SCIENCE CHINA
Information Sciences

• RESEARCH PAPER •

July 2021, Vol. 64 172209:1–172209:16
https://doi.org/10.1007/s11432-020-3056-5

Smooth quadrotor trajectory generation for tracking a moving target in cluttered environments

Lele XI[1,2,3], Zhihong PENG[1,2*], Lei JIAO[1,2] & Ben M. CHEN[4]

[1]*School of Automation, Beijing Institute of Technology, Beijing* 100081*, China;*
[2]*State Key Laboratory of Intelligent Control and Decision of Complex System, Beijing* 100081*, China;*
[3]*Peng Cheng Laboratory, Shenzhen* 518055*, China;*
[4]*Department of Mechanical and Automation Engineering, The Chinese University of Hong Kong, Hong Kong* 999077*, China*

Received 2 June 2020/Revised 23 July 2020/Accepted 1 September 2020/Published online 20 May 2021

Abstract In this paper, we present a trajectory generation method of a quadrotor, based on the optimal smoothing B-spline, for tracking a moving target with consideration of relative tracking pattern or limited field of view of the onboard sensor in cluttered environments. Compared to existing methods, safe flying zone, vehicle physical limits, and smoothness are fully considered to guarantee flight safety, kinodynamic feasibility, and tracking performance. To tackle the cluttered environments, a parallel particle swarm optimization algorithm is applied to find the feasible waypoints that the generated trajectory should be as close to as possible, with consideration of the target's future state as well as obstacles to trade off the tracking performance and flight safety. Then, a sequential motion planning method, considering the above constraints, is applied and embedded into a cost function for solving the problem of robust tracking trajectory generation for the quadrotor via a convex optimization approach. The feasibility and effectiveness of the proposed method are verified by numerical simulations.

Keywords quadrotor, target tracking, trajectory generation, B-spline, convex optimization

Citation Xi L L, Peng Z H, Jiao L, et al. Smooth quadrotor trajectory generation for tracking a moving target in cluttered environments. Sci China Inf Sci, 2021, 64(7): 172209, https://doi.org/10.1007/s11432-020-3056-5

1 Introduction

Target tracking plays an important role for autonomous aerial robots in both military and civil applications. The past decades have witnessed a rapid growth of unmanned aerial vehicles [1, 2], and the typical applications of UAVs include drug enforcement, surveillance, and rescue in urban or wild environments, just to name a few. In many application scenarios, as shown in Figure 1, it is necessary for a quadrotor to track a target of interest with standoff or persistent pattern, especially in complex environments [3]. Besides, limited to the physical performance and sensor saturation, the dynamics of the vehicle must be fully considered. Therefore, efficient trajectory generation is essential for the quadrotor to fulfill the task by trading off the tracking performance, kinodynamic feasibility, and flight safety on those occasions. There have been a variety of relevant studies on UAVs or other mobile robots to carry out dynamic games [4] or tracking mission, but most of the relevant work just focused on target following in general obstacle-free or obstacle-spare environments [5–8]. However, target tracking missions of the quadrotor are always carried out in cluttered environments, and the presence of obstacles poses more challenges including flight safety, kinodynamic feasibility, trajectory smoothness, and so forth.

Liu et al. [9] studied the problem of identifying and tracking a moving target using a quadrotor in forest environments. State prediction and vehicle's physical limits were not fully taken into account, which is insufficient for fast-tracking flights. Some relevant researches [10, 11] were also carried out on target pursuing of quadrotors in general complex environments. The work in [10] took advantage of quadratic programming (QP) to generate the polynomial trajectory of a quadrotor while tracking a moving target with many relative patterns. A contouring-control based method was adopted in [11] to calculate the

* Corresponding author (email: peng@bit.edu.cn)

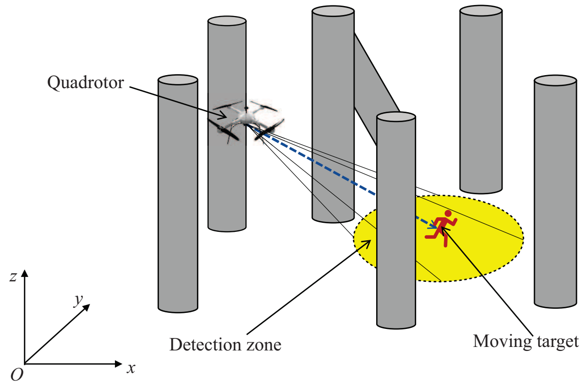

Figure 1 (Color online) A scenario of target tracking.

coefficient of polynomial trajectory for a quadrotor while keeping a mobile target in the field of view in 3D dense environments without the consideration of physical limits. In [12], a chasing task was studied for a quadrotor in image space. The control inputs were obtained to avoid occlusion from a single polyhedral occluder, which is of limited use for quadrotors in the real application. A similar image-based approach [13] was proposed to track an aggressive target while satisfying a set of actuation constraints and avoiding a few spherical obstacles. In [14], given the motion information of the target, vision-based moving ground target tracking by a quadrotor in obstacle-free environments was studied and simulations were performed in a robot operating system.

Many trajectory generation studies have been carried out to find feasible trajectories for the quadrotor in cluttered environments with the consideration of flight safety and physical limits to meet the real application. In [15–18], the quadrotors were considered as differential-flat holonomic vehicles, and free spaces in environments were represented by a series of convex shapes to formulate the problem of trajectory generation as a typical convex optimization that can be solved efficiently. In the above researches, the trajectories are formulated as polynomial curves or splines to satisfy the smoothness and kinodynamic feasibility. Ref. [19] presented an efficient technique to generate jerk limited trajectories for the quadrotor to travel inside the safe area. The work in [20] innovatively introduced a real-world challenging demonstration in which a quadrotor autonomously tracked trails in forest scenes. However, the obstacles avoidance issue is not investigated because the forest trail is wide enough for a quadrotor to navigate around.

Typically in cluttered environments, tracking a maneuvering target puts forward the need for active reactions to both target's constantly changing motion and obstacle avoidance under the threats of possible conflicts between flight safety and tracking performance [21]. In general scenarios, the motion information of the moving target can be obtained only if the target is in the field of view. Therefore, it is necessary to find the feasible flight trajectory for the quadrotor to keep the target detectable and improve robustness to occlusion and potential failure when the target temporarily leaves the field of view [22]. In many special scenarios, the quadrotor is also tasked to track a moving target with a relative pattern, and the efficient trajectory generation method is essential for real application. In this work, we focus on the trajectory generation for quadrotor based on the smooth B-spline [23–25], a generalized version of the Bézier curve, to trade off the maneuverable target tracking and flight safety in cluttered environments. A geometrical safe flight corridor (SFC), which is realistic on account of the lightweight but high-performance onboard computer, is formed and applied to provide a feasible flight space where the tracking trajectories have to be squeezed all the time to guarantee the flight safety of the task execution in our framework. For ease of research on smooth trajectory generation for the quadrotor while persistently tracking a maneuverable target, we assume that the quadrotor has full knowledge of obstacles and environment information, and the target's state can be observed or predicted. Meanwhile, the effects on the field of view from the coupling between translational and rotational dynamics are ignored.

Compared to the point-to-point planning work in [17, 26], in this paper, we propose a smooth trajectory generation method for the quadrotor to execute target tracking task with consideration of relative pattern or visibility in cluttered environments. In particular, the proposed method relaxes the waypoint constraints as an approximation problem instead of the hard equality constraint, which significantly improves flight efficiency. The main contributions of this work can be summarized as follows:

• We proposed a two-step method to find feasible trajectories of the quadrotor for target tracking in two usual patterns in cluttered environments. First, the feasible waypoints for the quadrotor are found by trading off the tracking performance and flight safety in a short moving horizon. Then, a sequential motion planning (SMP) method, which efficiently minimizes the control costs of the quadrotor and the deviation from trajectory to desired waypoints in a convex QP, is applied to guarantee the smoothness and tracking performance with consideration of safe flight zone and feasibility constraints.

• We represent the trajectory as axes-couple smoothing B-spline. Compared to axes-decoupled piecewise polynomials [10], B-spline addresses issues of axes-coupled and interval-wise constraints, guaranteeing the flight safety and kinodynamic feasibility with the constructed volume constraints, and increasing the solution quality. Moreover, different from the position contouring systems, we relax the hard waypoint constraints as an approximation problem, which makes a positive impact on flight efficiency.

The rest of the paper is organized as follows. In Section 2, the discussion on the method overview and trajectory's mathematical representation is covered. Whereas in Section 3, we present the proposed trajectory generation method of target tracking for the quadrotor in detail. In Section 4, results and analysis of simulations are emphatically provided. Finally, the conclusion and further work are drawn in Section 5.

2 Preliminaries

2.1 Differential flatness

A quadrotor has a six-dimensional configuration space, and the states and inputs can be written as algebraic functions of four carefully selected flat outputs and their derivatives [15, 26]. $\sigma = [x, y, z, \psi]$ is the proper choice of flat outputs where $[x, y, z]$ is vehicle's coordinates of the center of mass in the global frame and ψ is the yaw angle. Therefore, the vehicle can be seen as a point mass model decoupled in three degree-of-freedom (DOF) space with consideration of velocity, acceleration and jerk. On each axis of quadrotor, the outer control loop can be simplified as a triple integrator

$$\begin{cases} \dot{p} = v, \\ \dot{v} = a, \\ \dot{a} = j, \end{cases} \tag{1}$$

where p, v, a, j are its position, velocity, acceleration, and jerk, respectively [27]. The excellent robust perfect tracking (RPT) approach proposed in [28] can make the linear system track the given reference with arbitrarily fast enough settling time, thus accurately meet our requirements in this work.

2.2 Trajectory construction for quadrotor

To guarantee the smoothness and kinodynamic feasibility of the whole tracking process, a kth order clamped uniform B-spline S_k is chosen to represent the tracking trajectory where S_k is define as

$$S_k(s) = \sum_{i=0}^{M-1} c_i N_i^k(s), \quad c_i, S \in \chi, \ N_i^k \in \mathbb{R} \tag{2}$$

with N_i^k denoting the basis function and c_i is the control point. $C = [c_0, c_1, \ldots, c_{M-1}]$, $C \in \mathbb{R}^{d\times M}$ represents the control point matrix. The basis functions are defined over a knot vector $\mathcal{K} = [s_0, s_1, \ldots, s_{M+k}]$ and a path parameter s as

$$N_i^0 = \begin{cases} 1, & \text{if } s_i \leqslant s \leqslant s_{i+1}, \\ 0, & \text{otherwise}, \end{cases} \tag{3}$$

$$N_i^j(s) = N_A(s, i, j) N_i^{j-1}(s) + N_B(s, i, j) N_{i+1}^{j-1}(s), \tag{4}$$

where

$$N_A(s, i, j) = \begin{cases} 0, & \text{if } s_i = s_{i+j}, \\ \dfrac{s - s_i}{s_{i+j} - s_i}, & \text{otherwise}, \end{cases} \tag{5}$$

$$N_B(s,i,j)=\begin{cases}0, & \text{if } s_{i+1}=s_{i+j+1},\\ \dfrac{s_{i+j+1}-s}{s_{i+j+1}-s_{i+1}}, & \text{otherwise.}\end{cases} \tag{6}$$

For the spline to be clamped and uniform, there is

$$\begin{aligned}\mathcal{K}&=[s_0,s_1,\ldots,s_{M+k}]\\ &=[\underbrace{0,\ldots,0}_{k+1\text{ times}},1,2,\ldots,\underbrace{M-k,\ldots,M-k}_{k+1\text{ times}}].\end{aligned} \tag{7}$$

From [23], there is a linear relation between the path parameter s and time t:

$$\frac{s}{t}=\alpha. \tag{8}$$

And it gives $S_k(s)=S_k(\alpha t)$ where $t\in[0,T]$ and $T=\frac{M-k}{\alpha}$. In this paper, two excellent properties of B-splines are frequently used:

- Strong convex hull property. If all control points for a B-spline satisfy $Ac_i\leqslant b$, $i\in 0,1,\ldots,M-1$, so does the entire trajectory.
- The nth order derivative B-spline $S_k^{(n)}$ is the $k-n$ order B-spline with a new set of control points.

2.3 Method overview

In this paper, we formulate the trajectory generation for target tracking as an optimization problem:

$$\begin{aligned}&\min\ \int_{-\infty}^{+\infty}\left\|\frac{\mathrm{d}^nS_k}{\mathrm{d}t^n}\right\|^2\mathrm{d}t+\lambda\sum_{i=0}^{N-1}\|S_k(\alpha t_i)-d_i\|^2\\ &\text{s.t. } S_k(\alpha t_{\text{ini}})=s_{\text{ini}}\,\&\,S_k(\alpha t_{\text{end}})=s_{\text{end}},\\ &\qquad S_k(\alpha t)\in\text{SFC},\\ &\qquad \|S_k^{(n)}(\alpha t)\|\leqslant s_n^{\max},\end{aligned} \tag{9}$$

where the first term of cost function penalizes the aggressiveness of flight by minimize the integration of the square of nth $(i=1,2,3,4)$ derivatives of trajectory, and the second penalizes the deviation from the desire waypoints, calculated by moving target's future state and obstacles information. To improve the tracking efficiency, we relax the hard waypoint constraint with a weight λ. The first term of constraints represents the boundary conditions on a set of initial and end states. The second and third terms respectively indicate the safe constraints and kinodynamic feasibility conditions, which limit the trajectory and its derivatives in multiple convex polytopes. SFC represents the safe flight corridor mentioned in Subsection 3.2, and $s_n^{\max}$ is the maximum value of kinodynamic constraint of nth $(i=1,2,3,4)$ derivatives of trajectory. It should be emphasized that the boundary conditions in this paper include the initial state at current time and the nominal terminal state which is calculated by trading off the tracking performance and flight safety in short moving horizon.

The framework of the proposed method is shown in Figure 2. First, the future state of moving target and environment information are applied to find the feasible waypoints and generate a nominal path by a geometrical path finding algorithm for the quadrotor. Then, an SFC is generated to supply the safe geometrical constraints for the tracking trajectory generation module. Finally, the tracking trajectories for the quadrotor are generated based on smooth B-spline with consideration of physical limits and flight safety.

3 Generation of feasible trajectory

In this section, we give the detailed process of smooth trajectory generation of the quadrotor for tracking a moving target in two different patterns.

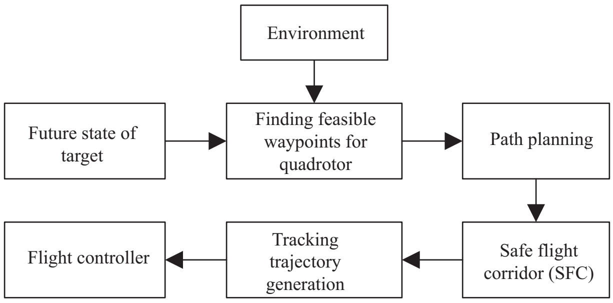

Figure 2 The framework of proposed method.

3.1 Feasible waypoint generation

To tackle the problem of robust target tracking in cluttered environments, it is crucial for the quadrotor to efficiently find feasible waypoints to navigate. As follows, we introduce the waypoint generation methods for the quadrotor in two target tracking patterns: (1) relative tracking pattern, to maintain a fixed distance and angle between the UAV and the moving target; (2) visibility maintenance pattern, to ensure that the moving target is in the field of view of the onboard sensor and there is no restriction on the relative angle between the UAV and the target.

3.1.1 *Relative tracking pattern*

When the quadrotor carries out target tracking while maintaining a fixed distance with respect to the moving target, the nominal terminal state at future time t_f can be obtained by

$$\begin{bmatrix} T_x(t_f) \\ T_y(t_f) \\ T_z(t_f) \end{bmatrix} = \begin{bmatrix} \sin\theta & & \\ & \cos\theta & \\ & & 1 \end{bmatrix} \begin{bmatrix} d_h \\ d_h \\ d_v \end{bmatrix} + \begin{bmatrix} P_x(t_f) \\ P_y(t_f) \\ P_z(t_f) \end{bmatrix}, \tag{10}$$

where θ represents the relative planar angle between the target and quadrotor in global frame. $P(t_f) = [P_x(t_f), P_y(t_f), P_z(t_f)]$ is the predicted future position of target at t_f and $T(t_f) = [T_x(t_f), T_y(t_f), T_z(t_f)]$ is the nominal waypoint of quadrotor. d_h is the desired planar distance between target and quadrotor, and d_v is the desired vertical height related to target. For common tracking applications, both d_h and d_v are preset to constant values.

Due to the existent infeasible region, it is possible that the solution calculated by Eq. (10) falls into the obstacles, which indicates that we have to find a feasible one in this case:

$$\begin{aligned} &\min \|W(t_f) - T(t_f)\|^2 \\ &\text{s.t. } W(t_f) \in \text{SFC}, \\ &\qquad W(t_f) \in V_f, \end{aligned} \tag{11}$$

where V_f is the bearing vector $T(t_f) - P(t_f)$. $W(t_f)$ is the desired feasible waypoint satisfying the constraints as illustrated in Figure 3.

3.1.2 *Visibility maintenance pattern*

In order to tackle the challenges with limited field of view of onboard sensor, we propose a novel and efficient method for trajectory generation while keeping target visible. Considering occlusion by obstacles, we obtain the waypoints through solving the following optimization problem:

$$\begin{aligned} &\min l_1 + \lambda_1 \mathrm{e}^{-(l_2+d_m)} + \lambda_2 \|l_3 - d_t\| \\ &\text{s.t. } W(t) \in \text{SFC}. \end{aligned} \tag{12}$$

The upper is divided into three parts: the first term, which directly affects flight trajectory length, penalizes the Euclidean distance l_1 from current position $W(t_c)$ at t_c to next desired waypoint $W(t_c+\Delta t)$ at $t_c+\Delta t$ with the predicting period Δt. The second term maximizes the closest distance l_2 from bearing

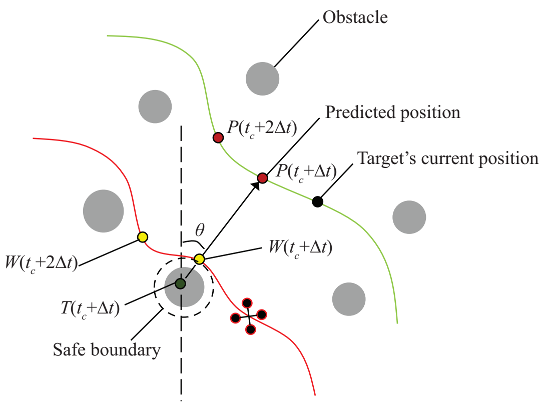

Figure 3 (Color online) Finding feasible waypoints in relative tracking pattern. t_c is current time and Δt is the predicting period. $T(t_c+\Delta t)$ is the solution calculated by Eq. (10) based on $P(t_c+\Delta t)$, d_p and d_v. $W(t_c+\Delta t)$ is the feasible waypoint calculated by Eq. (11).

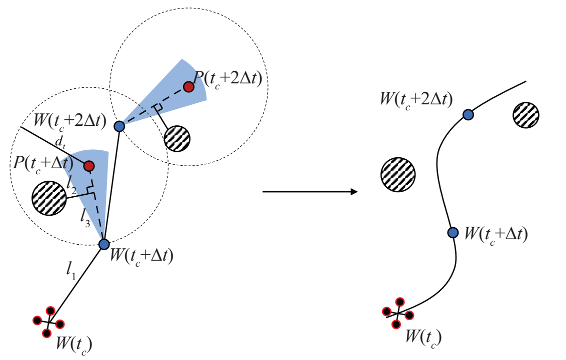

Figure 4 (Color online) Finding feasible waypoints in visibility maintenance pattern. The blue sector region represents the field of view (FOV) of onboard sensor. The red dots represent the moving target's predicted positions in the future, and blue dots are the desired waypoints of the quadrotor to navigate.

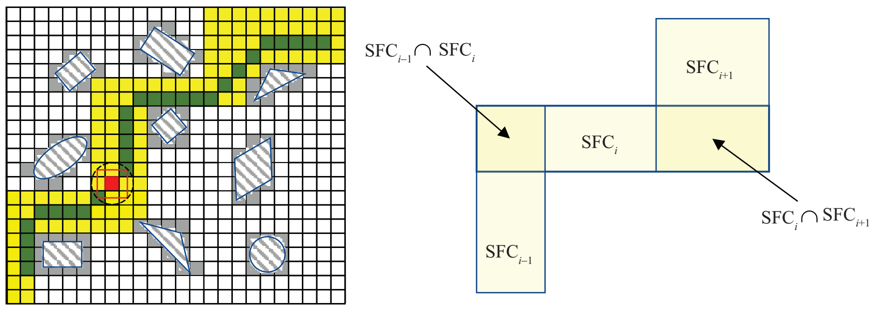

Figure 5 (Color online) The illustration of an SFC. (a) The scene diagram of SFC; (b) closed convex polytopes SFC_i. In (a), The path found by the nominal path-finding algorithm is shown as green grids. The flight corridor is marked in yellow. The dash black circle and solid red square respectively represent the sphere obtained by position from the node to nearest obstacle and the cube inscribed the sphere.

vector $P(t_c+\Delta t)-W(t_c+\Delta t)$ to obstacles, and d_m is a preset margin distance of map. Naturally, to achieve persistent tracking, we can ensure higher visibility once l_2 is no less than zero, and the larger the better. The third term penalizes the error between the distance from the desired waypoint $W(t_c+\Delta t)$ to target $P(t_c+\Delta t)$ and a preset distance threshold d_t, which maintains the relative tracking distance.

By solving Eq. (12), the desired waypoints are found. The above process is illustrated in Figure 4.

3.2 Safe corridor constraint

To tackle flight safety, we have to find a safe flight zone where the trajectories of the vehicle should be squeezed. There have been many representations of SFC [19, 29] to efficiently solve the problem of trajectory generation for single vehicle and multi-agents in cluttered environments. First, the safe path can be searched by a geometrical shortest path-finding algorithm in a 3D grid map. For each node of the path, the distance from the node to the nearest obstacle can be efficiently obtained in a preprocessed 3D Euclidean distance transform (EDT) map. Then, an initial sphere of safe space against the nearest obstacle and an inscribed cube are obtained respectively. Afterward, the initial SFC is formed by enlarging each initial cube along the positive and negative directions of x, y, z axes of the coordinate system until it hits an obstacle. Finally, the redundant cubes are eliminated to find the final SFC. The whole process described above is shown in Figure 5(a) and Algorithm 1.

As shown in Figure 5(b), the final SFC consists of multiple closed convex polytopes $\mathrm{SFC}_i, i \in 1,2,\ldots,m$ and for $i=1,2,\ldots,m-1$:

$$\mathrm{SFC}_i \cap \mathrm{SFC}_{i+1} \neq \emptyset. \tag{13}$$

Each interior space of polytopes can be represented as a linear inequal constraint ($Ac \leqslant b$). According

Algorithm 1 Safe flight corridor generation

Input: Map, waypoints.
Output: SFC.

Generate path from current position to waypoints;
for each node of path **do**
  Find distance d_s to the nearest obstacle;
  Generate initial sphere and inscribed cube based on d_s;
  Enlarge cube until it hits an obstacle;
end for
Generate initial $\mathrm{SFC}_{\mathrm{initial}}$ with enlarged cubes;
$m = \mathrm{size(cubes)}$;
for $i = m : 2$ **do**
  for $j = m - 1 : 1$ **do**
    if cube_j is contained by cube_i **then**
      Eliminate cube_j;
    else if cube_i is contained by cube_j **then**
      Eliminate cube_i;
    end if
  end for
end for
$n = \mathrm{size(cubes)}$;
for $i = n : 3$ **do**
  for $j = n - 2 : 1$ **do**
    if cube_i overlap with cube_j **then**
      Eliminate $\mathrm{cube}_{j+1}, \ldots, \mathrm{cube}_{i-1}$;
    end if
  end for
end for

to the excellent property mentioned in Subsection 2.2, we can confine the entire trajectory to the safe corridor by restricting all control points to the same region.

3.3 Problem formulation

3.3.1 *Cost function*

As described in Subsection 2.3, our method penalizes the integration of square of nth derivatives along the trajectory and deviation from the trajectory to desired waypoints, obtained by the predictive motion of moving target, at the corresponding time. The above two items are applied to trade off the tracking performance and smooth response from the quadrotor.

(1) Smooth response.

$$E_{\mathrm{smooth}} = \int_{-\infty}^{+\infty} \left\| \frac{\mathrm{d}^n S_k}{\mathrm{d}t^n} \right\|^2 \mathrm{d}t = \sum_{i=1}^{d} \int_{-\infty}^{+\infty} \left\| \frac{\mathrm{d}^n S_k^{(i)}}{\mathrm{d}t^n} \right\|^2 \mathrm{d}t. \tag{14}$$

Following [16, 23], Eq. (14) can be expressed as a quadratic form as

$$\widehat{C}^{\mathrm{T}} \left[V_n \otimes I_d \right] \widehat{C}, \tag{15}$$

where

$$V_{n,(i,j)} = \alpha^{2n-1} \int_{-\infty}^{+\infty} \frac{\mathrm{d}^n N_i^k(s)}{\mathrm{d}s^n} \frac{\mathrm{d}^n N_j^k(s)}{\mathrm{d}s^n} \mathrm{d}s, \tag{16}$$

and $\widehat{C}$ is the vectorized form of C. The symbol $\otimes$ represents the Kronecker product and I_d is the identity matrix of dimension $\mathbb{R}^{d\times d}$. $V_n \otimes I_d$ is positive semi-definite, which indicates it is the Hessian matrix. Due to the linear relationship between the control input of quadrotor and the snap (the fourth derivative of position) [15], we can minimize the snap cost to find the efficient trajectory and save control effort. In this paper, the integration of square of snap along the whole trajectory, which is called the snap cost, is used to evaluate the aggressiveness of trajectory. Furthermore, as described in [15], the higher the snap cost, the more aggressive the trajectory is.

(2) Waypoint approximation.

$$\mathrm{Con}_{(\mathrm{waypoints})} = \sum_{i=0}^{N-1} \| S_k(\alpha t_i) - d_i \|^2. \tag{17}$$

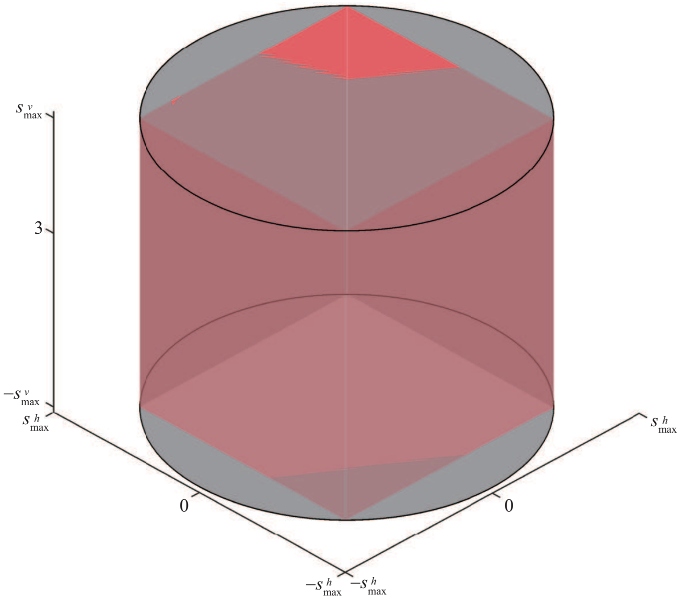

Figure 6 (Color online) The illustration of constraint volume. The gray cylinder represents the volume expressed by Eqs. (19) and (20) while the red volume is the embedded cuboid of cylinder.

It indicates that the waypoints $D = [d_0, d_1, \ldots, d_{N-1}]^{\mathrm{T}}$ generated in Subsection 3.1 are to be reached or approached at $T = [t_0, t_1, \ldots, t_N - 1]^{\mathrm{T}}$. Following [16], Eq. (17) can be expressed as

$$\begin{aligned}\mathrm{Con}_{(\text{waypoints})} &= (HC - D)^{\mathrm{T}}(HC - D)\\ &= \widehat{C}^{\mathrm{T}}(H^{\mathrm{T}}H \otimes I_d)\widehat{C} - 2\widehat{D}^{\mathrm{T}}(H \otimes I_d)\widehat{C} + \widehat{D}^{\mathrm{T}}\widehat{D},\end{aligned} \tag{18}$$

and $H^{\mathrm{T}}H \otimes I_d$ is positive semi-definite. To exactly reach the waypoints, we can set $HC - D = 0$ for all dimensions, thus generally resulting in an aggressive flight.

3.3.2 *Feasibility constraints*

Considering the sensor capability and state saturation, we must enforce a number of constraints to fit the physical limits on the thrust, body rate and speed when performing the tracking task. The thrust limits can be satisfied by

$$\|f\| = \sqrt{\ddot{x}^2 + \ddot{y}^2 + (\ddot{z} + g)^2} \leqslant \frac{f_{\max}}{m}, \tag{19}$$

$$\ddot{z} \geqslant \ddot{z}_{\min} \geqslant \frac{f_{\min}}{m} - g, \tag{20}$$

where $[x, y, z]$ denotes the position of quadrotor, f is the total thrust of vehicle, $f_{\max}$ and $f_{\min}$ respectively represent the maximum and minimum values of thrust, m is the mass of quadrotor, g is gravity, and $\ddot{z}_{\min}$ is a pre-fixed design variable. The maximum body rate $\omega_{\max}$ should be limited by

$$\sqrt{\dddot{x}^2 + \dddot{y}^2 + \dddot{z}^2} \leqslant (\ddot{z}_{\min} + g)\omega_{\max}. \tag{21}$$

From [16], the axes-coupled dynamical limits are transformed into linear constraints by finding the embedded convex polytopes. Meanwhile, the horizontal and vertical axes are assigned with different limits in terms of their dynamical differences.

In practice, a cylinder (Figure 6) which satisfies Eqs. (19) and (20) is used to cover a larger volume

$$\begin{gathered}\sqrt{s_x^2 + s_y^2} \leqslant s^h_{\max},\\ s^v_{\min} \leqslant s_z \leqslant s^v_{\max},\end{gathered} \tag{22}$$

where $[s_x, s_y, s_z]$ represents the vehicle's state including the velocity, acceleration and jerk. $s^h_{\max}$ and $s^v_{\max}$ are the maximum along horizontal and vertical direction respectively. In this paper, the embedded cuboid is selected as the constraint space to improve efficiency of calculation. Since the nth order derivative B-spline $S_k^{(n)}$ is another B-spline curve with a new set of control points, we can limit the velocity, acceleration and jerk by the same method as safe corridor constraint does.

3.3.3 *Sequential planning*

Once the feasible waypoints are obtained, the whole target tracking process can be reformulated as a sequential motion planning problem for the quadrotor with boundary state constraints:

$$\begin{aligned} &\min \ J = \widehat{C}^{\mathrm{T}}(V_n \otimes I_d + H^{\mathrm{T}}H \otimes I_d)\widehat{C} - 2\widehat{D}^{\mathrm{T}}(H \otimes I_d)\widehat{C} + \widehat{D}^{\mathrm{T}}\widehat{D} \\ &\text{s.t. } A_{\mathrm{eq}}\widehat{C} = b_{\mathrm{eq}}, \quad A_{\mathrm{ieq}}\widehat{C} \leqslant b_{\mathrm{ieq}}, \end{aligned} \tag{23}$$

where the cost function is formulated with consideration of the smooth response and the waypoint approximation. In constraint conditions, the equation term represents boundary state constraints while the inequality includes dynamics and safety limits. Since the optimization problem is a typical convex quadratic program under linear constraints, the presented tracking trajectory generation method for quadrotor is numerically robust, which indicates that solution can be found efficiently by general off-the-shelf optimization solver in Matlab. To cope with the exception of Hessian matrix H, an efficient open-source C program, dynamically linked against Matlab using Mex file, is adopted. The whole algorithm for tracking trajectory generation is described as Algorithm 2.

Algorithm 2 The sequential planning

Input: Cost function J, waypoints $W(t_f)$.
Output: C, control point matrix of spline trajectory.

1: **for** each planning horizon **do**
2: Update the waypoints $W(t_f)$;
3: Generate SFC;
4: $C \leftarrow$ iterateQP$(J, C, W(t_f))$;
5: Return C;
6: **end for**

4 Simulations and analysis

This section presents the simulation results to demonstrate the feasibility and robustness of the proposed framework for target tracking in cluttered space. The original target tracking problem, which is formulated as the QP, is solved using a general convex solver Quadprog supplied by Matlab. In our simulations, the order of the spline trajectory is set as $k = 4$ while the predicting period Δt and planning frequency are set as 2 s and 1 Hz respectively.

4.1 Smooth trajectory generation

In this subsection, resultant results of smooth trajectory generation for quadrotor, fully considering the flight safety, physical limits, and boundary state, are studied. For comparison, an existing method [15] which relies on high order polynomial curve is adopted. The SFC consisting of several convex polytopes is generated by the method described in Subsection 3.2. The kinodynamic feasibility constraints, determined by Subsection 3.3.2, are specified by the cylinder on the trajectory's velocity, acceleration and jerk with $v = [3.0, -0.5, 2.0]$, $a = [3.0, -0.5, 2.0]$, and $j = [5.0, -5.0, 5.0]$. In this simulation, the initial and terminal positions are respectively set as $p_{\mathrm{init}} = [1.0\ 1.0\ 0]$ and $p_{\mathrm{term}} = [15.0\ 18.0\ 3.0]$, while velocity, acceleration and jerk are all set as zero. Figure 7 shows the compared trajectories of two methods and the resultant velocities and accelerations. As shown in Table 1, the above two methods, which have the same total time of the whole planning process and time allocation for each corridor, are compared in terms of the average snap cost and computational time. It is obvious that the generated B-spline trajectory gives a lower cost with a slightly longer computational time. Compared to polynomial curves, B-spline guarantees the entire safety and kinodynamic feasibility based on its excellent properties mentioned in Subsection 2.2 while polynomial curves may dissatisfy those constraints in many situations. To facilitate such situations, extra safe waypoint constraints must be added to make it feasible along the entire trajectory, which will additionally increase the computational cost.

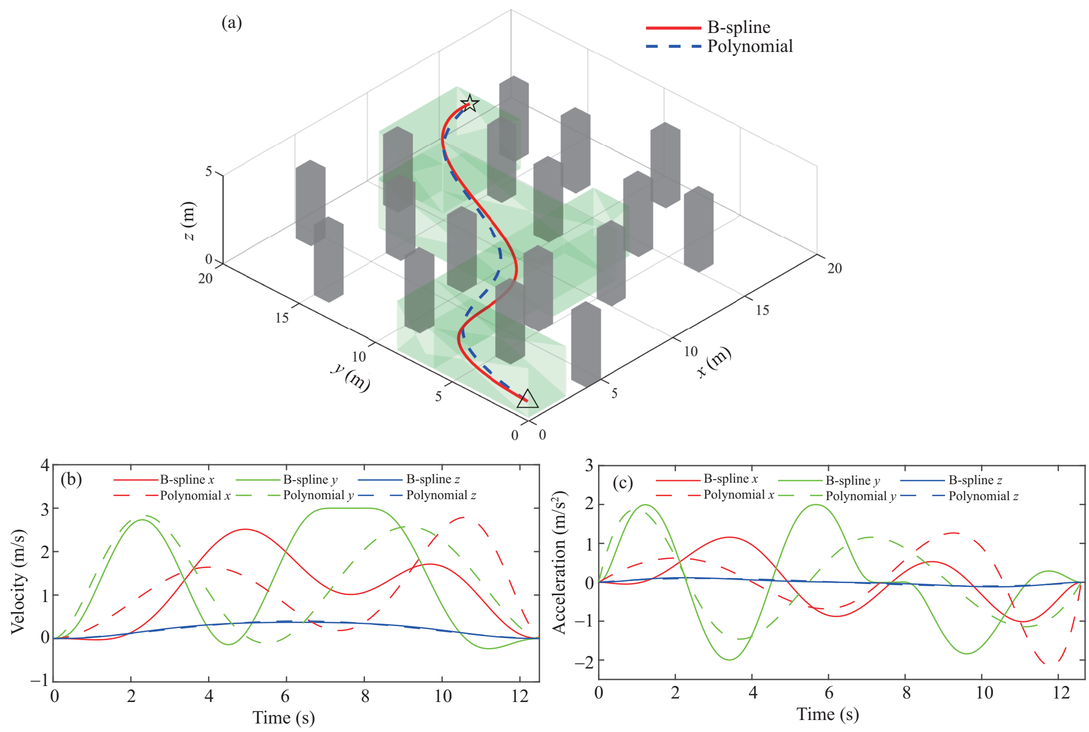

Figure 7 (Color online) Trajectory generation results in complex environment. (a) Trajectory generation in complex environment; (b) velocity; (c) acceleration.

Table 1 Time cost of trajectory generation in complex environment

Method	Snap cost	Time cost (ms)
B-spline	22.72	33.2
Polynomial	45.68	26.4

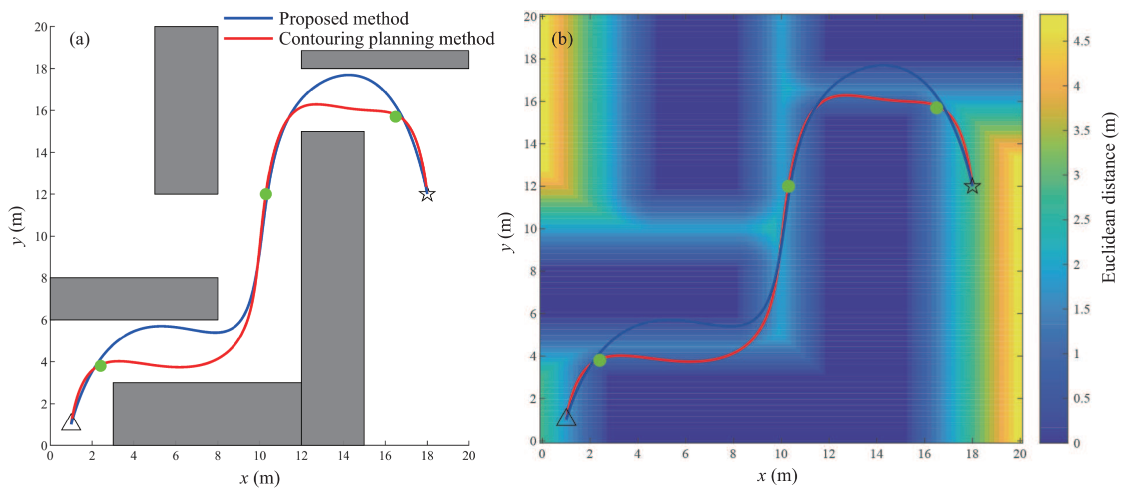

Figure 8 (Color online) Trajectory generation results in narrow corridor. The green dots are the waypoints to approximate or pass through. (a) 2D illustration; (b) Euclidean distance map illustration.

As shown in Figure 8, different from the general B-spline-based contouring planning methods, we relax the waypoint constraints as an approximation problem instead of the hard equality constraint, which significantly saves the control cost. The comparison of smoothness is given in Table 2 to demonstrate the efficiency of the proposed method. Both two methods represent the trajectories as B-splines, and the contouring planning should go through the waypoints while the proposed method approximates them. It is clear that the relaxed method has a lower snap cost.

Table 2 Snap cost of trajectory generation in narrow corridor

Method	Snap cost
Proposed method	48.40
Contouring planning method	76.20

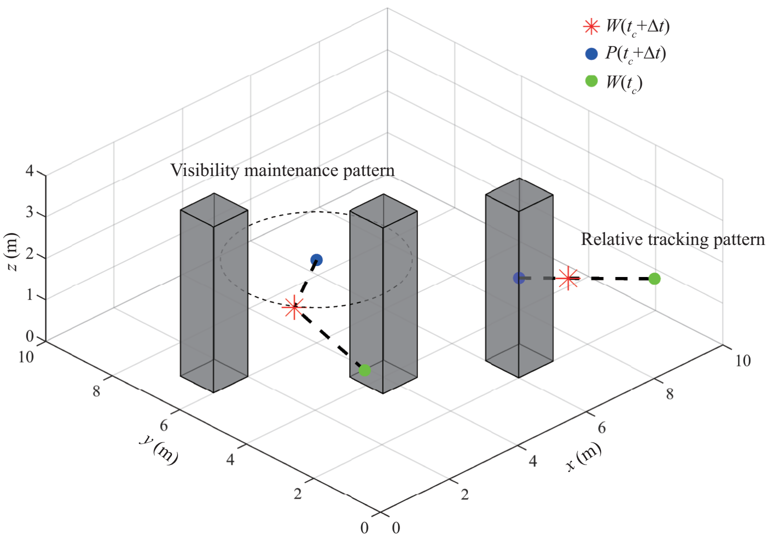

Figure 9 (Color online) Results of finding feasible waypoints in two different tracking patterns.

4.2 Tracking trajectory generation

In this subsection, we present the tracking trajectory generation for the quadrotor in two patterns. The quadrotor is tasked to track a moving target in 3D narrow corridor or complex environment. With consideration of physical performance and constraints, the velocity, acceleration and jerk limits of horizon are set as ± 3 m/s, ± 3 m/s^2 and $\pm$ 8 m/s^3, respectively while the vertical limits are set as $[-0.5, 2]$ m/s, $[-0.5, 2]$ m/s^2, and $[-5, 5]$ m/s^3.

4.2.1 *Results of waypoint generation*

To efficiently find feasible waypoints for safe tracking trajectory generation, a fast parallel particle swarm optimization (PSO) algorithm is employed to efficiently derive a proper solution $W(t)$ which satisfies Eq. (11) or (12). PSO is almost intrinsically parallel, which means that it can be efficiently implemented using parallel computing architecture. In this paper, compute unified device architecture (CUDA), a parallel computing platform, is adopted to save computational cost. Every particle is associated to a CUDA block while every dimension of the particle is associated to a thread of the corresponding block. The whole process is described as follows: First, a random initial swarm is defined. Then, all particles update their positions and velocities in parallel. Thereafter, every particle updates its local best position through evaluating a Fitness function and the best global position is determined. Afterward, each particle calculates its new velocity using Eq. (24) as well as new position using Eq. (25).

$$v_i^{j+1} = w \cdot v_i^j + c_1 \cdot \mathrm{rand}_1 \cdot (\alpha_{pi}^j - \alpha_i^j) + c_2 \cdot \mathrm{rand}_2 \cdot (\alpha_g^j - \alpha_i^j), \tag{24}$$

$$\alpha_i^{j+1} = \alpha_i^j + v_i^{j+1}, \tag{25}$$

where v_i^j is the velocity of particle i while α_i^j is its position at the jth iteration. α_{pi}^j is the best position of the particle and α_g^j is the best position of the swarm at the jth iteration. N_p is population size and N_c is the maximum value of iteration ($1 \leqslant i \leqslant N_p, 1 \leqslant j \leqslant N_c$). c_1 and c_2 are the social influence factors while rand_1 and rand_2 are random values between 0 and 1. w is the inertia personal influence factor. In this paper, the parameters are set as follows: $N_p = 20, N_c = 20, w = 0.4, c_1 = c_2 = 2$.

As shown in Figure 9, in relative tracking pattern, the proper solution $W(t_c + \Delta t)$ is obtained using target's predicted position $P(t_c + \Delta t)$, $d_h = 2.8$ m, and $\theta = \pi/4$. Meanwhile, in visibility maintenance pattern, the feasible waypoint $W(t_c + \Delta t)$ is also found with $d_t = 2.0$ m, $d_m = 0.02$ m and target's future

Table 3 Parameter configurations

Parameter	Value (narrow corridor)	Value (complex environment)
d_h	2.0 m	2.0 m
d_v	3.0 m	2.0 m
θ	$\pi/4$	$\pi/3$
λ	100	100

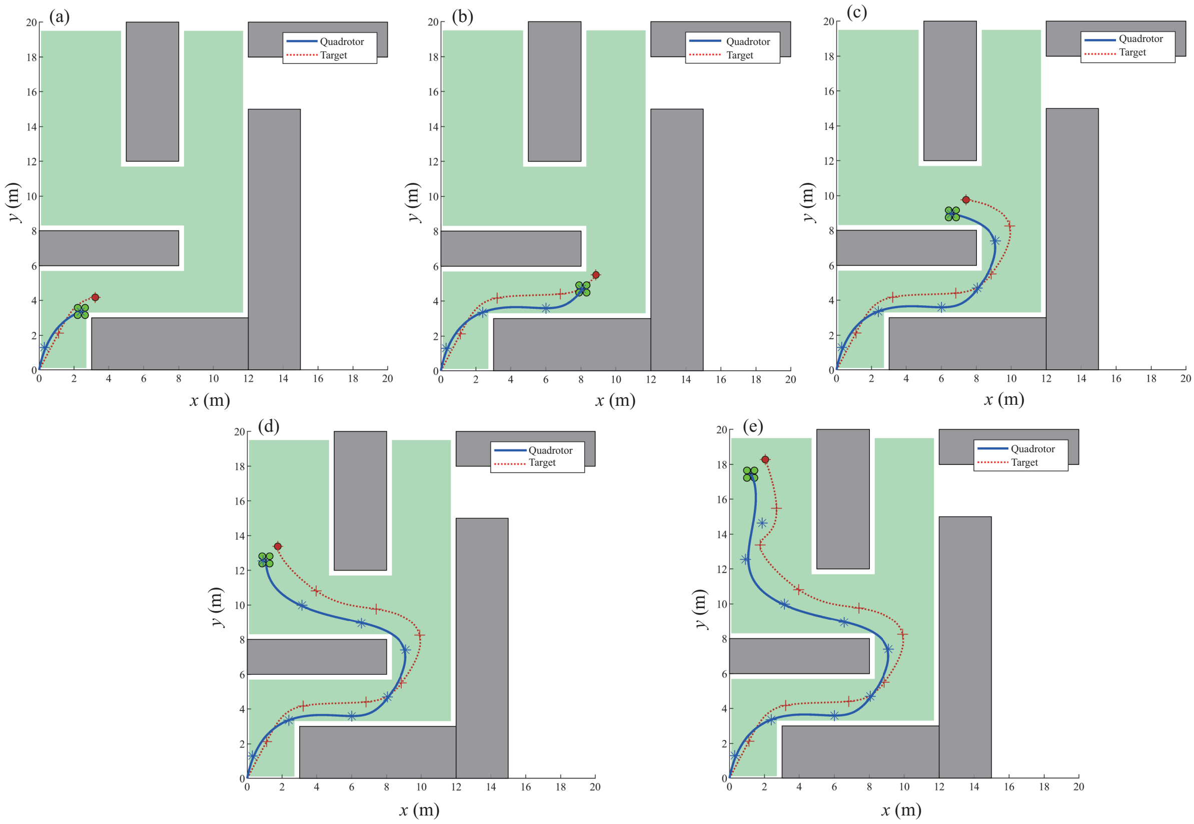

Figure 10 (Color online) Trajectory generation results of tracking a moving target in relative tracking pattern in narrow corridor. The red pluses represent waypoints of moving target while the blue stars are the corresponding waypoints of the quadrotor calculated by Subsection 3.1.1. (a) $t = 2.0$ s; (b) $t = 4.0$ s; (c) $t = 6.0$ s; (d) $t = 8.0$ s; (e) $t = 10.0$ s.

position $P(t_c + \Delta t)$ with consideration of limited filed of view. In those simulations, λ_1 and λ_2 are given as 0.8 and 10^4, respectively.

4.2.2 *Results in relative tracking pattern*

In this subsection, we present our simulation results of smooth trajectory generation for the quadrotor when tracking a moving target in relative tracking pattern in a 3D narrow corridor or complex environment. Some relevant simulation parameters are given in Table 3.

The simulation results in narrow corridor are plotted in Figures 10 and 11. Meanwhile, the resultant trajectory, as well as the planned velocity and acceleration in the complex environment are shown in Figure 12. First, as illustrated, the feasible waypoints are found by Subsection 3.1.1 with full consideration of safety and tracking performance. Then, the tracking trajectory of the quadrotor is generated as expected, and guaranteeing flight safety, physical limits, and smoothness.

4.2.3 *Results in visibility maintenance pattern*

In this subsection, we present the simulation results of safe trajectory generation for the quadrotor when tracking a moving target with consideration of the limited physical properties and field of view of the onboard sensor in two different environments. The desired distance d_t is set as 2 m. The quadrotor is tasked to execute target tracking in cluttered environments while keeping the target unobstructed as

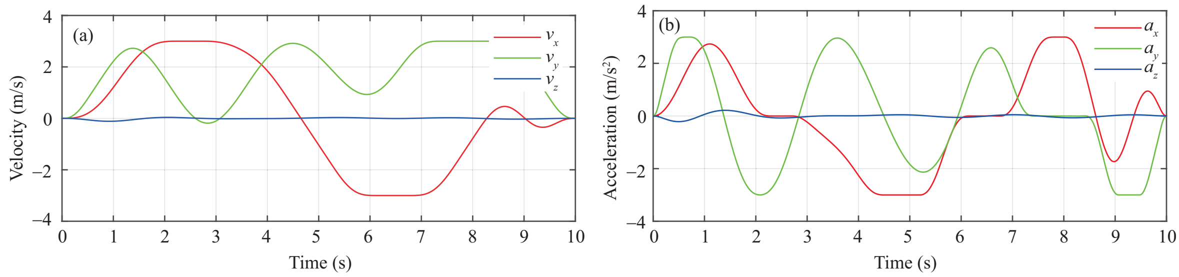

Figure 11 (Color online) The planned velocities and accelerations when the quadrotor tracks a moving target in relative tracking pattern in narrow corridor. (a) Velocity; (b) acceleration.

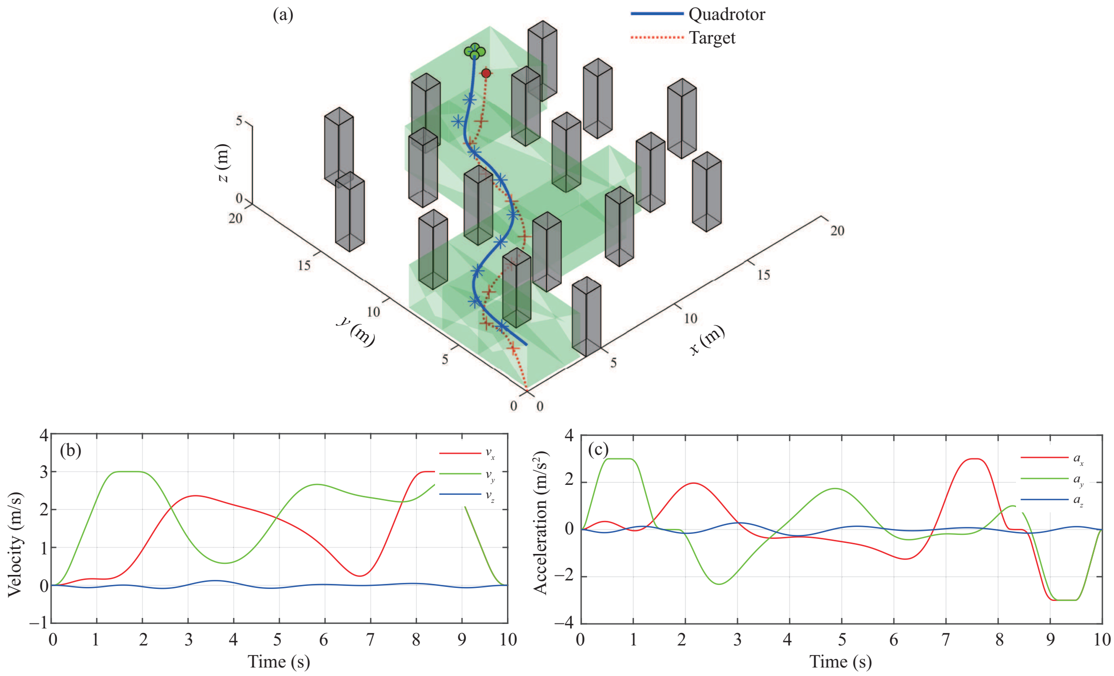

Figure 12 (Color online) Trajectory generation results of tracking a moving target in relative tracking pattern in complex environment. (a) Trajectory; (b) velocity; (c) acceleration.

Table 4 Average time cost

Process	Time cost (ms)
Path planning	8.2
Waypoint found	1.8
SFC generation	18.6
Trajectory generation	6.8

expected. The simulation results in narrow corridor are shown in Figures 13 and 14. Meanwhile, the resultant trajectory, planned velocity, and acceleration in the complex environment are shown in Figure 15. We see some exciting results. First, feasible waypoints for tracking trajectory generation are found by the method described in Subsection 3.1.2. Then, safe and dynamically feasible trajectories of the quadrotor are generated to fulfill the tracking task and keep the sight unobstructed.

The target-tracking trajectory generation method proposed in this work has efficient performance using a parallel PSO algorithm and convex optimization solver. The average time costs in each planning horizon of the above tracking simulations are shown in Table 4. It is clear that the proposed method makes it feasible to efficiently generate the tracking trajectory for the quadrotor in cluttered environments. Benefiting from the excellent properties of B-splines and proper convex polytopes, the quadrotor is always capable of tracking target and maintaining flight safety and smoothness.

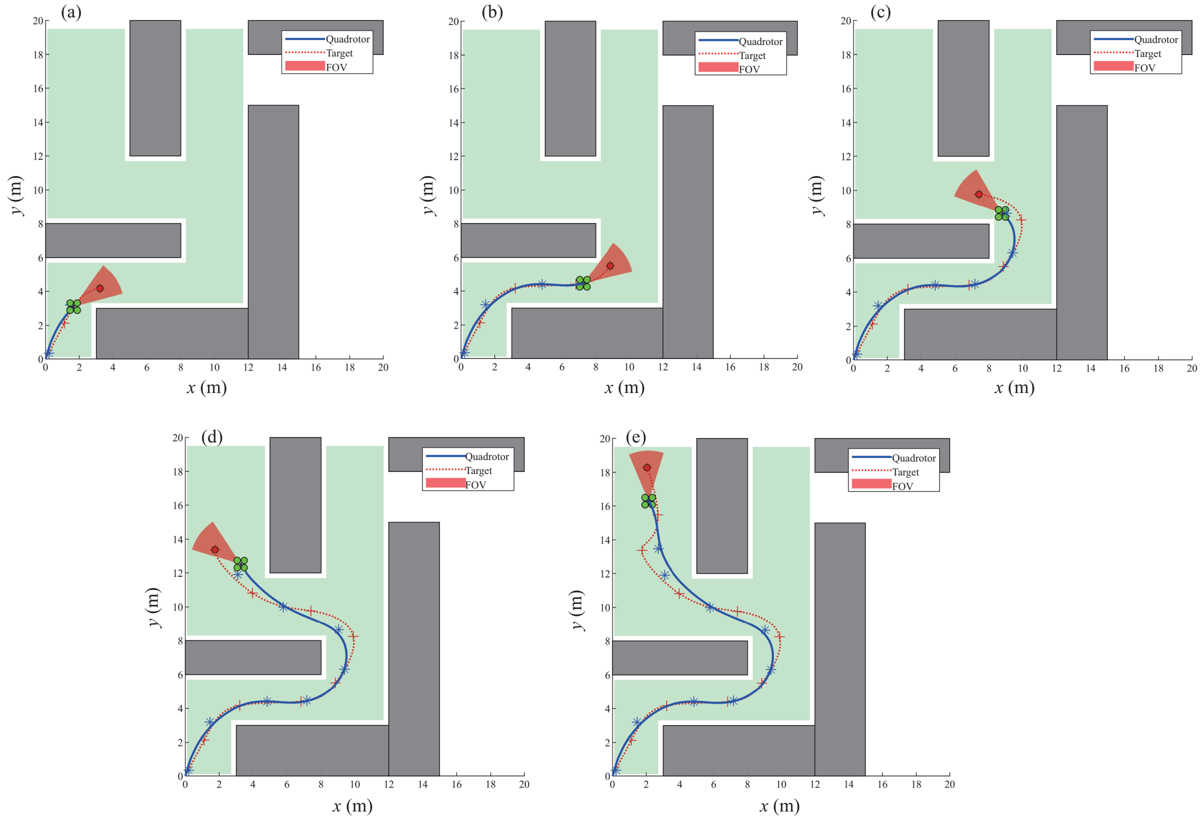

Figure 13 (Color online) Trajectory generation results of tracking a moving target in visibility maintenance pattern in narrow corridor. The red pluses represent waypoints of moving target while the blue stars are the corresponding waypoints obtained by trading off the flight length and visibility. The red sector represents the FOV. (a) $t = 2.0$ s; (b) $t = 4.0$ s; (c) $t = 6.0$ s; (d) $t = 8.0$ s; (e) $t = 10.0$ s.

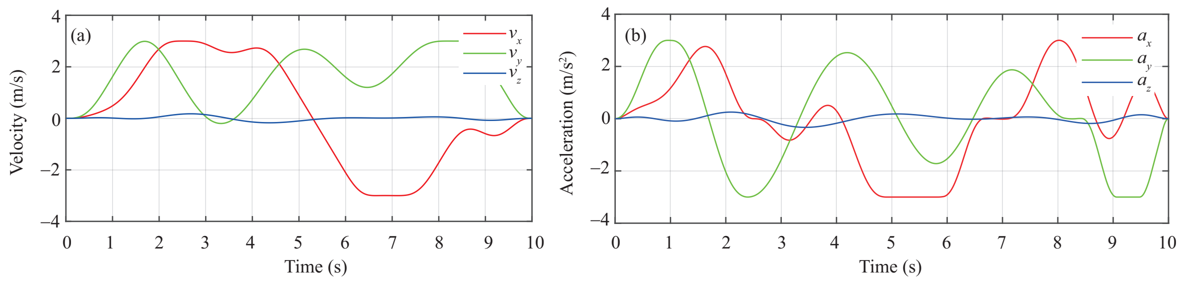

Figure 14 (Color online) The planned velocities and accelerations when the quadrotor tracks a moving target in visibility maintenance pattern in narrow corridor. (a) Velocity; (b) acceleration.

5 Conclusion

In this paper, we present a real-time smooth B-spline-based trajectory generation strategy for the quadrotor when tracking a moving target in two different patterns with consideration of physical limits and safety in complex environments. The dynamic and safe constraints are linearized by respectively forming geometrical cuboids where the entire flight trajectory should be squeezed. A parallel PSO method is applied to find feasible waypoints which the generated splines should be as close to as possible. Then, the whole problem is formulated as a general convex QP which can be solved efficiently. Simulation results demonstrate that the proposed method successfully tracks a moving target in different complex environments while guaranteeing flight safety, kinodynamic feasibility, and tracking performance. In the future, we will conduct further research on maneuvering target-tracking trajectory generation for quadrotors in dynamic complex environments.

Acknowledgements This work was supported by the Key Program of National Natural Science Foundation of China (NSFC) (Grant No. U1613225). The authors would like to thank Unmanned System Research Groups at the Chinese University of Hong

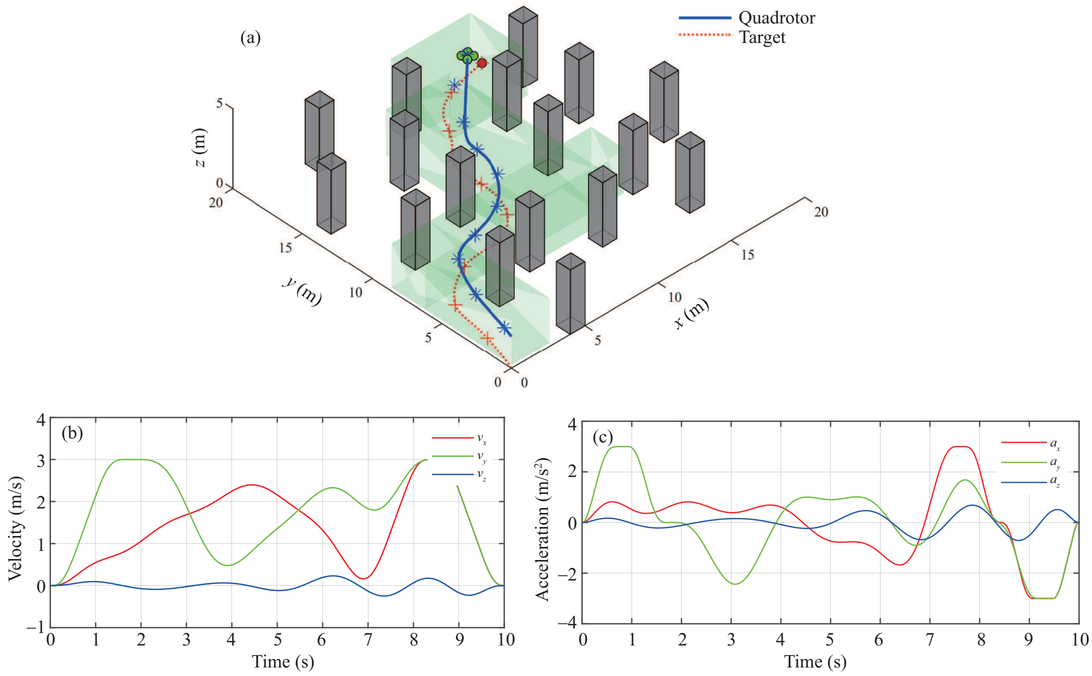

Figure 15 (Color online) Trajectory generation results of tracking a moving target in visibility maintenance pattern in complex environment. (a) Trajectory; (b) velocity; (c) acceleration.

Kong (CUHK) and Peng Cheng Laboratory (PCL) at Shenzhen, China.

References

1 He W, Huang H F, Chen Y N, et al. Development of an autonomous flapping-wing aerial vehicle. Sci China Inf Sci, 2017, 60: 063201
2 Liu S H, Wang S Q, Shi W H, et al. Vehicle tracking by detection in UAV aerial video. Sci China Inf Sci, 2019, 62: 024101
3 Hu C, Wang Y X, Wang R, et al. An improved radar detection and tracking method for small UAV under clutter environment. Sci China Inf Sci, 2019, 62: 029306
4 Li X X, Peng Z H, Liang L, et al. Policy iteration based Q-learning for linear nonzero-sum quadratic differential games. Sci China Inf Sci, 2019, 62: 052204
5 Thomas J, Welde J, Loianno G, et al. Autonomous flight for detection, localization, and tracking of moving targets with a small quadrotor. IEEE Robot Autom Lett, 2017, 2: 1762–1769
6 Zheng D L, Wang H S, Chen W D, et al. Planning and tracking in image space for image-based visual servoing of a quadrotor. IEEE Trans Ind Electron, 2018, 65: 3376–3385
7 Mueller M, Sharma G, Smith N, et al. Persistent aerial tracking system for UAVs. In: Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems, Daejeon, 2016. 1562–1569
8 Tan R Y, Kumar M. Tracking of ground mobile targets by quadrotor unmanned aerial vehicles. Unmanned Syst, 2014, 2: 157–173
9 Liu Y S, Wang Q X, Hu H S, et al. A novel real-time moving target tracking and path planning system for a quadrotor UAV in unknown unstructured outdoor scenes. IEEE Trans Syst Man Cybern Syst, 2019, 49: 2362–2372
10 Chen J, Liu T B, Shen H J. Tracking a moving target in cluttered environments using a quadcopter. In: Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems, Daejeon, 2016. 446–453
11 Jeon B F, Kim H J. Online trajectory generation of a MAV for chasing a moving target in 3D dense environments. In: Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems, Macau, 2019. 1115–1121
12 Nicolis D, Palumbo M, Zanchettin A M, et al. Occlusion-free visual servoing for the shared autonomy teleoperation of dual-arm robots. IEEE Robot Autom Lett, 2018, 3: 796–803
13 Penin B, Giordano P R, Chaumette F. Vision-based reactive planning for aggressive target tracking while avoiding collisions and occlusions. IEEE Robot Autom Lett, 2018, 3: 3725–3732
14 Wei Y S, Lin Z L. Vision-based tracking by a quadrotor on ROS. Unmanned Sys, 2019, 07: 233–244
15 Mellinger D, Kumar V. Minimum snap trajectory generation and control for quadrotor. In: Proceedings of IEEE International Conference on Robotics and Automation, Shanghai, 2011. 2520–2525
16 Lai S P, Lan M L, Chen B M. Optimal constrained trajectory generation for quadrotors through smoothing splines. In: Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems, Madrid, 2018. 4743–4750
17 Liu S K, Watterson M, Mohta K, et al. Planning dynamically feasible trajectories for quadrotors using safe flight corridors in 3-D complex environments. IEEE Robot Autom Lett, 2017, 2: 1688–1695
18 Landry B, Deits R, Florence P R, et al. Aggressive quadcopter flight through cluttered environments using mixed integer programming. In: Proceedings of IEEE International Conference on Robotics and Automation, Stockholm, 2016. 1469–1475
19 Lai S P, Lan M L, Chen B M. Efficient safe corridor navigation with jerk limited trajectory for quadcopters. In: Proceedings of the 37th Chinese Control Conference, Wuhan, 2018. 10065–10070
20 Giusti A, Guzzi J, Ciresan D C, et al. A machine learning approach to visual perception of forest trails for mobile robots. IEEE Robot Autom Lett, 2016, 1: 661–667

21 Chen J, Shen S J. Using a quadrotor to track a moving target with arbitrary relative motion pattern. In: Proceedings of IEEE International Conference on Intelligent Robots and Systems, Vancouver, 2017. 5310–5317
22 Parikh A, Kamalapurkar R, Dixon W E. Target tracking in the presence of intermittent measurements via motion model learning. IEEE Trans Robot, 2018, 34: 805–819
23 Kano H, Fujioka H, Martin C. Optimal smoothing and interpolating splines with constraints. Appl Math Comput, 2011, 218: 1831–1844
24 Zhang M, Yan W, Yuan C M, et al. Curve fitting and optimal interpolation on CNC machines based on quadratic B-splines. Sci China Inf Sci, 2011, 54: 1407–1418
25 Ding W T, Gao W L, Wang K X, et al. An efficient B-spline-based kinodynamic replanning framework for quadrotors. IEEE Trans Robot, 2019, 35: 1287–1306
26 Wang K L, Ke Y J, Chen B M. Autonomous reconfigurable hybrid tail-sitter UAV U-Lion. Sci China Inf Sci, 2017, 60: 033201
27 Yuan Y, Cheng L, Wang Z D, et al. Position tracking and attitude control for quadrotors via active disturbance rejection control method. Sci China Inf Sci, 2019, 62: 010201
28 Chen B M, Lee T H, Venkataramanan V. Hard disk drive servo system. In: Advances in Industrial Control Series. Berlin: Springer, 2002
29 Park J, Kim H J. Fast trajectory planning for multiple quadrotors using relative safe flight corridor. In: Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems, Macau, 2019. 596–603