Simultaneous H_2/H_∞ Optimal Control for Discrete-Time Systems: The State Feedback Case

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Abstract: This paper presents a simultaneous H_2/H_∞ optimal control problem for discrete-time systems in the state-feedback case. By the use of dynamic state feedback controllers, the design seeks to minimize the H_2 norm of a closed-loop transfer matrix while simultaneously satisfying a prescribed H_∞ norm bound on some other closed-loop transfer matrix. The class of problems addressed here is relatively general and consists of systems which have left invertible transfer function matrix from the control input to the controlled output. Necessary and sufficient conditions are established so that the posed simultaneous H_2/H_∞ problem is solvable with state feedback controllers.

Key words: simultaneous H_2/H_∞ optimal control; robust control; control for discrete-time systems; state feedback control

1 Introduction

In multivariable control theory, optimization of a nominal performance measure with robust stability is becoming a standard mode operation. H_2-norm is found to be the most appropriate measure in the characterization of nominal performance while the H_∞-norm is to identify robustness to unstructured plant uncertainties. H_2-norm minimization problems were heavily studied in 1960's and early 1970's as Linear Quadratic Gaussian (LQG) optimal control problems. More recently these problems have been studied in a generalized setting of minimizing the H_2-norm of a transfer function matrix from an exogenous disturbance to the controlled output of a given linear time-invariant system by an appropriate selection of an internally stabilizing controller (see e.g., [1] and [2]). On the other hand, since the seminal work of [3], H_∞-norm optimization problems have been heavily studied, and are continuing to be developed. In H_∞-norm optimization, one seeks a control law which stabilizes a given plant, and also makes the H_∞-norm of a selected closed-loop transfer function smaller than a priori given number. The H_∞-norm optimization deals with the worst-case objective in contrast

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with the common mean square objective of the traditional LQG(H_2) optimal control. Recently, problems where both H_2 and H_∞-norm performance measures are mixed, have received attention as they show a potential to achieve optimal nominal performance with some robust stability (see e. g. [4~6]). A typical problem in this connection, called a simultaneous H_2/H_∞ optimal control problem, has been formulated for continuous-time systems in [6] and later extended in [7]. This problem seeks to minimize the H_2-norm of a closed-loop transfer matrix while simultaneously satisfying a prescribed H_∞-norm bound on some other closed-loop transfer matrix. The intent of this paper is to look at the parallel problem in discrete-time systems. A set of necessary and sufficient conditions under which a simultaneous H_2/H_∞ optimal control problem is solvable for a class of singular problems for discrete-time systems are developed. The class of problems we consider have a left-invertible transfer function matrix from the control input to controlled output which is used for the H_2-norm performance measure. This class of problems subsumes the class of regular H_2 optimization problems. The development given here for discrete-time systems is analogous to but not quite the same as that for continuous-time systems in [7]. The differences reflect the specific nature and characteristics of the discrete-time systems.

This paper is organized as follows. Section 2 gives a clear mathematical statement of the problem, while Section 3 recalls several pertinent preliminary results. Section 4 develops the necessary and sufficient conditions under which the posed simultaneous H_2/H_∞ optimal control problem for discrete-time systems is solvable. Finally, Section 5 draws the conclusions of our current work.

Throughout this paper, Ker[V] and Im [V] denote respectively the kernel and the image of V. Also, ρ (M) denotes the spectral radius of matrix M, while normrank denotes the rank of a matrix with entries in the field of rational functions. Given a stable and strictly proper transfer function $G(z)$, as usual, its H_2-norm is denoted by $\| G \|_2$; and given a proper stable transfer function $G(z)$, its H_∞-norm is denoted by $\| G \|_\infty$. Also, $\mathbb{R}H^s$ denotes the set of real-rational transfer functions which are stable and strictly proper. Similarly, $\mathbb{R}H_\infty$ denotes the set of real-rational transfer functions which are stable and proper. Finally, $\mathbb{C}^{\bigcirc}$ and $\mathbb{C}^{\otimes}$ denote respectively the unit circle and the set of complex numbers outside the unit circle.

2 Problem Statement and Definitions

Consider the following system,

$$\Sigma: \begin{cases} x(k+1) = Ax(k) + Bu(k) + E_2 w_2(k) + E_\infty w_\infty(k), \\ y(k) = x(k), \\ z_2(k) = C_2 x(k) + D_2 u(k), \\ z_\infty(k) = C_\infty x(k) + D_\infty u(k), \end{cases} \tag{2.1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input, $w_\infty \in \mathbb{R}^{l_2}$ and $w_\infty \in \mathbb{R}^{l_\infty}$ are the disturbance inputs, and $z_2 \in \mathbb{R}^{q_2}$ and $z_\infty \in \mathbb{R}^{q_\infty}$ are the controlled outputs. Also, consider an arbitrary proper controller,

$$u = \boldsymbol{K}(z)x. \tag{2.2}$$

A controller $u = \boldsymbol{K}(z)x$ is said to be admissible if it provides internal stability of the resulting closed-loop system. Let $T_2(\boldsymbol{K})$ denote the closed-loop transfer functions from w_2 to z_2 and from w_∞ to z_∞, respectively, under the feedback control law $u = \boldsymbol{K}(z)x$. Moreover, let the infimum of the H_2 norm of the closed-loop transfer function $T_2(\boldsymbol{K})$ over all the stabilizing proper controllers $\boldsymbol{K}(z)$ be denoted by γ_2^*; that is,

$$\gamma_2^* := \inf\{ \| T_2(\boldsymbol{K}) \|_2 \,|\, u = \boldsymbol{K}(z)x \text{ internally stabilizes } \Sigma\}. \tag{2.3}$$

The simultaneous H_2/H_∞ optimal control problem is defined as follows:

Definition 2.1 (The simultaneous H_2/H_∞ optimal control problem). For the given plant Σ and a scalar $\gamma > 0$, find an admissible controller $\boldsymbol{K}(z)$ such that $\| T_2(\boldsymbol{K}) \|_2 = \gamma_2^*$ and $\| T_\infty(\boldsymbol{K}) \|_\infty < \gamma$.

Definition 2.2 The following definitions will also be convenient in the sequel.

1) (The H_2 optimal controller): An admissible controller $\boldsymbol{K}(z)$ is said to be an H_2 optimal controller if $\| T_2(\boldsymbol{K}) \|_2 = \gamma_2^*$.

2) (The H_∞ γ- suboptimal controller): An admissible controller $\boldsymbol{K}(z)$ is said to be an H_∞ γ- suboptimal controller if $\| T_\infty(\boldsymbol{K}) \|_\infty < \gamma$.

3) (Stabilizable weakly unobservable subspace) Given a system Σ_* characterized by a matrix quadruple (A,B,C,D), we define the stabilizable weakly unobservable subspace $\nu_g(\Sigma_*)$ as the largest subspace ν for which there exists a mapping F such that the following subspace inclusions are satisfied:

$$(A + BF)\nu \subseteq \nu \text{ and } (C + DF)\nu = \{0\},$$

and such that $A + BF|\nu$ is asymptotically stable.

Our goal in this paper is to derive a set of necessary and sufficient conditions under which the simultaneous H_2/H_∞ optimal control problem is solvable. To achieve this, we first, following [8], parameterize the set of all H_2 optimal dynamic state feedback controllers for general singular problems, and then utilize a theorem of [9] which studies the existence conditions for the γ- suboptimal strictly proper controller for discrete-time systems.

3 Preliminaries

In this section, we recall several preliminary results needed to establish the necessary and sufficient conditions under which the simultaneous H_2/H_∞ optimal control problem is solvable, while at the same time we also introduce some new results.

3.1 Review of H_2-optimal Control

In this subsection, we recall from [10] the necessary and sufficient conditions under which an H_2-optimal state feedback control law of either static or dynamic type for discrete-time systems exists. We also recall a recent result of [8] which characterizes all the possible H_2 optimal state feedback laws.

The conditions under which an optimal controller exists for the discrete-time system

$$\Sigma_2: \begin{cases} x(k+1) = Ax(k) + Bu(k) + E_2 w_2(k), \\ y(k) = x(k), \\ z_2(k) = C_2 x(k) + D_2 u(k), \end{cases} \tag{3.1}$$

can be formulated in terms of an auxiliary system Σ_{au2} constructed from the data of (3.1). The auxiliary system Σ_{au2} is as given below:

$$E_{au2}:\begin{cases} x_P(k+1) = Ax_P(k) + Bu_P(k) + E_2w_2(k), \\ y_P(k) = x_P(k), \\ z_P(k) = C_Px_P(k) + D_Pu_P(k). \end{cases} \tag{3.2}$$

Here C_P and D_P satisfy

$$F_2(P_2) = \begin{bmatrix} C_P' \\ D_P' \end{bmatrix}[C_P\ D_P],$$

where

$$F_2(P_2) := \begin{bmatrix} A'P_2A - P_2 + C_2'C_2 & A'P_2B + C_2'D_2 \\ B'P_2A + D'C & B'P_2B + D_2'D_2 \end{bmatrix}, \tag{3.3}$$

and where P_2 is the largest solution of the matrix inequality $F_2(P_2) \geqslant 0$. It is known that under the condition that (A,B) is stabilizable, such a solution P_2 exists and is unique.

We have the following theorem.

Theorem 3.1 Consider the given system Σ_2 as in (3.1), and the auxiliary system Σ_{au2} as in (3.2). Define a subsystem Σ_P of Σ_{au2} as that characterized by the quadruple (A,B,C_P,D_P). Then, the infimum, γ_2^*, can be attained by a static as well as by a dynamic stabilizing state feedback controller if and only if the pair (A,B) is stabilizable and $\mathrm{Im}(E_2) \subseteq \nu_g(\Sigma_P)$.

Proof See [10].

We know that whenever an optimal solution to the original H_2 problem exists, there exists a constant gain F such that $A_F := A + BF$ is stable and that

$$\|(C_2 + D_2F)(zI - A_F)^{-1}E_2\|_2 = \gamma_2^* \tag{3.4}$$

or equivalently (see [8]),

$$(C_P + D_PF)(zI - A_F)^{-1}E_2 = 0.$$

It can be easily shown that any proper dynamic controller $\boldsymbol{K}(z)$ that stabilizes the system Σ_{au2} can be written in the following form,

$$\begin{cases} \xi(k+1) = A_F\xi(k) + By_1(k), \\ u(k) = Fx(k) + y_1(k), \end{cases} \tag{3.5}$$

where

$$y_1(k) = Q(z)[x(k) - \xi(k)] \tag{3.6}$$

for some proper and stable $Q(z)$, i.e., $Q(z) \in \mathbb{R}H_\infty$, with appropriate dimensions. The following theorem qualifies $Q(z)$ so that the controller $\boldsymbol{K}(z)$ is H_2 optimal for the given system Σ_2.

Theorem 3.2 Consider the given system Σ_2 as in (3.1). Let the system characterized by the matrix quadruple (A,B,C_2,D_2) be left invertible. Also, assume that the pair (A,B) is stabilizable, and that $\mathrm{Im}(E_2) \subseteq \nu_g(\Sigma_P)$. Define a set $\boldsymbol{Q}$ as,

$$\boldsymbol{Q} := \{Q(z) \in \mathbb{R}H_\infty \mid Q(z) = W(z)(I - E_2E_2^+)(zI - A_F), W(z) \in \mathbb{R}H^S\}. \tag{3.7}$$

Then a proper dynamic controller $\boldsymbol{K}(z)$ stabilizes Σ_2 and achieves the infimum, γ_2^*, if and only if $\boldsymbol{K}(z)$ can be written in the form of (3.5) and (3.6) for some $Q(z) \in \boldsymbol{Q}$. Moreover, if (A_w,B_w,C_w) is a state space realization of $W(z)$, then $Q(z) = W(z)(I - E_2E_2^+)(zI - A_F)$

can be written as,

$$Q(z) = C_w(zI - A_w)^{-1}[A_wB_w(I - E_2E_2^+) - B_w(I - E_2E_2^+)A_F] + C_wB_w(I - E_2E_2^+). \tag{3.8}$$

Proof It follows from [8].

3.2 Existence of H_∞-suboptimal Controllers

We recall in this subsection a theorem of [9] which gives a set of necessary and sufficient conditions under which the following auxiliary system has an H_∞ γ-suboptimal strictly proper controller,

$$\Sigma_{au\infty}: \begin{cases} x(k+1) = Ax(k) + Bu(k) + E_\infty w_\infty(k), \\ y(k) = C_1x(k) + D_1w_\infty(k), \\ z_\infty(k) = C_\infty x(k) + D_\infty u(k). \end{cases} \tag{3.9}$$

For future use, let us define the following matrices. Given any symmetric positive semi-definite matrices $P_\infty \in \mathbb{R}^{n\times n}$ and $Q_\infty \in \mathbb{R}^{n\times n}$ which satisfy

$$R_\infty := I - E_\infty' P_\infty E_\infty > 0 \quad \text{and} \quad Y_\infty := (I - Q_\infty P_\infty)^{-1}Q_\infty \geqslant 0,$$

we define

$$V_\infty := B'P_\infty B + D_\infty' D_\infty,$$
$$A_x := A - BV_\infty^+(B'P_\infty A + D_\infty' C_\infty),$$
$$C_{1P} := C_1 + D_1R_\infty^{-1}E_\infty' P_\infty A_x,$$
$$C_{2P} := (V_\infty^{1/2})^+ (B'P_\infty A + D_\infty' C_\infty + B'P_\infty E_\infty R_\infty^{-1}E_\infty' P_\infty A_x),$$
$$D_{12P} := D_1R_\infty^{-1/2},$$
$$D_{22P} = (V_\infty^{1/2})^+ B'P_\infty E_\infty R_\infty^{-1/2},$$
$$W_P := D_{12P}D_{12P}' + C_{1P}Y_\infty C_{1P}',$$
$$S_P := I - D_{22P}D_{22P}' - C_{2P}Y_\infty C_{2P}' + (C_{2P}Y_\infty C_{1P}' + D_{22P}D_{12P}')W_P^+(C_{1P}Y_\infty C_{2P}' + D_{12P}D_{22P}').$$

Finally, we define

$$D_{22PY} := S_P^{-1/2}(C_{2P}Y_\infty C_{1P}' + D_{22P}D_{12P}')(W_P^{1/2})^+. \tag{3.10}$$

We have the following result.

Theorem 3.3 Consider the auxiliary system $\Sigma_{ua\infty}$ as in (3.9). Assume that two systems one characterized by $(A, B, C_\infty, D_\infty)$ and the other by (A, E_∞, C, D) have no invariant zeros on the unit circle. Then the following statements are equivalent:

1) There exists a linear, time-invariant and strictly proper dynamic compensator $\boldsymbol{K}_o(z)$ such that when the control law $u(z) = \boldsymbol{K}_o(z)y(z)$ is applied to $\Sigma_{au\infty}$, the resulting closed-loop system is internally stable. Moreover, the H_∞-norm of the closed-loop transfer function from the disturbance input w_∞ to the controlled output z_∞ is less than 1.

2) There exists symmetric matrices $P_\infty \geqslant 0$ and $Q_\infty \geqslant 0$ such that

a) We have $R_\infty := I - E_\infty' P_\infty E_\infty > 0$.

b) P_∞ satisfies the discrete algebraic Riccati equation:

$$P_\infty = A'P_\infty A + C_\infty' C_\infty - \begin{bmatrix} B'P_\infty A + D_\infty' C_\infty \\ E_\infty' P_\infty A \end{bmatrix}' G(P_\infty)^+ \begin{bmatrix} B'P_\infty A + D_\infty' C_\infty \\ E_\infty' P_\infty A \end{bmatrix}, \tag{3.11}$$

where

$$G(P_\infty) := \begin{bmatrix} D_\infty' D_\infty + B'P_\infty B & B'P_\infty E_\infty \\ E_\infty' P_\infty B & E_\infty' P_\infty E_\infty - I \end{bmatrix}. \tag{3.12}$$

c) For all $z \in \mathbb{C}^{\bigcirc} \cup \mathbb{C}^{\otimes}$, we have

$$\text{normrank}\begin{bmatrix} zI - A & -B & -E_\infty \\ B'P_\infty A + D_\infty' C_\infty & B'P_\infty B + D_\infty' D_\infty & B'P_\infty E_\infty \\ E_\infty' P_\infty A & E_\infty' P_\infty B & E_\infty' P_\infty E_\infty - I \end{bmatrix}$$
$$= n + l_\infty + \text{normrank}\{C_\infty (zI - A)^{-1} B + D_\infty\}.$$

d) We have $S_\infty := I - C_\infty Q_\infty C_\infty' > 0$.

e) Q_∞ satisfies the following discrete algebraic Riccati equation:

$$Q_\infty = AQ_\infty A' + E_\infty E_\infty' - \begin{bmatrix} C_1 Q_\infty A' + D_1 E_\infty' \\ C_\infty Q_\infty A' \end{bmatrix}' H(Q_\infty)^+ \begin{bmatrix} C_1 Q_\infty A' + D_1 E_\infty' \\ C_\infty Q_\infty A' \end{bmatrix}, \tag{3.13}$$

where
$$H(Q_\infty) = \begin{bmatrix} D_1 D_1' + C_1 Q_\infty C_1' & C_1 Q_\infty C_\infty' \\ C_\infty Q_\infty C_1' & C_\infty Q_\infty C_\infty' - I \end{bmatrix}. \tag{3.14}$$

f) For all $z \in \mathbb{C}^{\bigcirc} \cup \mathbb{C}^{\otimes}$, we have

$$\text{normrank}\begin{bmatrix} zI - A & AQ_\infty C_1' + E_\infty D_1' & AQ_\infty C_\infty' \\ -C_1 & C_1 Q_\infty C_1' + D_1 D_1' & C_1 Q_\infty C_\infty' \\ -C_\infty & C_\infty Q_\infty C_1' & C_\infty Q_\infty C_\infty' - I \end{bmatrix}$$
$$= n + q_\infty + \text{normrank}\{C_1 (zI - A)^{-1} E_\infty + D_1\}.$$

g) $\rho(P_\infty Q_\infty) < 1$.

h) $\| D_{22PY} \| < 1$, where D_{22PY} is as defined in (3.10) with P_∞ and Q_∞ satisfying the above conditions a) ~g).

Proof See[9].

4 The Simultaneous H_2/H_∞ Problem

In this section, we give our main result regarding the simultaneous H_2/H_∞ problem. We have the following theorem.

Theorem 4.1 Consder the given system Σ as in (2.1). Assume that the pair (A,B) is stabilizable and the system characterized by the quadruple (A,B,C_2,D_2) is left invertible. Also, assume that the quadruple (A,B,C_∞,D_∞) has no invariant on the unit circle. Then there exists an internally stabilizing control law $u = \boldsymbol{K}(z)x$ such that $\| T_2 \boldsymbol{K} \|_2 = \gamma_2^*$ and $\| T_\infty(\boldsymbol{K}) \|_\infty < 1$ if and only if the following conditions hold:

1) $\text{Im}\,(E_2) \subseteq \nu_g(\Sigma_P)$, which is equivalent to the fact that there exists an F such that $A_F := A + BF$ is stable and (3.4) holds. Also, let $C_{\infty F} := C_\infty + D_\infty F$ and $M_\infty = (1 - E_2 E_2^+) E_\infty$.

2) There exists symmetric matrices $P_\infty \geqslant 0$ and $Q_\infty \geqslant 0$ such that

a) We have $R_\infty := I - E_\infty' P_\infty E_\infty > 0$.

b) P_∞ satisfies the discrete algebraic Riccati equation:

$$P_\infty = A'P_\infty A + C_\infty' C_\infty - \begin{bmatrix} B'P_\infty A + D_\infty' C_\infty \\ E_\infty' P_\infty A \end{bmatrix}' G(P_\infty)^+ \begin{bmatrix} B'P_\infty A + D_\infty' C_\infty \\ E_\infty' P_\infty A \end{bmatrix}, \tag{4.1}$$

where
$$G(P_\infty) := \begin{bmatrix} D_\infty' D_\infty + B'P_\infty B & BP_\infty E_\infty \\ E_\infty' P_\infty B & E_\infty' P_\infty E_\infty - I \end{bmatrix}. \tag{4.2}$$

c) For all $z \in \mathbb{C}^{\bigcirc} \cup \mathbb{C}^{\otimes}$, we have

$$\text{normrank}\begin{bmatrix} zI - A & -B & -E_\infty \\ B'P_\infty A + D_\infty' C_\infty & B'P_\infty B + D_\infty' D_\infty & B'P_\infty E_\infty \\ E_\infty' P_\infty A & E'P_\infty B & E_\infty' P_\infty E_\infty - I \end{bmatrix}$$
$$= n + l_\infty + \text{normrank}\{C_\infty (zI - A)^{-1} B + D_\infty\}.$$

d) We have $S_\infty := I - C_{\infty F} Q_\infty C_{\infty F}' > 0$.

e) Q_∞ satisfies the following discrete algebraic Riccati equation:

$$Q_\infty = A_F Q_\infty A_F' + E_\infty E_\infty' - E_\infty M_\infty' (M_\infty M_\infty')^+ M_\infty E_\infty' + A_F Q_\infty C_{\infty F} S_\infty^+ C_{\infty F}' Q_\infty A_F'. \quad (4.3)$$

f) For all $z \in \mathbb{C}^{\bigcirc} \cup \mathbb{C}^{\otimes}$, we have

$$\text{normrank}\begin{bmatrix} zI - A_F & E_\infty M_\infty' & A_F Q_\infty C_{\infty F}' \\ 0 & M_\infty M_\infty' & 0 \\ -C_{\infty F} & 0 & C_{\infty F} Q_\infty C_{\infty F}' - I \end{bmatrix} = n + q_\infty + \text{rank}(M_\infty).$$

g) $\rho(P_\infty Q_\infty) < 1$.

h) $\| D_{22PY} \| < 1$, where D_{22PY} is as defined in (3.10) with P_∞ and Q_∞ satisfying the above conditions a)~g), $C_1 = 0$ and $D_1 = M_\infty$.

Proof At first, let us note that $T_\infty(\boldsymbol{K})$, the closed-loop transfer function from w_∞ to z_∞ under the controller of (3.5) and (3.6) with $Q(z) \in \boldsymbol{Q}$, is given by

$$T_\infty(\boldsymbol{K}) = C_{\infty F}(zI - A_F)^{-1} E_\infty + [C_{\infty F}(zI - A_F)^{-1} B + D_\infty] W(z) M_\infty. \quad (4.4)$$

It can be simply verified that $T_\infty(\boldsymbol{K})$ is equivalent to the closed-loop transfe function from w_∞ to z_∞ of the following auxiliary feedback system,

$$\Sigma_\infty: \begin{cases} x(k+1) = A_F x(k) + Bu(k) + E_\infty w_\infty(k), \\ y(k) = M_\infty w_\infty(k), \\ z_\infty(k) = C_{\infty F} x(k) + D_\infty u(k). \end{cases} \quad (4.5)$$

$$u = W(z) y. \quad (4.6)$$

Furthermore, let us observe that the system characterized by the quadruple $(A_F, E_\infty, 0, M_\infty)$ has no invariant zeros on $\mathbb{C}^{\bigcirc}$ due to the fact that A_F is stable. We are now ready to prove the theorem.

($\Rightarrow$): For the given system Σ, if there exists a stabilizing proper controller $u = \boldsymbol{K}(z)x$ such that the corresponding $\| T_2(\boldsymbol{K}) \|_2 = \gamma_2^*$ and $\| T_\infty(\boldsymbol{K}) \|_\infty < 1$, then by Theorem 3.1 we have $\text{Im}(E_2) \subseteq \nu_g(\Sigma_g)$, which is equivalent to the fact that there exists a constant gain F such that $A_F := A + BF$ is stable and (3.4) holds. Next, $\| T_\infty(\boldsymbol{K}) \|_\infty < 1$ implies that there exists a $Q(z) \in \boldsymbol{Q}$ such that the corresponding $W(z)$ is an H_∞ suboptimal controller to the auxiliary system Σ_∞ of (4.5). We also observe that Conditions 2 a)~c) in Theorem 4.1 are the conditions under which there exists a state feedback H_∞ suboptimal law to the following system,

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + E_\infty w_\infty(k), \\ y(k) = x(K), \\ z_\infty(k) = C_\infty x(k) + D_\infty u(k). \end{cases} \quad (4.7)$$

Then, from Theorem 3.3 and some simple algebra, it follows that Conditions in Item 2 hold.

($\Leftarrow$): Conversely, we assume that Conditions in Items 1 and 2 in Theorem 4. 1 hold. Then Conditions in Item 2 imply that there exists a strictly proper controller $W(z) \in \mathbb{R}H^S$ such that when it is applied to Σ_∞ the resulting closed-loop transfer function from w_∞ to z_∞ has H_∞ norm less than 1. We first note that due to the special structure of Σ_∞, all the internally stabilizing controllers must themselves be stable. Hence $W(z)$ is stable. Then it is straightforward to verify that the controller (3. 5) and (3. 6) with $Q(z) = W(z)(I - E_2E_2^+)(zI - A_F)$ achieves $\| T_2(K) \|_2 = \gamma_2^*$ and $\| T_\infty(K) \|_\infty < 1$. This completes the proof of Theorem 4. 1.

Remark 4. 1 Necessary and sufficient conditions for the existence of an internally stabilizing simultaneous H_2/H_∞ optimal compensator which makes the H_∞ norm of the closed loop system from w_∞ to z_∞ less than some ,a priori given, upper bound $\gamma > 0$ can be easily derived from Theorem 4. 1 scaling.

5 Conclusions

Necessary and sufficient conditions are established so that a simultaneous H_2/H_∞ problem for discrete-time systems is solvable using dynamic state-feedback controllers. The class of singular problems considered have a left invertible transfer function matrix from the control input to the controlled output which is used for the H_2 norm performance measure. The results extend the work of [7] in the continuous-time setting to the discrete-time setting.

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离散系统状态反馈下 H_2/H_∞ 同步优化控制问题

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摘要：本文提出一个离散系统状态反馈下 H_2/H_∞ 同步优化控制问题.本文提出的问题是利用设计动态状态反馈控制器来使某闭环传递函数的 H_2 范数达到最优，同时也使另一闭环传递函数满足预先给定的 H_∞ 范数值.本文所考虑的系统有相当之普遍性，只要求其中一个从控制输入到被控制输出的开环传递函数是左可逆.本文也建立了所提出的 H_2/H_∞ 同步优化控制问题有解之充分和必要条件.

关键词：H_2/H_∞ 同步优化控制；鲁棒控制；离散系统控制；状态反馈

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