Improving Transient Performance in Tracking General References Using Composite Nonlinear Feedback Control and Its Application to High-Speed XY-Table Positioning Mechanism

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***Abstract*—We adopt in this paper the newly developed composite nonlinear feedback (CNF) control method to track general target references for systems with input saturation. The original formulation of the CNF control technique is only applicable to set-point tracking, in which the target reference is set to be a constant. In this paper, a reference generator, which is able to produce more general reference signals such as sinusoidal and other waves, will be proposed to supplement the CNF control technique to yield a good performance. The resulting control law comprises the reference generator and a modified CNF control law, which is proven to be capable of tracking a target reference with fast settling time and minimal overshoot. Simulation and experimental results on an XY-table show that the proposed technique gives a very satisfactory performance.**

***Index Terms*—Actuator saturation, control applications, motion control, nonlinearities, robust control, servo systems, tracking control.**

I. Introduction

One of the important issues in tracking control is the transient performance. Short settling time and small overshoot are two typical specifications in desirable transient performance. Another major concern is the capability of tracking various references. However, contradiction exists between these specifications, especially for systems whose control input is limited. For example, quick response results in a large overshoot. Usually, tradeoffs have to be made in tracking controller design.

Much research work has been carried out in the literature to improve tracking performance for systems with input nonlinearities. For example, Lin *et al.* [10] proposed the idea of using a nonlinear feedback term to improve tracking performance for a class of second-order linear systems under state feedback. Turner *et al.* [19] later extended the results of [10] to higher order and multiple-input systems under a restrictive assumption on the system. However, both [10] and [19] considered only the state feedback case. Recently, Chen *et al.* [2], [3] developed a composite nonlinear feedback (CNF) control technique for a more general class of systems with measurement feedback and successfully applied the technique to solve a hard-disk-drive servo problem. The CNF control consists of a linear feedback law and a nonlinear feedback law without any switching element. The linear part is designed to yield a closed-loop system with a small damping ratio for a quick response, and the nonlinear part is used to increase the damping ratio of the closed-loop system as the system output approaches the target reference to reduce the overshoot caused by the linear part. Nonetheless, none of the aforementioned results considers the case when the systems have external disturbances. More recently, the CNF control technique has successfully been upgraded in [14] and [15] to deal with systems with external disturbances. In [14] and [15], an integrator is integrated to the control system design to attenuate steady-state bias caused by external disturbances. The overall design retains the fast rise-time property of the original CNF control.

Unfortunately, in all the formulations of the CNF control technique mentioned previously, the target reference has always been assumed to be the step function, which gives rise to the doubt as to whether the technique is capable of tracking a general nonstep reference. This motivates us to develop a more complete result. In this paper, we adopt the CNF control technique to track general target references for a class of linear systems with input saturation. In particular, a reference generator, which produces more general signals such as sinusoidal and other waves, will be proposed to supplement the CNF control technique to yield a good performance for tracking general nonstep references. As a result, the resulting control law comprises the reference generator and a CNF control law. Simulation and experimental results on an XY-table show that the proposed method yields a very satisfactory performance.

The outline of this paper is given as follows. In Section II, the theory of the generalized CNF control technique for tracking general nonstep references will be presented. In particular, a reference generator will be designed and integrated as part

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of the controller. Some illustrative examples will be given in Section III, while experimental tests on an XY-table will be carried out in Section IV. Finally, we make some concluding remarks in Section V.

II. Fast Tracking of General Target References

A generalized version of the CNF control design will be introduced in this section, where a reference generator is included to produce the desired reference signal for the CNF control to track. The new approach will retain the fast settling property of the original CNF control and the capacity of the enhanced CNF control to eliminate steady-state bias due to disturbances, and, at the same time, is capable of tracking nonstep references. More specifically, we consider a linear system with an amplitude-constrained actuator, which is characterized by

$$\Sigma_P : \begin{cases} \dot{x} = Ax + B\text{sat}(u) + Ew, \quad x(0) = x_0 \\ y = C_1 x \\ h = C_2 x \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}$, $y \in \mathbb{R}^p$, $h \in \mathbb{R}$, and $w \in \mathbb{R}$ are the state, control input, measurement output, controlled output, and disturbance input of system Σ_P, respectively. A, B, C_1, C_2, and E are appropriate dimensional constant matrices. The function $\text{sat} : \mathbb{R} \rightarrow \mathbb{R}$ represents the actuator saturation defined as

$$\text{sat}(u) = \text{sgn}(u) \min \{u_{\max}, |u|\} \tag{2}$$

with $u_{\max}$ being the saturation level of the input. The following assumptions on the given system are made.

1) (A, B) is stabilizable.
2) (A, C_1) is detectable.
3) (A, B, C_2) is invertible with no invariant zero at $s = 0$.
4) w is a bounded unknown constant disturbance.
5) h is a subset of y, i.e., h is also measurable.

Note that all these assumptions are fairly standard for tracking control. We aim to design a generalized CNF control law for the system with disturbances such that the resulting controlled output would track an arbitrary reference, e.g., r, as fast and as smooth as possible without having steady-state bias. We first design a reference generator, which can produce any nonstep signal as the reference to be tracked. Then, we follow the given procedure in [14] to design a modified enhanced CNF control law. The generalized CNF control law consists of the reference generator and the modified enhanced CNF control law.

A. Reference Generator

A reference generator, which will produce reference r to be tracked, will be designed in this section. The reference generator is constructed based on the nominal plant. Consider an auxiliary plant characterized by

$$\Sigma_{\text{aux}} : \begin{cases} \dot{x}_e = Ax_e + Bu_e, \quad x_e(0) = x_{e0} \\ r = C_2 x_e \end{cases} \tag{3}$$

where $x_e \in \mathbb{R}^n$, $u_e \in \mathbb{R}$, and $r \in \mathbb{R}$ are the state, control input, and output of the auxiliary system Σ_{aux}, respectively. r is the reference produced to be tracked. A, B, and C_2 are appropriate dimensional constant matrices of the system Σ_P.

Next, we design a linear control law for the auxiliary system Σ_{aux} as follows:

$$u_e = F_e x_e + r_s \tag{4}$$

where F_e is the feedback gain matrix and r_s is an external signal source. The auxiliary system (3) and the linear control law (4) are combined to form the reference generator as follows:

$$\Sigma_{\text{Ref}} : \begin{cases} \dot{x}_e = (A + BF_e)x_e + Br_s, \quad x_e(0) = x_{e0} \\ u_e = F_e x_e + r_s \\ r = C_2 x_e. \end{cases} \tag{5}$$

The reference generator (5) can generate an arbitrary type of output signal, such as the step signal, ramp signal, and sinusoidal signal by designing F_e, setting the initial value x_{e0}, and choosing r_s. For example, to generate a polynomial signal

$$r(t) = a_0 + a_1 t + \cdots + a_{n-1} t^{n-1}$$

we just set $r_s = 0$ and choose an F_e such that the eigenvalues of $A + BF_e$ are all zero, and let

$$x_{e0} = \begin{bmatrix} C_2 \\ C_2(A + BF_e) \\ \vdots \\ C_2(A + BF_e)^{n-1} \end{bmatrix}^{-1} \begin{pmatrix} a_0 \\ 1! a_1 \\ \vdots \\ (n-1)! a_{n-1} \end{pmatrix}.$$

To generate a simple sinusoidal signal $r(t) = a_1 \sin(\omega_1 t + \phi)$, we again set $r_s = 0$ and choose an F_e such that two eigenvalues of $A + BF_e$ are at $\pm j\omega_1$ and the rest are at zero, and

$$x_{e0} = \begin{bmatrix} C_2 \\ C_2(A + BF_e) \\ \vdots \\ C_2(A + BF_e)^{n-1} \end{bmatrix}^{-1} \begin{pmatrix} a_1 \sin\phi \\ a_1\omega_1 \cos\phi \\ 0 \\ \vdots \\ 0 \end{pmatrix}.$$

For more general signals $r(t)$, we might need a nonzero external signal r_s. For some cases, r_s can come from another auxiliary linear system. For the circumstance when $r(t)$ can be generated by an autonomous linear exosystem, similar ideas have been adopted to reformulate the tracking problem into an equivalent output regulation problem by augmenting the exosystem (see, e.g., [8]). Actually, here, we use the reference generator to construct target state x_e, which is an important variable in the CNF control technique.

B. CNF Control System Design

In this section, we will design a generalized CNF control law using the reference generator presented in the previous section. The procedure presented here is similar to the design of the enhanced CNF control law [14], except for a slight difference. However, the generalized CNF control law is capable of tracking nonstep references.

We follow the usual practice to augment an integrator into the given system. Such an integrator will eventually become part of the final control law. To be more specific, we define an auxiliary state variable

$$\dot{x}_i := e := h - r = C_2 x - r \tag{6}$$

which is implementable as h is assumed to be measurable, and augment it into the given system as follows:

$$\begin{cases} \dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}\mathrm{sat}(u) + \bar{B}_r r + \bar{E}w \\ \bar{y} = \bar{C}_1 \bar{x} \\ h = \bar{C}_2 \bar{x} \end{cases} \tag{7}$$

where

$$\bar{x} = \begin{pmatrix} x_i \\ x \end{pmatrix} \quad \bar{x}_0 = \begin{pmatrix} 0 \\ x_0 \end{pmatrix} \quad \bar{y} = \begin{pmatrix} x_i \\ y \end{pmatrix} \tag{8}$$

$$\bar{A} = \begin{bmatrix} 0 & C_2 \\ 0 & A \end{bmatrix} \quad \bar{B} = \begin{bmatrix} 0 \\ B \end{bmatrix} \quad \bar{B}_r = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \tag{9}$$

and

$$\bar{E} = \begin{bmatrix} 0 \\ E \end{bmatrix} \quad \bar{C}_1 = \begin{bmatrix} 1 & 0 \\ 0 & C_1 \end{bmatrix} \quad \bar{C}_2 = [0 \quad C_2]. \tag{10}$$

We note that it is straightforward to verify that under assumptions 1 and 3, the pair $(\bar{A}, \bar{B})$ is stabilizable.

Next, we proceed to carry out the design of modified enhanced CNF control laws for two different cases, i.e., the state feedback case and the reduced-order measurement feedback case. The full-order measurement feedback case is straightforward once the result for the reduced-order case is established.

1) State Feedback Case: We first investigate the case when all the state variables of the plant (7) are measurable, i.e., $\bar{y} = \bar{x}$. In conformity with the augmented system (7), the reference generator can be rewritten as

$$\begin{cases} \dot{\bar{x}}_e = \bar{A}\bar{x}_e + \bar{B}u_e + \bar{B}_r r \\ u_e = [0 \quad F_e]\bar{x}_e + r_s \\ r = \bar{C}_2 \bar{x}_e \end{cases} \tag{11}$$

where

$$\bar{x}_e = \begin{pmatrix} 0 \\ x_e \end{pmatrix} \quad \bar{x}_{e0} = \begin{pmatrix} 0 \\ x_{e0} \end{pmatrix}.$$

Defining $\tilde{x} = \bar{x} - \bar{x}_e$, then, from (7) and (11), we obtain

$$\dot{\tilde{x}} = \bar{A}\tilde{x} + \bar{B}\left\{\mathrm{sat}(u) - u_e\right\} + \bar{E}w. \tag{12}$$

This error equation will be used in the design of the modified enhanced CNF control law. The procedure that generates a modified enhanced CNF state feedback law will be done in three steps. In the first step, a linear feedback control law will be designed; in the second step, the design of nonlinear feedback control will be carried out; and, lastly, in the final step, the linear and nonlinear feedback laws will be combined to form a generalized CNF control law.

Step s.1) Design a linear feedback control law based on (12), i.e.,

$$u_L = F\tilde{x} + u_e \tag{13}$$

where F is chosen such that 1) $\bar{A} + \bar{B}F$ is an asymptotically stable matrix and 2) the closed-loop system $\bar{C}_2(sI - \bar{A} - \bar{B}F)^{-1}\bar{B}$ has certain desired properties. Let us partition $F = [F_i \quad F_x]$ in conformity with x_i and x. The general guideline in designing such an F is to place the closed-loop pole of $\bar{A} + \bar{B}F$ corresponding to the integration mode x_i to be sufficiently closer to the imaginary axis compared to the rest of the eigenvalues, which implies that F_i is a relatively small scalar. The remaining closed-loop poles of $\bar{A} + \bar{B}F$ should be placed to have a dominating pair with a small damping ratio, which in turn would yield a fast rise time in the closed-loop system response.

Step s.2) Given a positive definite symmetric matrix $W \in \mathbb{R}^{(n+1)\times(n+1)}$, we solve the following Lyapunov equation:

$$(\bar{A} + \bar{B}F)'P + P(\bar{A} + \bar{B}F) = -W \tag{14}$$

for $P > 0$. Such a solution is always existent as $(\bar{A} + \bar{B}F)$ is asymptotically stable. The nonlinear feedback portion of the modified enhanced CNF control law u_N is given by

$$u_N = \rho(e)\bar{B}'P\tilde{x} \tag{15}$$

where $\rho(e)$, with $e = h - r$ being the tracking error, is a nonpositive function of $|e|$, which is to be used to gradually change the system closed-loop damping ratio to yield better tracking performance. The choice of design parameter W is the same as that presented in [14]. The choice of design parameter $\rho(e)$ will be presented later in Section III.

Step s.3) The linear feedback control law and nonlinear feedback portion derived in the previous steps are now combined to form a generalized CNF control law

$$u = u_L + u_N = F\tilde{x} + u_e + \rho(e)\bar{B}'P\tilde{x}. \tag{16}$$

The generalized CNF control law comprises the reference generator (5) and the modified enhanced CNF control law (16).

Theorem 2.1: Consider the given system (1), with $y = x$ and disturbance w being bounded by a nonnegative scalar τ_w, i.e., $|w| \le \tau_w$. Let

$$\gamma := 2\tau_w \lambda_{\max}(PW^{-1})(\bar{E}'P\bar{E})^{1/2}. \tag{17}$$

Then, for any $\rho(e)$, which is a nonpositive function of $|e|$ and tends to a constant as $t \to \infty$, the generalized CNF control law comprising (5) and (16) will drive system-controlled output h to track arbitrary reference r from an initial state $\bar{x}_0$ asymptotically without steady-state bias, provided that the following conditions are satisfied.

1) There exist scalars $\delta \in (0,1)$ and $c_\delta > \gamma^2$ such that

$$\forall \bar{x} \in \boldsymbol{X}(F, c_\delta) := \{\bar{x} : \bar{x}' P \bar{x} \le c_\delta\} \Rightarrow |F\bar{x}| \le (1-\delta) u_{\max}. \tag{18}$$

2) Initial condition $\bar{x}_0$ satisfies

$$\bar{x}_0 - \bar{x}_e \in \boldsymbol{X}(F, c_\delta). \tag{19}$$

3) Control signal u_e to construct the target reference satisfies

$$|u_e| \le \delta u_{\max} \tag{20}$$

where u_e is defined in (5).

Proof: For simplicity, we drop the variable e in $\rho(e)$ throughout this proof. Based on (12), the closed-loop system comprising the augmented system (7) and the generalized control law composed of (5) and (16) can be expressed as

$$\dot{\tilde{x}} = (\bar{A} + \bar{B}F)\tilde{x} + \bar{B}v + \bar{E}w \tag{21}$$

where

$$v := \mathrm{sat}(u) - F\tilde{x} - u_e \tag{22}$$

and

$$u = F\tilde{x} + u_e + \rho \bar{B}' P \tilde{x}. \tag{23}$$

Next, for $\tilde{x} \in \boldsymbol{X}(F, c_\delta)$ and $|u_e| \le \delta u_{\max}$, we have

$$|F\tilde{x} + u_e| \le |F\tilde{x}| + |u_e| \le u_{\max}.$$

Depending on the range of u, the range of v can be estimated from (22) and (23) in three cases, i.e.,

$$\begin{cases} \rho \bar{B}' P \tilde{x} < v < 0, & u < -u_{\max} \\ v = \rho \bar{B}' P \tilde{x}, & |u| \le u_{\max} \\ 0 < v < \rho \bar{B}' P \tilde{x}, & u > u_{\max} \end{cases}. \tag{24}$$

Obviously, for all possible situations, we can always write v as

$$v = q\rho \bar{B}' P \tilde{x} \tag{25}$$

for some nonnegative variable $q \in [0,1]$. Thus, for the case when $\tilde{x} \in \boldsymbol{X}(F, c_\delta)$ and $|u_e| \le \delta u_{\max}$, the closed-loop system comprising the given augmented plant (7) and the generalized CNF control law of (5) and (16) can be expressed as the following:

$$\dot{\tilde{x}} = (\bar{A} + \bar{B}F + q\rho \bar{B}\bar{B}' P)\tilde{x} + \bar{E}w. \tag{26}$$

Let us define a Lyapunov function

$$V = \tilde{x}' P \tilde{x}. \tag{27}$$

Following the same line of reasoning as those in [14], we can show that (26) is stable, provided that the initial condition $\bar{x}_0$, the control signal to construct the target reference u_e, and disturbance w satisfy those conditions listed in the theorem. Furthermore, the closed-loop system, in the absence of disturbance w, has $\dot{V} < 0$ and is thus asymptotically stable. With the presence of disturbance w and with $\tilde{x}(0) = \bar{x}_0 - \bar{x}_e \in \boldsymbol{X}(F, c_\delta)$, where $c_\delta > \gamma^2$, the corresponding trajectory of (26) will remain in $\boldsymbol{X}(F, c_\delta)$ and converge to a point on a ball characterized by $\{\tilde{x} : \tilde{x}' P \tilde{x} \le \tilde{\gamma}^2\}$, with $\tilde{\gamma} \le \gamma$ when ρ trends to a constant as $t \to \infty$. Since $x_i(t) = \int_0^t e(\tau) d\tau$ converges to a constant, it is clear that the tracking error $e(t) \to 0$ as $t \to \infty$. This completes the proof of Theorem 2.1. ■

2) Measurement Feedback Case: In practical situations, it is unrealistic to assume all the state variables of a given plant to be measurable. In what follows, we will design an enhanced CNF control law using only information measurable from the plant. In principle, we can design either a full-order measurement feedback control law, for which its dynamical order will be identical to that of the given plant, or a reduced-order measurement feedback control law, in which we make full use of the measurement output and estimate only the unknown part of the state variable. As such, the dynamical order of the controller will be reduced. It is more feasible to implement controllers with smaller dynamical order. The development of this section follows pretty closely from that of [3].

For simplicity of presentation, we assume that C_1 in the measurement output of the given plant (1) is already in the form

$$C_1 = [I_p \quad 0]. \tag{28}$$

The augmented plant (7) can then be partitioned as the following:

$$\begin{cases} \begin{pmatrix} \dot{x}_i \\ \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{bmatrix} 0 & C_{21} & C_{22} \\ 0 & A_{11} & A_{12} \\ 0 & A_{21} & A_{22} \end{bmatrix} \begin{pmatrix} x_i \\ x_1 \\ x_2 \end{pmatrix} + \begin{bmatrix} 0 \\ B_1 \\ B_2 \end{bmatrix} \mathrm{sat}(u) \\ \qquad + \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} r + \begin{bmatrix} 0 \\ E_1 \\ E_2 \end{bmatrix} w \\ \bar{y} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & I_p & 0 \end{bmatrix} \begin{pmatrix} x_i \\ x_1 \\ x_2 \end{pmatrix} \\ h = [0 \quad C_{21} \quad C_{22}] \begin{pmatrix} x_i \\ x_1 \\ x_2 \end{pmatrix} \end{cases} \tag{29}$$

where

$$\begin{pmatrix} x_i \\ x_1 \\ x_2 \end{pmatrix} = \bar{x} \quad \begin{pmatrix} x_i(0) \\ x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 0 \\ x_{10} \\ x_{20} \end{pmatrix} = \bar{x}_0 \quad \bar{y} = \begin{pmatrix} x_i \\ y \end{pmatrix} = \begin{pmatrix} x_i \\ x_1 \end{pmatrix}.$$

Clearly, x_i and x_1 are readily available and need not be estimated. We only need to estimate x_2. There are two main steps in designing a reduced-order measurement feedback control laws, namely 1) the construction of a full-state feedback gain matrix F and 2) the construction of a reduced-order observer gain matrix K_R. The construction of gain matrix F is totally identical to that given in the previous section. The reduced-order observer gain matrix K_R is chosen such that the poles of $A_{22}+K_RA_{12}$ are placed in appropriate locations in the open left-half plane. Now, given a positive definite matrix $W\in\mathbb{R}^{(n+1)\times(n+1)}$, let $P>0$ be the solution to the Lyapunov equation, i.e.,

$$(\bar{A}+\bar{B}F)'P+P(\bar{A}+\bar{B}F)=-W. \quad (30)$$

The reduced-order enhanced CNF control law is then given by

$$\begin{aligned}\dot{x}_c=(A_{22}+K_RA_{12})x_c+[A_{21}+K_RA_{11}\\ -(A_{22}+K_RA_{12})K_R]\,y+(B_2+K_RB_1)\,\mathrm{sat}(u)\end{aligned} \quad (31)$$

and

$$u=\left(F+\rho(e)\bar{B}'P\right)\left[\begin{pmatrix}x_i\\ x_1\\ x_c-K_Ry\end{pmatrix}-\bar{x}_e\right]+u_e \quad (32)$$

where $\rho(e)$ is a nonpositive function of $|e|$, which is to be chosen to yield a desired performance.

Next, let us partition matrices F and P in conformity with x_i, x_1 and x_2 as follows:

$$F=[F_i\quad F_1\quad F_2]\quad P=[P_i\quad P_1\quad P_2]\,. \quad (33)$$

Given another positive definite matrix $W_R\in\mathbb{R}^{(n-p)\times(n-p)}$ with

$$W_R>F_2'\bar{B}'PW^{-1}P\bar{B}F_2 \quad (34)$$

let $Q_R>0$ be the solution to the Lyapunov equation

$$(A_{22}+K_RA_{12})'Q_R+Q_R(A_{22}+K_RA_{12})=-W_R. \quad (35)$$

Note that such a Q_R exists as $A_{22}+K_RA_{12}$ is asymptotically stable.

We have the following result.

Theorem 2.2: Consider the given system (1), with disturbance w being bounded by a nonnegative scalar τ_w, i.e., $|w|\le\tau_w$. Let

$$\begin{aligned}\gamma_R:=2\tau_w\lambda_{\max}\left(\begin{bmatrix}P&0\\0&Q_R\end{bmatrix}\begin{bmatrix}W&-P\bar{B}F_2\\-F_2'\bar{B}'P&W_R\end{bmatrix}^{-1}\right)\\ \times\sqrt{\left[\bar{E}'P\bar{E}+(E_2+K_RE_1)'Q_R(E_2+K_RE_1)\right]}.\end{aligned} \quad (36)$$

Then, there exists a scalar $\rho^*>0$ such that for any $\rho(e)$, a nonpositive function of $|e|$ with $|\rho(e)|\le\rho^*$ while trending to a constant as $t\to\infty$, the generalized reduced-order CNF control law comprising (5), (31), and (32) will drive the system-controlled output h to track arbitrary reference asymptotically without steady-state bias, provided that the following conditions are satisfied:

1) There exist positive scalars $\delta\in(0,1)$ and $c_{\mathrm{R}\delta}>\gamma_R^2$ such that

$$\begin{aligned}\forall\bar{x}\in\boldsymbol{X}(F,c_{\mathrm{R}\delta}):=\left\{\bar{x}:\bar{x}'\begin{bmatrix}P&0\\0&Q_R\end{bmatrix}\bar{x}\le c_{\mathrm{R}\delta}\right\}\\ \Rightarrow|[F\quad F_2]\bar{x}|\le(1-\delta)u_{\max}.\end{aligned} \quad (37)$$

2) The initial conditions $\bar{x}_0$ and $x_{c0}=x_c(0)$ satisfy

$$\begin{pmatrix}\bar{x}_0-\bar{x}_e\\ x_{c0}-x_{20}-K_Rx_{10}\end{pmatrix}\in\boldsymbol{X}(F,c_{\mathrm{R}\delta}). \quad (38)$$

3) The control signal u_e to construct the target reference satisfies

$$|u_e|\le\delta u_{\max} \quad (39)$$

where u_e is defined in (5).

Proof: Again, we drop variable e in $\rho(e)$ throughout this proof for simplicity. Let

$$\tilde{x}=\bar{x}-\bar{x}_e\quad\tilde{x}_c=x_c-K_Ry-x_2.$$

Then, the closed-loop system comprising the augmented system (7) and the generalized reduced-order control law comprising (5), (31), and (32) can be expressed as

$$\begin{aligned}\begin{pmatrix}\dot{\tilde{x}}\\ \dot{\tilde{x}}_c\end{pmatrix}=\begin{bmatrix}\bar{A}+\bar{B}F&\bar{B}F_2\\0&A_{22}+K_RA_{12}\end{bmatrix}\begin{pmatrix}\tilde{x}\\ \tilde{x}_c\end{pmatrix}\\ +\begin{bmatrix}\bar{B}\\0\end{bmatrix}v+\begin{bmatrix}\bar{E}\\-(E_2+K_RE_1)\end{bmatrix}w\end{aligned} \quad (40)$$

where

$$v:=\mathrm{sat}(u)-[F\quad F_2]\begin{pmatrix}\tilde{x}\\ \tilde{x}_c\end{pmatrix}-u_e \quad (41)$$

and

$$u=[F\quad F_2]\begin{pmatrix}\tilde{x}\\ \tilde{x}_c\end{pmatrix}+u_e+\rho\bar{B}'[P\quad P_2]\begin{pmatrix}\tilde{x}\\ \tilde{x}_c\end{pmatrix}. \quad (42)$$

Next, for $\binom{\tilde{x}}{\tilde{x}_c}\in\boldsymbol{X}(F,c_{\mathrm{R}\delta})$ and $|u_e|\le\delta u_{\max}$, we have

$$\left|[F\quad F_2]\begin{pmatrix}\tilde{x}\\ \tilde{x}_c\end{pmatrix}+u_e\right|\le\left|[F\quad F_2]\begin{pmatrix}\tilde{x}\\ \tilde{x}_c\end{pmatrix}\right|+|u_e|\le u_{\max}.$$

Similarly, as in the proof of Theorem 2.1, we can rewrite v as

$$v=q\rho\bar{B}'[P\quad P_2]\begin{pmatrix}\tilde{x}\\ \tilde{x}_c\end{pmatrix} \quad (43)$$

for some nonnegative variable $q\in[0,1]$. Thus, for the case when

$$\begin{pmatrix}\tilde{x}\\ \tilde{x}_c\end{pmatrix}\in\boldsymbol{X}(F,c_{\mathrm{R}\delta})$$

and $|u_e|\le\delta u_{\max}$, the closed-loop system comprising the given augmented plant (7) and the generalized reduced-order CNF control law composed of (5), (31), and (32) can be expressed as the following:

$$\begin{pmatrix} \dot{\tilde{x}} \\ \dot{\tilde{x}}_c \end{pmatrix} = \begin{bmatrix} \bar{A}+\bar{B}F+q\rho\bar{B}\bar{B}'P & \bar{B}F_2+q\rho\bar{B}\bar{B}'P_2 \\ 0 & A_{22}+K_RA_{12} \end{bmatrix} \begin{pmatrix} \tilde{x} \\ \tilde{x}_c \end{pmatrix} + \begin{bmatrix} \bar{E} \\ -(E_2+K_RE_1) \end{bmatrix} w. \quad (44)$$

The rest of the proof follows along similar lines of reasoning as those given in Theorem 2.1 and those for the measurement feedback case in [3]. ■

C. Selection of W and Nonlinear Gain $\rho(e)$

The procedures for selecting design parameter W and nonlinear gain $\rho(e)$ are the same as those given in [3]. Basically, the poles of the closed-loop system approach the locations of the invariant zeros of an auxiliary system $G_{\text{aux}}(s) := \bar{B}'P(sI - \bar{A} - \bar{B}F)^{-1}\bar{B}$ as $|\rho|$ becomes larger and larger. According to [3], $G_{\text{aux}}(s)$ is stable and invertible with a relative degree equal to 1, and is of minimum phase with n stable invariant zeros. The locations of the invariant zeros of $G_{\text{aux}}(s)$ can actually be manipulated by selecting an appropriate $W > 0$. In general, we should try to deploy the invariant zeros of $G_{\text{aux}}(s)$, which are corresponding to the closed-loop poles for larger $|\rho|$, such that the dominant ones have a large damping ratio, which in turn will yield a smaller overshoot. We refer interested readers to [3].

The selection of nonlinear function $\rho(e)$ is relatively simple. One possible choice of $\rho(e)$ is given as follows:

$$\rho(e) = -\beta \left| e^{-\alpha|e|} - e^{-\alpha|h(0)-r|} \right| \quad (45)$$

where α and β are appropriate positive scalars that can be chosen to yield a desired performance, i.e., fast settling time and small overshoot. This function $\rho(e)$ changes from 0 to $\rho_0 = -\beta|1 - e^{-\alpha|h(0)-r|}|$ as the tracking error approaches zero. In general, parameter ρ_0 should be chosen such that the poles of $\bar{A} + \bar{B}F + \rho_0\bar{B}\bar{B}'P$ are in the desired locations. Finally, we note that the choice of $\rho(e)$ is nonunique. Any function would work so long as it has similar properties of that given in (45).

III. Illustrative Examples

The proposed generalized CNF control technique will be verified in this section to track the step reference, ramp reference, sinusoidal references, and transcendental reference.

We consider a second-order system characterized by

$$\begin{cases} \dot{x} = \begin{bmatrix} 0 & 1 \\ -10 & 5 \end{bmatrix} x + \begin{bmatrix} 0 \\ 100 \end{bmatrix} \text{sat}(u) + \begin{bmatrix} 0 \\ 100 \end{bmatrix} w \\ y = h = [1 \quad 0]x \end{cases} \quad (46)$$

where $u_{\max} = 2$ and the disturbance is assumed to be $w = -0.1$ for the simulation. Our task here is to design a control law such that the system output can track a sinusoidal reference fast and accurately

$$r(t) = a_0 + a_1 \cdot \sin(\omega_1 t + \phi) + a_2 \cdot \sin(\omega_2 t). \quad (47)$$

For the preceding signal, a reference generator can be designed as follows:

$$\Sigma_{\text{aux}} : \begin{cases} \dot{x}_e = \begin{bmatrix} 0 & 1 \\ -\omega_1^2 & 0 \end{bmatrix} x_e + \begin{bmatrix} 0 \\ 100 \end{bmatrix} r_s \\ x_e(0) = \begin{pmatrix} a_0 + a_1 \sin\phi \\ a_2\omega_2 + a_1\omega_1\cos\phi \end{pmatrix} \\ r = [1 \quad 0]x_e \end{cases} \quad (48)$$

where r_s is an external signal given by

$$r_s(t) = \frac{1}{100}\left[a_0\omega_1^2 + a_2\left(\omega_1^2 - \omega_2^2\right)\sin(\omega_2 t)\right] \cdot 1(t)$$

with $1(t)$ being the unit step signal. Moreover, we have

$$u_e = \left[\, 0.1 - \tfrac{\omega_1^2}{100} \quad -0.05 \,\right] x_e + r_s. \quad (49)$$

Since there is disturbance in the system, we introduce an integration term $\dot{x}_i = h - r$ and obtain the corresponding augmented plant as in (7). Defining

$$\tilde{x} = \bar{x} - \begin{pmatrix} 0 \\ x_e \end{pmatrix}$$

and following the procedures given in the previous section, we first obtain a linear state feedback law given by

$$u_L = F\tilde{x} + u_e \quad (50)$$

where

$$F = -[0.0158 \quad 1.4799 \quad 0.1255] \quad (51)$$

which places the poles of $\bar{A} + \bar{B}F$ at -0.01 and a conjugate pair with a damping ratio of 0.3 and natural frequency of 4π. Next, we choose W to be a diagonal matrix with the diagonal elements being 0.085, 5, and 0.003, respectively. Solving the Lyapunov equation of (14), we obtain

$$P = \begin{bmatrix} 4.2791 & 2.0709\times10^{-1} & 2.6913\times10^{-2} \\ 2.0709\times10^{-1} & 4.9240\times10^{-1} & 1.7135\times10^{-2} \\ 2.6913\times10^{-2} & 1.7135\times10^{-2} & 2.4682\times10^{-3} \end{bmatrix}$$

which is indeed positive definite. The nonlinear feedback gain matrix is then given by

$$F_n = \bar{B}'P = [2.6913 \quad 1.7135 \quad 0.2468]. \quad (52)$$

The reduced-order observer gain matrix is selected as

$$K_R = -25 \quad (53)$$

which places the observer pole at -20, and the nonlinear gain function is selected as follows:

$$\rho(e) = -2\left| e^{-|e|} - e^{-1} \right|. \quad (54)$$

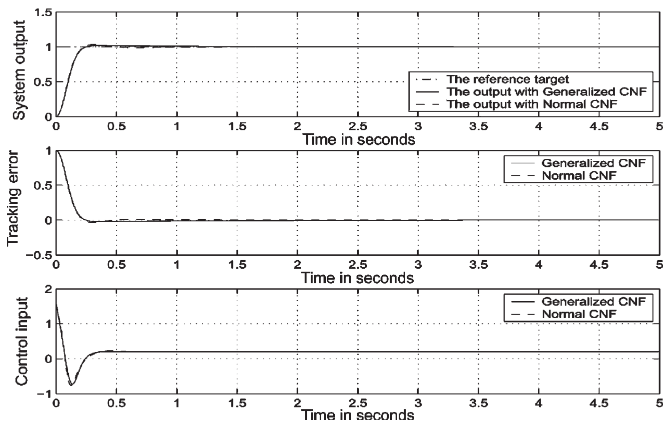

Fig. 1. Tracking a unit step reference.

Finally, the reduced-order generalized CNF control law is given by

$$\begin{pmatrix} \dot{x}_i \\ \dot{x}_v \end{pmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -20 \end{bmatrix} \begin{pmatrix} x_i \\ x_v \end{pmatrix} + \begin{bmatrix} 1 \\ -510 \end{bmatrix} y + \begin{bmatrix} 0 \\ 100 \end{bmatrix} \mathrm{sat}(u) - \begin{bmatrix} 1 \\ 0 \end{bmatrix} r \quad (55)$$

and

$$u = u_e + (F + \rho(e)F_n) \left[\begin{pmatrix} x_i \\ y \\ x_v + 25y \end{pmatrix} - \begin{pmatrix} 0 \\ x_e \end{pmatrix} \right] \quad (56)$$

where x_e is given in (48) and u_e is given in (49).

For comparison, we also design an enhanced CNF controller following the procedure given in [14], i.e.,

$$u = (F + \bar{\rho}(e)F_n) \begin{pmatrix} x_i \\ y - r \\ x_v + 25y \end{pmatrix} + 0.1r \quad (57)$$

with

$$\bar{\rho}(e) = -5 \left| e^{-|e|} - e^{-1} \right| \quad (58)$$

where F and F_n are given in (51) and (52), respectively, and x_i and x_v are given in (55).

A. Step Reference

We first check the performance of the generalized CNF control law in tracking a unit step reference. Obviously, we can just let $a_0 = 1$, $a_1 = 0$, $\omega_1 = 0$, $\phi = 0$, $a_2 = 0$, and $\omega_2 = 0$. Simulations are carried out in MATLAB. Simulation results for unit step reference are shown in Fig. 1. Judging from the figure, the output responses settle into the target fast and smoothly, and there is barely any difference between the output response of the generalized CNF control and that of the normal CNF control. In other words, the generalized CNF control can achieve the same performance as the normal CNF control for step reference target.

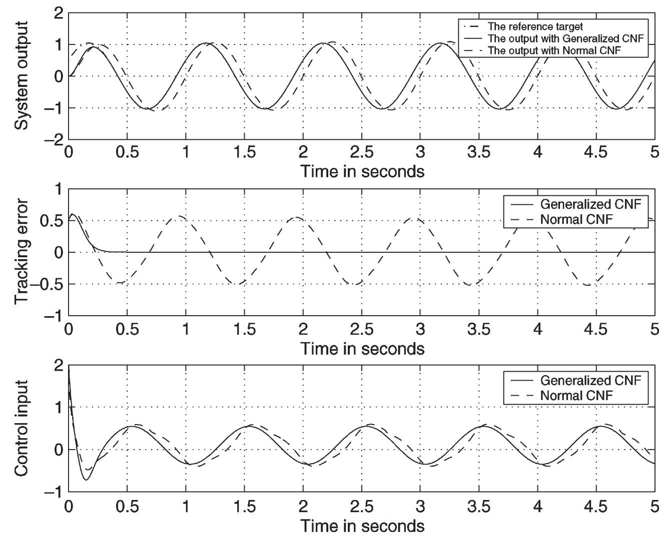

Fig. 2. Tracking a sinusoidal reference.

B. Sinusoidal Reference With Single Frequency

Next, we test the tracking performance with a sinusoidal reference

$$r(t) = \sin\left(2\pi t + \frac{\pi}{6}\right).$$

Fig. 2 shows the simulation results. It can be seen that the output response with normal CNF control lags behind the reference signal; there is steady-state tracking error. In contrast, the output response with generalized CNF control almost perfectly tracks the target.

C. Sinusoidal Reference With Multiple Frequencies

Next, we test the tracking performance for a sinusoidal reference with two frequency components

$$r(t) = 1 + 0.3\sin\left(2\pi t + \frac{\pi}{4}\right) + 0.1\sin(6\pi t).$$

Simulation results are shown in Fig. 3. It is obvious that the output response with normal CNF control lags behind the reference signal; hence, there is tracking error. In contrast, the output response with generalized CNF control can still perfectly track the target reference.

D. Ramp Reference

We now test the tracking performance with a ramp reference $r(t) = a_0 + a_1 t$. For this reference signal, the reference generator can be designed as follows:

$$\Sigma_{\mathrm{aux}} : \begin{cases} \dot{x}_e = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_e, \quad x_e(0) = \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} \\ r = [1 \quad 0] x_e \end{cases} \quad (59)$$

with $u_e = [0.1 \quad -0.05] x_e$.

Fig. 4 shows the simulation results for $r(t) = 0.1 + 0.3t$. It is clear that the generalized CNF control achieves almost perfect

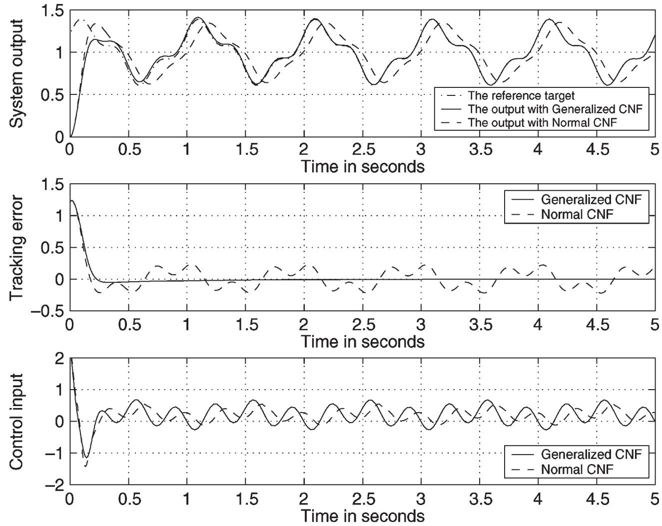

Fig. 3. Tracking a sinusoidal reference with multiple frequencies.

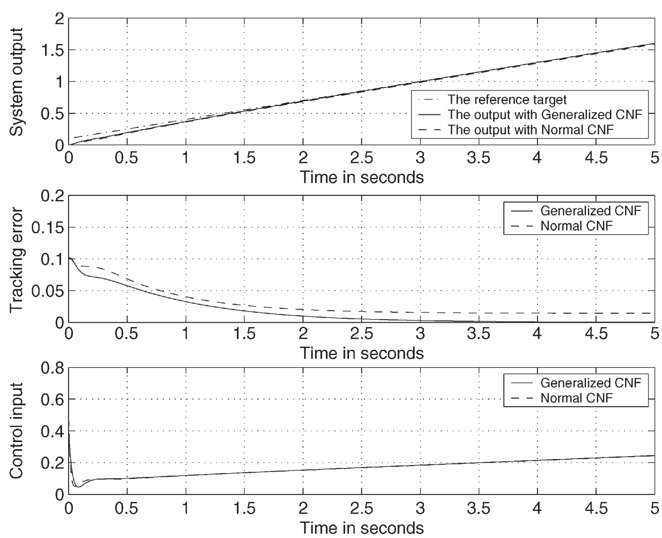

Fig. 4. Tracking a ramp reference.

tracking without steady-state error, whereas a constant bias occurs at the steady-state output with the normal CNF control.

E. Transcendental Reference

Finally, we test the tracking performance with a transcendental reference $r(t) = e^{\sin(\omega_1 t)}$ with $\omega_1 = 2\pi$. For this reference signal, the reference generator can be designed as follows:

$$\Sigma_{\text{aux}} : \begin{cases} \dot{x}_e = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_e + \begin{bmatrix} 0 \\ 100 \end{bmatrix} r_s \\ r = [1 \quad 0] x_e \end{cases} \tag{60}$$

with

$$x_e(0) = \begin{pmatrix} 1 \\ \omega_1 \end{pmatrix}$$

and r_s being an external signal given by

$$r_s(t) = \frac{1}{100} \omega_1^2 e^{\sin(\omega_1 t)} \left[\frac{1 + \cos(2\omega_1 t)}{2} - \sin(\omega_1 t) \right] \tag{61}$$

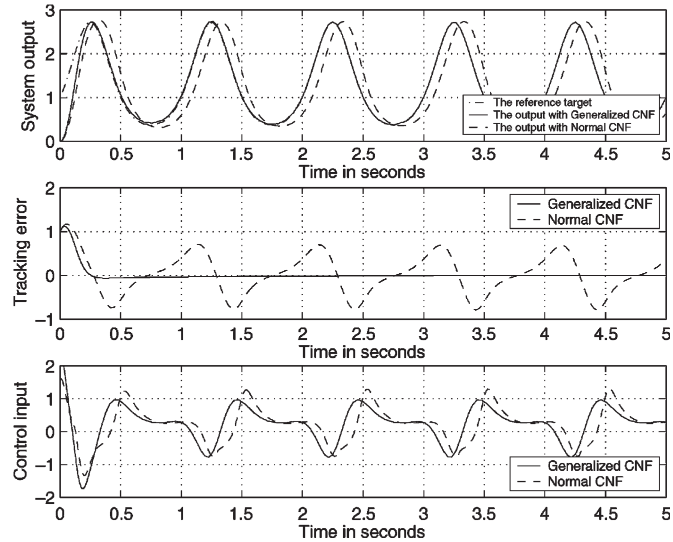

Fig. 5. Tracking a transcendental reference.

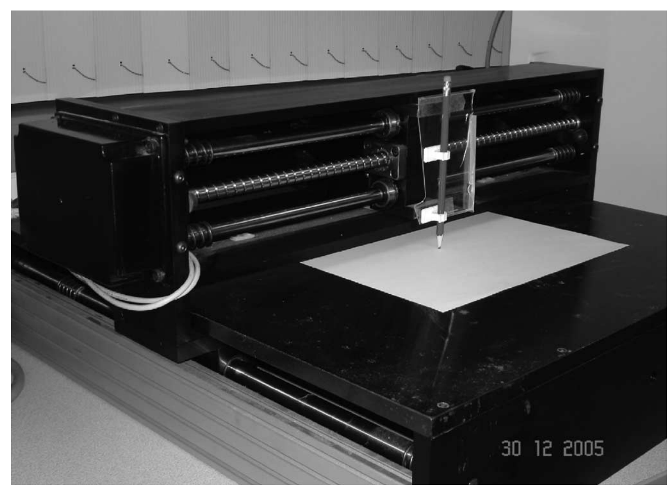

Fig. 6. XY-table used in experiments.

and

$$u_e = [0.1 \quad -0.05] x_e + r_s. \tag{62}$$

Simulation results in Fig. 5 clearly show that the output under the generalized CNF control almost perfectly tracks the transcendental signal, whereas the output under the normal CNF control always lags behind the target.

IV. Application in an XY-Table Trajectory Tracking Control

In this section, we demonstrate the application of the proposed control technique in solving the trajectory tracking control problem in an XY-table. The likes of XY-tables, e.g., machine tools, are commonly used in the manufacturing industry. The precision control of an XY-table has been widely studied (see, e.g., [4], [6], [9], [11]–[13], [17], and [18]).

Fig. 6 is a photograph of the XY-table we are working with. In each axis of the XY-table, there is a brush-type dc servomotor (model MT22G2-10) that drives its load with a ball screw. The XY-table has a maximum speed of 5000 r/min and a maximum travel of 0.5 m (or 0.25 m in both directions) in each axis, and the displacement of each axis is measured by an optical encoder with 4000 pulse/revolution. A pulsewidth-modulation (PWM) power amplifier is used in the current mode to drive the two motors of the XY-table. A pencil is attached to the mover (as the load) of the XY-table, which in turn is driven by the servomotors to move along the X-axis and Y-axis and, thus, can produce or draw any desirable 2-D trajectory onto the paper underneath.

The relation between the linear motion of the XY-table along each axis and the motor input current (before the PWM amplifier) has been identified as the following transfer functions [17]:

$$G_x(s) = \frac{8.034}{s^2 + 2.825s} \tag{63}$$

and

$$G_y(s) = \frac{6.774}{s^2 + 3.226s}. \tag{64}$$

The amplitude of the control input (electric current expressed in amperes) to the motor is limited by 1 A, i.e., $u_{\max} = 1$ A. The output displacement is expressed in meters.

The preceding two models can be cast into the state-space form

$$\Sigma_x : \begin{cases} \dot{x}_x = \begin{bmatrix} 0 & 1 \\ 0 & -2.825 \end{bmatrix} x_x + \begin{bmatrix} 0 \\ 8.034 \end{bmatrix} \mathrm{sat}(u_x) \\ h_x = [1 \quad 0] x_x \end{cases} \tag{65}$$

and

$$\Sigma_y : \begin{cases} \dot{x}_y = \begin{bmatrix} 0 & 1 \\ 0 & -3.226 \end{bmatrix} x_y + \begin{bmatrix} 0 \\ 6.774 \end{bmatrix} \mathrm{sat}(u_y) \\ h_y = [1 \quad 0] x_y \end{cases} \tag{66}$$

where (u_x, x_x, h_x) and (u_y, x_y, h_y) correspond to the control input, state vector, and output displacement of the two axes of the XY-table.

In what follows, controllers are designed for the two subsystems (axes) to track some target trajectory $r_x(t)$ and $r_y(t)$, using the generalized CNF control technique.

We first consider the subsystem for the X-axis. Given a sinusoidal target reference $r_x(t) = a_1 \cdot \sin(\omega_1 t + \phi)$, a reference generator can be designed as follows:

$$\Sigma_{\mathrm{rx}} : \begin{cases} \dot{x}_{\mathrm{ex}} = \begin{bmatrix} 0 & 1 \\ -\omega_1^2 & 0 \end{bmatrix} x_{\mathrm{ex}} \\ x_{\mathrm{ex}}(0) = \begin{pmatrix} a_1 \sin\phi \\ a_1\omega_1 \cos\phi \end{pmatrix} \\ r_x = [1 \quad 0] x_{\mathrm{ex}} \end{cases} \tag{67}$$

with

$$u_{\mathrm{ex}} = \left[-\tfrac{\omega_1^2}{8.034} \quad 0.3516 \right] x_{\mathrm{ex}}. \tag{68}$$

We introduce an integration term $\dot{x}_{\mathrm{ix}} = h_x - r_x$ into the system Σ_x and obtain the corresponding augmented system. We choose the preliminary conjugate poles with a damping ratio of 0.3, natural frequency of 6 rad/s and the integration pole at -0.01, the linear state feedback gain matrix is then given by

$$F_x = -[0.0448 \quad 4.4854 \quad 0.0977].$$

Next, we choose matrix $W_x = \mathrm{diag}(0.2, 40, 0.06)$ and solve the related Lyapunov equation; the nonlinear feedback gain matrix is then obtained as

$$F_{\mathrm{nx}} = [2.2317 \quad 4.6961 \quad 1.3676].$$

Now, a reduced-order observer is designed with the observer pole placed at -15. The nonlinear gain function is chosen as

$$\rho_x(h_x, r_x) = -3.5 \left| e^{-3|h_x - r_x|} - e^{-3|h_x(0) - r_x(0)|} \right|. \tag{69}$$

Note that friction exists in all mechanical systems where there is relative motion and becomes more influential at the beginning of motion or at low velocity when the contact surfaces seem to get stuck. As a result, tracking error will occur if friction is not compensated. Inspired by the idea of [1] and [16], we propose the following friction compensation term for the two axes of the XY-table:

$$u_f = \gamma \cdot \tanh\left(\lambda(r - h)\right) \cdot e^{-\eta|\hat{v}|} \tag{70}$$

where γ corresponds to (or can be a bit larger than) the static friction (i.e., the break-away force), λ and η are positive tuning parameters, and $\hat{v}$ is the estimated velocity of motion. Obviously, the compensation term is bounded by γ and will become influential only when the motion is slow while the tracking error $r - h$ is relatively large.

The generalized CNF control law for the X-axis is given by

$$\begin{pmatrix} \dot{x}_{\mathrm{ix}} \\ \dot{x}_{\mathrm{cx}} \end{pmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -15 \end{bmatrix} \begin{pmatrix} x_{\mathrm{ix}} \\ x_{\mathrm{cx}} \end{pmatrix} + \begin{bmatrix} 1 \\ -182.625 \end{bmatrix} h_x + \begin{bmatrix} 0 \\ 8.034 \end{bmatrix} \mathrm{sat}(u_x) - \begin{bmatrix} 1 \\ 0 \end{bmatrix} r_x \tag{71}$$

and

$$u_x = [F_x + \rho_x(h_x, r_x) F_{\mathrm{nx}}] \left[\begin{pmatrix} x_{\mathrm{ix}} \\ h_x \\ x_{\mathrm{cx}} + 12.175 h_x \end{pmatrix} - \begin{pmatrix} 0 \\ x_{\mathrm{ex}} \end{pmatrix} \right] + u_{\mathrm{ex}} + 0.18 \tanh\left(2000(r_x - h_x)\right) e^{-5|x_{\mathrm{cx}} + 12.175 h_x|}. \tag{72}$$

The controller design for the Y-axis is similar. For a sinusoidal target reference $r_y(t) = a_1 \cdot \sin(\omega_1 t)$, the reference generator can be designed as follows:

$$\Sigma_{\mathrm{ry}} : \begin{cases} \dot{x}_{\mathrm{ey}} = \begin{bmatrix} 0 & 1 \\ -\omega_1^2 & 0 \end{bmatrix} x_{\mathrm{ey}} \\ x_{\mathrm{ey}}(0) = \begin{pmatrix} 0 \\ a_1\omega_1 \end{pmatrix} \\ r_y = [1 \quad 0] x_{\mathrm{ey}} \end{cases} \tag{73}$$

with

$$u_{\mathrm{ey}} = \left[-\frac{\omega_1^2}{6.774} \quad 0.4762 \right] x_{\mathrm{ey}}. \tag{74}$$

We again introduce an integral augmentation and choose the preliminary conjugate poles with a damping ratio of 0.3 and natural frequency of 6 rad/s and the integration pole at -0.01. Furthermore, we choose matrix $W_y = \mathrm{diag}(0.3, 56, 0.08)$, and the nonlinear gain function

$$\rho_y(h_y, r_y) = -2 \left| e^{-2|h_y - r_y|} - e^{-2|h_y(0) - r_y(0)|} \right|. \tag{75}$$

Again, an observer is designed with a pole at -15. Finally, we obtain a generalized CNF control law for the Y-axis as follows:

$$\begin{pmatrix} \dot{x}_{\mathrm{iy}} \\ \dot{x}_{\mathrm{cy}} \end{pmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -15 \end{bmatrix} \begin{pmatrix} x_{\mathrm{iy}} \\ x_{\mathrm{cy}} \end{pmatrix} + \begin{bmatrix} 1 \\ -176.61 \end{bmatrix} h_y + \begin{bmatrix} 0 \\ 6.774 \end{bmatrix} \mathrm{sat}(u_y) - \begin{bmatrix} 1 \\ 0 \end{bmatrix} r_y \tag{76}$$

and

$$\begin{aligned} u_y = u_{\mathrm{ey}} &+ \big([-0.0531 \quad -5.3198 \quad -0.0567] \\ &+ \rho_y(h_y, r_y)\, [3.3475 \quad 6.6886 \quad 1.7901] \big) \\ &\times \left[\begin{pmatrix} x_{\mathrm{iy}} \\ h_y \\ x_{\mathrm{cy}} + 11.774 h_y \end{pmatrix} - \begin{pmatrix} 0 \\ x_{\mathrm{ey}} \end{pmatrix} \right] \\ &+ 0.2 \tanh\left(2000(r_y - h_y)\right) e^{-5|x_{\mathrm{cy}} + 11.774 h_y|}. \end{aligned} \tag{77}$$

For comparison, we present the following finely tuned modified proportional–integral differential control laws for the X- and Y-axis of the XY-table, respectively:

$$\begin{cases} u_x = \left(\frac{0.2211}{s} + 1.3552 + \frac{0.3744s}{1000s+1} \right)(r_x - h_x) \\ u_y = \left(\frac{0.4432}{s} + 2.6574 + \frac{0.6356s}{1000s+1} \right)(r_y - h_y) \end{cases}. \tag{78}$$

The parameters in the preceding PID control laws are tuned through simulation to obtain the best possible performance.

Simulation and experiments are carried out for the XY-table to move in a circle with a radius of 0.1 m, just like a graphic plotter drawing a circle. For this purpose, we set the target trajectory for the X-axis to be $r_x(t) = 0.1\cos(0.4\pi t) = 0.1\sin(0.4\pi t + \pi/2)$ m and the trajectory for the Y-axis to be $r_y(t) = 0.1\sin(0.4\pi t)$ m. Note that the frequency (or the period) of the reference trajectory determines the time it takes for the XY-table to finish drawing a full circle.

Simulation is done in MATLAB with SIMULINK. Note that, in simulation, the friction effects are not included in the plant models; hence, the term for friction compensation should be ignored in the control laws. The results are shown in Figs. 7–9. The simulation results show that the generalized CNF control yields a much better performance compared to that of PID control.

In the experiments, controllers are implemented on a dSpace digital signal processor board installed in a personal computer

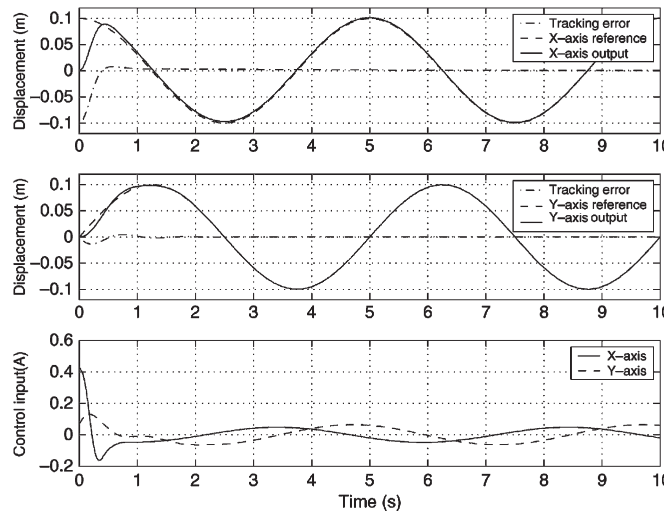

Fig. 7. Simulation: circular motion with generalized CNF control.

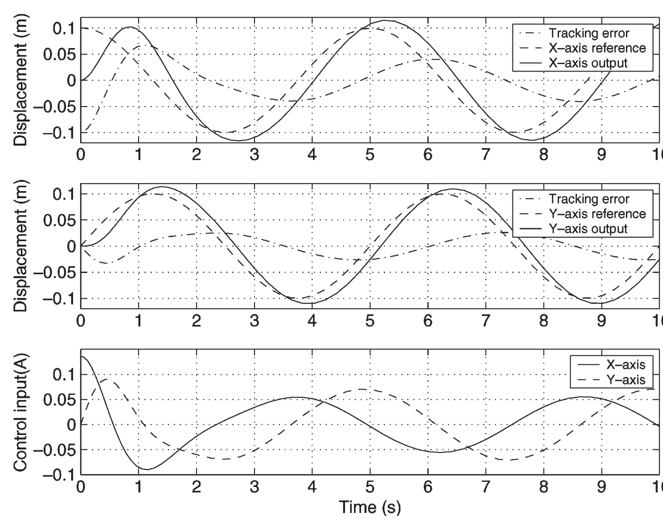

Fig. 8. Simulation: circular motion with PID control.

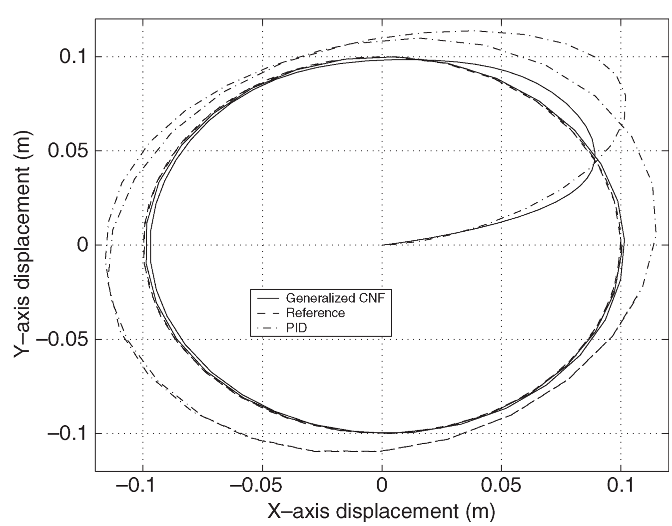

Fig. 9. Simulation: the drawn circle.

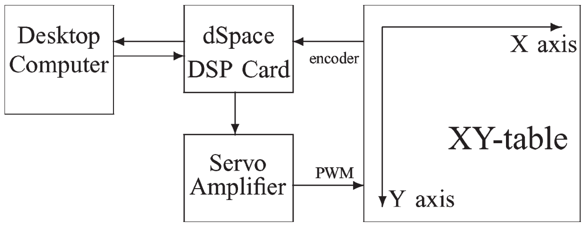

Fig. 10. Block diagram of the XY-table servo system.

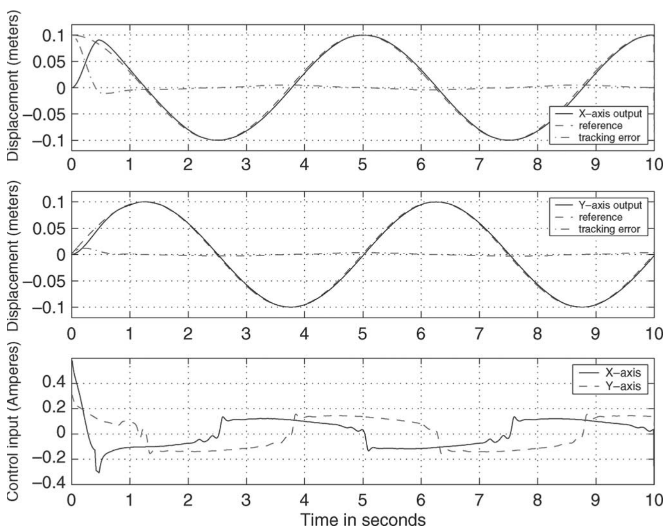

Fig. 11. XY-table experiment: circular motion with generalized CNF control.

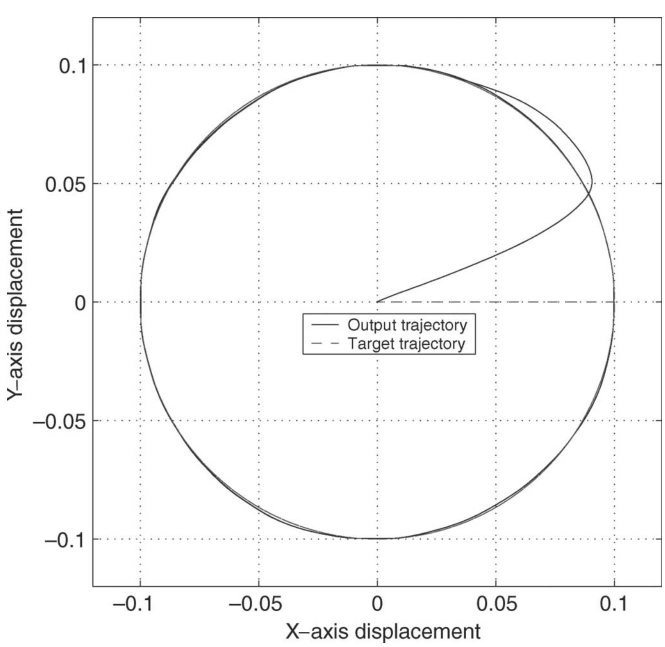

Fig. 12. XY-table experiment: the drawn circle with generalized CNF control.

(see Fig. 10). The sampling frequency is 100 Hz. The experimental results are shown in Figs. 11 and 12. Note that, in the figures, two cycles of motion are displayed for better illustration, although it only takes one cycle to draw a circle. It is clear that the position error converges to zero after a brief transient period, and, afterward, both the X-axis and the Y-axis can track the target references accurately, even at the neighborhood of zero velocity, when the static friction in the physical plant is very influential with respect to the control action. As a result, the circle shown in Fig. 12 is almost perfect.

Next, we let the XY-table draw a lemniscate of Bernoulli characterized by the following parametric equations (see, e.g., [5]):

$$x = \frac{a \cos \omega t}{1 + \sin^2 \omega t} \tag{79}$$

and

$$y = \frac{a \sin \omega t \cos \omega t}{1 + \sin^2 \omega t} \tag{80}$$

where $a = 0.1$ m is related to the torus radius of the lemniscate by a factor of $\sqrt{2}$, and $\omega = 0.2\pi$ is a timescale factor.

The reference generator for X-axis can be designed as follows:

$$\Sigma_{\rm rx} : \begin{cases} \dot{x}_{\rm ex} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_{\rm ex} + \begin{bmatrix} 0 \\ 8.034 \end{bmatrix} r_{\rm sx} \\ r_x = [1 \quad 0] x_{\rm ex} \\ u_{\rm ex} = [0 \quad 0.3516] x_{\rm ex} + r_{\rm sx} \end{cases} \tag{81}$$

with

$$x_{\rm ex}(0) = \begin{pmatrix} a \\ 0 \end{pmatrix}$$

and $r_{\rm sx}$ being given by

$$r_{\rm sx}(t) = -\frac{a\omega^2 \cos \omega t (\sin^4 \omega t - 12 \sin^2 \omega t + 3)}{8.034(1 + \sin^2 \omega t)^3}.$$

Similarly, the reference generator for Y-axis is designed as follows:

$$\Sigma_{\rm ry} : \begin{cases} \dot{x}_{\rm ey} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_{\rm ey} + \begin{bmatrix} 0 \\ 6.774 \end{bmatrix} r_{\rm sy} \\ r_y = [1 \quad 0] x_{\rm ey} \\ u_{\rm ey} = [0 \quad 0.4762], x_{\rm ey} + r_{\rm sy} \end{cases} \tag{82}$$

with

$$x_{\rm ey}(0) = \begin{pmatrix} 0 \\ a\omega \end{pmatrix}$$

and the external signal $r_{\rm sy}$ being given by

$$r_{\rm sy}(t) = -\frac{a\omega^2 (0.75 \sin 4\omega t + 3.5 \sin 2\omega t)}{6.774(1 + \sin^2 \omega t)^3}.$$

The control laws for the two axes are basically same as those given in (72) and (77), except that $u_{\rm ex}$ and $u_{\rm ey}$ now come from (80) and (82), respectively. The modified PID control law (78) is also applied for comparison. Simulation and experiments have been carried out. The results are shown in Figs. 13–17. It can be seen that the tracking performance in the first half cycle is not quite satisfactory, but it gets better after the transient process dies out. The generalized CNF control can obtain better tracking performance than the PID control.

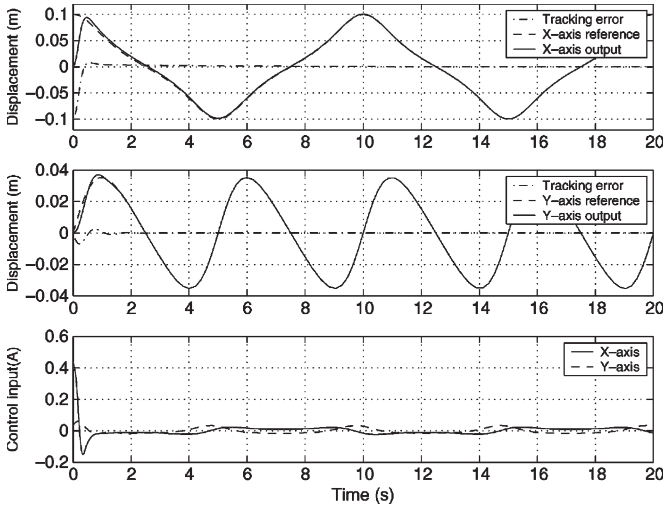

Fig. 13. Simulation: tracking a lemniscate with generalized CNF control.

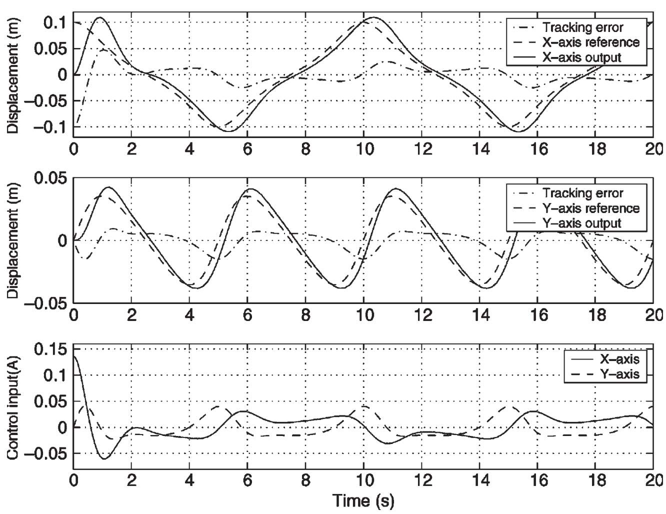

Fig. 14. Simulation: tracking a lemniscate with PID control.

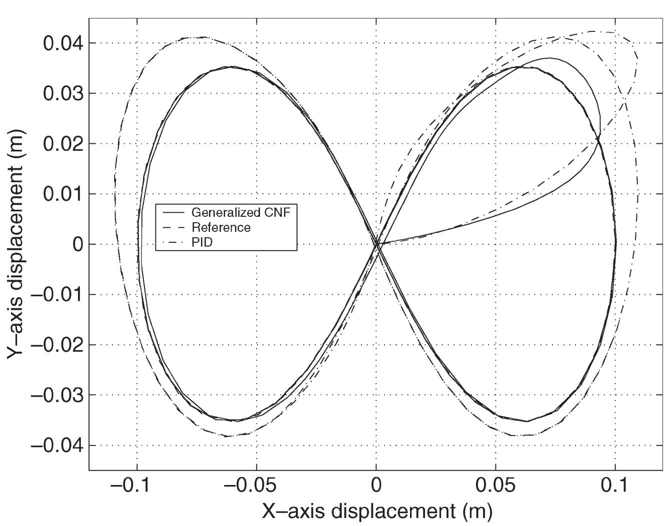

Fig. 15. Simulation: the drawn lemniscate.

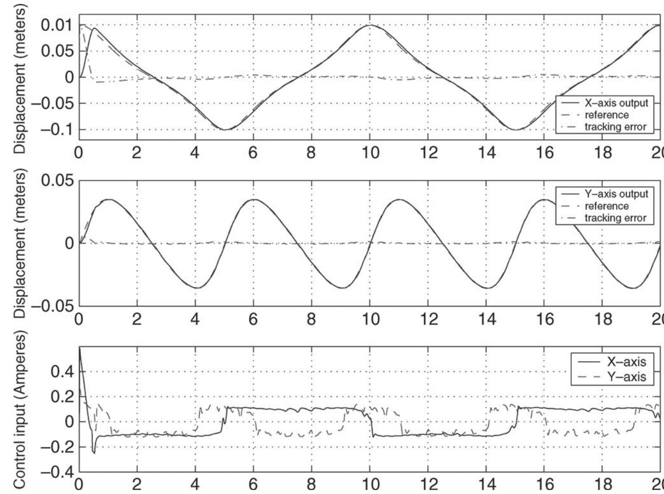

Fig. 16. XY-table experiment: tracking a lemniscate with generalized CNF control.

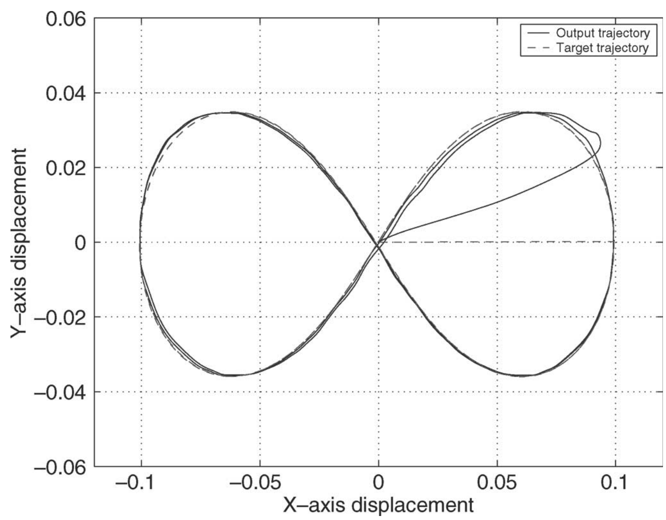

Fig. 17. XY-table experiment: the drawn lemniscate with generalized CNF control.

V. Concluding Remarks

A generalized CNF control technique has been presented to track nonstep references. A reference generator, which is capable of producing various references, has been adopted to work together with the CNF control technique. The generalized CNF control law is composed of the reference generator and an enhanced CNF control law. Thus, it retains the advantages of the CNF control technique such as fast rising time, small overshoot, and without steady-state bias while being capable of tracking various references. Illustrative examples and experiments on an XY-table have been provided to demonstrate the effectiveness of this control technique.

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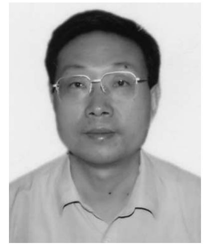

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