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Linear systems toolkit in Matlab: structural decompositions and their applications

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Abstract: This paper presents a brief description of the software toolbox, linear systems toolkit, developed in Matlab environment. The toolkit contains 66 m-functions, including structural decompositions of linear autonomous systems, unforced/unsensed systems, proper systems, and singular systems, along with their applications to system factorizations, sensor/actuator selection, H-two and H-infinity control, and disturbance decoupling problems.

Keywords: Linear systems; Structural decompositions; Linear control; Software development

1 Introduction

The state space representation of linear multivariable systems is fundamental to the analysis and design of dynamical systems. Modern control theory relies heavily on the state space representation of dynamical systems, which facilitates characterization of the inherent properties of dynamical systems. Since the introduction of the concept of the state, the study of linear systems in the state space representation itself has emerged as an ever active research area, covering a wide range of topics from the basic notions of stability, controllability, observability, redundancy and minimality to more intricate properties of finite and infinite zero structures, invertibility, and geometric subspaces. A deeper understanding of linear systems facilitates the development of modern control theory. The demanding expectations from modern control theory impose an ever increasing demand for the understanding and utilization of subtler properties of linear systems.

Structural properties play an important role in our understanding of linear systems in the state space representation. The structural canonical form representation of linear systems not only reveals the structural properties but also facilitates the design of feedback laws that meet various control objectives. In particular, it decomposes the system into various subsystems. These subsystems, along with the interconnections that exist among them, clearly show the structural properties of the system. The simplicity of the subsystems and their explicit interconnections with each other lead us to a deeper insight into how feedback control would take effect on the system, and thus to the explicit construction of feedback laws that meet our design specifications. The search for structural canonical forms and their applications in feedback design for various performance specifications has been an active area of research for a long time. The effectiveness of the structural decomposition approach has also been extensively explored in nonlinear systems and control theory in the recent past.

In this article, we present a MATLAB toolkit, linear systems toolkit, for realizing various structural decomposition techniques recently reported in a monograph by Chen, Lin and Shamash [1]. The toolkit [2], which has been built upon the earlier versions [3,4], is able to efficiently compute the structural decompositions of autonomous systems, unforced/unsensed systems, proper systems, and singular systems, along with their properties, such as finite and infinite zero structures, invertibility structures and geometric subspaces. The applications of these decomposition techniques to system factorizations, structural assignments via sensor or actuator selection, and H_2 and H_∞ control are also included. It is now used extensively in the education and research of control theory. Its rich collection of linear algebra functions are immediately useful to the control engineer and system analyst. Its easy-to-use environment allows the problems and solutions to be expressed in familiar mathematical notation.

The detailed algorithms and proofs of the functions reported in the toolkit can be found in the monograph of

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Chen, Lin and Shamash [1], and the beta version of this toolkit is currently available on the website at http://linearsystemskit.net or http://hdd.ece.nus.edu.sg/~bmchen. Readers who have our earlier versions [3,4] of the software realization of the special coordinate basis of Sannuti and Saberi [6] are strongly encouraged to update to the new toolkit. The special coordinate basis, implemented in the new toolkit, is based on a numerically stable algorithm recently reported in Chu, Liu and Tan [5], together with an enhanced procedure reported in [1].

The paper is organized as follows. Section 2 provides the detailed list of m-functions in the toolkit. Section 3 describes some key functions of the toolkit, while Section 4 demonstrates some m – functions with numerical examples. Finally, Section 5 draws a brief conclusion to the paper.

2 Contents of tookit

We list in this section the detailed contents of the toolkit. We note that some m-functions in the toolkit are interactive, which require users to enter desired parameters during execution. Others are implemented in a way that can return results either in a symbolic or numerical form.

The current version of the toolkit consists of the following m-functions:

A) Decompositions of autonomous systems.

1) ssd: continuous-time stability structural decomposition;

2) dssd: discrete-time stability structural decomposition;

3) jcf: Jordan canonical form;

4) rjd: real Jordan decomposition.

B) Decompositions of unforced and unsensed systems.

1) osd: observability structural decomposition;

2) obvidx: observability index;

3) bdosd: block diagonal observable structural decomposition;

4) csd: controllability structural decomposition;

5) ctridx: controllability index;

6) bdcsd: block diagonal controllable structural decomposition.

C) Decompositions & structural properties of proper system.

1) scbraw: raw decomposition without integration chains;

2) scb: decomposition of a continuous-time system;

3) dscb: decomposition of a discrete-time system;

4) kcf: Kronecker canonical form for system matrices;

5) morseidx: Morse indices;

6) blkz: blocking zeros;

7) invz: invariant zero structure;

8) infz: infinite zero structure;

9) l_invt: left invertibility structure;

10) r_invt: right invertibility structure;

11) normrank: normal rank;

12) v_star: veakly unobservable subspace;

13) v_minus: stable weakly unobservable subspace;

14) v_plus: unstabl weakly unobservable subspace;

15) s_star: strongly controllable subspace;

16) s_minus: stable strongly controllable subspace;

17) s_plus: unstable weakly unobservable subspace;

18) r_star: strongly controllable weakly unobservable subspace;

19) n_star: distributionally weakly unobservable subspace;

20) s_lambda: geometric subspace S_λ;

21) v_lambda: geometric subspace V_λ;

D) Operations of vector subspaces.

1) ssorder: ordering of vector subspaces;

2) ssintsec: intersection of vector subspaces;

3) ssadd: addition of vector subspaces.

E) Decompositions and properties of descriptor systems.

1) ea_ds: decomposition of a matrix pair (E, A);

2) sd_ds: decomposition for descriptor systems;

3) invz_ds: descriptor system invariant zero structure;

4) infz_ds: descriptor system infinite zero structure;

5) l_invt_ds: descriptor system left invertibility structure;

6) r_invt_ds: descriptor system right invertibility structure.

F) System factorizations.

1) mpfact: continuous minimum-phase/all-pass factorization;

2) iofact: continuous-time inner-outer factorization;

3) gcfact: continuous generalized cascade factorization;

4) dmpfact: discrete minimum-phase/all-pass factorization;

5) diofact: discrete-time inner-outer factorization.

G) Structural assignment via sensor/actuator selection.

1) sa_sen: structural assignment via sensor selection;

2) sa_act: structural assignment via actuator selection.

H) Asymptotic time-scale and eigenstructure assignment.

1) atea: continuous-time ATEA;

2) gm2star: infimum for continuous-time H_2 control;
3) h2care: solution to continuous-time H_2 ARE;
4) h2state: solution to continuous-time H_2 control;
5) gm8star: infimum for continuous-time H_∞ control;
6) h8care: solution to continuous-time H_∞ ARE;
7) h8state: solution to continuous-time H_∞ control;
8) addps: solution to continuous disturbance decoupling;
9) datea: discrete-time ATEA;
10) dare: solution to general discrete-time ARE;
11) dgm2star: infimum for discrete-time H_2 control;
12) h2dare: solution to discrete-time H_2 ARE;
13) dh2state: solution to discrete-time H_2 control;
14) dgm8star: infimum for discrete-time H_∞ control;
15) h8dare: solution to discrete-time H_∞ ARE;
16) dh8state: solution to discrete-time H_∞ control;
17) daddps: solution to discrete-time disturbance decoupling.

I) Disturbance decoupling with static output feedback.

1) ddpcm: solution to disturbance decoupling problem with static output feedback (DDPCM);
2) rosys4ddp: irreducible reduced-order system that can be used to solve DDPCM.

3 Descriptions of key m-functions

We briefly describe some key m-functions of the toolkit. In particular, we will present the well-known real Jordan decomposition for autonomous systems, a so-called observability structural decomposition for unforced systems, a block diagonal controllable structural decomposition for unsensed systems, the special coordinate basis, Morse invariance indices and weakly unobservable geometric subspace for proper systems, a structural decomposition technique for singular systems, the inner-outer system factorization, structural assignments through sensor or actuator selection, the asymptotic time-scale and eigenstructure assignment, the computation of the best achievable disturbance attenuation level in H_∞ control, and the solution to the problem of disturbance decoupling through static output feedback.

RJD Real Jordan decomposition.

$$[J, T] = \mathrm{RJD}(A)$$

generates a transformation that transforms a square matrix into its real Jordan canonical form, i.e.,

$$T^{-1}AT = J = \begin{bmatrix} J_1 & & & \\ & J_2 & & \\ & & \ddots & \\ & & & J_k \end{bmatrix},$$

where each block J_i, $i = 1, 2, \cdots, k$, has the following form:

$$J_i = \begin{bmatrix} \lambda_i & 1 & & \\ & \ddots & \ddots & \\ & & \lambda_i & 1 \\ & & & \lambda_i \end{bmatrix}$$

or

$$J_i = \begin{bmatrix} \Lambda_i & I & & \\ & \ddots & \ddots & \\ & & \Lambda_i & I \\ & & & \Lambda_i \end{bmatrix}$$

and λ_i is a real eigenvalue and

$$\Lambda_i = \begin{bmatrix} \mu_i & \omega_i \\ -\omega_i & \mu_i \end{bmatrix}$$

contains a pair of complex eigenvalues $\mu_i \pm j\omega_i$.

OSD Observability structural decomposition.

$$[At, Ct, Ts, To, \mathrm{uom}, \mathrm{Oidx}] = \mathrm{OSD}(A, C)$$

returns an observability structural decomposition for the matrix pair (A, C), i.e.,

$$At = Ts^{-1}ATs = \begin{bmatrix} A_0 & * & 0 & \cdots & * & 0 \\ 0 & * & I_{k_1-1} & \cdots & * & 0 \\ 0 & * & 0 & \cdots & * & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & * & 0 & \cdots & * & I_{k_p-1} \\ 0 & * & 0 & \cdots & * & 0 \end{bmatrix},$$

$$Ct = To^{-1}CTs = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \end{bmatrix},$$

$$\mathrm{nom} := n - \sum_{i=1}^{p} k_i, \quad \mathrm{Oidx} := \{k_1, k_2, \cdots, k_p\}.$$

We note that nom is the number of unobservable modes and the set Oidx is the observability index of (C, A).

BDCSD Block diagonal controllable structural decomposition.

$$[At, Bt, Ts, Ti, ks] = \mathrm{BDCSD}(A, B)$$

transforms a controllable pair (A, B) into the block diagonal controllable structureal decomposition form, i.e.,

$$At = Ts^{-1}ATs = \begin{bmatrix} A_1 & 0 & \cdots & 0 \\ 0 & A_2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & A_k \end{bmatrix},$$

$$Bt = Ts^{-1}BT_i = \begin{bmatrix} B_1 & * & \cdots & * & * \\ 0 & B_2 & \cdots & * & * \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & B_k & * \end{bmatrix},$$

$$ks = \{k_1, k_2, \cdots, k_k\},$$

where A_i and B_i, $i = 1, 2, \cdots, k$, are in the form of

$$A_i = \begin{bmatrix} 0 & I_{k_i - 1} \\ * & * \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

SCB Special coordinate basis of contionuous-time system.

$$[As, Bt, Ct, Dt, \mathrm{Gms}, \mathrm{Gmo}, \mathrm{Gmi}, \mathrm{dim}] = \mathrm{SCB}(A, B, C, D)$$

decomposes a continuous-time system into the standard SCB form with state subspaces x_a being separated into stable, marginally stable and unstable parts, and x_d being decomposed into chains of integrators.

$$At = \mathrm{Gms}^{-1} A \ \mathrm{Gms} = As + B_0 C_0$$

$$= \begin{bmatrix} A_{\mathrm{aa}} & L_{\mathrm{ab}} C_{\mathrm{b}} & 0 & L_{\mathrm{ad}} C_{\mathrm{d}} \\ 0 & A_{\mathrm{bb}} & 0 & L_{\mathrm{bd}} C_{\mathrm{d}} \\ E_{\mathrm{ca}} & L_{\mathrm{cb}} C_{\mathrm{b}} & A_{\mathrm{cc}} & L_{\mathrm{cd}} C_{\mathrm{d}} \\ B_d E_{\mathrm{da}} & B_{\mathrm{d}} E_{\mathrm{db}} & B_{\mathrm{d}} E_{\mathrm{dc}} & A_{\mathrm{dd}} \end{bmatrix} + \begin{bmatrix} B_{0\mathrm{a}} \\ B_{0\mathrm{b}} \\ B_{0\mathrm{c}} \\ B_{0\mathrm{d}} \end{bmatrix} [C_{0\mathrm{a}} \quad C_{0\mathrm{b}} \quad C_{0\mathrm{c}} \quad C_{0\mathrm{d}}],$$

$$Bt = \mathrm{Gms}^{-1} B \ \mathrm{Gmi} = [B_0 \quad B_{\mathrm{s}}] = \begin{bmatrix} B_{0\mathrm{a}} & 0 & 0 \\ B_{0\mathrm{b}} & 0 & 0 \\ B_{0\mathrm{c}} & 0 & B_{\mathrm{c}} \\ B_{0\mathrm{d}} & B_{\mathrm{d}} & 0 \end{bmatrix},$$

$$Ct = \mathrm{Gmo}^{-1} C \ \mathrm{Gms} = \begin{bmatrix} C_0 \\ C_{\mathrm{s}} \end{bmatrix} = \begin{bmatrix} C_{0\mathrm{a}} & C_{0\mathrm{b}} & C_{0\mathrm{c}} & C_{0\mathrm{d}} \\ 0 & 0 & 0 & C_{\mathrm{d}} \\ 0 & C_{\mathrm{b}} & 0 & 0 \end{bmatrix},$$

$$Dt = \mathrm{Gmo}^{-1} \mathrm{D} \ \mathrm{Gmi} = D_{\mathrm{s}} = \begin{bmatrix} I_{m_0} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

and

$$\mathrm{dim} = [n_a^-, n_a^0, n_a^+, n_b, n_c, n_d],$$

where

$$A_{\mathrm{dd}} = A_{\mathrm{dd}}^* + B_{\mathrm{d}} E_{\mathrm{dd}} + L_{\mathrm{dd}} C_{\mathrm{d}},$$

for some constant matrices L_{dd} and E_{dd} of appropriate dimensions, and

$$A_{\mathrm{dd}}^* = \mathrm{blkdiag}\{A_{q_1}, A_{q_2}, \cdots, A_{q_{md}}\},$$

$$B_{\mathrm{d}} = \mathrm{blkdiag}\{B_{q_1}, B_{q_2}, \cdots, B_{q_{md}}\},$$

$$C_{\mathrm{d}} = \mathrm{blkdiag}\{C_{q_1}, C_{q_2}, \cdots, C_{q_{md}}\},$$

with $(A_{q_i}, B_{q_i}, C_{q_i})$ being defined as

$$A_{q_i} = \begin{bmatrix} 0 & I_{q_i - 1} \\ 0 & 0 \end{bmatrix},$$

$$B_{q_i} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$C_{q_i} = [1, 0, \cdots, 0].$$

MORSEIDX Morse invariance indices of proper systems.

$$[\mathrm{I1}, \mathrm{I2}, \mathrm{I3}, \mathrm{I4}] = \mathrm{MORSEIDX}(A, B, C, D)$$

returns Morse structural invariance list:

I1 = zero dynamics matrix in Jordan form;

I2 = right invertibility structure;

I3 = left invertibility structure;

I4 = infinite zero structure.

V_STAR Weakly unobservable geometric subspace.

$$V = \text{V-STAR}(A, B, C, D)$$

computes a matrix whose columns span the geometric subspace V^*.

SD_DS Structural decomposition of continuous-time descriptor system.

$$[\mathrm{Es}, \mathrm{As}, \mathrm{Bs}, \mathrm{Cs}, \mathrm{Ds}, \mathrm{Ez}, \mathrm{Psi}, \mathrm{Psc}, \mathrm{Psd}, \mathrm{Psr}, \mathrm{Gme}, \mathrm{Gms}, \mathrm{Gmo}, \mathrm{iGmi}, \mathrm{dim}] = \mathrm{SD\_DS}(E, A, B, C, D)$$

generates the structural decomposition of a descriptor system. The state x are decomposed as

$$\tilde{x} = [x_z^{\mathrm{T}} \quad x_e^{\mathrm{T}} \quad x_a^{\mathrm{T}} \quad x_b^{\mathrm{T}} \quad x_c^{\mathrm{T}} \quad x_d^{\mathrm{T}}]^{\mathrm{T}}.$$

The quadruple (Es, As, Bs, Cs, Ds) has the same transfer function as that of the original system. Ez, Psi, Psc, Psd and Psr are some matrices or vectors whose elements are either polynomials or rational functions of s. In particular,

$$\mathrm{Psc}\ \tilde{x} + \mathrm{Psd}\ \tilde{u} = \mathrm{Psr}\ x_z.$$

See [2] for detail.

IOFACT Inner-outer factorization of continuous-time systems.

$$[\mathrm{Ai}, \mathrm{Bi}, \mathrm{Ci}, \mathrm{Di}, \mathrm{Ao}, \mathrm{Bo}, \mathrm{Co}, \mathrm{Do}] = \mathrm{IOFACT}(A, B, C, D)$$

computes an inner-outer factorization for a stable proper transfer function matrix $G(s)$ with a realization (A, B, C, D), in which both $[B^{\mathrm{T}}\ D^{\mathrm{T}}]$ and $[C \quad D]$ are assumed to be of full rank. The inner-outer factorization is given as

$$G(s) = G_i(s) G_o(s),$$

where

$$G_i(s) = \mathrm{Ci}(sI - \mathrm{Ai})^{-1} \mathrm{Bi} + \mathrm{Di}$$

is an inner, and

$$G_o(s) = C_o(sI - \mathrm{Ao})^{-1} \mathrm{Bo} + \mathrm{Do}$$

is an outer.

SA_SEN Structural assignment via sensor selection.

$$C = \mathrm{SA\_SEN}(A, B)$$

For a given unsensed system (A, B), the function returns a measurement output matrix C such that the resulting system characterized by (A, B, C) has the pre-specified desired structural properties.

ATEA Asymptotic time-scale and eigenstructure assignment.

$$F = \text{ATEA}(A, B, C, D[, \text{option}])$$

produces a state feedback law $u = Fx$ using the asymptotic time-scale structure and eigenstructure assignment design method for a continuous-time system.

Users have the 'option' to choose the result either in a numerical or in a symbolic form parameterized by a tuning parameter 'epsilon'. The latter is particularly useful in solving control problems, such as H_2 and H_∞ sub-optimal control as well as disturbance decoupling problem.

GM8STAR Infimum or optimal value for continuous H-infinity control.

$$\text{gms8} = \text{GM8STAR}(A, B, C, D, E)$$

calculates the infimum or the best achievable performance of the H_∞ suboptimal control problem for the plant,

$$\dot{x} = Ax + Bu + Ew,$$
$$h = Cx + Du,$$

under all possible stabilizing state feedbacks.

DDPCM Disturbance decoupling with static output feedback.

$$K = \text{DDPCM}(A, B, E, C1, D1, C2, D2, D22)$$

computes a solution to the disturbance decoupling problem with a constant (static) measurement output feedback forthe following system:

$$\dot{x} = Ax + Bu + Ew,$$
$$y = C1x + D1w,$$
$$h = C2x + D2u + D22w,$$

when the solution exists. Otherwise, the program will return an empty matrix for K.

4 Numerical examples

In this section, we illustrate a few key m-functions of Linear Systems Toolkit with several numerical examples. Some are straightforward. Others requires reference to the monograph [2] for detailed information.

Example 4.1 (JCF) Consider a Hilbert matrix A given by

$$A = \begin{bmatrix} 1 & 1/2 & 1/3 & 1/4 & 1/5 \\ 1/2 & 1/3 & 1/4 & 1/5 & 1/6 \\ 1/3 & 1/4 & 1/5 & 1/6 & 1/7 \\ 1/4 & 1/5 & 1/6 & 1/7 & 1/8 \\ 1/5 & 1/6 & 1/7 & 1/8 & 1/9 \end{bmatrix}.$$

The m-function $[J, T] = \text{JCF}(A)$ returns the Jordan form of A with

$$J = \text{blkdiag}\{3.2879 \times 10^{-6}, 3.0590 \times 10^{-4}, 1.1407 \times 10^{-2}, 2.0853 \times 10^{-1}, 1.5671\},$$

$$T = \begin{bmatrix} -0.0062 & -0.0472 & -0.2142 & 0.6019 & -0.7679 \\ 0.1167 & 0.4327 & 0.7241 & -0.2759 & -0.4458 \\ -0.5062 & -0.6674 & 0.1205 & -0.4249 & -0.3216 \\ 0.7672 & -0.2330 & -0.3096 & -0.4439 & -0.2534 \\ -0.3762 & 0.5576 & -0.5652 & -0.4290 & -0.2098 \end{bmatrix}.$$

The computing error is given by

$$\|(T^{-1}AT - J)\|_2 = 4.0773 \times 10^{-16}.$$

We note that it is hard to obtain a diagonal form for a Hilbert matrix. This example shows that our m-function JCF is capable of handling some rather ill-conditioned matrices.

Example 4.2 (RJD) Consider another constant matrix

$$A = \begin{bmatrix} 3 & 2 & 10 & 0 & 4 & 4 & 4 \\ 2 & 2 & 3 & 1 & 0 & 2 & 1 \\ -1 & 0 & -2 & -1 & 0 & -1 & 0 \\ 2 & 0 & 4 & 4 & -2 & 1 & -2 \\ 3 & 2 & 10 & 2 & 3 & 4 & 2 \\ -1 & -2 & -4 & 0 & -2 & -1 & -2 \\ -3 & -2 & -8 & -1 & -2 & -4 & -2 \end{bmatrix}. \tag{1}$$

The m-function $[J, T] = \text{RJD}(A)$ returns a real Jordan form of A with

$$J = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}.$$

The tramsformation matrix T is omitted here.

Example 4.3 (OSD) Consider an unforecd linear system characterized by a matrix pair (A, C) with

$$A = \begin{bmatrix} 0 & 2 & -1 & 1 & 2 & -2 \\ 2 & 8 & 3 & 2 & 9 & 1 \\ -4 & -12 & -6 & -5 & -13 & -2 \\ 2 & 8 & 4 & 3 & 10 & 3 \\ -1 & -6 & -1 & -1 & -7 & 0 \\ 1 & 5 & 1 & 1 & 5 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 3 & 1 & 1 & 3 & 0 \end{bmatrix}.$$

The function $[At, Ct, Ts, To, \text{uom}, \text{Oidx}] = \text{osd}(A, C)$ returns the following results:

$$At = \begin{bmatrix} -.1 & -2 & 0 & -2 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 3 & 0 & -4 & 0 & 1 \\ 0 & 5 & 0 & -1 & 0 & 0 \end{bmatrix},$$

$$Ct = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix},$$

$$Ts = \begin{bmatrix} 0.50 & -1.50 & -0.50 & 0.75 & -0.75 & 0.75 \\ -0.50 & 0.50 & 0.50 & 0.25 & -0.25 & 0.25 \\ -0.00 & -0.00 & 2.00 & -1.00 & -0.00 & -1.00 \\ -0.50 & -1.50 & -1.50 & 1.25 & 0.75 & 0.25 \\ 0.50 & 0.50 & -0.50 & -0.25 & 0.25 & -0.25 \\ 0.00 & 0.00 & -1.00 & -0.00 & 0.00 & 1.00 \end{bmatrix},$$

and

$$To = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \text{ uom} = 1, \text{ oidx} = [2 \quad 3].$$

There is one unobservable model at -1, and the observability index of (A, C) is given by {2, 3}.

Example 4.4(BDCSD) Consider an unsensed linear system characterized by (A, B) with A as given in (1) and

$$B = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}^{\mathrm{T}}.$$

The m-function [At, Bt, Ts, Ti, ks] = BDCSD (A, B) returns

$$At = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -4 & 16 & -28 & 28 & -17 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad Bt = \begin{bmatrix} 0 & 0.1688 \\ 0 & -0.0998 \\ 0 & -0.0836 \\ 0 & -0.2141 \\ 0 & -0.4276 \\ 1 & 0.0365 \\ 0 & 1.000 \end{bmatrix},$$

$$Ts = \begin{bmatrix} 97.5852 & -211.6782 & 208.9166 & -92.5697 & 15.7616 & 1.0000 & -0.3006 \\ 2.2539 & -15.0155 & 29.3963 & -22.6347 & 5.8731 & 0.1269 & -0.0000 \\ -10.5077 & 33.5232 & -42.7771 & 28.0155 & -9.2539 & 1.0000 & 0.0000 \\ -34.0310 & 61.5542 & -50.7926 & 11.7616 & 1.8731 & 0.1269 & 0.4658 \\ 47.0465 & -107.1085 & 113.6162 & -58.0620 & 13.0155 & 1.0000 & -0.1503 \\ -38.5387 & 77.5852 & -78.0930 & 39.5542 & -10.1424 & 0.1269 & -0.1652 \\ -57.5542 & 140.3778 & -159.8856 & 88.4428 & -21.8886 & 1.0000 & 0.4658 \end{bmatrix}, T_i = \begin{bmatrix} 1.0000 & 0 \\ 0.1269 & 1 \end{bmatrix},$$

and

$$ks = [6 \quad 1].$$

Example 4.5(SCB) Consider a continuous-time proper system characterized by (A, B, C, D) with

$$A = \begin{bmatrix} 1 & 2 & 1 & 4 & 2 \\ -3 & -2 & -1 & -6 & -2 \\ 0 & -1 & 0 & -1 & -1 \\ 4 & 3 & 1 & 7 & 3 \\ -3 & -3 & -1 & -6 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ -4 & -1 \\ 1 & 0 \\ 3 & 1 \\ -2 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 3 & 1 & 7 & 4 \\ 5 & 5 & 2 & 11 & 6 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

The m-function $[As, Bt, Ct, Dt,$ Gms, Gmo, Gmi, dim$]$ = sub (A, B, C, D) returns the following results:

$$As = \begin{bmatrix} -1.0000 & 0 & 0.1291 & 0 & 0.4518 \\ 0 & 1.0000 & -2.9439 & 0 & -1.4720 \\ 0 & 0 & 0.5000 & 0 & 0.2500 \\ -1.2910 & 1.0190 & -4.5000 & -2 & -4.7500 \\ 1.0328 & -0.9058 & 3.8000 & 2 & 4.5000 \end{bmatrix}, \quad Bt = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad Ct = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \quad Dt = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$Gms = \begin{bmatrix} 0.7746 & 0 & 0.6 & 0 & 0 \\ -0.5164 & -0.5661 & 0.6 & -1 & -2 \\ 0 & 0.6794 & 1 & 0 & 0.5 \\ -0.2582 & -0.1132 & -0.2 & 1 & 1.5 \\ 0.2582 & 0.4529 & -0.8 & -1 & -1 \end{bmatrix}, \quad Gmo = \begin{bmatrix} 1 & 0 \\ 1.5 & 1 \end{bmatrix}, \quad Gmi = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix},$$

and

$$\dim = [1 \quad 0 \quad 1 \quad 1 \quad 1 \quad 1].$$

It is straightforward to verify that the system has two invariant zeros at 1 and -1, respectively, and has a left invertibility index {1}, a right invertibility index {1} and infinite zero structure {1}. Thus, the system is neither left nor right invertible.

Using the m-functions V_STAR and V_MINUS, we can obtain the weakly unobservable subspace V^* and stable weakly unobservable subsepace V^- of the given system, which are respectively given by

$$V^* = \begin{bmatrix} 0.0129 & 0.0421 & -0.8608 \\ -0.7000 & 0.3978 & 0.3077 \\ 0.6515 & 0.1578 & 0.3494 \\ 0.0356 & -0.5977 & 0.2037 \\ 0.2902 & 0.6766 & -0.0289 \end{bmatrix},$$

and

$$V^- = \begin{bmatrix} 0.0375 & 0.7737 \\ -0.6016 & -0.4879 \\ 0.0000 & 0.0000 \\ 0.5642 & -0.2858 \\ -0.5642 & 0.2858 \end{bmatrix}.$$

5 Conclusion

In the paper, we have presented the contents and descriptions of the software toolbox Linear Systems Toolkit [2]. The package is an effective tool for identifying structural properties of linear systems and it can be used for many applications. We are currently expending the toolkit. More features are being added to the toolkit. Interested readers might access to the most up-to-date information about the toolkit through the website http://linearsystemskit.net or http://hdd.ece.nus.edu.sg/~bmchen.

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