

COMPUTER PROJECT NO: ONE

SOLVING LINEAR SYSTEMS USING HOUSEHOLDER TRANSFORMATIONS

CPT S 531 / MATH 544

INSTRUCTOR: DR. ALAN GENZ

BEN M CHEN

60883883

10-01-88

10/10 Good work

Due 10/10/88

Write a subroutine called HSOLVE to "solve" a linear system $AX = B$ using Householder transformations. The subroutine should take as input integers M and N ($M \geq N$), an $M \times N$ matrix A , an M -vector B and an integer JOB . If $JOB = 0$ then HSOLVE should reduce A to upper triangular form using a sequence of N Householder transformations, overwriting A and saving the betas and diagonal elements in vectors $BETA$ and R (see pages 148-149 in the text). If $JOB = 1$ then HSOLVE should reduce A , transform B using the same sequence of Householder transformations that were used for A and solve for X using the first N components of the transformed B . If $JOB = 2$ then HSOLVE should assume that A has already been transformed using a previous call of HSOLVE and use the entries in A , R and $BETA$ to transform B and solve for X . When $JOB = 1$ or 2 HSOLVE should return the solution in an output N -vector output parameter X . HSOLVE should begin like:

```
SUBROUTINE HSOLVE(LDA, A, M, N, B, BETA, R, X, JOB)
  INTEGER LDA, M, N, JOB
  REAL A(LDA,N), B(M), BETA(N), R(N), X(N)
```

HSOLVE should be clearly written with good structure and appropriate comments.

Use your subroutine in a main program to:

a) solve the Hilbert system $H^{(n)}x = b^{(n)}$, where $h_{ij}^{(n)} = 1/(i+j-1)$ and $b_i^{(n)} = \sum_{j=1}^n 1/(i+j-1)$, for $n = 2, 3, 4,$

5, 6,

b) solve the system:

$$\begin{bmatrix} 1 & -3 & 9 & -27 \\ 1 & -2 & 4 & -8 \\ 1 & -1 & 1 & -1 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \end{bmatrix} x = b,$$

for $b^T = (13, 7, 3, 1, 1, 3, 7)$ and $(10.57, 6.68, 2.99, 1, 1.01, 3.32, 9.43)$.

c) for $n = 10, 20, 40$: generate 100 random $n \times n$ matrices and use HSOLVE to compute their solutions when the right-hand side b vectors are produced using true solutions $x^T = (1, 1, \dots, 1)$. Determine the average ∞ norm error for each n , and compare this with the average error obtained using the LINPACK subroutines SGEFA and SGESL. The entries in the matrices should be uniform random numbers from $[-1, 1]$.

The program listing and relevant output should be handed in, along with a brief discussion of the results that you obtained.

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C -----PR000010
C Computer Project 1:      CPT S 531/MATH 544      Ben M. Chen PR000020
C -----PR000030
C DIMENSION A(40,40),B(40),BETA(40),X(40),R(40),C(40,40),D(40) PR000040
C DIMENSION IPVT(40) PR000050
C INTEGER LDA, M, N, JOB, INFO, IPVT PR000060
C REAL AVERRH, AVERRL, MAX PR000070
C -----PR000080
C Parameters and variables in main program: PR000090
C     Refer the subroutine HSOLVE for more information about the PR000100
C     parameters and variables, except PR000110
C         C -- copy of A & used in Part C for comparision PR000120
C         D -- copy of B & used in Part C for comparision PR000130
C         INFO -- creat by LINPACK subroutine. PR000140
C         IPVT -- pivoting information created by LINPACK. PR000150
C         MAX -- maximun value of a vector. PR000160
C         AVERRH -- average infinite norm error from HSOLVE. PR000170
C         AVERRL -- average infinite norm error from LINPACK. PR000180
C         VNI(0) -- random number uniformly in (-1, 1) PR000190
C -----PR000200
C Part A: Using HSOLVE to solve the Hilbert system Hx=b for PR000210
C n=2,3,4,5,6 PR000220
C -----PR000230
C DO 440 I=2,6 PR000240
C     DO 420 J=1,I PR000250
C         B(J)=0. PR000260
C         DO 410 K=1,I PR000270
C             A(J,K)=1./((J+K-1)) PR000280
C             B(J)=B(J)+A(J,K) PR000290
410             CONTINUE PR000300
420     CONTINUE PR000310
C     LDA=40 PR000320
C     M=I PR000330
C     N=I PR000340
C     JOB=1 PR000350
C     CALL HSOLVE(LDA,A,M,N,B,BETA,R,X,JOB) PR000360
C -----PR000370
C     *** P R I N T I N G *** PR000380
C -----PR000390
C     WRITE(6,*) ' ' PR000400
C     WRITE(6,*) 'Solution For Hilbert System; N=',N PR000410
C     WRITE(6,*) ' ' PR000420
C     DO 430 J=1,N PR000430
C         WRITE(6,*) X(J) PR000440
430     CONTINUE PR000450
440     CONTINUE PR000460
C -----PR000470
C Part B: Using HSOLVE to solve the given systems. PR000480
C -----PR000490
C M=7 PR000500
C N=4 PR000510
C WRITE(6,*) ' ' PR000520
C WRITE(6,*) 'Reading the matrix A by row-wise.' PR000530
C WRITE(6,*) ' ' PR000540
C     READ *, ((A(I,J),J=1,N),I=1,M) PR000550

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WRITE(6,*) ' ' PR000560
WRITE(6,*) 'Reading the first vector B:' PR000570
WRITE(6,*) ' ' PR000580
      READ *, (B(I),I=1,M) PR000590
JOB=1 PR000600
CALL HSOLVE(LDA,A,M,N,B,BETA,R,X,JOB) PR000610
C ----- PR000620
C      *** PRINTING *** PR000630
C ----- PR000640
WRITE(6,*) ' ' PR000650
WRITE(6,*) 'Solution For The First System of Part B: X=' PR000660
WRITE(6,*) ' ' PR000670
DO 510 I=1,N PR000680
      WRITE(6,*) X(I) PR000690
510 CONTINUE PR000700
C ----- PR000710
WRITE(6,*) ' ' PR000720
WRITE(6,*) 'Reading the second vector B:' PR000730
      READ *, (B(I),I=1,M) PR000740
JOB=2 PR000750
CALL HSOLVE(LDA,A,M,N,B,BETA,R,X,JOB) PR000760
C ----- PR000770
C      *** PRINTING *** PR000780
C ----- PR000790
WRITE(6,*) ' ' PR000800
WRITE(6,*) 'Solution For the Second System of Part B: X=' PR000810
WRITE(6,*) ' ' PR000820
DO 520 I=1,N PR000830
      WRITE(6,*) X(I) PR000840
520 CONTINUE PR000850
C ----- PR000860
C Part C: Using HSOLVE to determine the average infinite norm PR000870
C errors and comparing to the average errors obtained PR000880
C from LINPACK subroutine SGEFA and SGESL. PR000890
C PR000900
C * Because both HSOLVE and SGEFA overwrite the matrix A and B, PR000910
C we introduce C & D to momerize A & B, respectively. PR000920
C ----- PR000930
C PR000940
DO 660 I=0,2 PR000950
      N=(2**I)*10 PR000960
      M=N PR000970
      AVERRH=0. PR000980
      AVERRL=0. PR000990
      DO 650 II=1,100 PR001000
          DO 620 J=1,N PR001010
              B(J)=0. PR001020
          DO 610 K=1,N PR001030
              A(J,K)=VNI(0) PR001040
              C(J,K)=A(J,K) PR001050
              B(J)=B(J)+A(J,K) PR001060
          CONTINUE PR001070
          C(J,K)=A(J,K) PR001080
          B(J)=B(J)+A(J,K) PR001090
          CONTINUE PR001100
      CONTINUE
610

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        D(J)=B(J)                                PR001110
620      CONTINUE                                PR001120
        JOB=1                                    PR001130
        CALL HSOLVE(LDA,A,M,N,B,BETA,R,X,JOB)    PR001140
C      -----                                PR001150
C      Finding the infinite norm error in HSOLVE. PR001160
C      -----                                PR001170
        MAX=0.                                    PR001180
        DO 630 K=1,N                              PR001190
            IF (ABS(X(K)-1.).GT.MAX) MAX=ABS(X(K)-1.) PR001200
630      CONTINUE                                PR001210
        AVERRH=AVERRH+MAX/100.                    PR001220
C      -----                                PR001230
        CALL SGEFA(C,LDA,N,IPVT,INFO)             PR001240
        CALL SGESL(C,LDA,N,IPVT,D,0)             PR001250
C      -----                                PR001260
C      Finding the infinte norm error in LINPACK. PR001270
C      -----                                PR001280
        MAX=0.                                    PR001290
        DO 640 K=1,N                              PR001300
            IF (ABS(D(K)-1.).GT.MAX) MAX=ABS(D(K)-1.) PR001310
640      CONTINUE                                PR001320
        AVERRL=AVERRL+MAX/100.                    PR001330
C      -----                                PR001340
650      CONTINUE                                PR001350
C      -----                                PR001360
C      *** P R I N T I N G ***                    PR001370
C      -----                                PR001380
        WRITE(6,*) ' '                            PR001390
        WRITE(6,*) 'For N=',N,',the HSOLVE average norm error = ' PR001400
        WRITE(6,*) ' '                            PR001410
        WRITE(6,*) AVERRH                          PR001420
        WRITE(6,*) ' '                            PR001430
        WRITE(6,*) 'For N=',N,',the LINPACK average norm error = ' PR001440
        WRITE(6,*) ' '                            PR001450
        WRITE(6,*) AVERRL                          PR001460
660      CONTINUE                                PR001470
        STOP                                       PR001480
        END                                       PR001490
C      -----                                PR001500
C      -----                                PR001510
C      This subroutine, named HSOLVE, is written for solving a linear PR001520
C      system AX=B using Householder transformations, where A is PR001530
C      an M-by-N matrix and B is an M-vector.    PR001540
C      -----                                PR001550
C      Parameters & variables in this subroutine: PR001560
C      LDA  --  leading dimesion of the matrix A . PR001570
C      A    --  an M-by-N matrix.                PR001580
C      M    --  number of rows in matrix A and vector B. PR001590
C      N    --  number of columns in matrix A .   PR001600
C      B    --  column vector with M elements.    PR001610
C      BETA --  storing the betas in Householder transformation PR001620
C      R    --  storing the diagonal elements of trasformed A PR001630
C      X    --  storing the solution of the linear system. PR001640
C      JOB  --  JOB=0, HSOLVE only reduces matrix A . PR001650

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	JOB=1, HSOLVE reduce A and solve for X	PR001660
	JOB=2, HSOLVE assumes that A has been reduced,	PR001670
	and return the solution in X.	PR001680
	-----	PR001690
	SUBROUTINE HSOLVE(LDA,A,M,N,B,BETA,R,X,JOB)	PR001700
	INTEGER LDA,M,N,JOB	PR001710
	REAL A(LDA,N),B(M),BETA(N),R(N),X(N),ALFA,MAX,SIG	PR001720
	JOB=JOB-1	PR001730
	IF (JOB) 100,100,200	PR001740
	-----	PR001750
	Following is the program to reduce matrix A to a upper tran-	PR001760
	gular form using a sequence of N householder transformations.	PR001770
	-----	PR001780
	This for JOB=0 and JOB=1	PR001790
	-----	PR001800
100	DO 35 I=1,N	PR001810
	MAX=0.	PR001820
	DO 10 J=I,M	PR001830
	IF (ABS(A(J,I)).GT.MAX) MAX=ABS(A(J,I))	PR001840
10	CONTINUE	PR001850
	ALFA=0.	PR001860
	DO 15 J=I,M	PR001870
	A(J,I)=A(J,I)/MAX	PR001880
	ALFA=ALFA+A(J,I)**2	PR001890
15	CONTINUE	PR001900
	ALFA=SQRT(ALFA)	PR001910
	BETA(I)=1./(ALFA*(ALFA+ABS(A(I,I))))	PR001920
	IF (A(I,I).LT.0.) THEN	PR001930
	SIG=-1.	PR001940
	ELSE	PR001950
	SIG=1.	PR001960
	ENDIF	PR001970
	A(I,I)=A(I,I)+ SIG*ALFA	PR001980
	R(I)=- SIG*ALFA*MAX	PR001990
	DO 30 K=I+1,N	PR002000
	ALFA=0.	PR002010
	DO 20 J=I,M	PR002020
	ALFA=ALFA+A(J,I)*A(J,K)	PR002030
20	CONTINUE	PR002040
	ALFA=ALFA*BETA(I)	PR002050
	DO 25 J=I,M	PR002060
	A(J,K)=A(J,K)-ALFA*A(J,I)	PR002070
25	CONTINUE	PR002080
30	CONTINUE	PR002090
35	CONTINUE	PR002100
	IF (JOB.EQ.-1) GOTO 300	PR002110
	-----	PR002120
	Following program is to transform B using the same sequences	PR002130
	of Householder transformations that were used for A .	PR002140
	-----	PR002150
	This is for JOB=1 and JOB=2.	PR002160
	-----	PR002170
200	DO 50 I=1,N	PR002180
	ALFA=0.	PR002190
		PR002200

	DO 40 J=I,M	PR002210
	ALFA=ALFA+A(J,I)*B(J)	PR002220
40	CONTINUE	PR002230
	DO 45 J=I,M	PR002240
	B(J)=B(J)-ALFA*BETA(I)*A(J,I)	PR002250
45	CONTINUE	PR002260
50	CONTINUE	PR002270
C	-----	PR002280
C	Solving for X. (Note: A has been reduced in upper triangular.)	PR002290
C	-----	PR002300
	DO 60 I=N,1,-1	PR002310
	X(I)=B(I)	PR002320
	DO 55 J=I+1,N	PR002330
	X(I)=X(I)-A(I,J)*X(J)	PR002340
55	CONTINUE	PR002350
	X(I)=X(I)/R(I)	PR002360
60	CONTINUE	PR002370
300	RETURN	PR002380
	END	PR002390
		PR002400

1. Results For Part A: Hilbert Systems

Solution For Hilbert System; N = 2

0.999995708
1.00000477

Solution For Hilbert System; N = 3

1.00000763
0.999951303
1.00004578

Solution For Hilbert System; N = 4

0.999977589
1.00019169
0.999580860
1.00025749

Solution For Hilbert System; N = 5

0.999479711
1.00935936
0.960700333
1.05814934
0.972006679

Solution For Hilbert System; N = 6

0.999503314
1.01546097
0.894065619
1.27455330
0.699263990
1.11751842

2. Results For Part B: Solving The Given Systems

Solution For The First System of Part B: X =

1.00000000
-1.00000286
0.999998808
0.446724812E-06

Solution For the Second System of Part B: X =

0.999998152
-1.24952698
0.999998212
0.116667390

3. Results For Part C: Determine The Infinite Norm Errors

For N = 10 ,the HSOLVE average norm error =

0.572258868E-04

For N = 10 ,the LINPACK average norm error =

0.288394076E-04

For N = 20 ,the HSOLVE average norm error =

0.967520464E-04

For N = 20 ,the LINPACK average norm error =

0.517345616E-04

For N = 40 ,the HSOLVE average norm error =

0.129582477E-02

For N = 40 ,the LINPACK average norm error =

0.365520362E-03

DISCUSSION:

This program is written and run on CMS in FORTRAN.

Although we have 10 digits in each number we obtained, it is easy to check that only the first seven digits are useful, that is $t=7$.

From the results we obtained, we see that:

1. In Part A, the true solution for any Hilbert system must be the form of $\{1,1,\dots,1\}'$. In our solution, however, only the lower order systems ($n=2,3,4$) have the close results. For the higher order system, say $n=6$, the results are useless.
2. The computer gave quite nice solutions for the system given in Part B.
3. For Part C, we see that the subroutines in LINPACK gave us a little better results than that we obtained from HSOLVE (the average infinite norm errors from HSOLVE are about twice much the average infinite norm errors form LINPACK. Also, we note that the average infinite norm errors are increasing while the order of system increased.

Ben M Chen

Oct. 1, 88

Finding Eigenvalues and Eigenvectors of a Symmetric Matrix

by Using Serial Jacobi Algorithm

Ben M Chen

10/10
Good work

COMPUTER PROJECT NO: 2

Computer Science 531 / Mathematics 544

November 25, 1988

Homework 6: Due 11/11/88: 7.1-3, 7.1-8, 7.2-5, 7.3-3, 7.3-4, 7.4-3, 7.4-8

Computer Project 2: Due 11/30/87

Write a subroutine called JACOBI to find all of the eigenvalues of an $N \times N$ symmetric matrix A , using the algorithm given in the text on page 299. The subroutine should have for input parameters, LDA (the actual leading dimension of A and an output matrix U), N , A and DELTA (required accuracy). The output should be a vector EIGENS of eigenvalues, and a matrix U that has the eigenvectors of A for its columns. An additional output parameter COUNT, should give the number of iterations that were required for convergence. The matrix A should be overwritten during the calculations.

Test your subroutine with $\text{DELTA} = 0.00001$ using 20 randomly generated matrices for $N = 10, 20, 40$. For each N you should compute the average number of flops (using COUNT) for convergence, and the average Frobenius norm of $AU - UD$, where D is a diagonal matrix containing the computed eigenvalues of A . The entries in the random symmetric matrices should be taken from the interval $[-1,1]$.

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C -----JAC00010
C COMPUTER PROJECT NO:2      CPT S 531/MATH 544      BEN M CHEN JAC00020
C -----JAC00030
  DIMENSION A(40,40),U(40,40),D(40,40),EIGENS(40) JAC00040
  INTEGER LDA,N,COUNT,P,AFLOPS JAC00050
  REAL FNORM,FNORM1,DELTA JAC00060
  LDA=40 JAC00070
  DELTA=0.00001 JAC00080
  DO 160 K=0,2 JAC00090
    N=(2**K)*10 JAC00100
    FNORM=0. JAC00110
    AFLOPS=0 JAC00120
    DO 150 L=1,20 JAC00130
      DO 110 I=1,N JAC00140
        DO 100 J=I,N JAC00150
          A(I,J)=VNI(0) JAC00160
          A(J,I)=A(I,J) JAC00170
          JAC00180
          D(I,J)=A(I,J) JAC00190
          D(J,I)=A(J,I) JAC00200
          JAC00210
          CONTINUE JAC00220
        CONTINUE JAC00230
        CALL JACOBI(LDA,D,N,DELTA,EIGENS,U,COUNT) JAC00240
        AFLOPS=AFLOPS+COUNT*4*N*N*N JAC00250
        FNORM1=0. JAC00260
        DO 140 I=1,N JAC00270
          DO 130 J=1,N JAC00280
            D(I,J)=0. JAC00290
            DO 120 P=1,N JAC00300
              D(I,J)=D(I,J)+A(I,P)*U(P,J) JAC00310
            CONTINUE JAC00320
            D(I,J)=D(I,J)-EIGENS(J)*U(I,J) JAC00330
            FNORM1=FNORM1+D(I,J)*D(I,J) JAC00340
          CONTINUE JAC00350
        CONTINUE JAC00360
        FNORM=FNORM+SQRT(FNORM1) JAC00370
      CONTINUE JAC00380
      AFLOPS=AFLOPS/20 JAC00390
      FNORM=FNORM/20. JAC00400
    CONTINUE JAC00410
    WRITE(6,*)'F O R   N = ',N JAC00420
    WRITE(6,*)'AVERAGE NUMBER OF FLOPS FOR CONV. = ',AFLOPS JAC00430
    WRITE(6,*)'AVERAGE FROBENIUS NORM OF AU - UD = ',FNORM JAC00440
  CONTINUE JAC00450
  STOP JAC00460
  END JAC00470
C JAC00480
C -----JAC00490
C SUBROUTINE JACOBI(LDA,A,N,DELTA,EIGENS,U,COUNT) JAC00500
C INTEGER LDA,N,COUNT JAC00510
C REAL A(LDA,N),U(LDA,N),DELTA,EIGENS(N),FNORM,OFFA JAC00520
C -----JAC00530
C THIS SUBROUTINE IS FOR FINDING THE EIGENVALUES & EIGENVECTORS JAC00540
C OF AN N-BY-N SYMMETRIC MATRIX USING SERIAL JACOBI ALGORITHM. JAC00550

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C          * * * PARAMETERS & VARIABLES : * * * JAC00560
C      A      --- AN N-BY-N SYMMETRIC MATRIX. JAC00570
C      U      --- AN N-BY-N MATRIX CONSISTS OF EIGENVECTORS OF A. JAC00580
C      LDA    --- LEADING DIMENSION OF A AND U. JAC00590
C      DELTA  --- REQUIRED ACCURACY FOR THE COMPUTATION. JAC00600
C      EIGENS --- AN N-VECTOR CONSISTS OF EIGENVALUES OF A. JAC00610
C      COUNT  --- NUMBER OF ITERATIONS REQUIRED FOR CONVERGENCE. JAC00620
C      ----- JAC00630
C      DO 15 I=1,N JAC00640
C          DO 10 J=1,N JAC00650
C              IF (I.EQ.J) THEN JAC00660
C                  U(I,J)=1. JAC00670
C              ELSE JAC00680
C                  U(I,J)=0. JAC00690
C              ENDIF JAC00700
10      CONTINUE JAC00710
15      CONTINUE JAC00720
C      COUNT=0 JAC00730
C      FNORM=0. JAC00740
C      DO 20 I=1,N JAC00750
C          FNORM=FNORM+A(I,I)*A(I,I) JAC00760
20      CONTINUE JAC00770
C      OFFA=0. JAC00780
25      DO 35 I=1,N-1 JAC00790
C          DO 30 J=I+1,N JAC00800
C              OFFA=OFFA+A(I,J)*A(I,J) JAC00810
30      CONTINUE JAC00820
35      CONTINUE JAC00830
C      OFFA=OFFA*2. JAC00840
C      IF (COUNT.NE.0) GOTO 40 JAC00850
C      FNORM=FNORM+OFFA JAC00860
C      DELTA2=DELTA*DELTA*FNORM JAC00870
C      IF (OFFA.LE.DELTA2) GOTO 65 JAC00880
40      COUNT=COUNT+1 JAC00890
C      DO 60 I=1,N-1 JAC00900
C          DO 55 J=I+1,N JAC00910
C              IF (ABS(A(I,J)).LE.DELTA) GOTO 55 JAC00920
C              TAU=(A(I,I)-A(J,J))*0.5/A(I,J) JAC00930
C              IF (TAU.LT.0) THEN JAC00940
C                  SIGN=-1. JAC00950
C              ELSE JAC00960
C                  SIGN=1. JAC00970
C              ENDIF JAC00980
C              TAU2=1+TAU*TAU JAC00990
C              T=SIGN/(ABS(TAU)+SQRT(TAU2)) JAC01000
C              C=1+T*T JAC01010
C              C=1/SQRT(C) JAC01020
C              S=T*C JAC01030
C              DO 45 K=1,N JAC01040
C                  AKI=C*A(K,I)+S*A(K,J) JAC01050
C                  A(K,J)=-S*A(K,I)+C*A(K,J) JAC01060
C                  A(K,I)=AKI JAC01070
C              UKI=C*U(K,I)+S*U(K,J) JAC01080
C              JAC01090
C              JAC01100

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	U(K,J)=-S*U(K,I)+C*U(K,J)	JAC01110
	U(K,I)=UKI	JAC01120
45	CONTINUE	JAC01130
	A(I,I)=C*A(I,I)+S*A(J,I)	JAC01140
	A(J,J)=-S*A(I,J)+C*A(J,J)	JAC01150
	A(I,J)=0.	JAC01160
	DO 50 K=1,N	JAC01170
	A(I,K)=A(K,I)	JAC01180
	A(J,K)=A(K,J)	JAC01190
50	CONTINUE	JAC01200
55	CONTINUE	JAC01210
60	CONTINUE	JAC01220
	GOTO 25	JAC01230
65	DO 70 I=1,N	JAC01240
	EIGENS(I)=A(I,I)	JAC01250
70	CONTINUE	JAC01260
	RETURN	JAC01270
	END	JAC01280

COMPUTER PROJECT NO:2 -- SERIAL JACOBI ALGORITHM BEN M CHEN

R E S U L T S

1) F O R N = 10

AVERAGE NUMBER OF FLOPS FOR CONV. = 18600

AVERAGE FROBENIUS NORM OF AU - UD = 0.891845557E-04

2) F O R N = 20

AVERAGE NUMBER OF FLOPS FOR CONV. = 176000

AVERAGE FROBENIUS NORM OF AU - UD = 0.312740682E-03

3) F O R N = 40

AVERAGE NUMBER OF FLOPS FOR CONV. = 1548800

AVERAGE FROBENIUS NORM OF AU - UD = 0.972041860E-03
