eigenvalues on the unit circle are semisimple.

Assumption 3: For $i \in I[1, N]$, (A_i, B_i) is stabilizable, and all eigenvalues of A_i are located inside or on the unit circle.

Assumption 4: For $i \in I[1, N]$, the following regulator Equation

$$\begin{aligned} \Pi_i S &= A_i \Pi_i + B_i \Gamma_i + W_i \\ C_i \Pi_i &+ Q = 0 \end{aligned} \tag{4}$$

have a pair of solutions $\Pi_i \in \mathbb{R}^{n_i \times s}$ and $\Gamma_i \in \mathbb{R}^{q_i \times s}$.

Assumption 5: (S, Q) is detectable, and for $i \in I[1, N]$, (A_i, C_i) is detectable.

Assumption 6: For each $i \in I[1, N]$, there exists a time $T \geq 0$ such that $\|\Gamma_i w\|_{T,\infty} < p$ and $\|\Gamma_i S w\|_{T,\infty} < r$.

Remark 1: By Assumption 1, all eigenvalues of $\mathcal{L}_{ff}$ and $\mathcal{L}_5$ have positive real parts (see [31, Lemma 4] and [32, Lemma 4], respectively.) Under Assumption 2, $\|w\|$ is bounded. $\Gamma_i w$ and $\Gamma_i S w$ can be viewed as the generalized actuator position and rate of the leader. If the actuator position or rate of each follower is less than the actuator position or rate of the leader, it is impossible for the followers to catch up the leader when it moves at its maximal pace. Note that $\Gamma_i w(t+1) = \Gamma_i S w(t)$; therefore, $\|\Gamma_i w\|_{T,\infty} = \|\Gamma_i S w\|_{T,\infty}$ for a time $T \geq 0$. For simplicity, we denote $\delta = \min\{p, r\}$.

Lemma 1: (See Saberi et al. [33].) Let Assumption 3 hold. Then, for any $\epsilon > 0$, for $i \in I[1, N]$, there exists a unique positive definite matrix $P_i(\epsilon) \in \mathbb{R}^{n_i \times n_i}$, $i \in I[1, N]$, of the following parameteric algebraic Riccati equation:

$$P_i = A_i^{\mathrm{T}} P_i A_i - A_i^{\mathrm{T}} P_i B_i (B_i^{\mathrm{T}} P_i B_i + I)^{-1} B_i^{\mathrm{T}} P_i A_i + \epsilon I \tag{5}$$

and $A_i - B_i (B_i^{\mathrm{T}} P_i B_i + I)^{-1} B_i^{\mathrm{T}} P_i A_i$ is Schur. Moreover, $\lim_{\epsilon \to 0} P_i(\epsilon) = 0$.

For convenience, we denote $P_i := P_i(\epsilon)$ hereafter. Define

$$K_i = (B_i^{\mathrm{T}} P_i B_i + I)^{-1} B_i^{\mathrm{T}} P_i A_i. \tag{6}$$

Since $\lim_{\epsilon \to 0} P_i(\epsilon) = 0$, it is obvious that $\lim_{\epsilon \to 0} K_i(\epsilon) = 0$. Moreover, by (5), it can be verified that

$$(A_i - B_i K_i)^{\mathrm{T}} P_i (A_i - B_i K_i) - P_i = \epsilon I - K_i^{\mathrm{T}} K_i. \tag{7}$$

For real systems, the states of the leader w and followers x_i may not be measured and only the outputs y_i are available. In such a situation, an output feedback-based consensus problem is formulated instead. To estimate the state of the leader agent, we construct the following observers for the informed followers and uninformed followers:

$$\hat{w}_i(t+1) = S\hat{w}_i + L_S(y_0 + Q\hat{w}_i), \ i \in I[1, l] \tag{8}$$

$$\hat{w}_i(t+1) = S\hat{w}_i + \mu S \sum_{j=1}^{N} a_{ij}(\hat{w}_j - \hat{w}_i), \ i \in I[l+1, N] \tag{9}$$

where L_S is a matrix such that $S + L_S Q$ is Schur, and μ satisfies

$$0 < \mu < \frac{2}{\rho^*(\mathcal{L}_5)} \tag{10}$$

where $\mathcal{L}_5$ is defined in (3) and $\rho^*(\mathcal{L}_5) = \max_{\lambda \in \chi(\mathcal{L}_5)} \{\frac{|\lambda^2|}{Re(\lambda)}\}$, $\chi(\mathcal{L}_5)$ denotes the spectrum of the square matrix $\mathcal{L}_5$.

We also construct the following dynamic compensator for each follower to estimate the state of its own:

$$\hat{x}_i(t+1) = A_i\hat{x}_i + L_{A,i}(C_i\hat{x}_i - y_i) + Bv_i + W_i\hat{w}_i,\ i = 1,\ldots,N \tag{11}$$

where $L_{A,i}$ is a matrix such that $A_i + L_{A,i}C_i$ is Schur. Such a $L_{A,i}$ exists due to Assumption 5.

Problem 1: Consider a multiagent system consisting of the leader (1) and the followers (2). Assume that Assumptions 1–6 hold. For a priori given bounded sets $\mathcal{X}_{i,0} \subset \mathbb{R}^{n_i}$, $\mathcal{V}_{i,0} \subset \mathbb{R}^{q_i}$, $\mathcal{W}_0 \subset \mathbb{R}^s$, $\hat{\mathcal{X}}_{i,0} \subset \mathbb{R}^{n_i}$, and $\hat{\mathcal{W}}_{i,0} \subset \mathbb{R}^s$, construct an output feedback consensus protocol $u_i = f_i(\hat{x}_i, v_i, \hat{w}_i)$ for each follower, based on the distributed state observers (8)–(11), such that for $[x_i^{\mathrm{T}}(0), v_i^{\mathrm{T}}(0), w^{\mathrm{T}}(0), \hat{x}_i^{\mathrm{T}}(0), \hat{w}_i^{\mathrm{T}}(0)]^{\mathrm{T}} \in \mathcal{X}_{i,0} \times \mathcal{V}_{i,0} \times \mathcal{W}_0 \times \hat{\mathcal{X}}_{i,0} \times \hat{\mathcal{W}}_{i,0}$, the leader-following output consensus is achieved, that is, for any $i \in I[1,N]$, $\lim_{t\to\infty} e_i = 0$.

It is noted that the control law is not fully distributed because the parameter μ in the observer (9) is dependent of the communication topology, as showed in (10). Nonfully distributed control laws are also constructed in, for instance, [34] and [35].

III. Main Results

In this section, we propose the following output feedback consensus protocol for Problem 1. For $i \in I[1,N]$

$$u_i = -K_iA_i(\hat{x}_i - \Pi_i\hat{w}_i) - K_iB_i(v_i - \Gamma_i\hat{w}_i) + \Gamma_iS\hat{w}_i - \alpha_iv_i \tag{12}$$

where $\hat{w}_i$ is the state of the observers (8) and (9), $\hat{x}_i$ is the state of the observer (11), Π_i and Γ_i are a pair of solutions of the regulator (4), and K_i is defined in (6).

Then, we have the following theorem.

Theorem 1: Consider a multiagent system consisting of the leader (1) and the followers (2). Assume that Assumptions 1–6 hold. Then, the consensus protocol (12) solves Problem 1. That is, for any a priori given bounded sets $\mathcal{X}_{i,0}$, $\mathcal{V}_{i,0}$, $\mathcal{W}_0$, $\hat{\mathcal{W}}_{i,0}$, and $\hat{\mathcal{X}}_{i,0}$, there exists an $\epsilon^* \in (0,1]$ such that for any $\epsilon \in (0,\epsilon^*]$ and for all $[x_i^{\mathrm{T}}(0), v_i^{\mathrm{T}}(0), w^{\mathrm{T}}(0), \hat{w}_i^{\mathrm{T}}(0), \hat{x}_i^{\mathrm{T}}(0)]^{\mathrm{T}} \in \mathcal{X}_{i,0} \times \mathcal{V}_{i,0} \times \mathcal{W}_0 \times \hat{\mathcal{W}}_{i,0} \times \hat{\mathcal{X}}_{i,0}$, the output consensus error satisfies $\lim_{t\to\infty} e_i = 0$.

Proof: For $i \in I[1,N]$, let $\tilde{w}_i = \hat{w}_i - w$, $i \in I[1,N]$, represent the leader state estimation error for the ith follower. Denote $\tilde{w}_a = [\tilde{w}_1^{\mathrm{T}}, \tilde{w}_2^{\mathrm{T}}, \ldots, \tilde{w}_l^{\mathrm{T}}]^{\mathrm{T}}$ and $\tilde{w}_b = [\tilde{w}_{l+1}^{\mathrm{T}}, \tilde{w}_{l+2}^{\mathrm{T}}, \ldots, \tilde{w}_N^{\mathrm{T}}]^{\mathrm{T}}$. It follows from (8) and (9) that

$$\begin{aligned}\tilde{w}_a(t+1) &= (I_l \otimes (S + L_SQ))\,\tilde{w}_a\\ \tilde{w}_b(t+1) &= (I_{N-l} \otimes S - \mu\mathcal{L}_5 \otimes I_s)\tilde{w}_b - \mu(\mathcal{L}_4 \otimes I_s)\tilde{w}_a.\end{aligned} \tag{13}$$

Since $S + L_SQ$ is Schur and $0 < \mu < \frac{2}{\rho^*(\mathcal{L}_5)}$, we have $\lim_{t\to\infty}\tilde{w}_i = 0$ for any $\tilde{w}_i(0)$ and all $i \in I[1,N]$.

Let $\bar{x}_i = \hat{x}_i - x_i$, $i \in I[1,N]$, be the state estimation error of the i-th follower itself. By (2) and (11), we have

$$\bar{x}_i(t+1) = (A_i + L_{A,i}C_i)\bar{x}_i + B_i[v_i - \sigma_p(v_i)] + W_i\tilde{w}_i. \tag{14}$$

Because $A_i + L_{A,i}C_i$ is Schur, for any positive definite matrix $N_i \in \mathbb{R}^{n_i\times n_i}$, there exists a unique positive definite solution $M_i \in \mathbb{R}^{n_i\times n_i}$ of the following equation:

$$(A_i + L_{A,i}C_i)^{\mathrm{T}}M_i(A_i + L_{A,i}C_i) - M_i = -N_i.$$

Denoting $\tilde{x}_i = x_i - \Pi_iw$, it follows that:

$$\begin{aligned}\tilde{x}_i(t+1) &= A_ix_i + B_i\sigma_p(v_i) + W_iw - \Pi_iSw\\ &= A_i\tilde{x}_i + B_i\sigma_p(v_i) - B_i\Gamma_iw\end{aligned} \tag{15}$$

where the last equality holds due to Assumption 4.

For each follower, we define the following Lyapunov function candidate. For $i \in I[1,N]$:

$$\begin{aligned}V_i &= \tilde{x}_i^{\mathrm{T}}P_i\tilde{x}_i + \lambda_{\max}(P_i)\bar{x}_i^{\mathrm{T}}M_i\bar{x}_i\\ &\quad + 4\left(K_i\tilde{x}_i + v_i - \Gamma_iw\right)^{\mathrm{T}}\left(K_i\tilde{x}_i + v_i - \Gamma_iw\right).\end{aligned} \tag{16}$$

Along the trajectories of systems (1), (2), and (13)–(15), we have

$$\begin{aligned}\Delta V_i &= V_i(t+1) - V_i(t)\\ &= \tilde{x}_i^{\mathrm{T}}(t+1)P_i\tilde{x}_i(t+1) - \tilde{x}_i^{\mathrm{T}}P_i\tilde{x}_i\\ &\quad + \lambda_{\max}(P_i)\bar{x}_i^{\mathrm{T}}(t+1)M_i\bar{x}_i(t+1) - \lambda_{\max}(P_i)\bar{x}_i^{\mathrm{T}}M_i\bar{x}_i\\ &\quad + 4\left[K_i\tilde{x}_i(t+1) + v_i(t+1) - \Gamma_iw(t+1)\right]^{\mathrm{T}}\\ &\quad \times \left[K_i\tilde{x}_i(t+1) + v_i(t+1) - \Gamma_iw(t+1)\right]\\ &\quad - 4(K_i\tilde{x}_i + v_i - \Gamma_iw)^{\mathrm{T}}(K_i\tilde{x}_i + v_i - \Gamma_iw).\end{aligned} \tag{17}$$

Denote

$$\begin{aligned}\Theta_{i,1} &= \tilde{x}_i^{\mathrm{T}}(t+1)P_i\tilde{x}_i(t+1) - \tilde{x}_i^{\mathrm{T}}P_i\tilde{x}_i\\ \Theta_{i,2} &= \left[K_i\tilde{x}_i(t+1) + v_i(t+1) - \Gamma_iw(t+1)\right]^{\mathrm{T}}\\ &\quad \times \left[K_i\tilde{x}_i(t+1) + v_i(t+1) - \Gamma_iw(t+1)\right]\\ &\quad - (K_i\tilde{x}_i + v_i - \Gamma_iw)^{\mathrm{T}}(K_i\tilde{x}_i + v_i - \Gamma_iw)\\ \Theta_{i,3} &= \lambda_{\max}(P_i)\bar{x}_i^{\mathrm{T}}(t+1)M_i\bar{x}_i(t+1) - \lambda_{\max}(P_i)\bar{x}_i^{\mathrm{T}}M_i\bar{x}_i.\end{aligned} \tag{18}$$

Thus,

$$\Delta V_i = \Theta_{i,1} + 4\Theta_{i,2} + \Theta_{i,3}.$$

More specifically,

$$\begin{aligned}\Theta_{i,1} &= \tilde{x}_i^{\mathrm{T}}A_i^{\mathrm{T}}P_i\tilde{x}_i - \tilde{x}_i^{\mathrm{T}}P_i\tilde{x}_i + \tilde{x}_i^{\mathrm{T}}A_i^{\mathrm{T}}P_iB_i[\sigma_p(v_i) - \Gamma_iw]\\ &\quad + [B_i\sigma_p(v_i) - B_i\Gamma_iw]^{\mathrm{T}}P_i[A_i\tilde{x}_i + B_i\sigma_p(v_i) - B_i\Gamma_iw]\\ &= \tilde{x}_i^{\mathrm{T}}\left[-\epsilon I - K_i^{\mathrm{T}}K_i + A_i^{\mathrm{T}}P_iB_iK_i + K_i^{\mathrm{T}}B_i^{\mathrm{T}}P_iA_i\right.\\ &\quad \left.-K_i^{\mathrm{T}}B_i^{\mathrm{T}}P_iB_iK_i\right]\tilde{x}_i + \tilde{x}_i^{\mathrm{T}}A_i^{\mathrm{T}}P_iB_i[\sigma_p(v_i) - \Gamma_iw]\\ &\quad + [B_i\sigma_p(v_i) - B_i\Gamma_iw]^{\mathrm{T}}P_i[A_i\tilde{x}_i + B_i\sigma_p(v_i) - B_i\Gamma_iw]\end{aligned} \tag{19}$$

$$\begin{aligned}&= -\epsilon\tilde{x}_i^{\mathrm{T}}\tilde{x}_i - \tilde{x}_i^{\mathrm{T}}K_i^{\mathrm{T}}K_i\tilde{x}_i\\ &\quad + [\sigma_p(v_i) - \Gamma_iw + K_i\tilde{x}_i]^{\mathrm{T}}B_i^{\mathrm{T}}P_iB_i[\sigma_p(v_i) - \Gamma_iw + K_i\tilde{x}_i]\\ &\quad + 2\tilde{x}_i^{\mathrm{T}}(A_i - B_iK_i)^{\mathrm{T}}P_iB_i[\sigma_p(v_i) - \Gamma_iw + K_i\tilde{x}_i]\\ &= -\epsilon\tilde{x}_i^{\mathrm{T}}\tilde{x}_i - \tilde{x}_i^{\mathrm{T}}K_i^{\mathrm{T}}K_i\tilde{x}_i\\ &\quad + [\sigma_p(v_i) - \Gamma_iw + K_i\tilde{x}_i]^{\mathrm{T}}B_i^{\mathrm{T}}P_iB_i[\sigma_p(v_i) - \Gamma_iw + K_i\tilde{x}_i]\\ &\quad + 2\tilde{x}_i^{\mathrm{T}}K_i^{\mathrm{T}}[\sigma_p(v_i) - \Gamma_iw + K_i\tilde{x}_i]\end{aligned} \tag{20}$$

where (19) and (20) hold because of (7) and the definition of K_i in (6), respectively.

$$\begin{aligned}\Theta_{i,2} &= (K_iA_i\tilde{x}_i + K_iB_i\sigma_p(v_i) - K_iB_i\Gamma_iw + v_i\\ &\quad + \sigma_r(u_i + \alpha_iv_i - v_i) - \Gamma_iSw)^{\mathrm{T}}\\ &\quad \times (K_iA_i\tilde{x}_i + K_iB_i\sigma_p(v_i) - K_iB_i\Gamma_iw + v_i\\ &\quad + \sigma_r(u_i + \alpha_iv_i - v_i) - \Gamma_iSw)\\ &\quad - (K_i\tilde{x}_i + v_i - \Gamma_iw)^{\mathrm{T}}(K_i\tilde{x}_i + v_i - \Gamma_iw)\end{aligned} \tag{21}$$

where

$$\sigma_r(u_i+\alpha_i v_i - v_i) = \sigma_r\left(-K_iA_i\tilde{x}_i - K_iB_i(v_i-\Gamma_i w)+\Gamma_i Sw - v_i - K_iA_i\bar{x}_i + K_i(A_i\Pi_i+B_i\Gamma_i)\tilde{w}_i + \Gamma_i S\tilde{w}_i\right).$$

In addition,

$$\begin{aligned}\Theta_{i,3} = & -\bar{x}_i^{\mathrm{T}}N_i\bar{x}_i + 2\lambda_{\max}(P_i)\bar{x}_i^{\mathrm{T}}(A_i+L_{A,i}C_i)^{\mathrm{T}}M_i \\ & \times (B_i(v_i-\sigma_p(v_i))+W_i\tilde{w}_i) \\ & + \lambda_{\max}(P_i)\left(B_i(v_i-\sigma_p(v_i))+W_i\tilde{w}_i\right)^{\mathrm{T}}M_i \\ & \times (B_i(v_i-\sigma_p(v_i))+W_i\tilde{w}_i).\end{aligned} \tag{22}$$

Let $c_i>0$ be a constant such that

$$\sup_{[\tilde{x}_i(0),v_i(0),w(0),\bar{x}_i(0)]^{\mathrm{T}}\in\tilde{\mathcal{X}}_{i,0}\times\mathcal{V}_{i,0}\times\mathcal{W}_0\times\bar{\mathcal{X}}_{i,0}} V_i \le c_i. \tag{23}$$

The existence of such a $c_i>0$ is because of the boundedness of $\tilde{\mathcal{X}}_{i,0}$, $\mathcal{V}_{i,0}$, $\mathcal{W}_0$, and $\bar{\mathcal{X}}_{i,0}$. Define

$$L_{V_i}(c_i) := \left\{[\tilde{x}_i^{\mathrm{T}}, v_i^{\mathrm{T}}, w^{\mathrm{T}}, \bar{x}_i^{\mathrm{T}}]^{\mathrm{T}}\in\mathbb{R}^{2n_i+q_i+s} : V_i\le c_i\right\}.$$

Let $\epsilon^*\in(0,1]$ be such that, for all $\epsilon\in(0,\epsilon^*]$, $[\tilde{x}_i^{\mathrm{T}}, v_i^{\mathrm{T}}, w^{\mathrm{T}}, \bar{x}_i^{\mathrm{T}}]^{\mathrm{T}}\in L_{V_i}(c_i)$ implies that

$$\begin{aligned}&\|K_i\tilde{x}_i\|\le\frac{1}{60}\delta,\quad \|K_iA_i\tilde{x}_i\|\le\frac{1}{60}\delta \\ &\|K_iB_iv_i\|\le\frac{1}{60}\delta,\quad \|K_iB_i\sigma_p(v_i)\|\le\frac{1}{60}\delta \\ &\|K_iA_i\bar{x}_i\|\le\frac{1}{60}\delta,\quad \|B_i^{\mathrm{T}}P_iB_i\|\le\frac{12}{23}, \|K_iB_i\Gamma_i w\|\le\frac{1}{60}\delta \\ &\|K_i(A_i\Pi_i+B_i\Gamma_i)\tilde{w}_i\|\le\frac{1}{60}\delta, \|\Gamma_iS\tilde{w}_i\|\le\frac{1}{60}\delta \\ &\left\|\lambda_{\max}(P_i)\bar{x}_i^{\mathrm{T}}(A_i+L_{A,i}C_i)^{\mathrm{T}}M_i\left[B_i(v_i-\sigma_p(v_i))+W_i\tilde{w}_i\right]\right\| \\ &\qquad\le\frac{1}{10}\delta^2 \\ &\left\|\lambda_{\max}(P_i)\left[B_i(v_i-\sigma_p(v_i))+W_i\tilde{w}_i\right]^{\mathrm{T}}M_i \right. \\ &\qquad \left.\times\left[B_i(v_i-\sigma_p(v_i))+W_i\tilde{w}_i\right]\right\|\le\frac{1}{10}\delta^2.\end{aligned} \tag{24}$$

Let $k_i := K_i\tilde{x}_i$, $h_i := K_iA_i\tilde{x}_i$, $r_i := K_iB_iv_i$, $t_i := K_iB_i\sigma_p(v_i)$, $m_i := K_iB_i\Gamma_i w$, $n_i := K_i(A_i\Pi_i\tilde{w}_i + B_i\Gamma_i\tilde{w}_i)$, and $d_i := \Gamma_iS\tilde{w}_i$. We further define

$$\begin{aligned}a_i &:= 2\lambda_{\max}(P_i)\bar{x}_i^{\mathrm{T}}(A_i+L_{A,i}C_i)^{\mathrm{T}}M_iW_i\tilde{w}_i \\ &\quad + \lambda_{\max}(P_i)\tilde{w}_i^{\mathrm{T}}W_i^{\mathrm{T}}M_iW_i\tilde{w}_i \\ b_i &:= K_iA_i\bar{x}_i.\end{aligned}$$

Substituting (20)–(22) into (17), we obtain

$$\begin{aligned}\Delta V_i \le & -\epsilon\tilde{x}_i^{\mathrm{T}}\tilde{x}_i + \frac{12}{23}[\sigma_p(v_i)-\Gamma_i w + k_i]^2 - k_i^2 \\ & + 2k_i[\sigma_p(v_i)-\Gamma_i w+k_i] \\ & + 4\left([h_i+t_i-m_i+v_i-\Gamma_iSw\right. \\ & + \sigma_r(-h_i-r_i+m_i+\Gamma_iSw-v_i+n_i+d_i-b_i)]^2 \\ & \left.-(v_i-\Gamma_iw+k_i)^2\right) \\ & + \left(-\bar{x}_i^{\mathrm{T}}N_i\bar{x}_i + 2\lambda_{\max}(P_i)\bar{x}_i^{\mathrm{T}}(A_i+L_{A,i}C_i)^{\mathrm{T}}M_i\right. \\ & \times[B_iv_i - B_i\sigma_p(v_i)+W_i\tilde{w}_i] \\ & + \lambda_{\max}(P_i)[B_iv_i-B_i\sigma_p(v_i)+W_i\tilde{w}_i]^{\mathrm{T}}M_i \\ & \left.\times[B_iv_i-B_i\sigma_p(v_i)+W_i\tilde{w}_i]\right).\end{aligned} \tag{25}$$

1) If $\|v_i-\Gamma_iSw\|\le\frac{9}{10}\delta$ and $\|v_i\|\le\delta$, we have

$$\begin{aligned}& -k_i^2+2k_i[\sigma_p(v_i)-\Gamma_iw+k_i]+\frac{12}{23}[\sigma_p(v_i)-\Gamma_iw+k_i]^2 \\ & + 4\left([h_i+t_i-m_i+v_i-\Gamma_iSw\right. \\ & +\sigma_r(-h_i-r_i+m_i+\Gamma_iSw-v_i+n_i+d_i-b_i)]^2 \\ & \left.-(v_i-\Gamma_iw+k_i)^2\right) \\ = & -(v_i-\Gamma_iw)^2-\frac{57}{23}(v_i-\Gamma_iw+k_i)^2 \\ & + 4\bar{x}_i^{\mathrm{T}}A_i^{\mathrm{T}}K_i^{\mathrm{T}}K_iA_i\bar{x}_i + 4(n_i+d_i)^4 - 8b_i(n_i+d_i)\end{aligned}$$

which implies

$$\begin{aligned}\Delta V_i \le & -\epsilon\tilde{x}_i^{\mathrm{T}}\tilde{x}_i - \bar{x}_i^{\mathrm{T}}(N_i-4A_i^{\mathrm{T}}K_i^{\mathrm{T}}K_iA_i)\bar{x}_i \\ & -(v_i-\Gamma_iw)^2-\frac{57}{23}(v_i-\Gamma_iw+k_i)^2 \\ & + 4(n_i+d_i)^4-8b_i(n_i+d_i)+a_i.\end{aligned} \tag{26}$$

Because N_i is any positive definite matrix, there exists an N_i such that $N_i-4A_i^{\mathrm{T}}K_i^{\mathrm{T}}K_iA_i>0$. Then, since $\lim_{t\to\infty}n_i=0$, $\lim_{t\to\infty}d_i=0$ and $\lim_{t\to\infty}a_i=0$, according to the comparison lemma (see [36, Lemma 3.4]), it concludes that $\forall[\tilde{x}_i^{\mathrm{T}}, v_i^{\mathrm{T}}, w^{\mathrm{T}}, \bar{x}_i^{\mathrm{T}}]^{\mathrm{T}}\in L_{V_i}(c_i)\backslash\{0\}$

$$\Delta V_i<0. \tag{27}$$

2) Otherwise,

$$\begin{aligned}\Delta V_i \le & -\epsilon\tilde{x}_i^{\mathrm{T}}\tilde{x}_i - \bar{x}_i^{\mathrm{T}}N_i\bar{x}_i + \frac{3}{10}\delta^2 \\ & + \left[k_i^2 + 2k_i(\sigma_p(v_i)-\Gamma_iw)\right. \\ & + \frac{12}{23}(\sigma_p(v_i)-\Gamma_iw+k_i)^2 - 4(v_i-\Gamma_iw+k_i)^2 \\ & \left.+4(v_i-\Gamma_iSw+h_i+t_i-m_i-\delta)^2\right] \\ \le & -\epsilon\tilde{x}_i^{\mathrm{T}}\tilde{x}_i + -\bar{x}_i^{\mathrm{T}}N_i\bar{x}_i + \frac{3}{10}\delta^2 + \left[\frac{1}{60^2}\delta^2\right. \\ & + 2\cdot\frac{1}{60}\delta\cdot 1.9\delta + \frac{12}{23}\left(1+0.9+\frac{1}{60}\right)\delta^2 \\ & \left.+4\left(2\times\frac{9}{10}+\frac{1}{20}-1+\frac{1}{60}\right)\left(-1+\frac{1}{20}+\frac{1}{60}\right)\delta^2\right] \\ \le & -\epsilon\tilde{x}_i^{\mathrm{T}}\tilde{x}_i - \bar{x}_i^{\mathrm{T}}N_i\bar{x}_i - \delta^2\end{aligned} \tag{28}$$

which implies (27) also holds.

Therefore, we can conclude from (26) and (28) that $\lim_{t\to\infty}\tilde{x}_i=0$, $\lim_{t\to\infty}\bar{x}_i=0$ and $\lim_{t\to\infty}(v_i-\Gamma_iw)=0$. It means $\lim_{t\to\infty}e_i=\lim_{t\to\infty}(C_i(\tilde{x}_i+\Pi_iw)+Qw)=0$.

It is noted from (26) and (28) that the value of ϵ will affect the convergence rate of e_i. The smaller ϵ, the lower the convergence rate.

Remark 2: Compared with [26], this article deals with the problem of heterogeneous multiagent systems. The nontrivial control law is obtained by combining the control law in [26] and the output regulation theory, with the distributed observer approach. It is rather involved to prove the effectiveness of our control law due to some additional terms caused by heterogeneous systems.

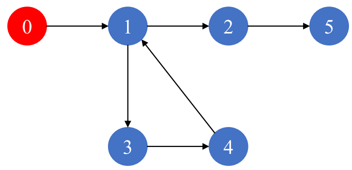


Fig. 1. Communication graph.

IV. Illustrative Example

The group of multiagent system consists of one leader and five followers. The communication graph $\mathcal{G}$ is shown in Fig. 1. The leader is described by (1) with

$$S = \begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix}, \quad Q = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$$

The followers are described by (2) with

$$A_i = \begin{bmatrix} 1 & 0.1 & 0 \\ 0 & 1 & 0.1 \\ 0 & 0 & 1 \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ 0 \\ 0.1 \end{bmatrix}$$

$$W_i = \begin{bmatrix} -0.2 & -0.7 \\ 0.6 & -0.2 \\ 0 & -0.1 \end{bmatrix}, \quad C_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \ i = 1, 2$$

$$A_i = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 1 & 0.1 \\ 0 & 0 & 1 \end{bmatrix}, \quad B_i = \begin{bmatrix} 0.1 \\ 0 \\ 0.1 \end{bmatrix}$$

$$W_i = \begin{bmatrix} 0.3 & -0.7 \\ -0.7 & 0.2 \\ -0.2 & -0.7 \end{bmatrix}, \quad C_i = \begin{bmatrix} -1 & 0 & 2 \\ 0 & -1 & 0 \end{bmatrix}, \ i = 3, 4, 5.$$

We assume in this example that each follower has the same actuator "time constants," that is, for $i = 1, 2, 3, 4, 5$, $\alpha_i = 0.5$. We also assume $\delta = 8$. Given the matrices of agents, it is easy to obtain the solutions of the regulator equation (4). One can verify that Assumptions 1–6 are satisfied.

The initial state of the leader is set as $w(0) = [3\,3]^{\mathrm{T}}$. The initial system states, actuator positions of the followers, and initial states of the observers (8)–(11) are chosen as

$$\begin{bmatrix} x_1(0) & x_2(0) & x_3(0) & x_4(0) & x_5(0) \end{bmatrix} = \begin{bmatrix} 2 & 2 & 0 & -2 & 0 \\ 3 & 2 & -1 & 0 & 2 \\ 4 & 0 & 3 & -1 & 1 \end{bmatrix}$$

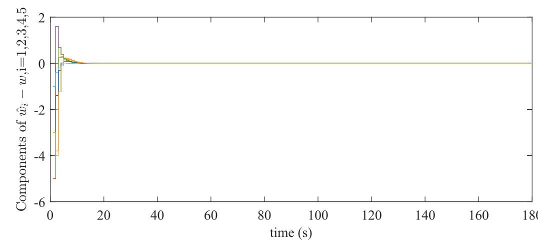
$$\begin{bmatrix} v_1(0) & v_2(0) & v_3(0) & v_4(0) & v_5(0) \end{bmatrix} = \begin{bmatrix} 0 & -1 & 2 & -2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \hat{w}_2(0) & \hat{w}_3(0) & \hat{w}_4(0) & \hat{w}_5(0) \end{bmatrix} = \begin{bmatrix} 2 & -1 & 2 & -1 & 0 \\ 2 & 1 & -1 & -2 & 0 \end{bmatrix}$$

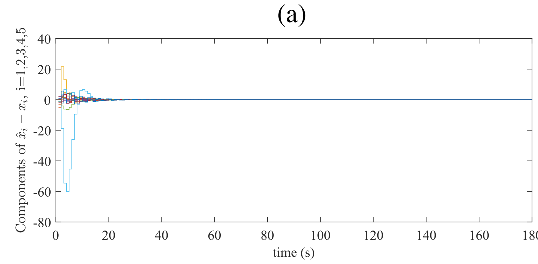
$$\begin{bmatrix} \hat{x}_1(0) & \hat{x}_2(0) & \hat{x}_3(0) & \hat{x}_4(0) & \hat{x}_5(0) \end{bmatrix} = \begin{bmatrix} 1 & 3 & 0 & 0 & -1 \\ -2 & 2 & 0 & 2 & 0 \\ 3 & -1 & 0 & -2 & 2 \end{bmatrix}.$$

For the informed follower 1, i.e., $i = 1$, the gain matrix L_S in distributed observer (8) is set as $L_S = \begin{bmatrix} 0.2 & 0 \\ 0.6 & 0.4 \end{bmatrix}$. The parameter μ defined in (9) is chosen as $\mu = 1$, and the gain matrices $L_{A,i}$ in distributed observers (11) are set as

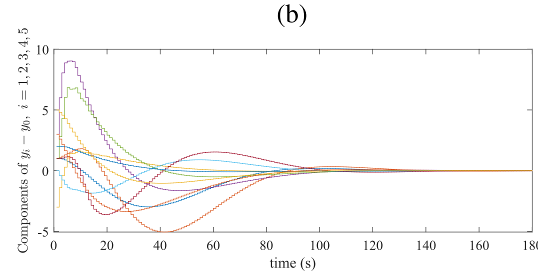
$$L_{A,(1,2)} = \begin{bmatrix} -1 & 0 \\ -2 & -1 \\ -18 & -0.9 \end{bmatrix}, \ L_{A,(3,4,5)} = \begin{bmatrix} 0.5 & 0.5 \\ 0.3 & 1 \\ -0.5 & 1.725 \end{bmatrix}.$$



(a)



(b)



(c)

Fig. 2. Simulation results. (a) Components of estimation errors $\hat{w}_i - w$. (b) Components of estimation errors $\hat{x}_i - x_i$. (c) Output consensus errors e_i under the output feedback consensus protocols (12) with $\epsilon = 0.1$, for $i = 1, 2, 3, 4$, and 5.

We consider the low gain parameter $\epsilon = 0.1$. One can conclude from Fig. 2 that estimation errors $\hat{w}_i - w$, $\hat{x}_i - x_i$ and output consensus errors e_i all converge to zero asymptotically.

V. Conclusion

In this article, we have investigated the semiglobal output consensus problem for discrete-time heterogeneous systems with actuator position and rate saturation. We have constructed the output feedback consensus protocol for each follower. It has been proved that given any bounded initial conditions, the problem can be solved by our consensus protocol, provided that the communication graph contains a spanning tree. Moreover, it is worth considering a switching communication topology for the problem, because this condition is milder.

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