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On improvement of transient performance in tracking control for a class of nonlinear systems with input saturation

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Abstract

This paper studies the technique of the composite nonlinear feedback (CNF) control for a class of cascade nonlinear systems with input saturation. The objective of this paper is to improve the transient performance of the closed-loop system by designing a CNF control law such that the output of the system tracks a step input rapidly with small overshoot and at the same time maintains the stability of the whole cascade system. The CNF control law consists of a linear feedback control law and a nonlinear feedback control law. The linear feedback law is designed to yield a closed-loop system with a small damping ratio for a quick response, while the nonlinear feedback law is used to increase the damping ratio of the closed-loop system when the system output approaches the target reference to reduce the overshoot. The result has been successfully demonstrated by numerical and application examples including a flight control system for a fighter aircraft.

Keywords: Composite nonlinear feedback control; Nonlinear systems; Input saturation; Tracking control; Control applications

1. Introduction

Transient performance is one of the important issues in the tracking control problems such as target tracking [5] and output regulation [7]. In general, quick response and small overshoot are desirable in most of the target tracking control problems. However, it is well known that quick response results in a large overshoot. Thus, most of the design schemes have to make a trade-off between these two transient performance indices. In this paper, we consider a tracking problem (or an equivalent output regulation) for partially linear composite systems with input saturation. Particular attention is paid to improve the transient performance of the closed-loop system by using a so-called composite nonlinear feedback (CNF) control technique. To improve the tracking performance, Lin et al. [12] proposed the CNF control technique in their pioneering work for a class of second order linear systems. Turner et al. [18] later extended the results of [12] to higher-order and multiple-input systems under a restrictive assumption on the system. However, both [12] and [18] considered only the state feedback case. Recently, Chen et al. [3] have developed a CNF control to a more general class of systems with measurement feedback, and successfully applied the technique to solve a hard disk drive servo problem. The CNF control consists of a linear feedback law and a nonlinear feedback law without any switching element. The linear feedback part is designed to yield a closed-loop system with a small damping ratio for a quick response, while at the same time not exceeding the actuator limits for desired command input levels. The nonlinear feedback law is used to increase the damping ratio of the closed-loop system as the system output approaches the target reference to reduce the overshoot caused by the linear part.

The partially linear composite system consists of two parts, a linear portion and a nonlinear portion with the output of the linear part connecting to the input of the nonlinear part and with the input of the given system saturated. Many nonlinear systems can be transformed into partially linear composite systems via a state-space diffeomorphism and/or a preliminary feedback transformations (see e.g., [8]). In

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recent two decades, the semi-global and global stabilization problems for partially linear composite systems have been extensively studied by many researchers [1,9,10,13–16], to name just a few. In particular, it was shown in [15] that a nonlinear system which is zero input globally asymptotically stable (GAS) will preserve its GAS property if its input decreases to zero with a very fast exponential rate. It is not difficult to make the output of the linear part, which is the input of the nonlinear part, to converge to zero with some exponential rate. However, the peaking phenomenon in linear systems may destroy the stability of the nonlinear systems before the output rapidly decays to zero [15]. This paper aims to design a CNF control law for partially linear composite systems with input saturation based on the linear part of the composite system such that the closed-loop system has desired performances, e.g., quick response and small overshoot, and the tracking error decays to zero with sufficiently large exponential rate to guarantee the stability of the whole system. The result will be illustrated by two examples, one is a numerical example of a target tracking problem and the other is a step tracking problem for a fighter aircraft.

It is worth noting that although the structure of our CNF control looks similar to the anti-windup control design, the philosophies of these two design methods are totally different. The anti-windup design aims to alleviate or eliminate 'windup', in which the original compensator (the linear part) is left alone as long as it does not encounter input saturation. Only when the input signal is saturated, the adding nonlinear modifications take effect to suppress undesirable oscillations and quicken transient responses (see e.g., [17]). The CNF control, on the other hand, aims to improve the closed-loop transient performance to get quick response and small overshoot, the nonlinear function is designed to be small when the output is far away from the reference input and become larger and larger when the output approaches the reference input. Thus, in general, when the nonlinear part become effective, the linear control input is unsaturated. We would like to further note that the anti-windup technique has primarily developed for linear systems. We are not aware of any result related to the anti-windup technique that is applicable to the nonlinear systems considered in this paper.

The remaining part of the manuscript is organized as follows. Section 2 describes our control problem and presents some relevant preliminary results. The CNF control law design for the partially linear composite systems is given in Section 3. Section 4 illustrates the proposed design technique with numerical and application examples where the performances of the closed-loop system are compared between the CNF control and the corresponding linear control. Finally, Section 5 draws some concluding remarks.

2. Problem description and preliminaries

Consider a partially linear composite systems with input saturation characterized by

$$\dot{\xi} = f(\xi, y), \quad \xi(0) = \xi_0, \tag{1}$$

$$\dot{x} = Ax + B\,\mathrm{sat}(u), \quad x(0) = x_0, \tag{2}$$

$$y = Cx, \tag{3}$$

where $(\xi, x) \in \mathbb{R}^m \times \mathbb{R}^n$ is the state, $u \in \mathbb{R}$ the control input, and $y \in \mathbb{R}$ the output of the system, f is a smooth (i.e., C^∞) function, A, B and C are appropriate dimensional constant matrices, and $\mathrm{sat} : \mathbb{R} \to \mathbb{R}$ represents the actuator saturation defined as

$$\mathrm{sat}(u) = \mathrm{sgn}(u) \min\{u_{\max}, |u|\} \tag{4}$$

with $u_{\max}$ being the saturation level of the input. We aim to design a CNF control law for (1)–(3) such that the resulting closed-loop system is stable and the output of the closed-loop system will track a step reference input r rapidly without experiencing large overshoot. This problem is an extension of the recent work of [3,12] on composite nonlinear feedback control for linear systems by connecting a nonlinear zero dynamics (1) to the linear system (2). The CNF control law consists of a linear feedback control and a nonlinear feedback control. The linear feedback law is designed to stabilize the system with a small closed-loop damping ratio for quick tracking. The nonlinear feedback law is to increase the closed-loop damping ratio as the system output approaches the reference input to reduce the overshoot while it keeps the closed-loop stability.

Without loss of generality, we assume $f(0, r) = 0$. In fact, if $f(\xi^*, r) = 0$ with $\xi^* \neq 0$, the state transformation $\tilde{\xi} = \xi - \xi^*$ gives

$$\dot{\tilde{\xi}} = f(\tilde{\xi} + \xi^*, r) := \tilde{f}(\tilde{\xi}, r),$$

then, we have $\tilde{f}(0, r) = 0$. Moreover, we assume that

A1: (A, B) is controllable,
A2: (A, B, C) is invertible and has no invariant zeros at $s = 0$, and
A3: there exists a C^1 positive definite function $V_\xi(\xi)$ and class K_∞ functions α_1 and α_2 such that

$$\alpha_1(\|\xi\|) \leqslant V_\xi(\xi) \leqslant \alpha_2(\|\xi\|), \tag{5}$$

$$\frac{\partial V_\xi(\xi)}{\partial \xi} f(\xi, r) < 0 \tag{6}$$

for all $\xi \in \Omega \subseteq \mathbb{R}^m$, where Ω is a compact set containing the origin.

Remark 2.1. Assumptions A1 and A2 are quite standard in the tracking control literature. Assumption A3 is to ensure that the nonlinear system (1) is asymptotically stable when the system output y tracks exactly the step command input r.

Remark 2.2 (*Sussmann and Kokotovic [15]*). Consider the nonlinear control system of the form:

$$\dot{\xi} = f(\xi, r + \eta(t)), \tag{7}$$

which satisfies Assumption A3. When $\Omega = \mathbb{R}^m$, Theorem 4.1 of [15] shows that given any $\gamma > 0$ and $\beta > 0$, there exists a scalar $a > 0$ such that for any

$$|\eta(t)| \leqslant \beta e^{-at}, \quad t \geqslant 0, \tag{8}$$

the solution $\xi(t)$ of (7) exists and is bounded for all $t \geqslant 0$ provided that $\xi(0) \in \{\xi\colon \|\xi\| \leqslant \gamma\}$. And such an a is called *good for* (γ, β) with respect to (7). For the case $\Omega \subset \mathbb{R}^m$, let $\gamma > 0$ such that

$$\{\xi : V_\xi(\xi) \leqslant c + 1\} \subseteq \Omega,$$

where $c = \max\{V_\xi(\xi)\colon \|\xi\| \leqslant \gamma\}$. From the proof of Theorem 4.1 of [15], it is clear that, for any given $a > 0$, there exists an $\beta > 0$ such that a is good for (γ, β).

3. Design of the composite nonlinear feedback control law

In this section, we proceed to design a CNF control law for system (1)–(3). We assume that the given system (1)–(3) satisfies Assumptions A1–A3, and all the states of the linear system (2) are available for feedback. The CNF control law can be constructed by the following step-by-step procedure.

Step S.1: Design a linear feedback law

$$u_L = Fx + Gr, \tag{9}$$

where r is a step command input and F is chosen such that

(1) $A + BF$ is Hurwitz and
(2) the closed-loop system $C(sI - A - BF)^{-1}B$ has certain desired properties, e.g., having a small damping ratio.

The existence of such an F is guaranteed by Assumption A1, i.e., (A, B) is controllable. In fact, it can be designed using methods such as the H_2 and H_∞ optimization approaches, as well as the robust and perfect tracking technique. G is a scalar given by

$$G = -[C(A + BF)^{-1}B]^{-1}. \tag{10}$$

Note that G is well defined since $A + BF$ is Hurwitz and the triple (A, B, C) is invertible and has no invariant zeros at $s = 0$. We also let

$$H := [1 - F(A + BF)^{-1}B]G \tag{11}$$

and

$$x_e := G_e r := -(A + BF)^{-1}BGr. \tag{12}$$

Step S.2: Given a positive-define matrix $W \in \mathbb{R}^{n\times n}$, solve the Lyapunov equation

$$(A + BF)'P + P(A + BF) = -W \tag{13}$$

for $P > 0$. Note that such a P exists since $A + BF$ is asymptotically stable. Then, the nonlinear feedback control law $u_N(t)$ is given by

$$u_N = \rho(r, y)B'P(x - x_e), \tag{14}$$

where $\rho(r, y)$ is any non-positive function locally Lipschitz in y. This nonlinear control law is used to change the system closed-loop damping ratio as the output approaches the step command input.

Step S.3: Let $\gamma > 0$ such that $\{\xi\colon V_\xi(\xi) \leqslant c + 1\} \subseteq \Omega$ with $c = \max\{V_\xi(\xi)\colon \|\xi\| \leqslant \gamma\}$, and $0 \leqslant a \leqslant \lambda_{\min}(W)/(2\lambda_{\max}(P))$. Then select an appropriate scalar $\beta > 0$ such that a is good for (γ, β) with respect to (7).

Step S.4: The CNF control law is given by combining the linear and nonlinear feedback law derived in the previous steps,

$$u = u_L + u_N = Fx + Gr + \rho(r, y)B'P(x - x_e). \tag{15}$$

Theorem 3.1. *Consider the given system* (1)–(3) *satisfies Assumptions* A1–A3. *Let* γ, β, a, F, H *and* P *be selected in above-described procedure*, *and let*

$$\mathscr{N} := \left\{ x \in \mathbb{R}^n\colon \|x\| \leqslant \frac{\beta}{\|C\|}\sqrt{\frac{\lambda_{\min}(P)}{\lambda_{\max}(P)}} \right\}. \tag{16}$$

For any $\delta \in (0, 1)$, *let* $c_\delta > 0$ *be the largest positive scalar satisfying the following condition*:

$$|Fx| \leqslant u_{\max}(1 - \delta) \tag{17}$$

for all $x \in \mathbf{X}_\delta$, *where*

$$\mathbf{X}_\delta := \{x\colon x'Px \leqslant c_\delta,\ x \in \mathscr{N}\}.$$

Then for any non-positive function $\rho(r, y)$, *locally Lipschitz in* y, *the state trajectory of the closed-loop system comprising the given system* (1)–(3) *and the CNF control law* (15) *is bounded for all* $t \geqslant 0$, *provided that the initial states* ξ_0 *and* x_0, *and amplitude of step input* r *satisfy*

$$\|\xi_0\| \leqslant \gamma, \quad \tilde{x}_0 := (x_0 - x_e) \in \mathbf{X}_\delta, \quad |Hr| \leqslant \delta u_{\max}. \tag{18}$$

Moreover, *the system output* y *tracks asymptotically the step command input of amplitude* r.

Proof. The closed-loop system comprising the given plant (1)–(3) and the CNF control law (15) is given by

$$\dot{\xi} = f(\xi, y), \tag{19}$$

$$\dot{x} = Ax + B\,\mathrm{sat}(Fx + Gr + \rho(r, y)B'P(x - x_e)), \tag{20}$$

$$y = Cx. \tag{21}$$

Let $\tilde{x} = x - x_e$. The closed-loop system (19)–(20) can be expressed as

$$\dot{\xi} = f(\xi, r + C\tilde{x}), \tag{22}$$

$$\dot{\tilde{x}} = (A + BF)\tilde{x} + Bw, \tag{23}$$

where

$$w = \text{sat}(F\tilde{x} + Hr + \rho(r, y)B'P\tilde{x}) - F\tilde{x} - Hr. \quad (24)$$

Define a Lyapunov function $V_{\tilde{x}}(\tilde{x}) = \tilde{x}'P\tilde{x}$, then we have

$$\lambda_{\min}(P)\|\tilde{x}\|^2 \leqslant V_{\tilde{x}}(\tilde{x}) \leqslant \lambda_{\max}(P)\|\tilde{x}\|^2, \quad (25)$$

where $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ are the minimal and maximal eigenvalues of P, respectively. Then,

$$\begin{aligned}\dot{V}_{\tilde{x}}(\tilde{x}) &= \frac{\partial V_{\tilde{x}}(\tilde{x})}{\partial \tilde{x}}((A+BF)\tilde{x} + Bw) \\ &= -\tilde{x}'W\tilde{x} + \frac{\partial V_{\tilde{x}}(\tilde{x})}{\partial \tilde{x}} Bw.\end{aligned}$$

It have been shown in [3] that

$$\frac{\partial V_{\tilde{x}}(\tilde{x})}{\partial \tilde{x}} Bw = 2\tilde{x}'PBw \leqslant 0$$

for all $\tilde{x} \in \mathbf{X}_\delta$ and $|Hr| \leqslant \delta u_{\max}$. Thus

$$\dot{V}_{\tilde{x}}(\tilde{x}) \leqslant -\tilde{x}'W\tilde{x}, \quad \tilde{x} \in \mathbf{X}_\delta, \quad (26)$$

i.e., $\mathbf{X}_\delta$ is an invariant set of the system (23). Therefore, the solution of (23) exists and is bounded for all $t \geqslant 0$ and $\tilde{x}_0 \in \mathbf{X}_\delta$. Noting that $x = x_e + \tilde{x}$, x exists and is bounded for all $t \geqslant 0$ and x_0 satisfies (18).

To show the existence and boundedness of the solution ξ of (22), it is sufficient to show that $\|\tilde{y}\| := \|C\tilde{x}\| \leqslant \beta \mathrm{e}^{-at}$. Noting that (26) gives

$$\dot{V}_{\tilde{x}}(\tilde{x}) \leqslant -\tilde{x}'W\tilde{x} \leqslant -\lambda_{\min}(W)\|\tilde{x}\|^2 \quad (27)$$

for all $\tilde{x} \in \mathbf{X}_\delta$. According to the proof of Theorem 4.10 of [11], (25) and (27) yield that

$$\begin{aligned}\|\tilde{x}(t)\| &\leqslant \left(\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}\right)^{1/2} \|\tilde{x}(0)\| \mathrm{e}^{-[\lambda_{\min}(W)/2\lambda_{\max}(P)]t} \\ &\leqslant \left(\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}\right)^{1/2} \|\tilde{x}(0)\| \mathrm{e}^{-at},\end{aligned}$$

since a is selected such that $0 < a \leqslant \lambda_{\min}(W)/(2\lambda_{\max}(P))$. Then

$$\begin{aligned}\|\tilde{y}(t)\| = \|C\tilde{x}(t)\| &\leqslant \|C\|\|\tilde{x}(t)\| \\ &\leqslant \|C\| \left(\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}\right)^{1/2} \|\tilde{x}(0)\| \mathrm{e}^{-at} \\ &\leqslant \beta \mathrm{e}^{-at}\end{aligned}$$

for all $\tilde{x}(0) \in \mathbf{X}_\delta$. Thus, by Remark 2.2, the solution of (22) exists and is bounded for all $t \geqslant 0$.

Moreover, noting that $W > 0$, all trajectories of (23) starting from $\mathbf{X}_\delta$ will converge to the origin. Thus,

$$\lim_{t\to\infty} x(t) = x_e$$

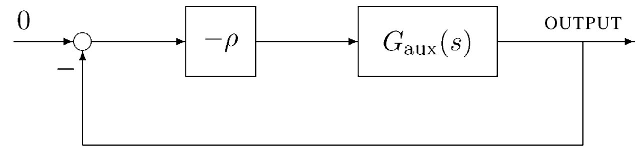

Fig. 1. Interpretation of the nonlinear function $\rho(r, y)$.

for all initial state x_0 and the step command input of amplitude r that satisfy (18). Therefore,

$$\lim_{t\to\infty} y(t) = Cx_e = -C(A+BF)^{-1}BGr = r.$$

This completes the proof of Theorem 3.1. □

Remark 3.1. The CNF control law (15) is reduced to the linear feedback control law (9) when the function $\rho(r, y) = 0$. Thus, Theorem 3.1 shows that the additional nonlinear feedback control law (14) does not affect the ability of the closed-loop system to track the command input. Any command input that can be asymptotically tracked by the linear control law (9) can also be asymptotically tracked by the CNF control law (15). However, this additional term u_N in the CNF control law can be used to improve the performance of the overall closed-loop system. This is the key property of the control technique studied in this manuscript.

Remark 3.2. The main purpose of adding the nonlinear part to the CNF control law is to speed up the settling time, or equivalently to contribute a significant value to the control input when the tracking error, $r - y$, is small. The nonlinear part, in general, will be in action when the control signal is far away from its saturation level and, thus, it will not cause the control input to hit its limits. Under such a circumstance, it is straightforward to verify that the closed-loop system comprising (2) and (15) can be expressed as

$$\dot{\tilde{x}} = (A+BF)\tilde{x} + \rho(r, y)BB'P\tilde{x}. \quad (28)$$

It is clear that eigenvalues of the closed-loop system (28) can be changed by the function $\rho(r, y)$. In fact, define the auxiliary system $G_{\text{aux}}(s)$ as

$$\begin{aligned}G_{\text{aux}}(s) &:= C_{\text{aux}}(sI - A_{\text{aux}})^{-1}B_{\text{aux}} \\ &:= B'P(sI - A - BF)^{-1}B. \quad (29)\end{aligned}$$

Then, system (28) can be expressed as Fig. 1. Using the well-known classical root-locus theory, the poles of the closed-loop system (28) approach the location of the invariant zeros of $G_{\text{aux}}(s)$ as $|\rho|$ becomes larger and larger.

Remark 3.3. It is shown in [3] that the auxiliary system G_{aux} is stable and invertible with a relative degree equal to 1, and is of minimum phase with $n-1$ stable invariant zeros. It should be noted that there is freedom in pre-selecting the locations of these invariant zeros by selecting an appropriate W in (13). In general, we should select the invariant zeros of G_{aux}, which are corresponding to the closed-loop poles of (28) for large $|\rho|$, such that the dominated ones have a large damping ratio, which in turn will yield a smaller overshoot. Interested readers are referred to [3] for the detailed procedure for the selecting of such a W. Another important step in designing the CNF control law is the selection of the non-positive nonlinear function $\rho(r, y)$. We usually choose $\rho(r, y)$ as a function of the tracking error $r - y$, which in most practical situations is known and available for feedback, such that $\rho(r, y)$ has the following two properties: (1) when the output y is far away from the final set point, $|\rho(r, y)|$ is small and thus the effect of the nonlinear part on the overall system is very limited, and (2) when the output approaches the set point, $|\rho(r, y)|$ becomes larger and larger, and the nonlinear control law will become effective. Of course, the choice of $\rho(r, y)$ is non-unique. The following choice is one of the suitable candidates:

$$\rho(r, y) = -\beta_n |e^{-\alpha_n |y(t)-r|} - e^{-\alpha_n |y(0)-r|}|, \tag{30}$$

where $\beta_n > 0$ and $\alpha_n > 0$ are tuning parameters.

4. Illustrative examples

In this section, we illustrate the CNF design method with two examples. To compare the performance of the CNF control law and the linear control law, we first take the example from [14] where the semi-global stabilization problem is solved by a linear state feedback. Based on the linear control law given by [14], we will design a CNF control law to improve the performance of the closed-loop system. The second example is the design of a flight control system for a simplified model of a fighter aircraft reported in [19].

Example 4.1. Consider a partially linear composite system (see [14]) characterized by

$$\dot{\xi} = -\xi + \xi^2 y, \tag{31}$$

$$\dot{x} = Ax + B\,\mathrm{sat}(u), \tag{32}$$

$$y = Cx \tag{33}$$

with

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C = [0\ \ 0\ \ 0\ \ 0\ \ 1], \tag{34}$$

and $u_{max} = 0.2$. For the stabilization problem of (31)–(33), we let $r = 0.5$. It is simple to verify that the triple (A, B, C) is controllable and has a relative degree of 1 and four invariant zeros at $\{j, -j, j, -j\}$. Thus, Assumptions A1 and A2 are satisfied. Assumption A3 is satisfied for $\{\xi\colon \|\xi\| \leqslant 1.5\}$. Let $\gamma = 0.5$ and $\beta = 1$, then it can be shown that any $a > 0$ is good for (γ, β). To design the CNF control law, we use the linear feedback control law

$$\begin{aligned} u_L &= Fx \\ &= [0.403\ \ -0.0001\ \ -0.204\ \ -4.06\ \ -10.4]x \end{aligned} \tag{35}$$

reported in [14]. Next, we select $W = I_5$ and solve the following Lyapunov equation:

$$(A + BF)'P + P(A + BF) = -W,$$

which yields a solution

$$P = \begin{bmatrix} 12.7439 & -0.5000 & -8.2902 & -25.8924 & -2.4813 \\ -0.5000 & 12.8221 & 26.6781 & 4.4923 & 0.1934 \\ -8.2902 & 26.6781 & 75.5045 & 26.7835 & 1.9341 \\ -25.8924 & 4.4923 & 26.7835 & 70.7732 & 6.7201 \\ -2.4813 & 0.1934 & 1.9341 & 6.7201 & 0.6942 \end{bmatrix} > 0.$$

The nonlinear function $\rho(r, y)$ is chosen as

$$\rho(r, y) = -25.5(e^{-0.8|y-r|} - e^{-0.8|y(0)-r|}). \tag{36}$$

Finally, the CNF control law is given by

$$u = Fx + Gr + \rho(r, y)B'P(x - G_e r). \tag{37}$$

The simulation result is shown in Fig. 2 where the transient performance is compared between the linear control law and the CNF control law under the same initial conditions $\xi(0) = -0.2$ and $x(0) = 0$. Clearly, the CNF control has outperformed the linear counterpart significantly. Comparing Figs. 2(a) and (b), we can see that all the states of the closed-loop system under the CNF control convergence to the steady state quickly, and their transient amplitudes are much smaller than the ones under the linear control law. Figs. 2(c) and (d) show the system output of the closed-loop system and the control inputs applied on the system under the linear control and the CNF control. The overshoot under the linear control is 19.19%, but under the CNF control, there is no overshoot at all.

Example 4.2. Consider a simplified model of a fighter aircraft reported in [19], which is characterized by

$$\begin{aligned} \dot{v} &= 1.8254\cos(0.0175(\alpha + 11.3404)) - 1.9821 \\ &\quad \times 10^{-3}(0.0886 + 1.75 \times 10^{-2}\alpha)v^2, \end{aligned} \tag{38}$$

$$\dot{\alpha} = -0.5923\alpha + 50.7296q - 0.1145\mathrm{sat}(u), \tag{39}$$

$$\dot{q} = -0.0178\alpha - 0.3636q - 0.0676\mathrm{sat}(u), \tag{40}$$

where the airspeed v (m/s), angle of attack α (deg), and pitch angular rate q (rad/s) are state variables, deflection of elevator u (deg) is control input with a saturation level $u_{max} = 10°$. The model is extracted from the nonlinear model of six degree of freedoms based on a steady flight condition with mach = 0.3, height = 1000 m, and with a straight and horizontal flight. The control objective is to set the angle of

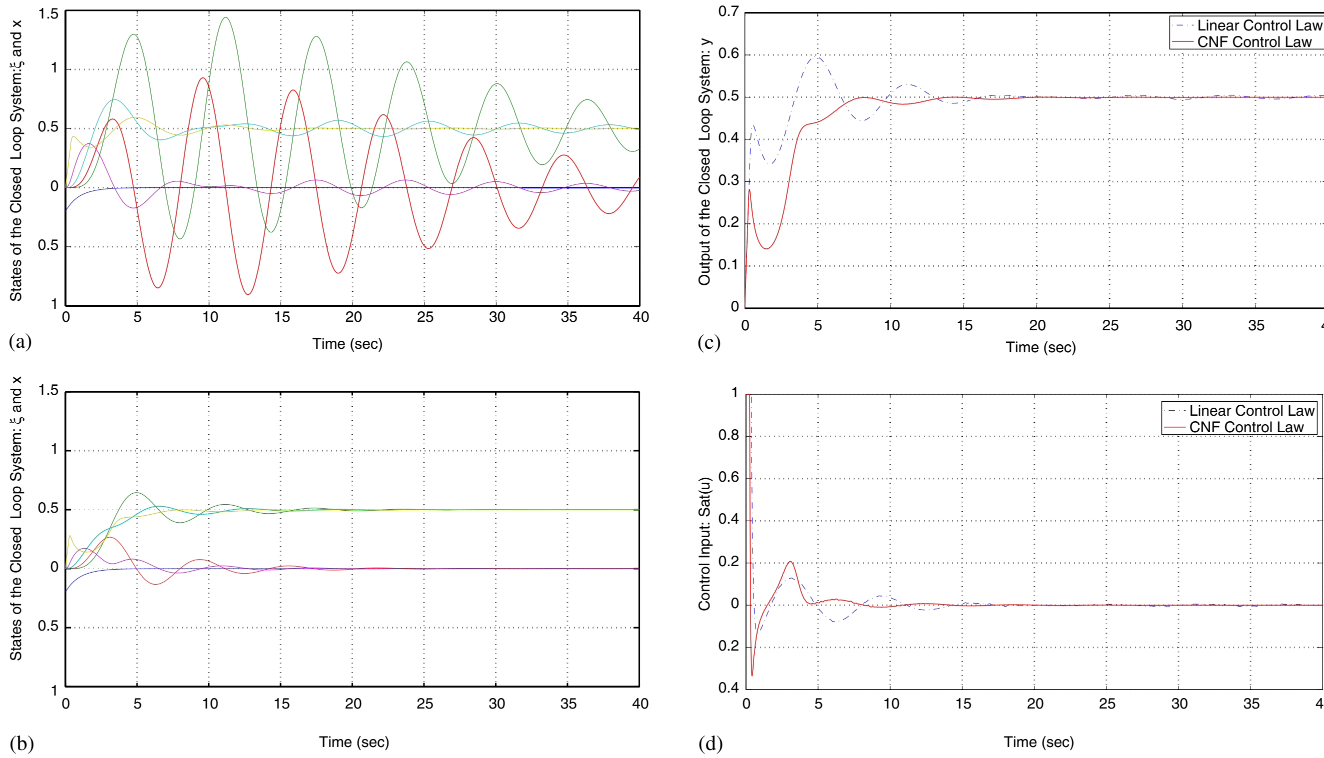

Fig. 2. State responses and control signals of the closed-loop systems. (a) State responses with the linear control law. (b) State responses with the CNF control law. (c) System output of the closed-loop system. (d) Control signals.

attack to a reference attitude 5° quickly without experiencing large overshoot.

Let $\xi = v$ and $x = (\alpha,\ q)'$, and let $y = \alpha$. Then, the dynamics in the aircraft can be rewritten in the form of (1)–(3), i.e.,

$$\dot{\xi} = 1.8254\cos(0.0175(y + 11.3404)) - 1.9821 \times 10^{-3}(0.0886 + 1.75 \times 10^{-2}y)\xi^2, \tag{41}$$

$$\dot{x} = Ax + B\ \text{sat}(u), \tag{42}$$

$$y = Cx, \tag{43}$$

where

$$A = \begin{bmatrix} -0.5923 & 50.7296 \\ -0.0178 & -0.3636 \end{bmatrix}, \quad B = \begin{bmatrix} -0.1145 \\ -0.0676 \end{bmatrix},$$
$$C = [1\ \ 0].$$

The triple (A, B, C) is controllable, and has a relative degree of 1 and an invariant zero at -30.3140. Thus, Assumptions A1 and A2 are satisfied. Let $r = 5$, then the nonlinear system (41) with $y = r$ has an equilibrium point $\xi = v_0 = 70.8328$. Let $\tilde{\xi} = \xi - v_0$. We have

$$\dot{\tilde{\xi}} = -0.0495\tilde{\xi} - 3.4912 \times 10^{-4}\tilde{\xi}^2. \tag{44}$$

It is simple to verify that (44) is regionally asymptotically stable, e.g., Assumption A3 is satisfied locally in $\{\tilde{\xi}\colon \|\tilde{\xi}\| \leqslant 60\}$. Thus, a CNF control law can be constructed, which is given as follows:

$$u = Fx + Gr + \rho(r, y)B'P(x - G_e r) \tag{45}$$

with $F = [0.9253,\ \ 35.5945]$ placing the eigenvalues of $A + BF$ at $-1.7677 \pm j1.7677$, $G = -1.5966$, $G_e = [1,\ \ 0.0097]'$,

$$\rho(r, y) = -(e^{-|y-r|} - e^{-|y_0 - r|}) \tag{46}$$

and P is the positive-define solution of the following Lyapunov equation

$$(A + BF)'P + P(A + BF) = -W,$$

where

$$W = \begin{bmatrix} 0.4 & 9.4 \\ 9.4 & 2568.7 \end{bmatrix} > 0 \tag{47}$$

is selected, according to [3,4], such that the invariant zeros of $G_{aux}(s) = B'P(sI - A - BF)^{-1}B$ is -0.5.

The simulation results shown in Fig. 3(a) shows the system output (angle of attack) under the CNF control law (45) and the linear control law which switches off the nonlinear part of the CNF control law (45) under the initial conditions $\xi(0) = 100$ and $x(0) = 0$. Thanks to the nonlinear part of the CNF control law, the output can track the reference command input rapidly, and the overshoot is reduced evidently, 4.31% for the linear control law, 0.26% for the CNF control law. Fig. 3(b) shows the control input applied to the system under these two control laws.

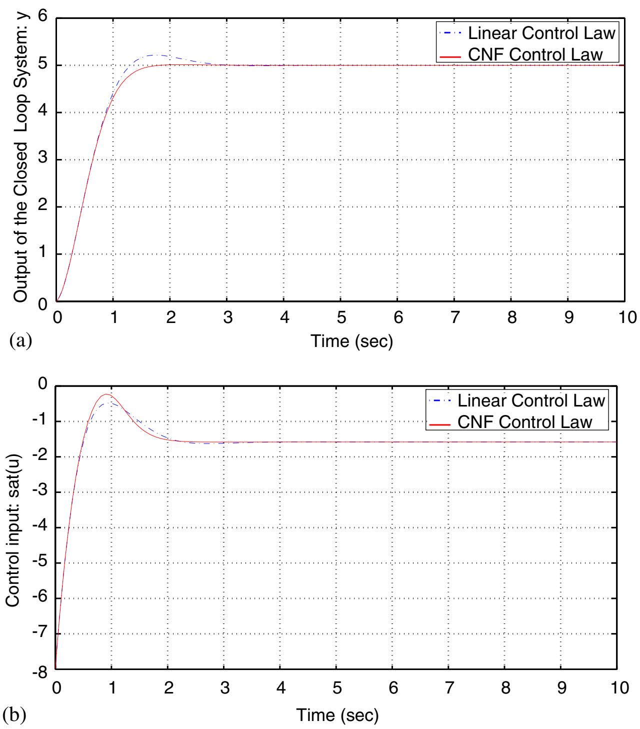

Fig. 3. Output responses and control signals of the flight control system. (a) Output responses. (b) Control signals.

5. Conclusions

The composite nonlinear feedback control technique is extended to the partially linear composite system with input saturation. Simulation result shows that the nonlinear control law greatly improved the performance of the closed-loop system. It should be noted that, in this paper, we have assumed that the linear part of the composite system is SISO, and all the states of the linear part are available to feedback. It should not be too difficult to extend the result of this paper to MIMO systems with state and measurement feedback using the result reported in [6].

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Further reading

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