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H-Infinity Static Output-feedback Control for Rotorcraft

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Abstract The problem of stabilization of an autonomous rotorcraft platform in a hover configuration subject to external disturbances is addressed. Necessary and sufficient conditions are presented for static output-feedback control of linear time-invariant systems using the H-Infinity approach. Simplified conditions are given which only require the solution of two coupled matrix design equations. This paper also proposes a numerically efficient solution algorithm for the coupled design equations to determine the output-feedback gain. A major contribution is that an initial stabilizing gain is not needed. The efficacy of the control law and the disturbance accommodation properties are shown on a rotorcraft design example. The helicopter dynamics do not decouple as in the fixed-wing aircraft case, so that the design of helicopter flight controllers with a desirable intuitive structure is not straightforward. In this paper an output feedback approach is given that allows one to selectively close prescribed multivariable feedback loops using a reduced set of the states. Shaping filters are added that improve performance and yield guaranteed robustness

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and speed of response. This gives direct control over the design procedure and performance. Accurate identification of the System parameters is a challenging task for rotorcraft control, addition of loop shaping facilitates implementation engineers to counteract unmodeled high frequency dynamics. The net result yields control structures that have been historically accepted in the flight control community.

Keywords H-Infinity control · Unmanned aerial vehicles

1 Introduction

This paper investigates the application of a novel restricted-measurement static output-feedback (OPFB) control methodology to control an unmanned rotorcraft. In the past few years, there has been a significant interest in using unmanned aerial vehicles for applications such as search and rescue, surveillance and remote inspection. Rotorcrafts (especially helicopters) have several significant advantages over conventional fixed wing platforms in conducting several of these tasks. The advantages are exemplified by certain unique capabilities of the rotorcraft e.g., they can hover and can take-off and land in very limited spaces. Moreover, helicopters are highly maneuverable making them preferable for such tasks.

Several linear as well as non-linear control strategies have been proposed for control of helicopters and can be found in [15, 23, 29] and references within. The methodologies used in [15] and [23] use an adaptive feedback linearization approach where in a neural network approximates the uncertainties and the network weights are updated adaptively based on the trajectory tracking errors. While the controllers are efficient, one introduces additional dynamics to synthesize the controller. Further this makes the controller of very high order and there is no optimality guaranteed against the specific classes of disturbances/uncertainties considered. Alternately, we propose a static output feedback control structure based on H-Infinity theory. We specifically focus on the problem of control in a hover configuration which in general is an unstable configuration. Further, in the presence of disturbances, the helicopter exhibits deviations in the dynamical states which complicate the control problem as the helicopter dynamical states are very tightly coupled. For example, in hover, pitch motion almost always is accompanied by forward and vertical motion and all three states need to be controlled simultaneously [16].

The static output-feedback problem is one of the most researched problems in systems and control theory. The use of output feedback allows flexibility and simplicity of implementation. Moreover, in practical applications, full state measurements are not usually possible. The restricted-measurement static output-feedback problem is of extreme importance in practical controller design applications including flight control [26], manufacturing robotics [17], and elsewhere where it is desired that the controller have certain pre-specified desirable structure, e.g., unity gain outer tracking loop and feedback only from certain available sensors. A survey of OPFB design results is presented in [27]. Finally, though many theoretical conditions have been offered for the existence of OPFB, there are few good solution algorithms. Most existing algorithms require the determination of an initial stabilizing gain, which can be extremely difficult.

It is well known that the OPFB optimal control solution can be prescribed in terms of three coupled matrix equations [21] namely two associated Riccati equations and a spectral radius coupling equation. A sequential numerical algorithm to solve these equations is presented in [22]. OPFB stabilizability conditions that only require the solution of two coupled matrix equations are given [11, 20] and [28]. Some recent LMI approaches for OPFB design are presented [2, 7] and [12]. These allow the design of OPFB controllers using numerically efficient software, e.g., the MATLAB LMI toolbox [10]. However several problems are still open. Most of the solution algorithms are hard to implement, are difficult to solve for higher order systems, may impose numerical problems and may have restricted solution procedures such as the initial stabilizing gain requirements.

H-Infinity design has played an important role in the study and analysis of control theory since its original formulation in an input–output setting [30]. It is well known that, though conservative, they provide better response in the presence of disturbance than H_2 optimal techniques. State-space H-Infinity solutions were rigorously derived for the linear time-invariant case that required solving several associated Riccati equations [6]. Later, more insight into the problem was given after the H-Infinity linear control problem was posed as a zero-sum two-player differential game [1]. A thorough treatment of H-Infinity design is given in [19], which also considers the case of OPFB using dynamic feedback. An excellent treatment of H2 and H-Infinity is given [5].

Static OPFB design, as opposed to dynamic output feedback with a regulator, is suitable for the design of aircraft controllers of prescribed structure. H-Infinity design has been considered for static OPFB, Hol and Scherer [14] addressed the applicability of matrix-valued sum-of-squares (sos) techniques for the computations of LMI lower bounds. Prempain and Postlethwaite [24] presented conditions for a static output loop shaping controller in terms of two coupled matrix inequalities. Recently, the application of loop shaping procedure in Helicopter control has lead to several improvements [25].

The aim of this paper is to demonstrate that high performance low order controllers can be easily and efficiently computed using H-Infinity Static Output Feedback Techniques given in [8] and [9]. In this paper, we show that the H-Infinity approach can be used for static OPFB design to yield a simplified solution procedure that only requires the solution of one associated Riccati equation and a coupled gain matrix condition. This explains and illuminates the results in [20]. That is, H-Infinity design provides more straightforward design equations than optimal control, which requires solving three coupled equations. We have two objectives. First, we give necessary and sufficient conditions for OPFB with H-Infinity design. Second, we suggest a less restrictive numerical solution algorithm with no initial stabilizing gain requirement. The design synthesis procedure is applied to the robust stabilization of an autonomous rotorcraft.

The paper is organized as follows. Section 2 details the formulation of Necessary and Sufficient Condition for H-Infinity OPFB Control. A solution Algorithm is proposed in Section 3. Section 4 illustrates the Unmanned Aerial Vehicle (UAV) model; controller structure, H-Infinity loop shaping design procedure, and simulation results with disturbance effects.

2 Necessary and Sufficient Condition for H-Infinity OPFB Control

In this section we present a method for finding H-Infinity static output feedback (OPFB) gains. It is seen that the H-Infinity OPFB gain is computed in terms of only two coupled matrix equations. This is a simpler problem to solve than the optimal OPFB problem given in terms of three coupled equations [26]. Moreover, a numerical algorithm is given to solve these equations that does not require an initial stabilizing OPFB gain.

2.1 System Description and Definitions

Consider the linear time-invariant system of Fig. 1 with control input $u(t)$, output $y(t)$, and disturbance $d(t)$ given by

$$\dot{x} = Ax + Bu + Dd, \quad y = Cx, \tag{1}$$

and a performance output $z(t)$ that satisfies

$$\|z(t)\|^2 = x^T Qx + u^T Ru. \tag{2}$$

A static output-feedback control is given by

$$u = -Ky = -KCx. \tag{3}$$

By definition the pair (A, B) is said to be *stabilizable* if there exists a real matrix K such that $A-BK$ is (asymptotically) stable. The pair (A, C) is said to be *detectable* if there exists a real matrix L such that $A-LC$ is stable. System (1) is said to be *output feedback stabilizable* if there exists a real matrix K such that $A-BKC$ is stable.

2.2 Bounded L_2 Gain Design Problem

The System L_2 gain is said to be *bounded or attenuated by* γ if

$$\frac{\int_0^\infty \|z(t)\|^2 dt}{\int_0^\infty \|d(x)\|^2 dt} = \frac{\int_0^\infty \left(x^T Qx + u^T Ru\right) dt}{\int_0^\infty \left(d^T d\right) dt} \le \gamma^2 \tag{4}$$

for any non-zero energy-bounded disturbance input d. Call γ^* the minimum gain for which this occurs. For linear systems, these are explicit formulae to compute γ^* [4]. Throughout this paper we shall assume that γ is fixed and $\gamma^* > \gamma$. The case when $\gamma = \gamma^*$ is called H_∞ Control. It is desired to find a static OPFB gain K such that the system is stable and the L_2 gain is bounded by a prescribed value γ.

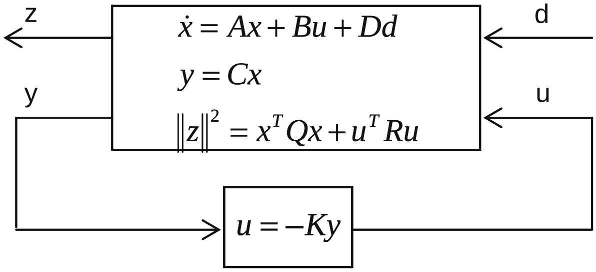

Fig. 1 System description

The next theorem gives necessary and sufficient conditions for existence of bounded L_2 gain static OPFB control, *c.f.* [9].

Theorem 1 *Necessary and Sufficient Conditions for Bounded L_2 gain Static OPFB Control:*

For a given $\gamma > \gamma^*$, there exists an OPFB gain such that $A_0 \equiv (A - BKC)$ is asymptotically stable with L_2 gain bounded by γ if and only if:

i. *(A, C) is detectable and there exist matrices L and $P = P^T \geq 0$ such that:*

ii.

$$KC = R^{-1}\left(B^T P + L\right). \tag{5}$$

iii.

$$PA + A^T P + C^T C + \frac{1}{\gamma^2} PDD^T P - PBR^{-1}B^T P + L^T R^{-1} L = 0 \tag{6}$$

3 Solution Algorithm

Most existing iterative algorithms for OPFB design require the determination of an initial stabilizing gain, which can be very difficult for practical aerospace systems such as the stabilization of an autonomous rotorcraft in hover. The following algorithm is proposed to solve the two coupled design equations in Theorem 1. Note that *it does not require an initial stabilizing gain* since, in contrast to Kleinman's state feedback Algorithm [18] and the OPFB algorithm of Moerder and Calise [22], it uses a Riccati equation solution, not a Lyapunov equation, at each step.

1. Initialize:
   Set $n = 0$, $L_0 = 0$, and select γ, Q and R.
2. *n-th* iteration:
   solve for P_n in

$$P_n A + A^T P_n + Q + \frac{1}{\gamma^2} P_n DD^T P_n - P_n BR^{-1}B^T P_n + L_n^T R^{-1} L_n = 0 \tag{7}$$

   Evaluate gain and update L

$$K_{n+1} = R^{-1}\left(B^T P_n + L_n\right) C^T \left(CC^T\right)^{-1} \tag{8}$$

$$L_{n+1} = RK_{n+1}C - B^T P_n \tag{9}$$

   If L_{n+1} and L_n are close enough to each other, go to 3 otherwise set $n = n + 1$ and go to 2.
3. Terminate:
   Set $K = K_{n+1}$

Lemma *If this algorithm converges, it provides the solution to Eqs. 5 and 6.*

Proof Clearly at convergence Eq. 7 holds for P_n. Note that substitution of Eq. 8 into Eq. 9 yields.

$$L_{n+1} = R\left[R^{-1}\left(B^T P_n + L_n\right) C^+\right] C - B^T P_n$$

At convergence $L_{n+1} = L_n \equiv L$, $P_n \equiv P$ so that

$$L = \left(B^T P + L\right) C^+ C - B^T P,$$

or

$$B^T P + L = \left(B^T P + L\right) C^+ C$$

This guarantees that there exists a solution K to Eq. 5 given by $K = R^{-1}\left(B^T P + L\right) C^+$. □

Note that this algorithm uses well-developed techniques for solving Riccati equations available, for instance, in MATLAB. It generalizes the algorithm in [11] to the case of nonzero initial gain. It is described in Section 4 that this algorithm is also suitable to find static output-feedback gains for loop-shaped plants.

4 Attitude Control Loop Design Example

4.1 System Description

The controller design is based on an 11-state linear model of a "Raptor-90" helicopter shown in Fig. 2.

The results are based on the model derived at National University of Singapore. A linearized model for hover operating point has been established. The model currently used is a state-space model which represents the helicopter as 6-degree-of-freedom (DOF) rigid body augmented with servo/rotor dynamics and artificial yaw damping dynamics [3]. The state vector physically shown in Fig. 3 contains eleven states and can be expressed as $X = \left[\, U \; V \; p \; q \; \phi \; \theta \; a_s \; b_s \; W \; r \; r_{fb} \,\right]^T$. Variables are described in the table below (Table 1).

The input vector can be written as $u = \left[\, \delta_{lati} \; \delta_{longi} \; \delta_{ped} \,\right]^T$. Where δ_{lati} is the lateral channel input and affects roll motion, δ_{longi} is longitudinal channel input and affects

Fig. 2 Raptor-90 helicopter

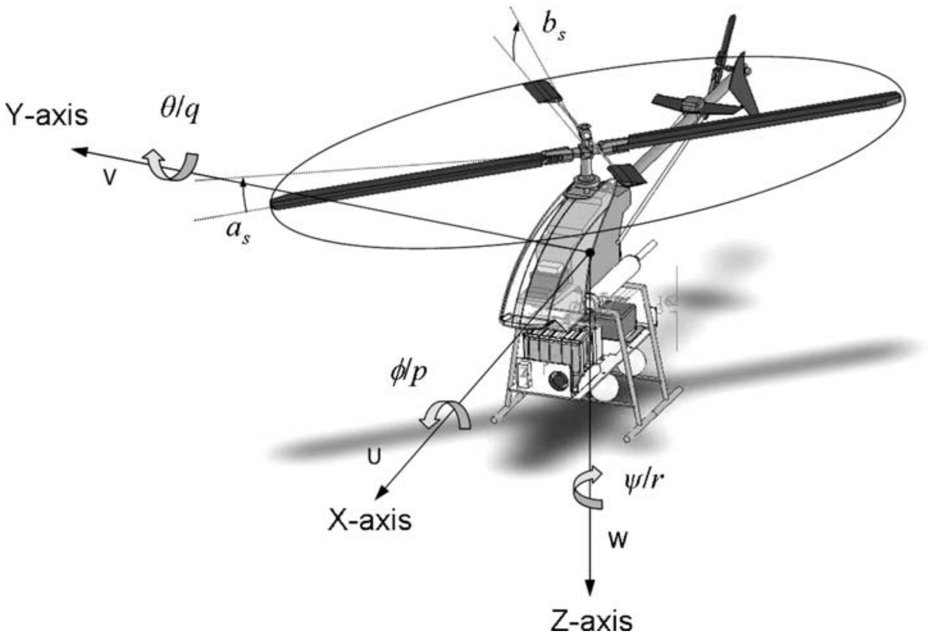

Fig. 3 Helicopter states in body frame coordinate system

pitch, δ_{ped} is pedal channel input of remote controller and affects yaw motion. In helicopters there is a high degree of coupling between lateral and longitudinal dynamics. In this paper the collective channel, the fourth actuator which produces lift, is left to be controlled manually.

The primary variables to be controlled are the pitch angle and roll angle. Two extra rate gyros measuring pitch angular rate and roll angular rate will also be used for feedback purposes. Five system states constitute the output vector $y = \left[\phi\ \theta\ r\ p\ q\right]^T$. The rotorcraft equations mentioned were trimmed in a hover configuration to obtain the reference trim condition. The nonlinear equations then

Table 1 Helicopter variables

Variables	
U	Velocity along the body frame x-axis
V	Velocity along the body frame y-axis
p	Roll rate in the body frame components
q	Pitch rate in the body frame components
ϕ	Roll angle
θ	Pitch angle
ψ	Yaw angle
a_s	Longitudinal blade angle
b_s	Lateral blade angle
W	Velocity along the body frame z-axis
r	Yaw rate in the body frame components
r_{fb}	Yaw rate feedback

linearized for the hover configuration based on the reference values obtained. The procedure is described in [26]. The plant linear matrices are as below

$$A=\begin{bmatrix} -0.1778 & 0 & 0 & 0 & 0 & -9.7807 & -9.7807 & 0 & 0 & 0 & 0 \\ 0 & -0.3104 & 0 & 0 & 9.7807 & 0 & 0 & 9.7807 & 0 & 0 & 0 \\ -0.3326 & -0.5353 & 0 & 0 & 0 & 0 & 75.7640 & 343.8600 & 0 & 0 & 0 \\ 0.1903 & -0.2940 & 0 & 0 & 0 & 0 & 172.6200 & -59.9580 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & -8.1222 & 4.6535 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -0.0921 & -8.1222 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 17.1680 & 7.1018 & -0.6821 & -0.1070 & 0 \\ 0 & 0 & -0.2834 & 0 & 0 & 0 & 0 & 0 & -0.1446 & -5.5561 & -36.6740 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.7492 & -11.1120 \end{bmatrix}$$

$$B=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.0632 & 3.3390 & 0 \\ 3.1739 & 0.2216 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -74.3640 \\ 0 & 0 & 0 \end{bmatrix}$$

4.2 Wind Turbulence Model

The disturbance vector d given in (10) has wind components along the $\left[x\ y\right]^T$ fuselage axes, disturbance input matrix D defines dynamics involved with body frame x, and y velocities. For this example D is a 11×2 matrix and is constituted from first two columns of the plant matrix A.

$$d = \left[\, d_U\ d_V \,\right]^T \tag{10}$$

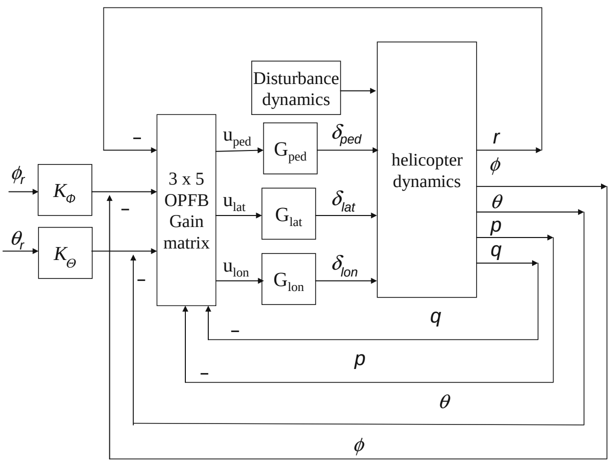

Fig. 4 Controller structure

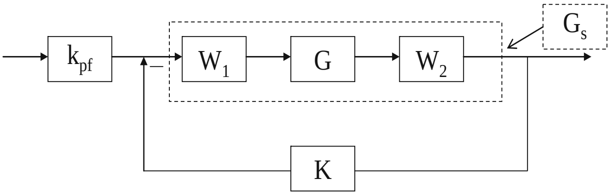

Fig. 5 Loop shaped plant with controller

In Hall and Bryson [13] the wind components along the fuselage axes are modeled by independently excited correlated Gauss-Markov processes

$$\begin{bmatrix} \dot{d}_U \\ \dot{d}_V \end{bmatrix} = \begin{bmatrix} -1/\tau_c & 0 \\ 0 & -1/\tau_c \end{bmatrix} \begin{bmatrix} d_U \\ d_V \end{bmatrix} + \rho^* B_w \begin{bmatrix} q_U \\ q_V \end{bmatrix} \tag{11}$$

Equation 11 is called a "shaping filter" for the wind, where q_U, and q_V are independent with zero mean, $\tau_c = 3.2$ s is the correlation time of the wind, $\sigma_{q_U}, \sigma_{q_V} = 20\ ft/s$, B_w is the turbulence input identity matrix, and $\rho = 1/2$ is the scalar weighting factor.

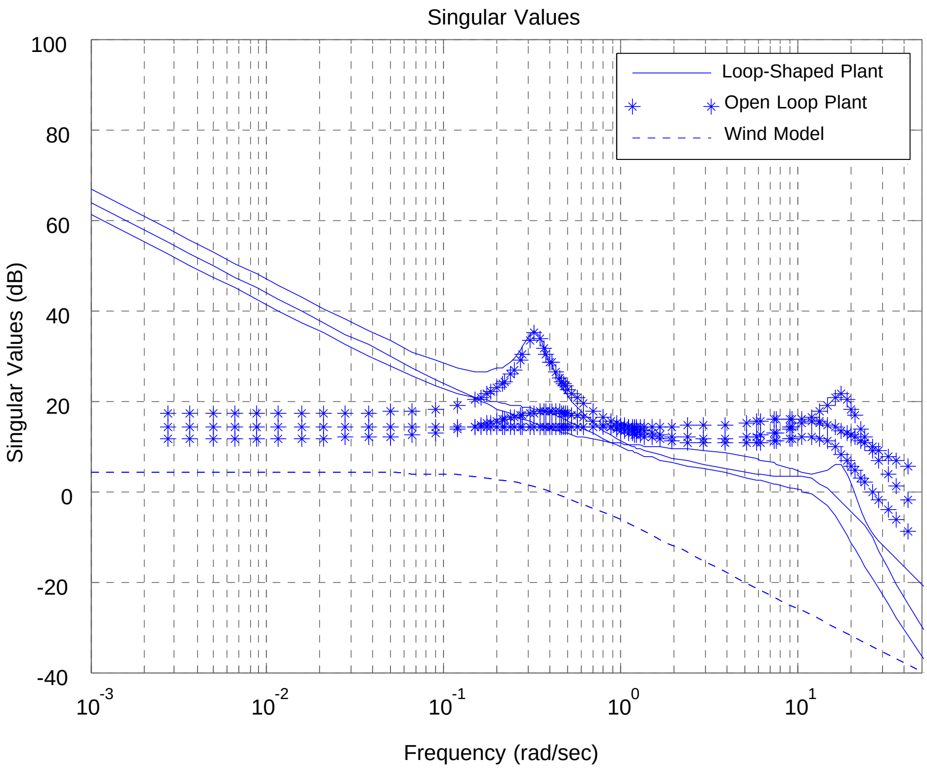

Fig. 6 Loop-gain singular value plots

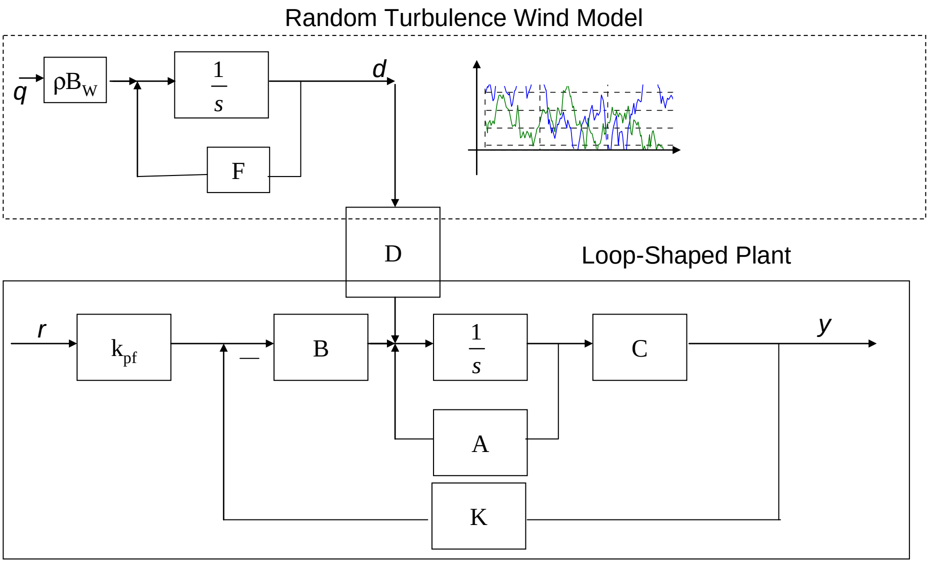

Fig. 7 Simulation with turbulence model

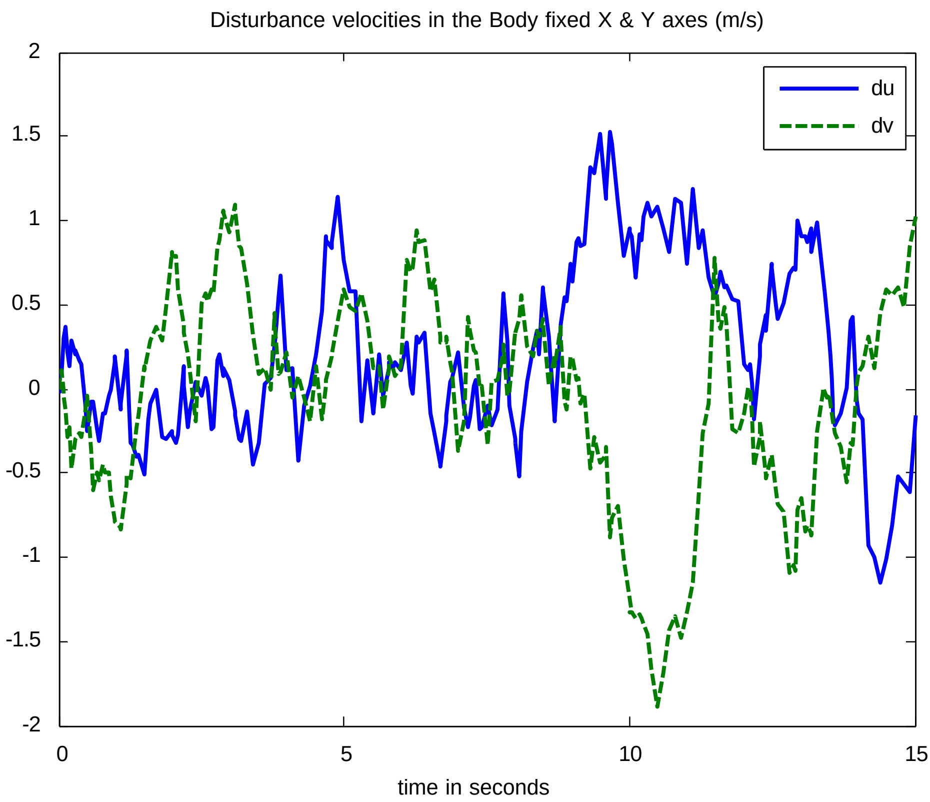

Fig. 8 Random velocity disturbance vector

4.3 Controller Structure

The control structure shown in Fig. 4 is basically an attitude control loop; each input channel is augmented with a compensator. Precompensators $G_{lat}(s)$, $G_{long}(s)$, and $G_{ped}(s)$ shape the plant prior to closing the loop (Fig. 5). The loop shaping procedure is explained in the next section.

4.4 H-Infinity Loop Shaping Design Procedure

We will now formally state the design procedure. The objective of this approach is to balance the tradeoff between performance and robustness in loop shaping. The procedure couples loop shaping design with H-Infinity output-feedback control techniques.

- Using a precompensator W_1 and a postcompensator W_2, the singular values of the nominal plant are shaped to achieve a desired open-loop shape.
- The nominal plant G and the compensators are combined to form the shaped plant G_s. Let (A, B, C, D) be a realization of G_s.
- Choose weighing matrices Q and R for G_s.
- Use H-Infinity static output feedback algorithm to find the static output-feedback gain. The algorithm is described in Section 3.
- Find prefilter gain K_{pf} for unity steady state gain between input and output pairs.

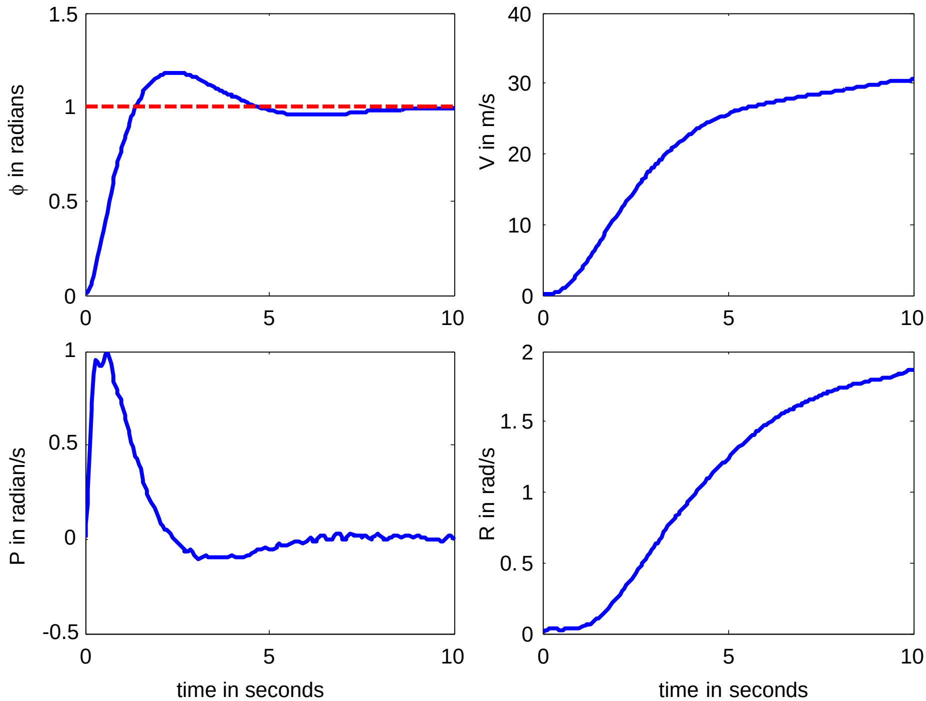

Fig. 9 Closed-loop lateral-directional state responses to a unity bank angle step demand

4.4.1 Loop Shaping

In this example precompensators $G_{lat}(s)$, $G_{lon}(s)$, and $G_{ped}(s)$ all are chosen as $G_{precomp}(s) = 2\left(\frac{s+0.5}{s(s+5)}\right)$ to shape the open loop plant. Additional dynamics in the Pre-Compensators is included to pull the cut-off to within the 2–5 rad/s region, which is typical with aircraft and rotorcraft controllers. There exists a large amount of published literature relating to loop shaping including [25]. In terms of the bode magnitude plot, it is known that the loop gain should be high at low frequencies for performance robustness, but low at high frequencies, where unmodeled dynamics may be present, for stability robustness. The classical frequency domain methods are extended to multi-input multi-output system in a rigorous fashion by a loop shaping procedure. The design was effective using only the Pre-Compensators, so no Post-Compensators were chosen, i.e., the Post-Compensator weights was set to the identity matrix. The singular value plots of the original loop-gain and the shaped loop-gain are shown in Fig. 6. Also shown is the wind gust spectrum.

4.4.2 Weighting Matrices

In this example the weighting matrices are taken as

$$Q = \text{diag}\left[0.25\ 0.25\ \ 0.01\ 0.01\ 100\ 100\ 1\text{E}-4\ 1\text{E}-4\ 0.25\ \ 0.01\ 0.01\ 0\ 0\ 0\ 0\ 0\ 0\right]$$
$$R = \text{diag}\left[169\ 169\ 0.78\right].$$

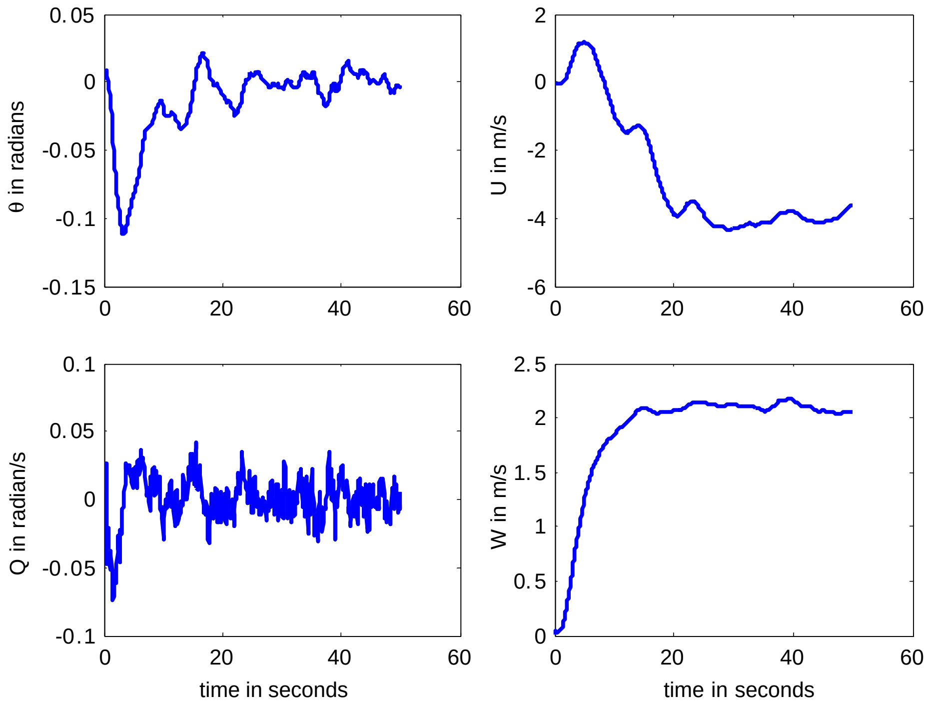

Fig. 10 Closed-loop longitudinal state responses to a unity bank angle step demand

The selection of Q and R is further discussed in the next subsection.

4.5 Simulation Results with Disturbance Effects

The static output feedback solution derived in Section 3 is applied to obtain an output feedback controller to stabilize the loop-shaped plant. The controller is then simulated subject to the wind disturbances to evaluate the efficacy of the proposed control law. The closed-loop system is shown in Fig. 7, where the exogenous disturbance input $d(t)$ is a random variable, shown in Fig. 8, generated in the time domain to match statistical properties of the turbulence model, as discussed in Section 4.5.2.

For the computation of the output-feedback gain K it is necessary to select weighting matrices Q and R. A diagonal structure is used for Q and R. The diagonal entries are tuned iteratively. That is, for a given selection of Q and R, our algorithm was run to find the OPFB gain K. Then, the closed-loop system was simulated. If the results are not satisfactory, Q and R were modified and the procedure was repeated. Our algorithm makes it very fast and easy to perform this procedure., To avoid the excitation of un-modeled high frequency dynamics, the control input and velocity states are heavily penalized.

The gain parameter γ defines the desired L_2 gain bound. For the initial design, a fairly large γ is selected. If the algorithm converges, the parameter γ may be reduced.

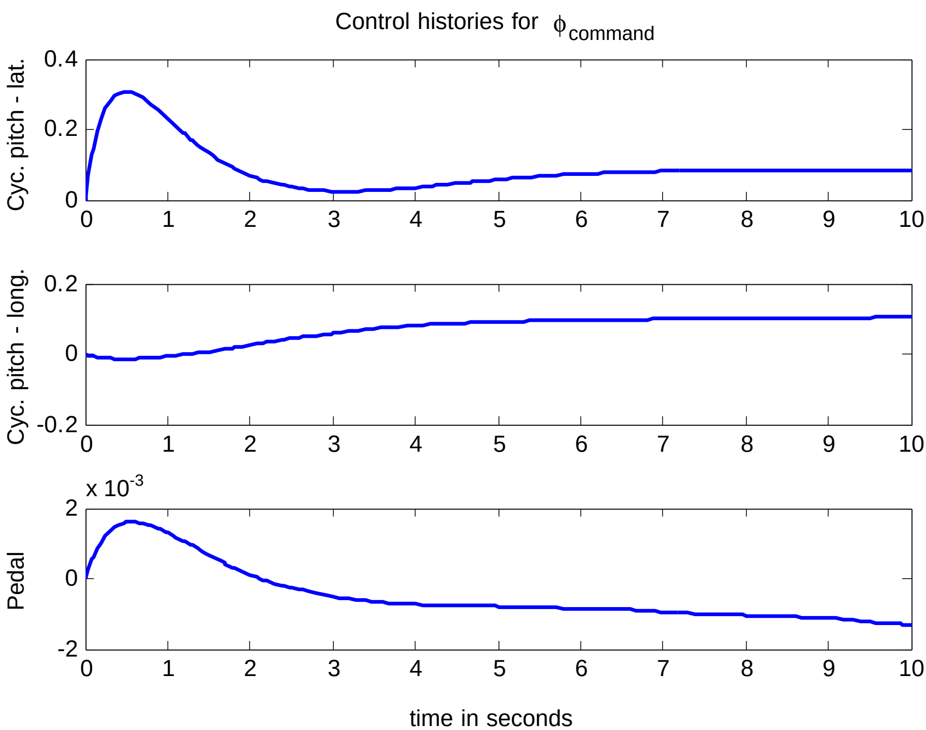

Fig. 11 Control history for a unity bank angle step demand

If γ is taken too small the algorithm will not converge since the Algebraic Riccati Equation has no positive semidefinite solution. After some design repetitions, which were performed very quickly using the algorithm; we found the smallest value of the gain to be 0.62.

Two particular cases were simulated to evaluate the closed loop system performance, namely bank angle command tracking i.e. $\phi_{command}$ and a pitch angle command tracking i.e. $\theta_{command}$

4.5.1 Bank Angle Command Tracking ($\phi_{command}$)

The step responses of the lateral-directional states for a unit bank angle command (equivalent of 1 radian) are shown in Fig. 9. The inner loop simulation is begun at a hover configuration at an altitude of 50 m and was subjected to a turbulent wind disturbance with peak amplitude of 4.0 m/s. Considering, that the helicopter is initially in the hover configuration, this is a significant perturbation. It is seen that the bank angle settles to less than 5% of the steady state value within 5 s. The overshoot is 18.5%. The roll rate does not peak beyond 1 rad/s, which is within acceptable limits. We also note that the yaw rate activity is consistent with the build up in the lateral velocity. It is to be mentioned that throughout this inner loop control design, the collective pitch is not utilized. The consequence of this is a velocity build-up that causes the helicopter to drift from its current position. It was seen that without the collective pitch being active, the helicopter loses altitude very rapidly as the main

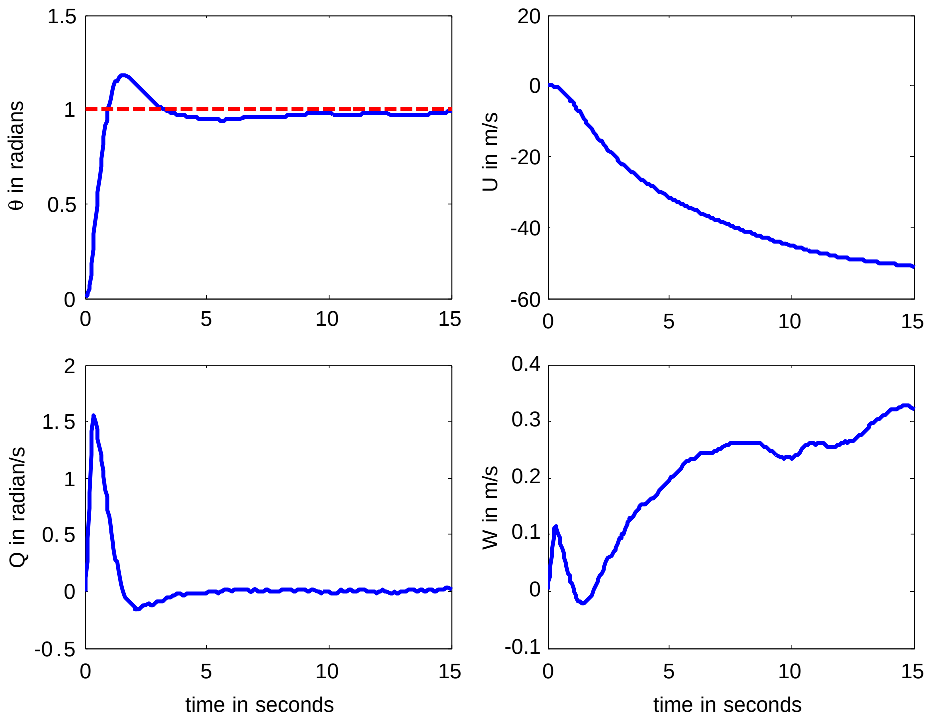

Fig. 12 Closed-loop longitudinal state responses to a unity pitch angle step demand

rotor thrust vector is no longer aligned along the inertial Z-axis. The only way to increase the component of the thrust along the inertial Z-axis to balance the weight of the helicopter is to use the collective pitch.

Figure 10 shows the longitudinal state responses for this case. It is seen that the states are all within acceptable limits. Note, the slight build up in the body axes U and W components of the velocities is attributed to the loss in lift due to the vectoring of the main rotor thrust to achieve the desired bank angle as well as the coupling between the longitudinal and lateral-directional dynamics. In addition, there is a velocity disturbance along the body X-axis due to turbulent wind.

Figure 11 shows the cyclic-pitch activity in the lateral as well as the longitudinal axes and the rudder pedal activity. As expected the activity in the rudder is minimal. The longitudinal cyclic-pitch responds to arrest the build up in the longitudinal states (pitch angle and pitch rate).

4.5.2 Pitch Angle Command Tracking ($\theta_{command}$)

The step responses of the longitudinal states for a unit pitch angle command (equivalent of 1 radian) are shown in Fig. 12. The helicopter configuration is identical to the earlier case, i.e. there is no collective pitch activity and similar turbulent wind disturbances are injected into the system. It is seen that the pitch angle settles to less than 6% of the steady state value within 5 s. The overshoot is <18.5%. The slight oscillations within the 5% settling band are due to the external state disturbance (due

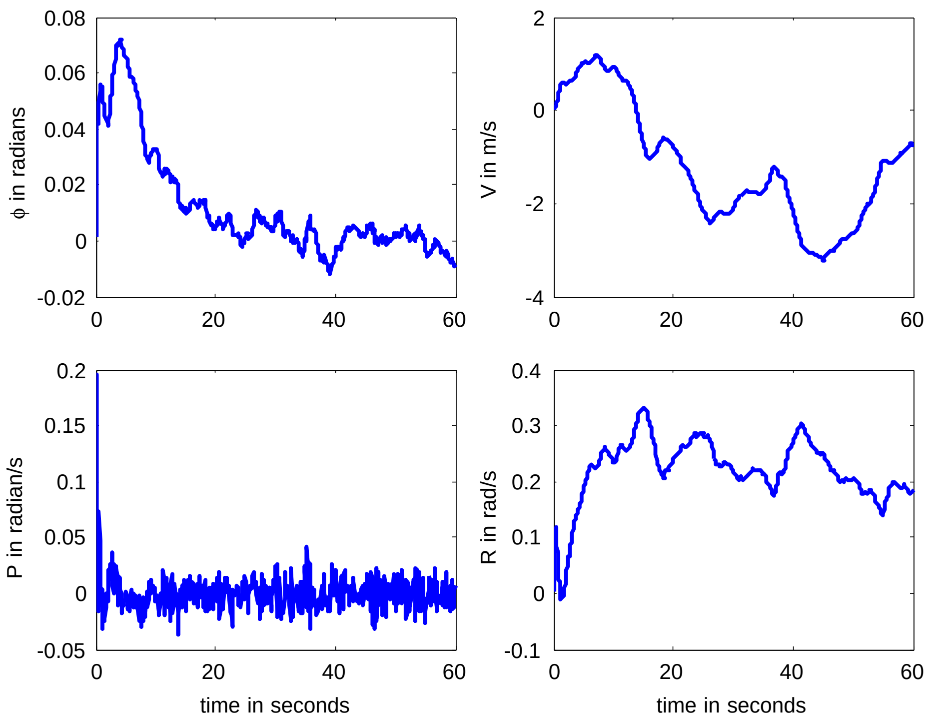

Fig. 13 Closed-loop lateral-directional state responses to a unity pitch angle step demand

to turbulent wind). The pitch rate does not peak beyond 1.5 rad/s, which is within acceptable limits (<90 deg/s). While there is a significant change in the horizontal velocity there isn't as much change in the vertical velocity. The pitch angle demand is very aggressive, almost 60 degrees whose primary effect is to drastically slow down the helicopter. In the hover configuration, this would mean that the helicopter moves backwards while losing altitude. There is also a lateral shift in the inertial position due to the external disturbance activity and the weak coupling inherent in the vehicle dynamics. One way to arrest the build up in the translational velocities is to include an inner-loop for the translational dynamics (velocity loops) and use collective pitch.

Figures 13 and 14 show the lateral-directional responses and the control activity for this maneuver (pitch angle command). As it is seen from the plots, the lateral-directional responses are within acceptable limits and the control histories are as expected.

We note from the plots for both the maneuvers the cross coupling between the longitudinal and lateral-directional modes is minimal. Additionally the roll rate and the pitch rate have low peaks for the respective maneuvers (<90 deg/s). Our objective was to reduce these rate peaks as much as possible and also obtain good step responses in the attitude variables.

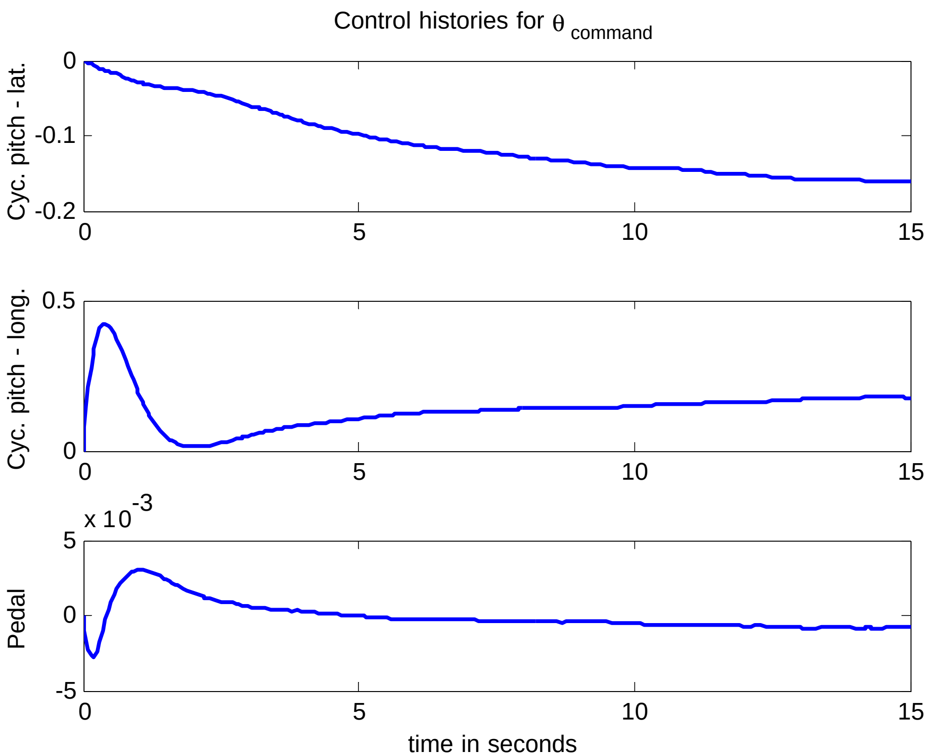

Fig. 14 Control history for a unity bank angle step demand

5 Conclusion

The problem of disturbance attenuation with stability using static output-feedback for linear time-invariant systems has been studied. Necessary and sufficient conditions were developed, which yield two coupled matrix design equations to be solved for the OPFB gain. A computational algorithm to solve for the output-feedback gain that achieves pre-specified disturbance attenuation was developed. The algorithm requires no initial stabilizing gain, in contrast to other existing recursive OPFB solution algorithms. This procedure allows output-feedback control design with pre-specified controller structures and guaranteed performance. A robust controller for stabilizing an autonomous rotorcraft in hover was designed using the algorithm highlighted in the paper.

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