Discrete-Time Composite Nonlinear Feedback Control With an Application in Design of a Hard Disk Drive Servo System

V. Venkataraman, Kemao Peng, Ben M. Chen, *Senior Member, IEEE*, and Tong H. Lee

***Abstract*—In a typical disk drive servo system, two or more types of controllers are used for track seeking, track following, and track settling modes. This leads to the problem of mode switching among these controllers. We present in this paper a unified control scheme, the discrete-time composite nonlinear feedback control, which can perform all the above functions in hard disk drive (HDD) servo systems with actuator saturation. The proposed scheme is composed by combining a linear feedback law and a nonlinear feedback law. The linear feedback law is designed to yield a fast response, while the nonlinear feedback law is used to increase the damping ratio of the closed-loop system as the system output approaches the command input. In the face of actuator saturation, this control law not only increases the speed of closed-loop response, but also improves the settling performance. Implementation results show that the proposed method outperforms the conventional proximate time-optimal servomechanism by about 30% in settling time.**

***Index Terms*—Actuator saturation, hard disk drives (HDDs), nonlinear control, servo systems.**

I. Introduction

HARD disk drives (HDDs) provide an important data-storage medium for computers and other data-processing systems. In most HDDs, rotating disks coated with a thin magnetic layer or recording medium are written with data, which are arranged in concentric circles or tracks. Data are read or written with a read/write (R/W) head, which consists of a small horseshoe-shaped electromagnet. Fig. 1 shows a simple illustration of a typical HDD servo system used in commercial products.

The two main functions of the R/W head positioning servomechanism in disk drives are track seeking and track following. Track seeking moves the R/W head from the present track to a specified destination track in minimum time using a bounded control effort. Track following maintains the head as close as possible to the destination track center while information is being read from or written to the disk (see Fig. 2 for a detailed illustration). Track density is the reciprocal of the track width. It is suggested that on a disk surface, tracks should be written as closely spaced as possible so that we can maximize the usage of the disk surface. This means an increase in the track density, which subsequently means a more stringent requirement on the allowable variations of the position of the heads from the true track center.

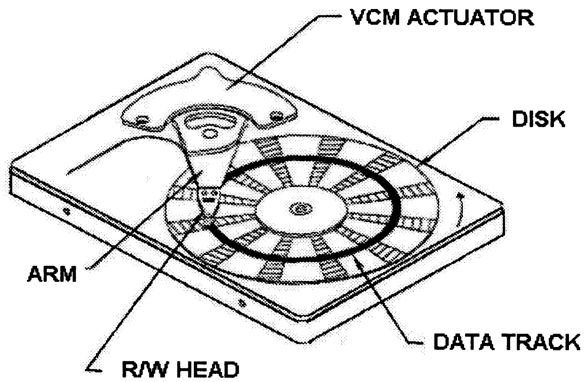

Fig. 1. Typical HDD with a VCM actuator servo system.

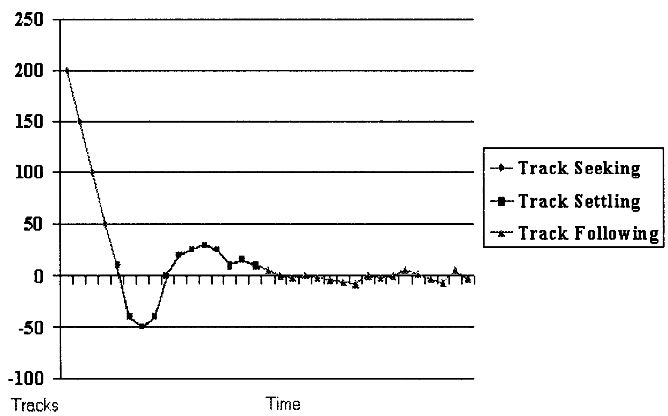

Fig. 2. Track seeking and following of an HDD servo system.

The prevalent trend in hard disk design is toward smaller hard disks with increasingly larger capacities. This implies that the track width has to be smaller leading to lower error tolerance in the positioning of the head. The controller for track following has to achieve tighter regulation in the control of the servomechanism. Controlling HDD servo systems is a challenging problem. Both speed and accuracy are required and these variables typically tradeoff, one against the other. Basically, the servo system of an HDD can be divided into three stages, i.e., the track seeking, track settling, and track following stages. Current HDDs use a combination of classical control techniques, such as proximate time optimal control technique in the track seeking stage, and lead-lag compensators or proportional integral derivative (PID) compensators in the track following stage,

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plus some notch filters to reduce the effects of high-frequency resonant modes (see, e.g., [6]–[9], [11], [14]–[17], [22]–[24], [26], [29], [31], [32], and references therein). These classical methods can no longer meet the demand for HDDs of higher performance. Thus, many control approaches have been tried, such as the linear quadratic Gaussian (LQG) with loop transfer recovery (LTR) approach (see, e.g., [10] and [27]), H_∞ control approach (see, e.g., [1], [2], [12], [13], [18], [19], and [25]), and adaptive control (see, e.g., [4], [21], [28], and [30]), and so on. Although much work has been done to date, more studies need to be conducted to use more control methods to achieve better performance of the HDDs.

Almost all HDDs have two or more different control structures to handle the two main requirements of HDD servo systems, i.e., high-speed head positioning in seeking, and highly accurate head positioning in tracking. In a conventional controller-switching method, a velocity controller is employed in seeking and a positioning feedback controller such as a PID controller, robust perfect tracking controller or an H_∞ controller can be used in tracking. But, all these types of controllers need careful design of mode-switching from track seeking to track following. To overcome the problem of mode-switching, this paper proposes a discrete-time composite nonlinear feedback (CNF) controller, which is a counterpart of the result reported in Lin *et al.* [20], which deals with a class of second-order continuous-time systems, and Chen *et al.* [3], which deals with a class of fairly general continuous-time systems. It consists of a linear feedback law and a nonlinear feedback law. The linear feedback part is designed to yield a closed-loop system with a small damping ratio to get a quick response, while at the same time not exceeding the actuator limits for the desired command input levels. The nonlinear feedback law is used to increase the damping ratio of the closed-loop system as the system output approaches the command input to reduce the overshoot caused by the linear part. Simulation and implementation results show that such a composite nonlinear feedback control law is capable of performing both track seeking and track following without a mode-switching but with much better performance compared to conventional approaches. Finally, we note that such a control scheme can actually be utilized in designing many other servo systems that deal with asymptotic target tracking or "point-and-shoot" fast-targeting.

The outline of the rest of the manuscript is: Section II deals with the modeling of a HDD, on which the actual experiment will be performed. The main part of this paper is Section III, where the composite nonlinear feedback control is derived for linear discrete-time systems. The results are then used to design a servo controller for a disk drive servo system and they are reported in Section IV. The simulation and experimental results are shown in Section V, which clearly show about 30% improvement in settling time from the conventional proximate time-optimal servomechanism (PTOS) for HDD. Finally, the concluding remarks are given in Section VI.

II. Dynamic Model of HDD

In this section, we present the model of a commercially available hard disk drive with a voice-coil motor (VCM) actuator as depicted in Fig. 1. On the surface of a disk, there are thousands of data tracks. A magnetic head is supported by a suspension and a carriage, and it is suspended several micro-inches above the disk surface. The actuator initiates the carriage and moves the head on a desired track. The mechanical part of the plant, that is, the controlled object, consists of the VCM, the carriage, the suspension, and the heads. The controlled variable is the head position. The plant model can be derived from the frequency response characteristics of the HDD servo system taken from an experiment. It is quite conventional to approximate the dynamics of the VCM actuator by a second-order state-space model as

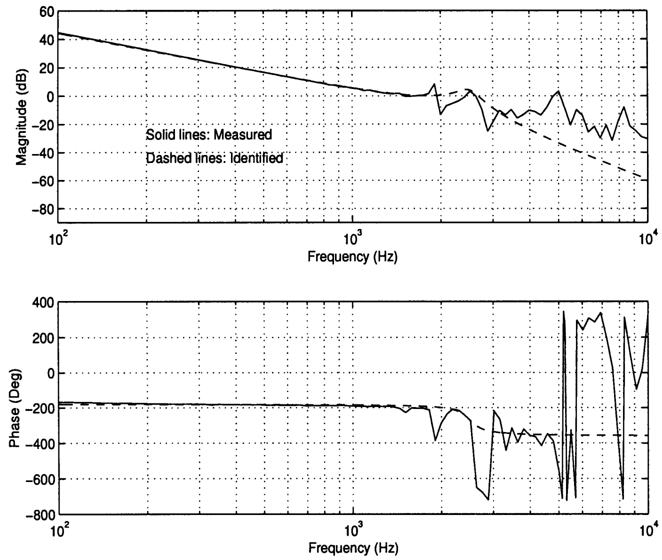

Fig. 3. Frequency response characteristics of HDD.

$$\begin{pmatrix} \dot{y} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} y \\ v \end{pmatrix} + \begin{pmatrix} 0 \\ a \end{pmatrix} u \tag{1}$$

where u is the actuator input (in volts) and is bounded as $u_{\min} \le u(t) \le u_{\max}$, y and v are the position (in μm) and the velocity (in μm/s) of the R/W head, and $a = K_t/J_a$ is the acceleration constant, with K_t being the torque constant and J_a being the moment of inertia of the actuator mass. Thus, the transfer function from u to y of the VCM model can be written as

$$G_{v1}(s) = \frac{a}{s^2}. \tag{2}$$

However, if we consider the high-frequency resonance modes also, a more realistic model for the VCM actuator would be

$$G_v(s) = \frac{a}{s^2}\,\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}. \tag{3}$$

To design and implement the proposed controller, an actual HDD was taken and the model was identified through frequency response test. Fig. 3 shows the frequency response characteristics of a Seagate HDD (ST 31 276A). These response characteristics were obtained using a laser Doppler vibrometer (LDV) and an HP made dynamic signal analyzer (HP35670A). Using these measured data from the actual system (see Fig. 3), and the algorithms of [5] and [26], we obtained a fourth-order model for the actuator

$$G_v(s) = \frac{6.4013 \times 2.467 \times 10^{15}}{s^2(s^2 + 2.513 \times 10^3 s + 2.467 \times 10^8)}. \quad (4)$$

This model will be used throughout the rest of the paper in designing appropriate controllers and simulations.

III. Discrete-Time CNF Control

We consider a linear discrete-time system Σ with an amplitude-constrained actuator characterized by

$$\begin{cases} x(k+1) = A\,x(k) + B\ \text{sat}[u(k)], & x(0) = x_0 \\ y(k) = C_2\,x(k) \end{cases} \quad (5)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}$, and $y \in \mathbb{R}$ are, respectively, the state, control input, and controlled output of Σ. A, B and C_2 are appropriate dimensional constant matrices, and sat: $\mathbb{R} \to \mathbb{R}$ represents the actuator saturation defined as

$$\text{sat}(u) = \text{sgn}(u) \min\{u_{\max},\ |u|\} \quad (6)$$

with $u_{\max}$ being the saturation level of the input. The following assumptions on the system matrices are required.

1) (A, B) is stabilizable.
2) (A, B, C_2) is invertible and has no invariant zeros at $z = 1$.

Note that the first assumption is necessary for any control problem, whereas the second assumption is fairly standard in problems dealing with asymptotic target tracking. It is to guarantee the existence of matrix G specified later in (8).

Inspired by the work of Lin *et al.* [20], which is applicable for a class of second-order continuous-time systems, we derive, in the sequel, a CNF method for general single-input–single-output (SISO) linear discrete-time systems with an amplitude constrained actuator. Our objective here is to design a discrete-time CNF law that will cause the output to track a high-amplitude step input rapidly without experiencing large overshoot and without the adverse actuator saturation effects. This can be done through the design of a discrete-time linear feedback law with a small closed-loop damping ratio and a nonlinear feedback law. Through the use of an appropriate Lyapunov function, the closed-loop system can be made to become highly damped as the system output approaches the command input. As such, the overshoot of the closed-loop response is reduced. We present, in what follows, a step-by-step design algorithm.

Step 1: Design a linear feedback law

$$u_{\mathrm{L}}(k) = Fx(k) + G\,r \quad (7)$$

where r is the input command, and F is chosen such that $A + BF$ has all its eigenvalues in the open unit disc and the closed-loop system $C_2(zI - A - BF)^{-1}B$ meets certain design specifications. Furthermore, let

$$G = \left[C_2(I - A - BF)^{-1}B\right]^{-1}. \quad (8)$$

We note that G is well defined because $A + BF$ has all its eigenvalues inside the unit circle, and (A, B, C_2) is invertible and has no invariant zeros at $z = 1$.

The following lemma determines the magnitude of r that can be tracked by such a control law without exceeding the control limits.

Lemma 1: Given a positive definite matrix $W \in \mathbb{R}^{n \times n}$, let $P > 0$ be the solution of the following Lyapunov equation:

$$P = (A + BF)' P (A + BF) + W. \quad (9)$$

Such a P exists as $A + BF$ is asymptotically stable. For any $\delta \in (0, 1)$, let $c_\delta > 0$ be the largest positive scalar such that

$$|Fx(k)| \le u_{\max}(1 - \delta), \qquad \forall\, x(k) \in \boldsymbol{X}_\delta \quad (10)$$

where

$$\boldsymbol{X}_\delta := \{x\colon x'Px \le c_\delta\}. \quad (11)$$

Also, let

$$H := \left[1 + F(I - A - BF)^{-1}B\right] G \quad (12)$$

and

$$x_{\mathrm{e}} := G_{\mathrm{e}}\, r := (I - A - BF)^{-1} BG\, r. \quad (13)$$

Then, the control law in (7) is capable of driving the system controlled output $y(k)$ to track asymptotically a step command input of amplitude r, provided that the initial state x_0 and r satisfy

$$(x_0 - x_{\mathrm{e}}) \in \boldsymbol{X}_\delta \quad \text{and} \quad |Hr| \le \delta \cdot u_{\max}. \quad (14)$$

Proof: Let $\tilde{x} = x - x_{\mathrm{e}}$. Then, the linear feedback control law u_{L} can be rewritten as

$$\begin{aligned} u_{\mathrm{L}}(k) &= F\tilde{x}(k) + \left[1 + F(I - A - BF)^{-1}B\right] Gr \\ &= F\tilde{x}(k) + Hr. \end{aligned} \quad (15)$$

Hence, for all

$$\tilde{x}(k) \in \boldsymbol{X}_\delta \Rightarrow |F\tilde{x}(k)| \le u_{\max}(1 - \delta) \quad (16)$$

and for any r satisfying

$$|Hr| \le \delta \cdot u_{\max} \quad (17)$$

the linear control law can be written as

$$|u_{\mathrm{L}}(k)| = |F\tilde{x}(k) + Hr| \le |F\tilde{x}(k)| + |Hr| \le u_{\max} \quad (18)$$

which indicates that the control signal $u_{\mathrm{L}}(k)$ will never exceed the saturation. Next, let us move to verify the asymptotic stability of the closed-loop system comprising the given plant in (5) and the linear feedback law in (7), which can be expressed as follows:

$$\tilde{x}(k+1) = (A + BF)\tilde{x}(k). \quad (19)$$

Let us define a Lyapunov function for the closed-loop system in (19) as

$$V(k) = \tilde{x}'(k) P \tilde{x}(k). \quad (20)$$

Along the trajectories of the closed-loop system in (19) the increment of the Lyapunov function in (20) is given by

$$\begin{aligned} \nabla V(k+1) &= \tilde{x}'(k+1) P \tilde{x}(k+1) - \tilde{x}'(k) P \tilde{x}(k) \\ &= \tilde{x}'(k)(A+BF)' P (A+BF)\tilde{x}(k) - \tilde{x}'(k) P \tilde{x}(k) \\ &= -\tilde{x}'(k) W \tilde{x}(k) \le 0. \end{aligned} \quad (21)$$

This shows that $\boldsymbol{X}_\delta$ is an invariant set of the closed-loop system in (19) and all trajectories of (19) starting from $\boldsymbol{X}_\delta$ will converge to the origin. Thus, for any initial state x_0 and the step command input r that satisfy (14), we have

$$\lim_{k\to\infty} x(k) = x_{\rm e} \tag{22}$$

and, hence

$$\lim_{k\to\infty} y(k) = \lim_{k\to\infty} C_2 x(k) = C_2 x_{\rm e} = r. \tag{23}$$

This completes the proof of Lemma 1.

Remark 1: We would like to note that, for the case when $x_0 = 0$, any step command of amplitude r can be tracked asymptotically provided that

$$|r| \le \left[c_\delta (G_{\rm e}' P G_{\rm e})^{-1}\right]^{1/2} \quad \text{and} \quad |Hr| \le \delta \cdot u_{\max}. \tag{24}$$

This input command amplitude can be increased by increasing δ and/or decreasing $G_{\rm e}' P G_{\rm e}$ through the choice of W. However, the change in F will, of course, affect the damping ratio of the closed-loop system and, hence, its rise time.

Step 2: The nonlinear feedback control law $u_{\rm N}(k)$ is given by

$$u_{\rm N}(k) = \rho(r, y) B' P (A + BF) \left[x(k) - x_{\rm e}\right] \tag{25}$$

where $\rho(r, y)$ is a nonpositive scalar function, locally Lipschitz in y, and is to be used to change the system closed-loop damping ratio as the output approaches the step command input. The choice of $\rho(r, y)$ will be discussed later in detail.

Step 3: The linear and nonlinear components derived above are now combined to form a discrete-time CNF law

$$u(k) = u_{\rm L}(k) + u_{\rm N}(k). \tag{26}$$

We have the following result.

Theorem 1: Consider the discrete-time system in (5). Then, for any nonpositive $\rho(r, y)$, locally Lipschitz in y and $|\rho(r, y)| \le \rho^* := 2(B'PB)^{-1}$, the CNF law in (26) is capable of driving the system controlled output $y(k)$ to track the step command input of amplitude r from an initial state x_0, provided that x_0 and r satisfy (14).

Proof: Let $\tilde{x} = x - x_{\rm e}$. Then, the closed-loop system can be written as

$$\tilde{x}(k+1) = (A + BF)\tilde{x}(k) + Bw(k) \tag{27}$$

where

$$w(k) = \operatorname{sat}\left[F\tilde{x}(k) + Hr + u_{\rm N}(k)\right] - F\tilde{x}(k) - Hr. \tag{28}$$

Equation (14) implies that $\tilde{x}_0 \in \boldsymbol{X}_\delta$. Define a Lyapunov function

$$V(k) = \tilde{x}'(k) P \tilde{x}(k). \tag{29}$$

Noting that

$$\tilde{x}(k) \in \boldsymbol{X}_\delta \;\Rightarrow\; |F\tilde{x}(k)| \le u_{\max}(1 - \delta) \tag{30}$$

we can evaluate the increment of $V(k)$ along the trajectories of the closed-loop system in (27) as follows:

$$\begin{aligned}
\nabla V&(k+1) \\
&= \tilde{x}'(k+1) P \tilde{x}(k+1) - \tilde{x}'(k) P \tilde{x}(k) \\
&= \tilde{x}'(k)(A+BF)' P (A+BF)\tilde{x}(k) - \tilde{x}'(k) P \tilde{x}(k) \\
&\quad + 2\,\tilde{x}'(k)(A+BF)' PBw(k) + w'(k) B' PBw(k) \\
&= -\tilde{x}'(k) W \tilde{x}(k) + 2\tilde{x}'(k)(A+BF)' PBw(k) \\
&\quad + w'(k) B' PBw(k).
\end{aligned} \tag{31}$$

Next, we proceed to find the increment of $V(k)$ for three different cases.

If $|F\tilde{x}(k) + Hr + u_{\rm N}(k)| \le u_{\max}$, then

$$w(k) = u_{\rm N}(k) = \rho B' P (A + BF)\tilde{x}(k). \tag{32}$$

Thus

$$\begin{aligned}
\nabla V&(k+1) \\
&= -\tilde{x}'(k) W \tilde{x}(k) + 2\tilde{x}'(k)(A+BF)' PBw(k) \\
&\quad + w'(k) B' PBw(k) \\
&= -\tilde{x}'(k) W \tilde{x}(k) + \rho\tilde{x}'(k)(A+BF)' PB(2 + \rho B' PB) \\
&\quad \cdot B' P (A+BF)\tilde{x}(k).
\end{aligned} \tag{33}$$

For any nonpositive $\rho(r, y)$ with $|\rho(r, y)| \le \rho^*$, it is clear that the increment $\nabla V(k+1) \le -\tilde{x}'(k) W \tilde{x}(k) \le 0$.

If $F\tilde{x}(k) + Hr + u_{\rm N}(k) > u_{\max}$, then $|F\tilde{x}(k) + Hr| \le u_{\max}$ implies that $0 < w(k) < u_{\rm N}(k)$ and $\rho(r, y) < 0$. Hence

$$\begin{aligned}
\nabla V&(k+1) \\
&= -\tilde{x}'(k) W \tilde{x}(k) + 2\tilde{x}'(k)(A+BF)' PBw(k) \\
&\quad + w'(k) B' PBw(k) \\
&= -\tilde{x}'(k) W \tilde{x}(k) + w'(k)\left[2\rho^{-1} u_{\rm N}(k) + B' PBw(k)\right] \\
&< -\tilde{x}'(k) W \tilde{x}(k) + w'(k)\left[2\rho^{-1} u_{\rm N}(k) + B' PBu_{\rm N}(k)\right] \\
&= -\tilde{x}'(k) W \tilde{x}(k) + w'(k)\left(2\rho^{-1} + B' PB\right) u_{\rm N}(k).
\end{aligned} \tag{34}$$

Thus, for all $-\rho^* \le \rho(r, y) < 0$, we have $2\rho^{-1} + B'PB \le 0$, and, hence

$$\nabla V(k+1) \le -\tilde{x}'(k) W \tilde{x}(k) \le 0. \tag{35}$$

Similarly, for the case when $F\tilde{x}(k) + Hr + u_{\rm N}(k) < -u_{\max}$, it can be shown that $\nabla V(k+1) \le -\tilde{x}'(k) W \tilde{x}(k) \le 0$.

Thus, $\boldsymbol{X}_\delta$ is an invariant set of the closed-loop system in (27) and all trajectories of (27) starting from $\boldsymbol{X}_\delta$ will remain there and converge to the origin. This, in turn, indicates that, for all initial states x_0 and the step command input of amplitude r that satisfy (14)

$$\lim_{k\to\infty} x(k) = x_{\rm e} \tag{36}$$

and

$$\lim_{k\to\infty} y(k) = \lim_{k\to\infty} C_2 x(k) = C_2 x_{\rm e} = r. \tag{37}$$

This completes the proof of Theorem 1.

Remark 2: Theorem 1 shows that the addition of the nonlinear feedback control law $u_{\rm N}$ as given in (25) does not affect the ability to track the class of command inputs. Any command input that can be tracked by the linear feedback law in (7) can also be tracked by the CNF control law (26). The composite feedback law in (26) does not reduce the level of the trackable command input for any choice of the function $\rho(r, y)$. This freedom can be used to improve the performance of the overall system. The choice of $\rho(r, y)$ is discussed in the next subsection.

A. Tuning of Composite Nonlinear Feedback Law

The freedom to choose the function $\rho(r, y)$ is used to tune the CNF controller so as to increase the damping ratio of the closed-loop system as the output approaches the command input. Since the nonlinear part of the CNF controller is designed to contribute a significant value to the CNF controller only when the tracking error is small, it is sufficient to discuss the choice of $\rho(r, y)$ for the case when the control signal is below its saturation level. In such a case, the closed-loop system (27) can be transformed as follows:

$$\tilde{x}(k+1) = \{A + B\,[F + \rho B'P(A+BF)]\}\,\tilde{x}(k). \tag{38}$$

It is now clear that eigenvalues of the closed-loop system (38) can be changed by modifying the value of the function ρ. In particular, when $\rho = 0$, the closed-loop system (38) becomes a purely linear system. In this paper, ρ is chosen as an exponential function of the tracking error

$$\rho(r, y) = -1.5820\,\beta\left(e^{-|1-y/r|} - 0.3679\right) \tag{39}$$

where y and r are the system output and the step command input, respectively. It is easy to determine the function $\rho(r, y)$ using a root-locus plot on the z-plane based on the required locus of the pole locations of the closed-loop system. In this paper, the function $\rho(r, y)$ changes from 0 to $-\beta$ as the tracking error approaches to zero.

IV. HDD Servo System Design Using the CNF Approach

The results of Section III are now used to design a unified track seeking and track following controller for an HDD identified in Section II. The CNF controller will provide a fast track seeking performance through its linear part and the precise head positioning in the track following stage through its nonlinear component. A simplified design procedure can be summarized as follows.

1) Determine a state feedback gain matrix F using any appropriate method (such as H_2 or H_∞ control) such that $A + BF$ is stable and the resulting closed-loop system has a quick rise time with control input below saturation.
2) Compute the gain matrices G, G_e, and x_e.
3) Choose an appropriate matrix $W > 0$ and solve Lyapunov equation (9) for $P > 0$.
4) Select the nonlinear function ρ as defined in (39). Select the value of β such that the eigenvalues of matrix $A + B[F - \beta B'P(A+BF)]$ will generate small overshoot in the closed-loop response.

It is noted that the control law depends on the size of the command input as the control signal generated by the linear part of the CNF controller must not exceed the actuator saturation level.

We now move to the design of the CNF controller for the HDD system identified in Section II. The fourth-order model of the HDD given in (4) can actually be approximated by a second-order model by ignoring the high-frequency resonant modes. The following discretized model is obtained with a sampling frequency of 10 kHz:

$$\begin{cases} x(k+1) = \begin{bmatrix} 1 & 0.0001 \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 0.3201 \\ 6401.3 \end{bmatrix} u(k) \\ y(k) = [1 \quad 0]\; x(k). \end{cases} \tag{40}$$

The constraint on the control input is $|u(k)| \le 3.0$ V. Following the design procedure, we obtain a CNF control law

$$u(k) = F[x(k) - x_e] + G_r r + \rho H[x(k) - x_e] \tag{41}$$

where ρ is as defined in (39), and

$$G_r = 0, \qquad x_e = \begin{pmatrix} r \\ 0 \end{pmatrix}. \tag{42}$$

To fully capitalize the capacity of the CNF control method, the gain matrices F and H should be parameterized as functions of the target reference r. For simplicity, we select the gain matrices for $r = 100$ and 300 μm. The corresponding simulation and implementation results will be given in the next section. For $r = 100$ μm, we have $\beta = 1.36$

$$\left.\begin{aligned} F &= [\,-0.013\,481 \quad -9.2629 \times 10^{-6}\,] \\ H &= [\,0.0135 \quad 2.585\,62 \times 10^{-5}\,] \end{aligned}\right\} \tag{43}$$

and for $r = 300$ μm, we have $\beta = 1.4$

$$\left.\begin{aligned} F &= [\,-9.7076 \times 10^{-3} \quad -7.7886 \times 10^{-6}\,] \\ H &= [\,9.6646 \times 10^{-3} \quad 2.1593 \times 10^{-5}\,]. \end{aligned}\right\} \tag{44}$$

To compare the results of the CNF controller with the conventional methods, a PTOS controller proposed by Workman [30] was used. The PTOS controller was designed for our HDD model following its design procedure and the PTOS control law was found as

$$u = u_{\max} \cdot \text{sat}\left(\frac{k_2\,[f(e) - v]}{u_{\max}}\right) \tag{45}$$

where $e = r - y$ and the function $f(e)$ is defined as

$$f(e) = \begin{cases} \dfrac{k_1}{k_2}\,(e), & \text{for } |e| \le y_\ell \\ \text{sgn}(e)\left[\sqrt{2u_{\max} a\alpha|e|} - \dfrac{u_{\max}}{k_2}\right], & \text{for } |e| > y_\ell. \end{cases} \tag{46}$$

The values of various parameters, which are used to obtain the best performance in both the track seeking and the track following modes with PTOS, are given by $a = 6.4013 \times 10^7$, $k_1 = 0.0178$, $k_2 = 2.997 \times 10^{-5}$, $\alpha = 0.62$ and $y_\ell = 168.32$ μm. Note that α is commonly regarded as the discount factor of the PTOS scheme, whereas y_ℓ is its linear region.

As there is only the displacement measurable in the implementations, the following estimator of the velocity v is used for both CNF and PTOS controllers:

$$\hat{v} = \frac{5121}{z - 0.6}\,\text{sat}(u) + \frac{4000(z-1)}{z - 0.6}\,y. \tag{47}$$

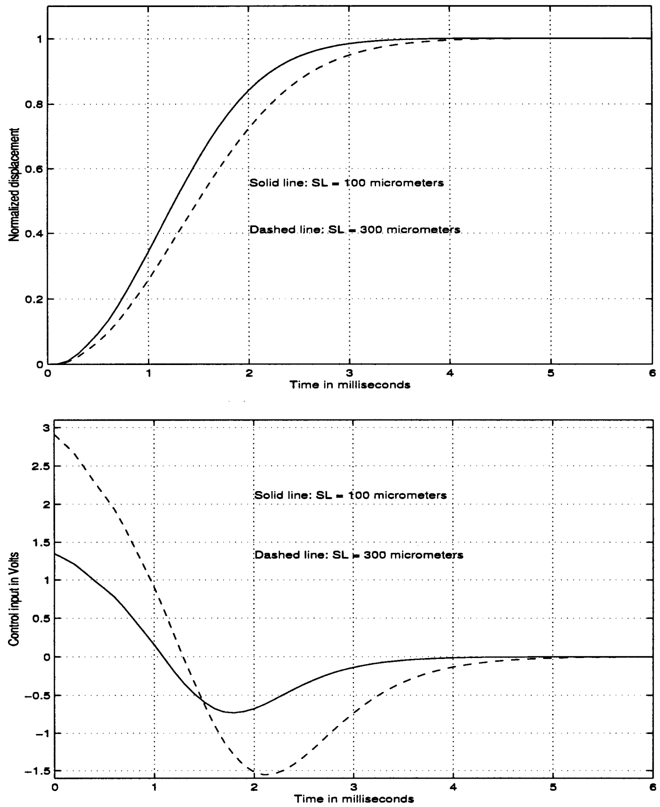

Fig. 4. Simulation result: Normalized responses with CNF.

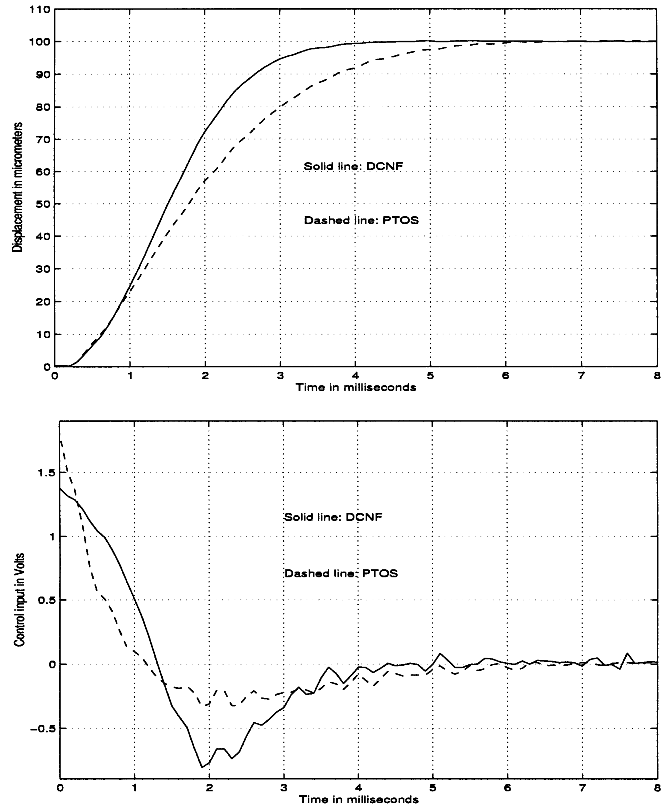

Fig. 6. Experimental results: Responses for $\mathrm{SL} = 100\ \mu\mathrm{m}$ using CNF and PTOS.

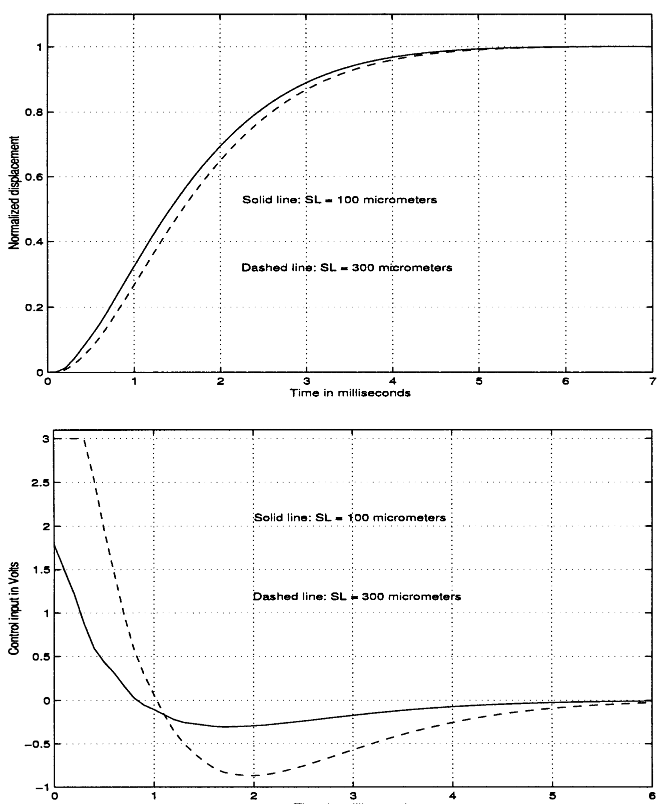

Fig. 5. Simulation result: Normalized responses with PTOS.

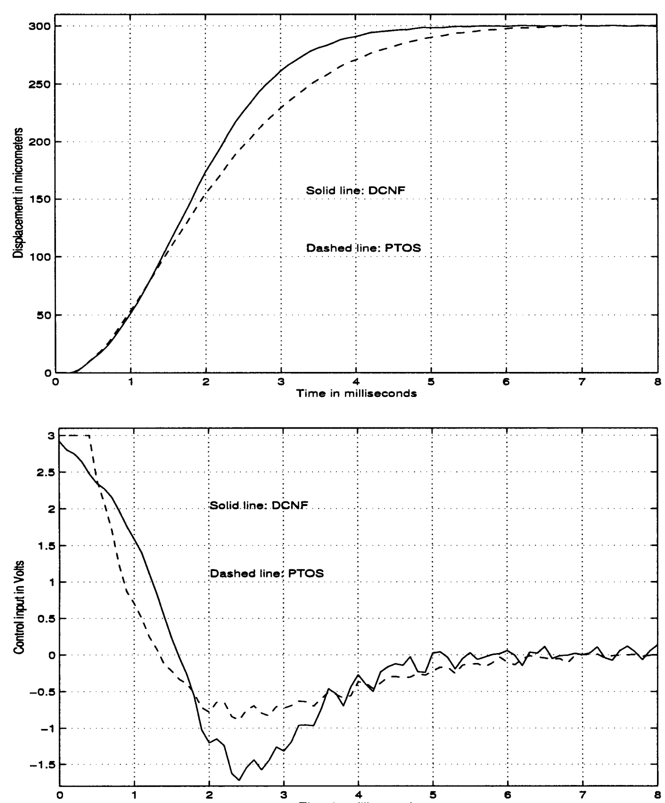

Fig. 7. Experimental results: Responses for $\mathrm{SL} = 300\ \mu\mathrm{m}$ using CNF and PTOS.

TABLE I
SETTLING TIME FROM SIMULATION RESULTS

Seek Length (μm)	Settling Time(ms)		Improvement (%)
	PTOS	CNF	
100	5.6	3.7	34
300	5.9	4.3	27

TABLE II
SETTLING TIME FROM EXPERIMENTAL RESULTS

Seek Length (μm)	Settling Time(ms)		Improvement (%)
	PTOS	CNF	
100	6.48	4.50	31
300	6.80	5.54	21

V. SIMULATION AND EXPERIMENTAL RESULTS

The simulation results for track seek lengths (SL) $r = 100$ and 300 μm using the CNF method are shown in Fig. 4 and the simulation results obtained from the conventional PTOS are shown in Fig. 5. All these simulation results were verified practically. The implementation results were obtained from the experiment on a typical 3.5-inh HDD, whose frequency response characteristics are shown in Fig. 3. The implementation results using the CNF controller and the PTOS controller for $r = 100$ and 300 μm are shown in Figs. 6 and 7, respectively. The settling time from simulation and implementation results are shown in Tables I and II, respectively. We note that the *settling time* is defined as the total time to move the R/W head from the initial position to within a steady error around the final target. The steady–state error is 0.2 μm for $r = 100$ μm and 0.5 μm for $r = 300$ μm, which is limited by the implementation setup. The controllers were implemented on an open HDD with a TMS320 digital signal processor (DSP) and with a sampling rate of 10 kHz. The R/W head position was measured using an LDV and the track width was assumed as 1 μm, and hence, the track density was assumed to be 25 000 TPI.

Clearly, our simulation and implementation results show that the newly proposed method has improved the performance in settling time by 20 to 30% compared to the conventional PTOS approach. This is mainly due to the fact that the composite nonlinear feedback control law unifies the nonlinear and linear components without switching, whereas the conventional PTOS involves a switching element in between the nonlinear to linear parts. It is our belief that this switching element disgrades the performance of PTOS a great deal.

VI. CONCLUDING REMARKS

A method to improve the servo performance in the HDD servo system has been proposed in this paper. The proposed method uses a linear state feedback control, which yields a low value of closed-loop damping ratio and, thus, a decrease in the rise time, and a nonlinear part of the control law, which increases the damping ratio as the system output approaches the reference input and, thus, reduces the overshoot. As such, the composition of these two parts can be used to achieve fast track seek and precise head positioning in HDD servo systems. The experimental and simulation results shows that the proposed method has less settling time than the conventional method. Furthermore, the problem of mode switching in the conventional HDD servo system was eliminated. Finally, we note that the proposed method of this paper can be applied to solve many other problems that deal with asymptotic tracking of certain target reference signals.

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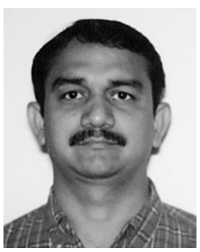

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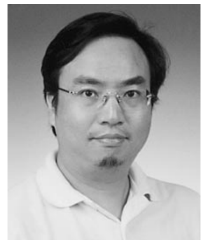

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