Ben Chen

EE 509, Homework #1

1. Examine the stability of the equilibrium states of the following differential equations:

   1. $\dot{x} = (\sin t)x$

   2. $\dot{x} = (3t\sin t - t)x$

   3. $\dot{x} = a(t)$ where $a(t)$ is a continuous function with $a(t) < 0$ $\forall t \geq t_o \geq 0$.

2. Find the equilibrium states of the scalar differential equation

$$\dot{x} = -(x - 1)(x - 2)^2$$

and examine their stability properties using

   1. Linearization

   2. An appropriate Lyapunov function

3. 1. Let

$$G_1(s) = \frac{1}{s + 1}, \quad L(s) = \frac{s - 1}{(s + 1)^2}$$

find a bound on $\varepsilon > 0$ for the transfer function:

$$G_\varepsilon(s) = G_1(s) + \varepsilon L(s)$$

to be PR, SPR.

   2. Repeat 1 when

$$G_1(s) = \frac{s + 5}{(s + 1)(s + 3)}, \quad L(s) = -\frac{1}{s + 1}$$

comment on your results.

(i.e., Find om $\varepsilon^* > 0$ such that for all ε $\varepsilon[0,\varepsilon^*]$) $G_\varepsilon(s)$ is PR, SPR)

4. Consider the following system:

$$\dot{e} = Ae + b\phi \sin t, \quad e_1 = h^T e$$

$$\dot{\phi} = -e_1 \sin t$$

where $\phi, e_1 \in R^1$, $e \in R^n$ and $W_m(s) = h^T(sI - A)^{-1}b$ is SPR. Use Lemma 1.3.3 to define an appropriate Lyapunov function and use H study the stability of the system. (i.e., of the equilibrium $\phi = 0$, $e = 0$)

5. Examine the stability of the equilibrium state of the system:

$$\dot{x}_1 = -2x_1 + x_1 x_2$$

$$\dot{x}_2 = -x_1^2 - \sigma x_2$$

where $\sigma > 0$ is a constant by using an appropriate Lyapunov function.

Problem 1: Examine the stability of the equilibrium states of the following differential equations:

1. $\dot{x} = (\sin t)\, x$

ⓐ Eequilibrium state $x_e = 0$.

ⓑ $V(t,x) \triangleq e^{2\cos t} x^2 \geq x^2$ is p.d.f. & decrescent.

$\dot{V}(t,x) = -2\sin t\, x^2 e^{2\cos t} + 2\sin t\, x^2 e^{2\cos t} = 0 \geq 0$ is p.s.d.f.

Hence x_e is stable, uniformly stable.

Note: for this example, it is easy to find the closed solution. And it is easy to check that x_e is not a.s.

2. $\dot{x} = (3t\sin t - t)\, x$

ⓐ Equilibrium state $x_e = 0$

ⓑ $V(t,x) \triangleq e^{2(0.4t^2 + 3t\cos t - 3\sin t)} x^2$ is p.d.f.

$$\dot{V}(t,x) = 2(0.8t + 3\cos t - 3t\sin t - 3\cos t)\, e^{*} x^2 + 2(3t\sin t - t)\, e^{*} x^2$$

$$= -0.4t\, e^{2(0.4t^2 + 3t\cos t - 3\sin t)} x^2 \leq 0$$

Hence, $x_e = 0$ is stable.

3. $\dot{x} = a(t)$ where $a(t)$ is a cont. function with $a(t) < 0$, $\forall\, t \geq t_0 \geq 0$.

No equilibrium point! (How nice!)

Problem 2. Find the equilibrium states of the scalar differential equation

$$\dot{x} = -(x-1)(x-2)^2$$

and examine their stability properties using

1. Linearization
2. An appropriate Lyapunov function

Clearly, the equilibrium states are $x_{e1} = 1$, $x_{e2} = 2$.

1.

$A_1 = \left.\frac{\partial f}{\partial x}\right|_{x=1} = -1$, then x_{e1} is locally e.s.

$A_2 = \left.\frac{\partial f}{\partial x}\right|_{x=2} = 0$, this implies there is no linear term for $x=2$, nothing could be conclude

2. ⓐ Define $y = x-1$

$$\dot{y} = -y(y-1)^2$$

$V(t,y) = y^2$ is p.d.f. and decrescent. $c_1 = 0.5$, $c_2 = 1.5$

$\dot{V}(t,y) = -2y^2(y-1)^2 \leq -\underbrace{2(1-h)^2}_{c_3>0} y^2$ for $y \in B_h$, $h<1$

Hence, $x_{e1} = 1$ is locally e.s.

ⓑ Define $y = x-2$

$$\dot{y} = -(y+1)y^2$$

$V(t,y) = y^2 - 2y$ is decrescent

① $\dot{V}(t,y) = 2(y+1)y^2 - 2y(y+1)y^2 = 2(1-y^2)y^2$ is l.p.d.f. $y \in B_h, h<$

② $V(t,0) = 0$ and $\exists$ a pnt is arbitrary closed to $y=0$ or $x_{e2} = 2$ $\ni V(t_0, y) > 0$

Hence $x_{e2} = 2$ is unstable.

Problem 3:

1. $G_1(s) = \dfrac{1}{s+1}$, $L(s) = \dfrac{s-1}{(s+1)^2}$

· find a bound on $\varepsilon > 0$ for the transfer function:

$$G_\varepsilon(s) = G_1(s) + \varepsilon L(s)$$

to be PR, SPR.

$$G_\varepsilon(s) = \frac{1}{s+1} + \varepsilon \frac{s-1}{(s+1)^2}$$

$$= \frac{s+1+\varepsilon s - \varepsilon}{(s+1)^2} = \frac{(1+\varepsilon)s + (1-\varepsilon)}{(s+1)^2}$$

. Condition $G_\varepsilon(s)$ is real whenever s is real (✓)

· $G_\varepsilon(s)$ has a zero at

$$s = \frac{\varepsilon - 1}{\varepsilon + 1} \leq 0 \quad \text{implies} \quad \varepsilon \leq 1$$

· $G_\varepsilon(j\omega) = \dfrac{(1-\varepsilon) + j\omega(1+\varepsilon)}{(1-\omega^2) + j2\omega} = \dfrac{1}{*}\left[2\omega^2 + 2\varepsilon\omega^2 + 1 - \omega^2 - \varepsilon + \varepsilon\omega^2\right] + \text{imag.}$

$$\mathrm{Re}\left[G_\varepsilon(j\omega)\right] = \frac{1}{*}\left[\omega^2 + 3\varepsilon\omega^2 + 1 - \varepsilon\right] \geq 0 \quad \text{for } \varepsilon \leq 1$$

Hence,

<u>for $0 < \varepsilon \leq 1$, $G_\varepsilon(s)$ is PR.</u>

<u>for $0 < \varepsilon < 1$, $G_\varepsilon(s)$ is SPR.</u>

(Note: for SPR, condition (iii) of Theorem 1.3.1 is easily to be checked).

Problem 3:

2. Repeat 1 when

$$G_1(s) = \frac{s+5}{(s+1)(s+3)}, \quad L(s) = -\frac{1}{s+1}$$

Comment on your results.

$$G_\varepsilon(s) = \frac{s+5}{(s+1)(s+3)} - \varepsilon\frac{1}{s+1}$$

$$= \frac{s+5 - \varepsilon(s+3)}{(s+1)(s+3)}$$

$$= \frac{(1-\varepsilon)s + (5-3\varepsilon)}{(s+1)(s+3)}$$

$G_\varepsilon(s)$ has a zero at

$$s = \frac{5-3\varepsilon}{\varepsilon-1} \leq 0 \quad \text{implies}$$

$$\varepsilon \geq \tfrac{5}{3} \quad \text{and} \quad \varepsilon < 1$$

$$G_\varepsilon(j\omega) = \frac{(5-3\varepsilon)+j(1-\varepsilon)\omega}{(j\omega+1)(j\omega+3)} = \frac{(5-3\varepsilon)+j(1-\varepsilon)\omega}{(3-\omega^2)+j4\omega}$$

$$= \frac{1}{(3-\omega^2)^2+16\omega^2}\cdot[15-9\varepsilon-\omega^2-\varepsilon\omega^2] + \text{imag.}$$

$\text{Re}[G_\varepsilon(j\omega)]$ is <0 for all $\omega \geq 3.873$ no matter what the ε is. This implies $G_\varepsilon(s)$ never is P.R or S.P.R.

(If ε is allowed to be negative, then

$G_\varepsilon(s)$ is P.R and SPR when $\varepsilon < -1$)

Problem 4: Consider the following system

$$\dot{e} = Ae + b\phi \sin t \,, \qquad e_1 = h^T e$$

$$\dot{\phi} = -e_1 \sin t$$

where, $\phi, e_1 \in R^1$, $e \in R^n$ and $W_m(s) = h^T(sI-A)^{-1}b$ is SPR. Use Lemma 1.3.3 to define an appropriate Lyapunov function and use it to study the stability of the system.

Solution. The equilibrum is $\phi = 0$ and $e = 0$

In order to apply Lemma 1.3.3, we must assume (A, b) is controllable. Then

$W_m(s) = h^T(sI-A)^{-1}b$ is SPR. implies

there exist for $Q > 0$, $\exists$ $P > 0$, $\varepsilon > 0$ and g $\ni$

$$A^TP + PA = -gg^T - \varepsilon Q \triangleq -Q^* < 0$$

$$Pb - h = 0 \quad \Rightarrow \quad e_1 = b^TPe$$

Now define a Lyapunov function as (Let $X = \begin{pmatrix} e \\ \phi \end{pmatrix}$)

$$V(X,t) = [e^T \;\; \phi] \cdot \begin{bmatrix} P & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} e \\ \phi \end{bmatrix}$$

$$= e^TPe + \phi^2$$

Hence.

$$\min\{1, \lambda_{min}(P)\} * |X|^2 \le V(X,t) \le \max\{\lambda_{max}(P), 1\} \cdot |X|^2$$

$$\dot{V}(X,t) = \dot{e}^TPe + e^TP\dot{e} + 2\phi\dot{\phi}$$

$$= (e^TA^T + \phi \sin t\, b^T)Pe + e^TP(Ae + \phi \sin t\, b)$$
$$- 2\phi \sin t\, e_1$$

$$= e^T(A^TP + PA)e + \phi \sin t \underbrace{(b^TPe + e^TPb - 2b^TPe)}_{=0}$$

$$= -e^TQ^*e \le 0$$

Now, define

$$S = \{ x \in \mathbb{R}^{n+1} \mid \dot{V}(x) = 0 \}$$

$$= \{ \binom{e}{\phi} \in \mathbb{R}^{n+1} \mid e = 0 \}.$$

Suppose $\binom{e}{\phi} \in S \Rightarrow e \equiv 0 \Rightarrow \dot{e} \equiv 0.$

$$\Rightarrow b\phi \sin t \equiv 0 \Rightarrow \phi \equiv 0.$$

Hence $x = \binom{e}{\phi} = 0$ is <u>G. U. a. S.</u> since $V(t, x)$ satisfies condition 1, 2, 3, 4 in Theorem 1.1.1.

Problem 5: *Examine the stability of the equilibrum state of system:*

$$\dot{x}_1 = -2x_1 + x_1 x_2$$
$$\dot{x}_2 = -x_1^2 - \sigma x_2$$

where $\sigma > 0$ is a constant by using an appropriate Lyap Fun.

$$V(t, x) = |x|^2 = x_1^2 + x_2^2 \qquad (x_e = 0)$$

$$0.5 |x|^2 < V(t, x) < 1.5 |x|^2 \quad , \quad C_1 = 0.5, \; C_2 = 1.5$$

$$\dot{V}(t, x) = 2x_1 \dot{x}_1 + 2x_2 \dot{x}_2 = 2x_1(-2x_1 + x_1 x_2) + 2x_2(-x_1^2 - \sigma x_2)$$

$$= -4x_1^2 - 2\sigma x_2^2 = -2(2x_1^2 + \sigma x_2^2)$$

$$\leq -2 \min(2, \sigma) (x_1^2 + x_2^2)$$

$$C_3 = 2 \min(2, \sigma) > 0.$$

Thus, $x_e = 0$ is <u>g. e. s.</u> ※

Ben M. Chen

EE 509 Homework #2

Problem 1:

Find adaptation law for the following cases:

a)

Model

$$\dot{x}_m = A_m x_m + B_m u \qquad x \varepsilon \mathbb{R}^n,\ u \varepsilon \mathbb{R}^m$$

Plant and Controller (Adjustable system):

$$\dot{x}_p = (A_p + B_p F) x_p + B_p u$$

Assumptions:

i) The elements of B_p can be adjusted directly

ii) $\exists$ a matrix F^* such that $A_p + B_m F^* = A_m$

b)

Model:

$$\dot{x}_m = A_m x_m + B_m u \qquad x \varepsilon \mathbb{R}^n,\ u \varepsilon \mathbb{R}^m$$

Plant and Controller:

$$\dot{x}_p = A_p x_p + B_p Q u$$

Assumptions:

i) $A_m = A_p$

ii) $\exists$ a matrix $Q^* \ni B_p Q^* = B_m$

iii) B_p & B_m are of full rank

Problem 2: (Identification)

Plant:

$$\dot{x}_p = A_p x_p + B_p u, \quad x \in \mathbb{R}^4, \quad u \in \mathbb{R}^2,$$

$$A_p = \begin{bmatrix} -0.0366 & 0.0271 & 0.15 & -0.19 \\ 0.0482 & -1.01 & 1.06 & -1.9 \\ a_{31}^* & 0.368 & a_{33}^* & -1.9 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B_p = \begin{bmatrix} 0.4422 & 0.1761 \\ b_{21}^* & 7.59 \\ -5.52 & 4.49 \\ 0 & 0 \end{bmatrix}$$

where a_{31}^*, a_{33}^*, b_{21}^* are unknown parameter ($a_{31}^* = 0.2$, $a_{33}^* = -1.95$, $b_{21}^* = 5$)

Model:

$$\dot{x}_m = C x_m + (A_m(t) - C) x_p + B_m(t) u$$

The matrices $A_m(t)$, $B_m(t)$ are the same as A_p and B_p with a_{31}^*, a_{33}^* and b_{21}^* replaced by $a_{31}(t)$, $a_{33}(t)$ and $b_{21}(t)$ respectively. $C = -10I$ where I is 4x4 identity matrix.

Design an identification scheme with all necessary simulations.

Problem 3: (Adaptive control)

Plant with Controller:

$$\dot{x}_p = (A_p + b_p q f)\, x_p + b_p q u$$

$$A_p = \begin{bmatrix} 2 & 0 \\ -6 & -7 \end{bmatrix}, \quad b_p = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

Model:

$$\dot{x}_m = A_m x_m + b_m u$$

$$A_m = \begin{bmatrix} 0 & 1 \\ -10 & -5 \end{bmatrix}, \quad b_m = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Design the adaptation scheme and simulate your report.

$$P = \begin{bmatrix} 13.5 & 0.5 \\ 0.5 & 1.1 \end{bmatrix}$$

Solution to Problem 1 (a)

Let $e = x_m - x_p$

$$\dot{e} = A_m x_m + B_m u - (A_p + B_p F) x_p - B_p u$$

$$= A_m e + (A_m - A_p - B_p F) x_p + (B_m - B_p) u$$

By assumption, i) B_p can be adjusted directly $\Rightarrow$ $B_p = B_m$

ii) $A_p + B_m F^* = A_m \Rightarrow A_m - A_p = B_m F^*$

Hence

$$\dot{e} = A_m e + B_m (F^* - F) x_p \triangleq A_m e - B_m \Phi x_p$$

where $\Phi = F - F^*$.

Define a Lyaponov function

$$V = \frac{1}{2} [e^T P e + tr\{\Phi^T \Phi\}$$

where $P = P^T > 0$, $A_m^T P + P A_m = -Q < 0$.

$$\dot{V} = \frac{1}{2} [\dot{e}^T P e + e^T P \dot{e}] + tr\{\Phi^T \dot{\Phi}\}$$

$$= \frac{1}{2} [e^T A_m^T P e - x_p^T \Phi^T B_m^T P e + e^T P A_m e - e^T P B_m \Phi x_p] + tr\{\Phi^T \dot{\Phi}\}$$

$$= -\frac{1}{2} e^T Q e + tr\{\Phi^T \dot{\Phi}\} - tr\{x_p^T \Phi^T B_m^T P e\}$$

$$= -\frac{1}{2} e^T Q e + tr\{\Phi^T \dot{\Phi}\} - tr\{\Phi^T B_m^T P e x_p^T\}$$

$$= -\frac{1}{2} e^T Q e + tr\{\Phi^T [\dot{\Phi} - B_m^T P e x_p^T]\}$$

Let

$$\boxed{\dot{\Phi} = \dot{F} = B_m^T P e x_p^T} \quad \text{Adaptation Law.}$$

Then we have $\dot{V} = -\frac{1}{2} e^T Q e \leq 0$. implies e, Φ are u.b.

$\Rightarrow \int \dot{V} < \infty \Rightarrow e \in L_2$, $\dot{e} \in L_\infty$ implies $e \to 0$.

This is good enough for the control problem.

If u is P.E. then $\Phi \to 0$ as well.

Solution to Problem 1 (b)

Something as we did anywhere, let

$$e = x_m - x_p$$

$$\dot{e} = A_m x_m + B_m u - A_p x_p - B_p Q u$$

$$= A_m e + (A_m - A_p) x_p + (B_m - B_p Q) u.$$

By the assumptions, we know

$$A_m = A_p \quad \text{and} \quad \exists\ Q^* \ni B_p Q^* = B_m$$

Thus, we have

$$\dot{e} = A_m e + B_m (Q^{-1} - Q^{*-1}) Q u$$

Let $\quad \Psi = Q^{-1} - Q^{*-1}$

$$\dot{e} = A_m e + B_m \Psi Q u.$$

Choose Lyaponov function,

$$V = \frac{1}{2}[e^T p e + tr(\Psi^T \Psi)]$$

where $\quad p = p^T > 0, \quad A_m^T p + p A_m = -Q < 0.$

$$\dot{V} = \frac{1}{2}[\dot{e}^T p e + e^T p \dot{e}] + tr(\Psi^T \dot{\Psi})$$

$$= \frac{1}{2}[e^T A_m^T p e + u^T Q^T \Psi^T B_m^T p e + e^T p A_m e + e^T p B_m \Psi Q u] + tr(\Psi^T$$

$$= -\frac{1}{2} e^T Q e + u^T Q^T \Psi^T B_m^T p e + tr(\Psi^T \dot{\Psi})$$

$$= -\frac{1}{2} e^T Q e + tr(\Psi^T B_m^T p e u^T Q^T) + tr(\Psi^T \dot{\Psi})$$

$$= -\frac{1}{2} e^T Q e \le 0 \text{ if we choose adaptation law.}$$

$$\boxed{\dot{\Psi} = \dot{Q}^{-1} = -B_m^T p e u^T Q^T}$$ (this requires u is P.E. $\ni \det(Q) \neq 0$)

$e \in L_\infty$, $\Psi \in L_\infty$, $\int \dot{V} < \infty \Rightarrow e \in L_2 \Rightarrow e \rightarrow 0.$

if u is P.E. then $\Psi \rightarrow 0$ as well.

Solution to Problem 2

Let $e = x_m - x_p$

$$\dot{e} = \dot{x}_m - \dot{x}_p = Cx_m + (A_m - C)x_p + B_m u - A_p x_p - B_p u$$

$$= Ce + (A_m - A_p)x_p + (B_m - B_p)u.$$

Let $\Phi = A_m(t) - A_p$, $\Psi = B_m(t) - B_p$.

$$\dot{e} = Ce + \Phi x_p + \Psi u.$$

Choose

$$V = \frac{1}{2}[e^T p e + tr(\Phi^T\Phi) + tr(\Psi^T\Psi)]$$

where $p > 0$.

$$\dot{V} = \frac{1}{2}[\dot{e}^T p e + e^T p \dot{e}] + tr(\Phi^T\dot{\Phi}) + tr(\Psi^T\dot{\Psi})$$

$$= \frac{1}{2}[e^T C^T p e + x_p^T \Phi^T p e + u^T \Psi^T p e + e^T p C e + e^T p \Phi x_p + e^T p \Psi u] + tr(\Phi^T\dot{\Phi}) + tr(\Psi^T\dot{\Psi})$$

$$= \frac{1}{2}e^T(C^T p + pC)e + tr(x_p^T \Phi^T p e) + tr(\Phi^T\dot{\Phi}) + tr(u^T\Psi^T p e) + tr(\Psi^T\dot{\Psi})$$

$$= \frac{1}{2}e^T[C^T p + pC]e + tr[\Phi^T(\dot{\Phi} + p e x_p^T)] + tr[\Psi^T(\dot{\Psi} + p e u^T)]$$

Let $p = I$. and the adaptation law then is given as

$$\boxed{\dot{\Phi} = \dot{A}_m(t) = -e x_p^T \quad , \quad \dot{\Psi} = \dot{B}_m(t) = -e u^T}$$

But since most elements of $A_m(t)$ and $B_m(t)$ are known, we only have to adapt $a_{31}(t)$, $a_{33}(t)$ and $b_{21}(t)$. Hence.

$$\dot{A}_m(t) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \dot{a}_{31}(t) & 0 & \dot{a}_{33}(t) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \times & \times & \times & \times \\ \times & \times & \times & \times \\ -e_3 x_p^1 & \times & -e_3 x_p^3 & \times \\ \times & \times & \times & \times \end{bmatrix},$$

$$\dot{B}_m(t) = \begin{bmatrix} 0 & 0 \\ \dot{b}_{21}(t) & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \times & \times \\ -e_2 u_1 & \times \\ \times & \times \\ \times & \times \end{bmatrix}$$

Problem 2 (cont.)

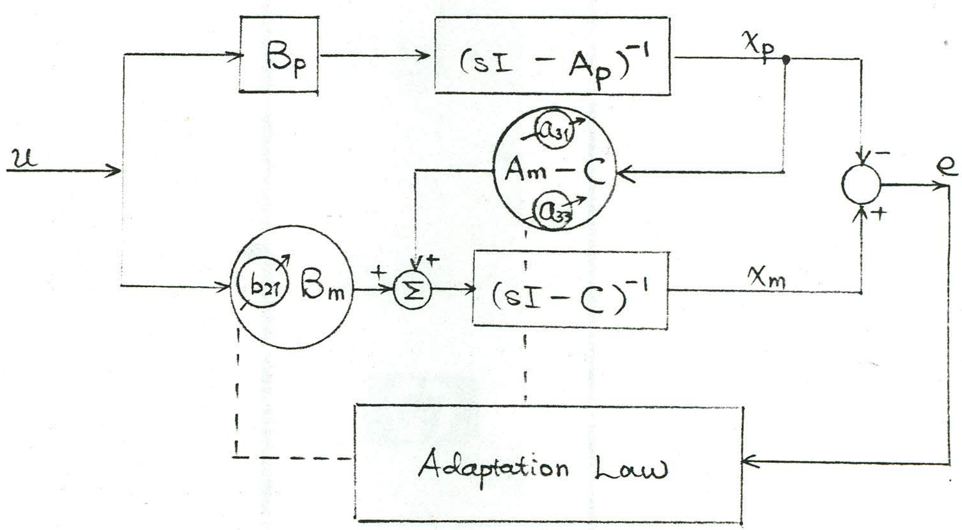
Actual Adaptation laws:

$$\dot{a}_{31}(t) = -e_3 x_p^1$$

$$\dot{a}_{33}(t) = -e_3 x_p^3$$

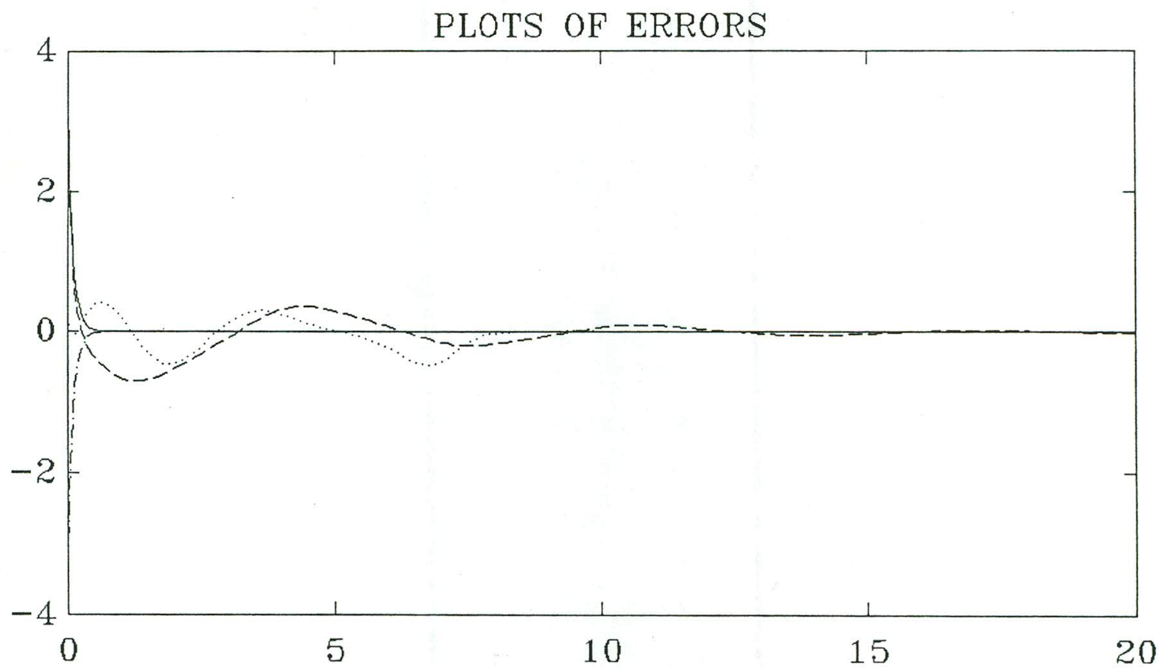
$$\dot{b}_{21}(t) = -e_2 u_1$$

Scheme:

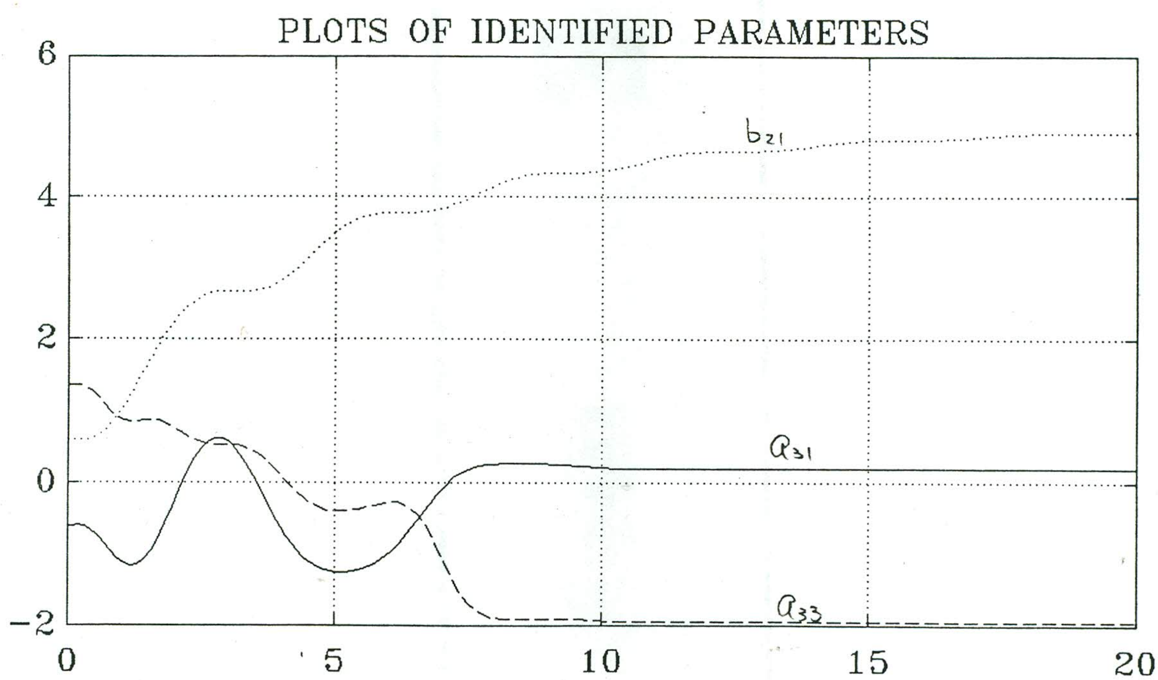


Simulation Equations:

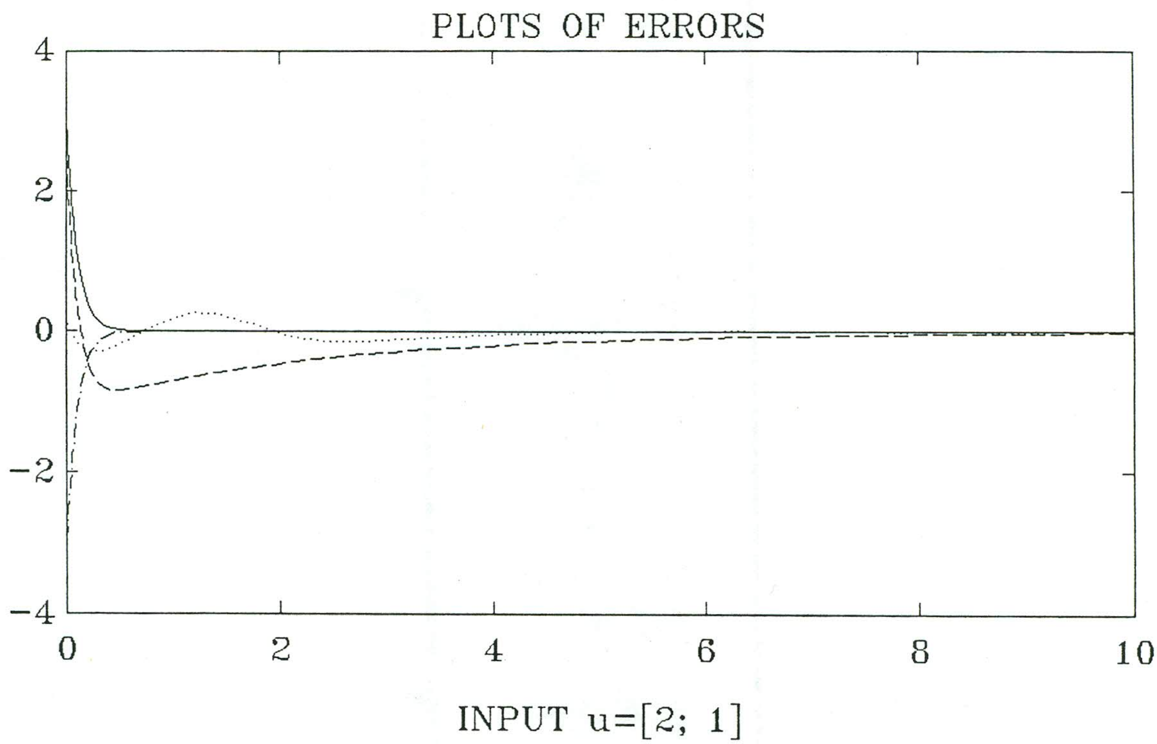
$$\begin{bmatrix} x_1 & x_p^1 \\ x_2 & x_p^2 \\ x_3 & x_p^3 \\ x_4 & x_p^4 \\ x_5 & a_{31} \\ x_6 & a_{33} \\ x_7 & b_{21} \\ x_8 & e_1 \\ x_9 & e_2 \\ x_{10} & e_3 \\ x_{11} & e_4 \end{bmatrix}
\quad
\left\{ \begin{aligned}
\dot{x}_p^1 &= -0.0366 x_p^1 + 0.0271 x_p^2 + 0.15 x_p^3 - 0.19 x_p^4 + 0.4422 u_1 + 0.1761 u_2 \\
\dot{x}_p^2 &= 0.0482 x_p^1 - 1.01 x_p^2 + 1.06 x_p^3 - 1.9 x_p^4 + 5 u_1 + 7.59 u_2 \\
\dot{x}_p^3 &= 0.2 x_p^1 + 0.368 x_p^2 - 1.95 x_p^3 - 1.9 x_p^4 - 5.52 u_1 + 4.49 u_2 \\
\dot{x}_p^4 &= x_p^3 \\
\dot{a}_{31}(t) &= -e_3 x_p^1 \\
\dot{a}_{33}(t) &= -e_3 x_p^3 \\
\dot{b}_{21}(t) &= -e_2 u_1 \\
\dot{e}_1 &= -10 e_1 \\
\dot{e}_2 &= -10 e_2 + b_{21}(t) u_1 - 5 u_1 \\
\dot{e}_3 &= -10 e_3 + a_{31}(t) x_p^1 + a_{33}(t) x_p^3 - 0.2 x_p^1 + 1.95 x_p^3 \\
\dot{e}_4 &= -10 e_4
\end{aligned} \right.$$

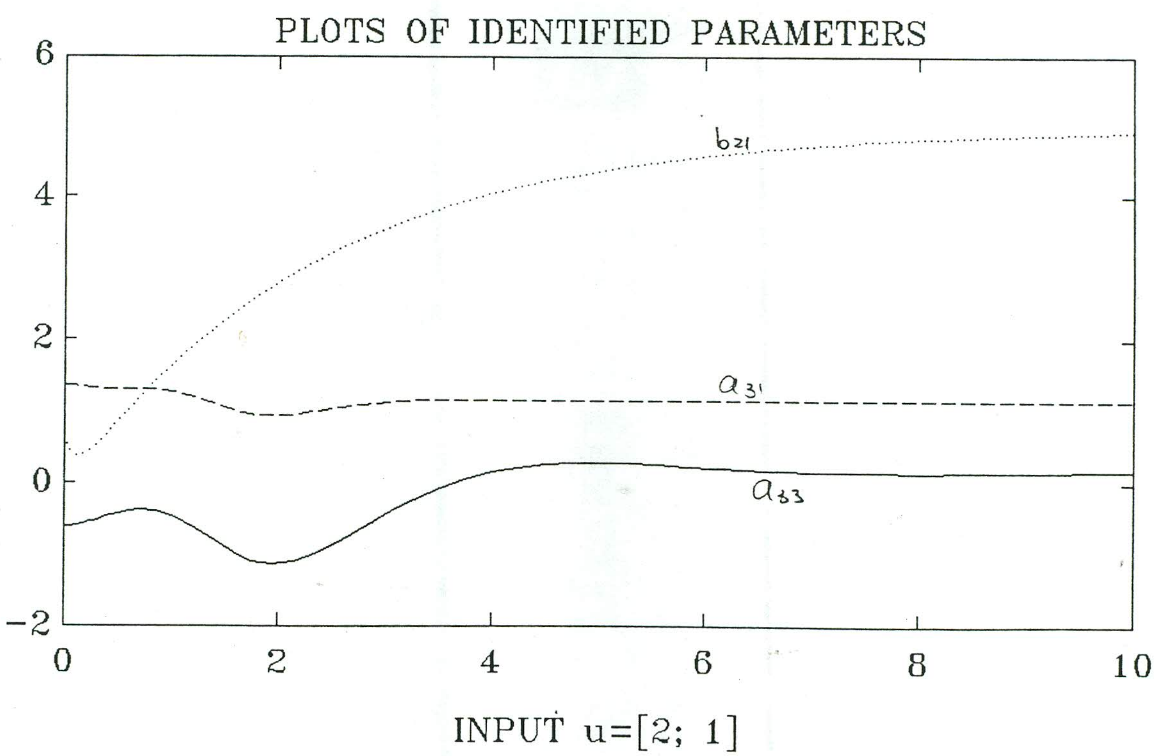


INPUT u=[2sin(t)+0.1cos(t);0.23sin(.2t)+cos(t)]

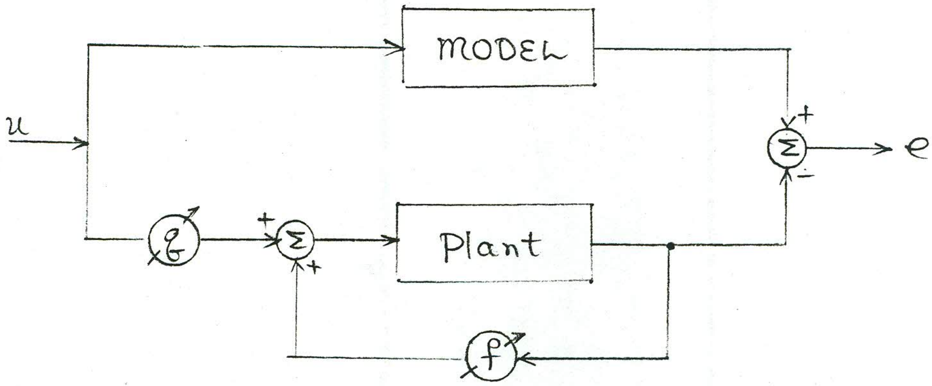


INPUT u=[2sin(t)+0.1cos(t);0.23sin(.2t)+cos(t)]


PLOTS OF ERRORS
4
2
0
−2
−4
0
2
4
6
8
10
INPUT u=[2; 1]


PLOTS OF IDENTIFIED PARAMETERS
6
4
2
0
−2
0
2
4
6
8
10
INPUT u=[2; 1]

Solution to Problem 3



As usual, let

$$e = x_m - x_p.$$

$$\dot e = A_m x_m + b_m u - (A_p + b_p g f) x_p - b_p g u$$

$$= A_m e + (A_m - A_p - b_p g f) x_p + (b_m - b_p g) u.$$

By assumptions, $A_m - A_p = b_p g^* f^*$, $b_p = b_m * g^{*-1}$

$$= b_m f^*$$

$$\dot e = A_m e + [A_m - A_p - b_p (g^* + g - g^*) f] x_p + b_m (g^{-1} - g^{*-1}) g u.$$

$$= A_m e + [b_m f^* - b_m f] x_p + [b_m - b_m g^{*-1} g] f x_p + b_m (g^{-1} - g^{*-1}) g$$

$$= A_m e + b_m [f^* - f] + b_m [g^{-1} - g^{*-1}] g \cdot [f x_p + u]$$

Let $\phi = f - f^*$, $\psi = g^{-1} - g^{*-1}$

$$\dot e = A_m e - b_m \phi x_p + b_m \psi g [f x_p + u]$$

Choose

$$V = \tfrac{1}{2}[e^T P e + tr(\Phi^T \Phi) + \psi^2]$$

$$\dot V = \tfrac{1}{2}[\dot e^T P e + e^T P \dot e] + tr(\Phi^T \dot\Phi) + \psi \dot\psi$$

$$= \tfrac{1}{2}[e^T A_m^T P e + e^T P A_m e] - \Phi^T b_m^T P e x_p + \psi g [f x_p + u] b_m^T + tr(\Phi^T \dot\Phi) + \psi \dot\psi$$

Adaptation laws:

$$\boxed{\begin{aligned} \dot\Phi &= \dot f = b_m^T P e x_p^T \\ \dot\psi &= \dot{g}^{-1} = -b_m^T P e (u + f x_p)^T g \end{aligned}}$$

Problem 3 (cont.)

Simulation equations $\quad g^* = 0.5, \quad f^* = [-2, 1]$

Choose

$$P = \begin{bmatrix} 1.35 & 0.05 \\ 0.05 & 0.11 \end{bmatrix} \times 0.1$$

$$\dot{x}_p = \left(\begin{bmatrix} 2 & 0 \\ -6 & -7 \end{bmatrix} + \begin{bmatrix} 2 \\ 4 \end{bmatrix} g \cdot [f_1, f_2] \right) x_p + \begin{bmatrix} 2 \\ 4 \end{bmatrix} \cdot g u$$

1 $\quad \dot{x}_p^1 = (2 + 2gf_1)\, x_p^1 + 2gf_2 \cdot x_p^2 + 2g\, u$

2 $\quad \dot{x}_p^2 = (-6 + 4gf_1)\, x_p^1 + (-7 + 4gf_2)\, x_p^2 + 4g\, u$

3 $\quad \dot{e}_1 = e_2 - (2 + 2gf_1)\, x_p^1 + (1 - 2gf_2)\, x_p^2 + (1 - 2g)\, u$

4 $\quad \dot{e}_2 = -10e_1 - 5e_2 - (4 + 2gf_1)\, x_p^1 + (2 - 4gf_2)\, x_p^2 + (2 - 4g)\, u$

5 $\quad \dot{g}^{-1} = -(1.45\, e_1 + 0.27\, e_2)(u + f_1 x_p^1 + f_2 x_p^2)\, g \times 0.1$

6 $\quad \dot{f}_1 = +(1.45\, e_1 + 0.27\, e_2)\, x_p^1 \times 0.1$

7 $\quad \dot{f}_2 = +(1.45\, e_1 + 0.27\, e_2)\, x_p^2 \times 0.1$

Initial values:

$x_{p0}^1 = 1.0$

$x_{p0}^2 = 1.0$

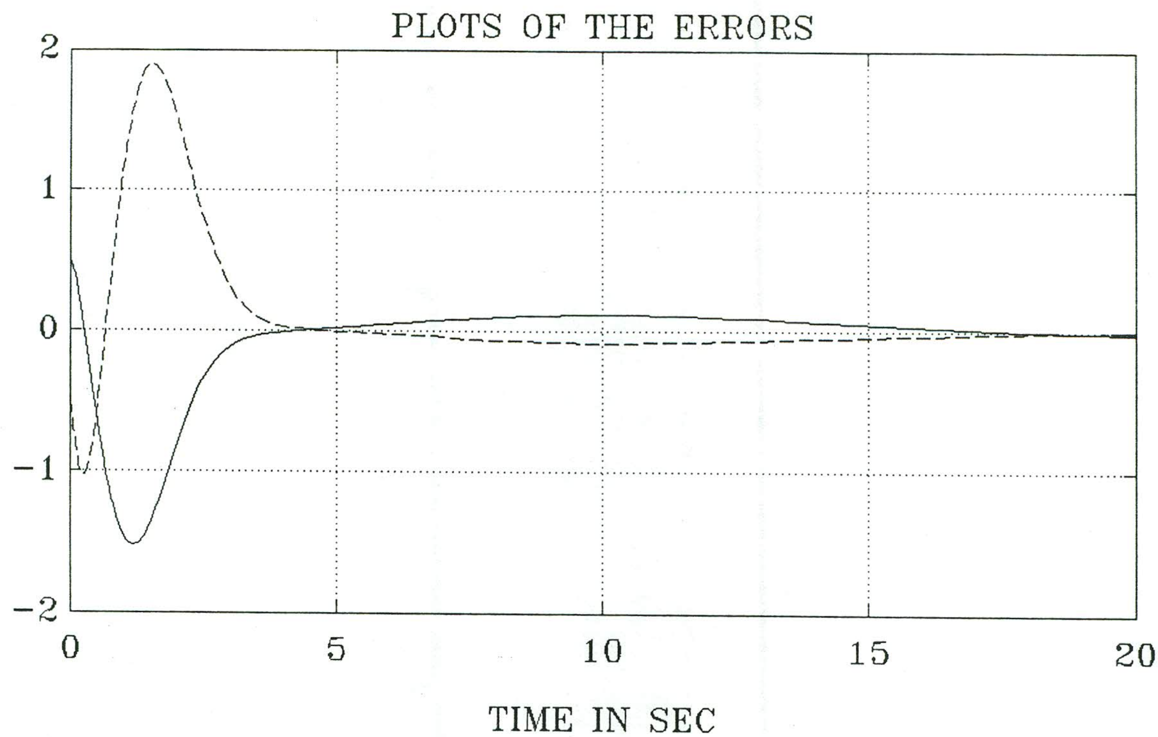
$e_{10} = 0.5$

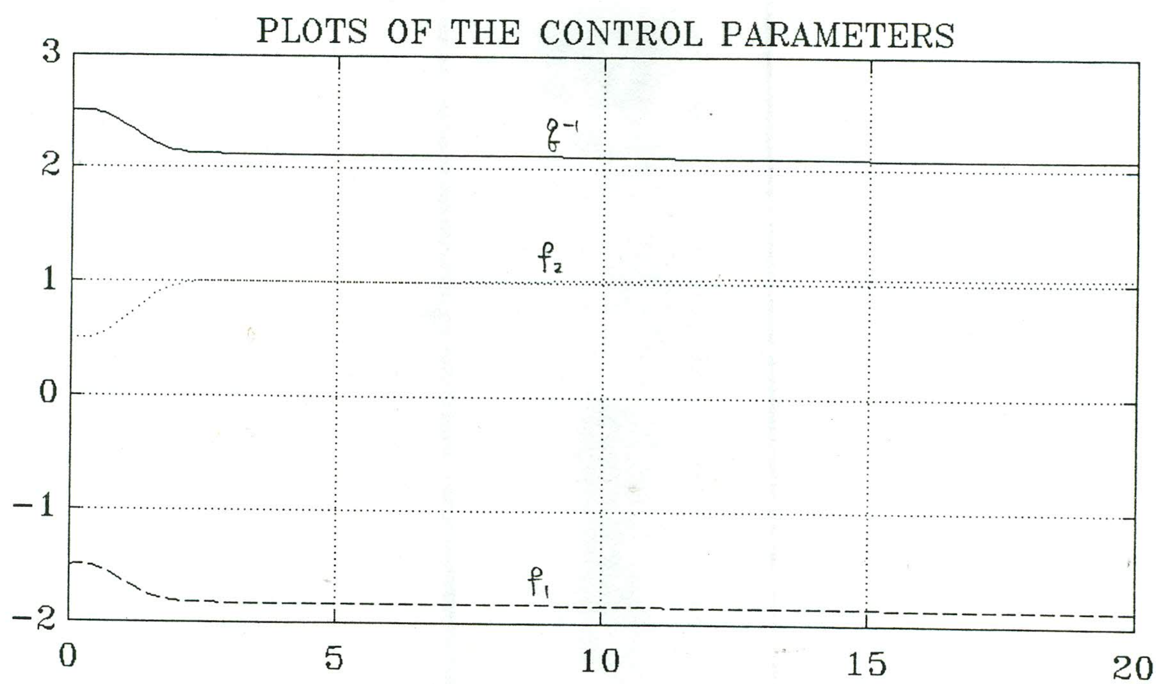
$e_{20} = -0.5$

$g_0^{-1} = 2.5$

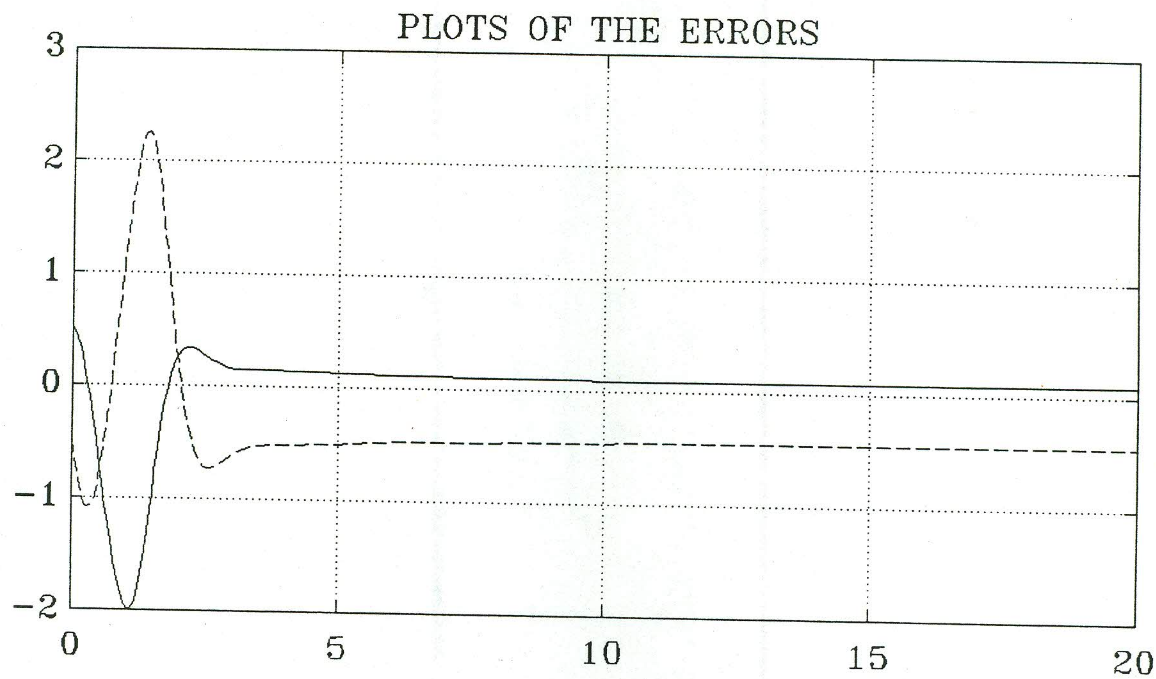
$f_{10} = -1.5$

$f_{20} = 0.5$

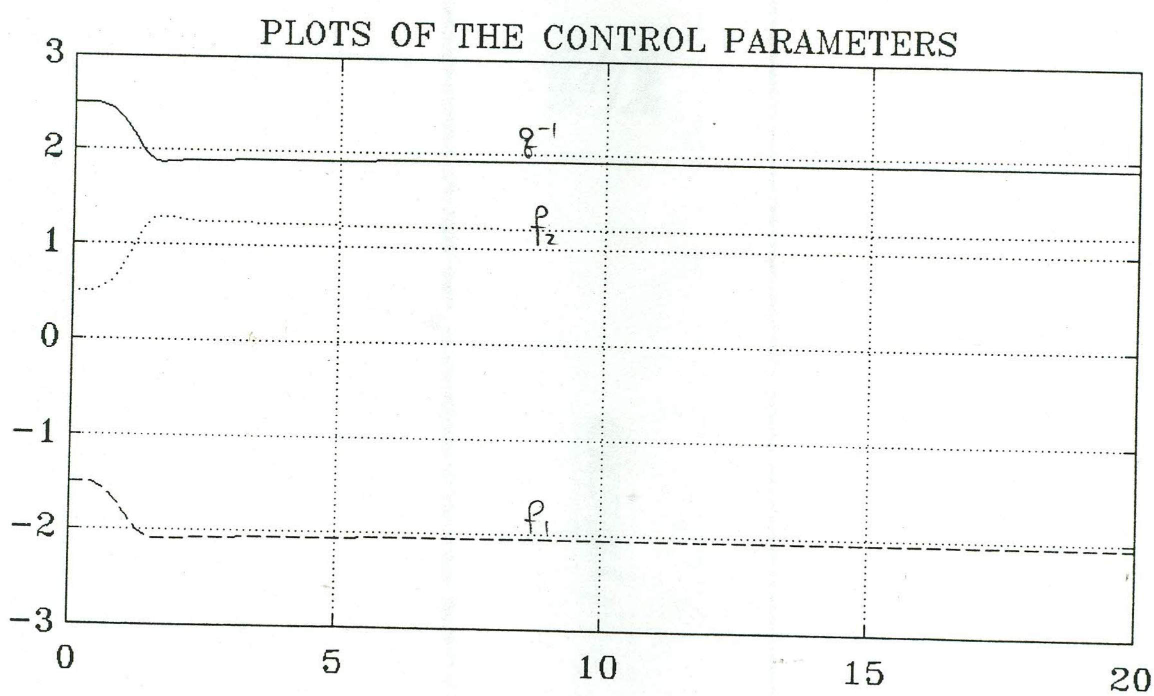




INPUT u=0.2cos(.33t)+0.23sin(−.132t)



TIME IN SEC



INPUT u=2

Problem 4.1 A linear time-invariant system has the transfer function.

$$W(s) = \frac{b_1 s + b_2}{s^2 + a_1 s + a_2}$$

Represent the plant in the following canonical forms:

(a) $\dot{x} = Ax + bu$; $y = h^T x$ where $a = [a_1, \cdots, a_n]^T$,

$$A = \left[-a \,\middle|\, \frac{I}{0} \right]$$

and $h^T = [1, 0, \cdots, 0]$.

Answer : $A = \begin{bmatrix} -a_1 & 1 \\ -a_2 & 0 \end{bmatrix}$, $b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

(b) h is the same as in (a), $\alpha = [\alpha_1, \cdots, \alpha_n]^T$,

$$A = \left[-\alpha \,\middle|\, \begin{matrix} 1 & 1 & \cdots & 1 \\ & \Lambda & & \end{matrix} \right]$$

where $\Lambda = \lambda I$.

Answer : $A = \left[\begin{array}{c|c} -(a_1+\lambda) & 1 \\ -a_2 & \lambda \end{array} \right]$, $b = \begin{bmatrix} b_1 \\ b_2 + b_1\lambda \end{bmatrix}$.

(c) nonminimal representation 1

Answer : choosing $\lambda = 1$ $\Lambda = -1$, $l = 1$,

$\Rightarrow R(s) = s+1$, $P(s) = b_1 s + b_2$,

$Q(s) = (s^2 + 2s + 1) - s^2 - a_1 s - a_2 = (2-a_1)s + (1-a_2)$

$\Rightarrow c_0 = b_1$, $\bar{c} = b_2 - b_1$, $d_0 = (2-a_1)$, $\bar{d} = a_2 - a_1 - 1$

Hence

$$\dot{x}_1 = -x_1 + [b_1, b_2 - b_1, (2-a_1), a_2 - a_1 - 1] \cdot (u, \omega_1, y_p, \omega_2)^T$$

$$\dot{\omega}_1 = -\omega_1 + u, \quad \dot{\omega}_2 = -\omega_2 + y_p, \quad y_p = x_1$$

Problem 4.1 (d) nonminimal representation 2.

Choosing $F = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$, $g = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\dot{\omega}_1 = F\omega_1 + g u$$

$$\dot{\omega}_2 = F\omega_2 + g y_p$$

$$y_p = c^T\omega_1 + d^T\omega_2$$

$$\Rightarrow \dot{\omega}_2 = F\omega_2 + g c^T\omega_1 + g d^T\omega_2$$

$$= (F + g d^T)\omega_2 + g c^T \omega_1$$

$$\Rightarrow \begin{pmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \end{pmatrix} = \begin{bmatrix} F & 0 \\ g c^T & F + g d^T \end{bmatrix} \cdot \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix} + \begin{pmatrix} g \\ 0 \end{pmatrix} u.$$

$$y_p = c^T\omega_1 + d^T\omega_2$$

$$W_p(s) = [c^T, d^T] \cdot \begin{bmatrix} sI - F & 0 \\ -g c^T & sI - F - g d^T \end{bmatrix}^{-1} \cdot \begin{bmatrix} g \\ 0 \end{bmatrix}$$

$$= [c^T, d^T] \cdot \begin{bmatrix} (sI-F)^{-1} & 0 \\ +(sI-F-gd^T)^{-1} g c^T (sI-F)^{-1} & (sI-F-gd^T)^{-1} \end{bmatrix} \cdot \begin{bmatrix} g \\ 0 \end{bmatrix}$$

$$= [I + d^T(sI-F-gd^T)^{-1} g] \cdot c^T (sI-F)^{-1} g$$

$$= [I + d^T(sI-F)^{-1} g]^{-1} \cdot c^T(sI-F)^{-1} g$$

Note: $(sI-F)^{-1} g = \dfrac{\begin{pmatrix} 1 \\ s+1 \end{pmatrix}}{(s+1)^2}$, Let $d^T = [d_1, d_2]$, $c^T = [c_1, c_2]$

we have

$$W_p(s) = \frac{c_2 s + (c_1 + c_2)}{s^2 + (d_2+2)s + (d_1+d_2+1)} = \frac{b_1 s + b_2}{s^2 + a_1 s + a_2}$$

$$\Rightarrow c_1 = b_2 - b_1, \quad c_2 = b_1$$

$$d_1 = a_2 - a_1 + 1, \quad d_2 = a_1 - 2$$

Problem 4.2 Determine an adaptive observer for identifying

(a) the unknown parameter vectors a and b in Problem 1 (a).

Choose : $K = \begin{bmatrix} -2 & 1 \\ -1 & 0 \end{bmatrix}$, $d^T = [1, d_2]$

$$h^T(sI-K)^{-1}d = (1 \quad 0)\cdot \begin{bmatrix} s+2 & -1 \\ 1 & s \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 \\ d_2 \end{bmatrix}$$

$$= (1 \quad 0)\cdot \begin{bmatrix} s & 1 \\ -1 & s+2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ d_2 \end{bmatrix} \Big/ (s^2+2s+1)$$

$$= \frac{s+d_2}{s^2+2s+1}$$

Choose : $d_2 = 1$, $\Rightarrow h^T(sI-K)^{-1}d$ is S.P.R.

Let

$$G_1 = \frac{s}{s+1}, \qquad G_2 = \frac{1}{s+1}$$

$$G(s) = \begin{bmatrix} \frac{s}{s+1} \\ \frac{1}{s+1} \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0 & -1 \\ 0 & 1 \end{bmatrix}$$

Hence, we have $[\phi(t) = g(t) - g,\ g = k - a,\ \psi(t) = b(t) - b]$

$$\dot{e} = Ke + \phi y_p + \psi u + v_1(t) + v_2(t), \quad e_1 = h^T e$$

$$\dot{\psi} = -e_1 \omega^1, \qquad \omega^1 = G(s)u(t)$$

$$\dot{\phi} = -e_1 \omega^2 \qquad \omega^2 = G(s) y_p(t)$$

$$v_1(t) = [0, A_2 \omega^1]^T \dot{\psi} = \begin{bmatrix} 0 & 0 \\ -\omega_2^1 & \omega_2^1 \end{bmatrix} \cdot (-e_1 \omega^1) = \begin{bmatrix} 0 \\ -e_1({\omega_2^1}^2 - \omega_1^1 \omega_2^1) \end{bmatrix}$$

$$v_2(t) = \begin{bmatrix} 0 \\ -e_1({\omega_2^2}^2 - \omega_1^2 \omega_2^2) \end{bmatrix}$$

Problem 4.2 (a) Simulation Equations.

Choose : $b_1 = 1$, $b_2 = 3$, $a_1 = 5$, $a_2 = 6$

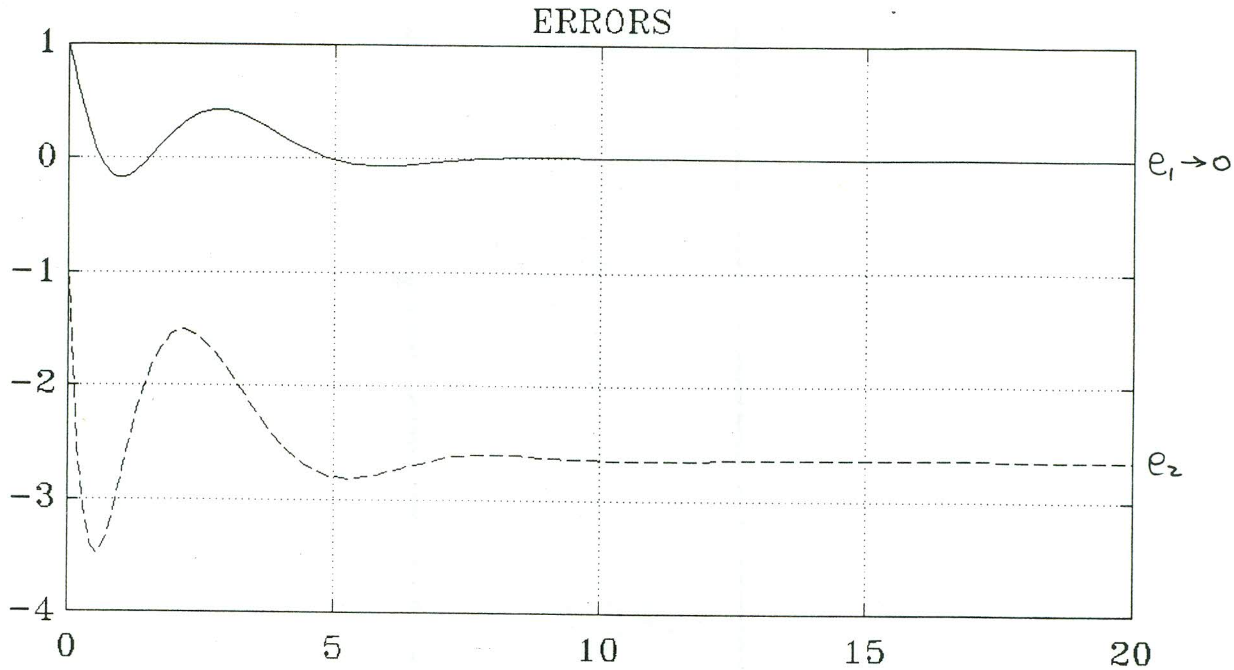
$\Rightarrow g_1 = -3$, $g_2 = -5$

$\dot{x}_1 = -5x_1 + x_2 + 1u$	$x(1) = x_1$	
$\dot{x}_2 = -6x_1 + 3u$	$x(2) = x_2$	
$\dot{x}_3 = -x_3 + u$, $\omega_1^1 = -x_3 + u$,	$x(3) = x_3$	
$\cancel{\dot{\omega}_2^1 = \omega_2^1 + u}$,	$x(4) = x_3$	
$\dot{x}_5 = -x_5 + x_1$, $\omega_1^2 = -x_5 + x_1$,	$x(5) = x_5$	
$\cancel{\dot{\omega}_2^2 = -\omega_2^2 + x_1}$	$x(6) = x_5$	
$\dot{\psi}_1 = -e_1\omega_1^1$	$x(7) = \psi_1$	— +1
$\dot{\psi}_2 = -e_1\omega_2^1$	$x(8) = \psi_2$	-- +3
$\dot{\phi}_1 = -e_1\omega_1^2$	$x(9) = \phi_1$	-· −3
$\dot{\phi}_2 = -e_1\omega_2^2$	$x(10) = \phi_2$	··· −5
$\dot{e}_1 = -2e_1 + e_2 + \phi_1 x_1 + \psi_1 u$	$x(11) = e_1$	····
$\dot{e}_2 = -e_1 + \phi_2 x_1 + \psi_2 u - e_1({\omega_2^1}^2 - \omega_1^1\omega_2^1) - e_1({\omega_2^2}^2 - \omega_1^2\omega_2^2)$	$x(12) = e_2$	—

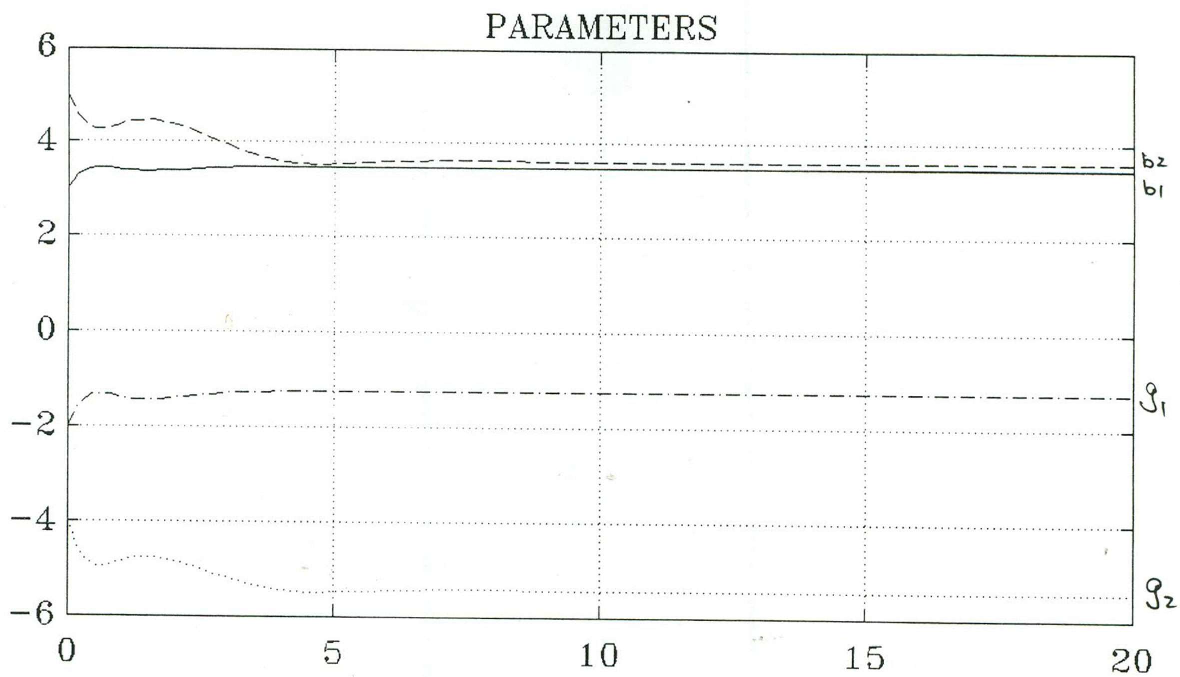
initial condition = Random number

p42a1.met

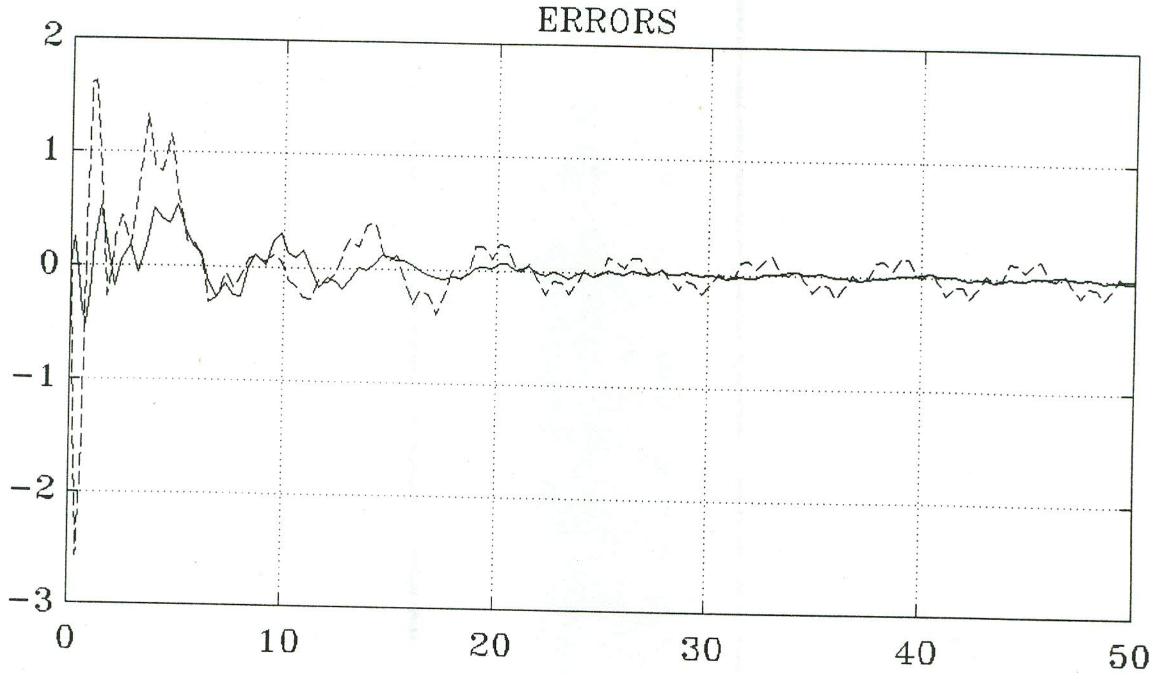
p42a2.met



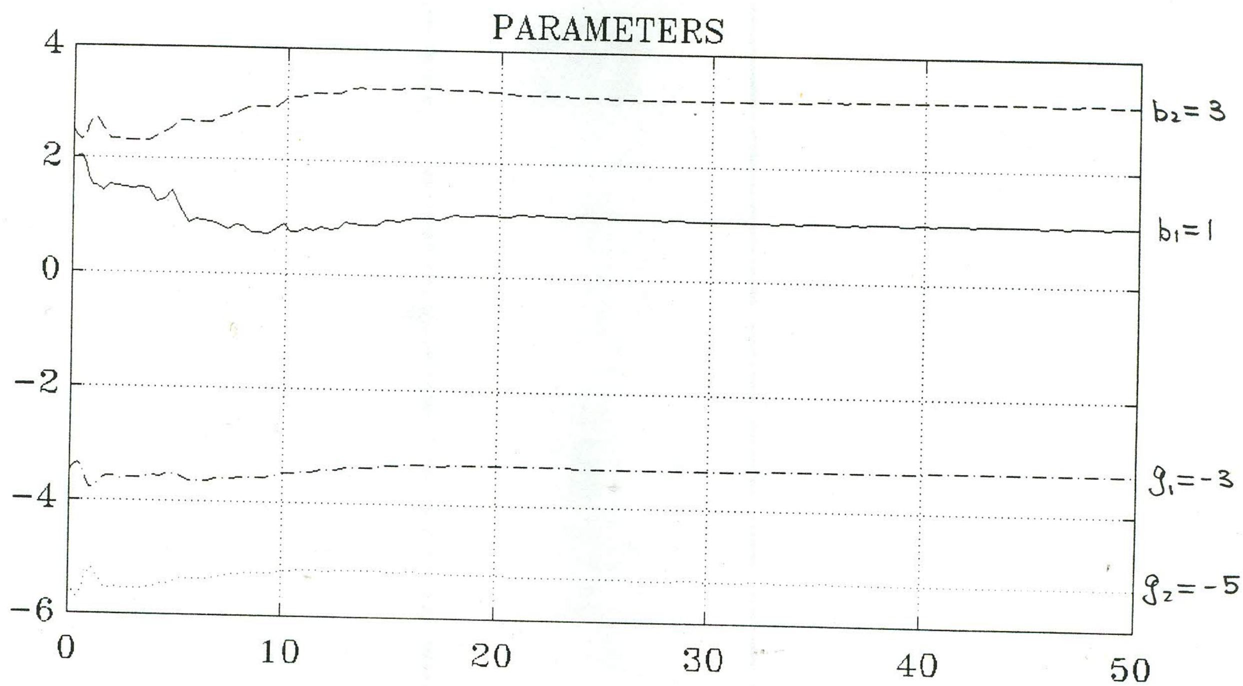
u=1



TIME IN SECONDS



u=sin(t)+2sin(2t)



TIME IN SECONDS

Problem 4.2 (b) the unknown parameters in Problem 1 (d)

Recall $F = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}, \quad g = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$c_1 = b_2 - b_1 = 9 - 4 = 5, \quad c_2 = b_1 = 4$$

$$d_1 = a_2 - a_1 + 1 = 6 - 5 + 1 = 2, \quad d_2 = a_1 - 2 = 3$$

Adaptive Laws: $e_1 = \hat{y}_p - y_p$

$$\dot{c}_1(t) = -e_1\omega_1^1, \quad \dot{c}_2(t) = -e_1\omega_1^2, \quad \dot{d}_1(t) = -e_1\omega_2^1, \quad \dot{d}_2(t) = -e\omega_2^2.$$

Simulation Equations:

Plant:

$$\dot{x}_1 = -5x_1 + x_2 + 4u \qquad x(1)$$

$$\dot{x}_2 = -6x_1 + 9u, \quad y_p = x_1 \qquad x(2)$$

Observer:

$$\dot{\omega}_1^1 = -\omega_1^1 + \omega_1^2 \qquad x(3)$$

$$\dot{\omega}_1^2 = -\omega_1^2 + u. \qquad x(4)$$

$$\dot{\omega}_2^1 = -\omega_2^1 + \omega_2^2 \qquad x(5)$$

$$\dot{\omega}_2^2 = -\omega_2^2 + y_p \qquad x(6)$$

Error:

$$e_1 = \hat{y}_p - y_p = c_1\omega_1^1 + c_2\omega_1^2 + d_1\omega_2^1 + d_2\omega_2^2 - y_p$$

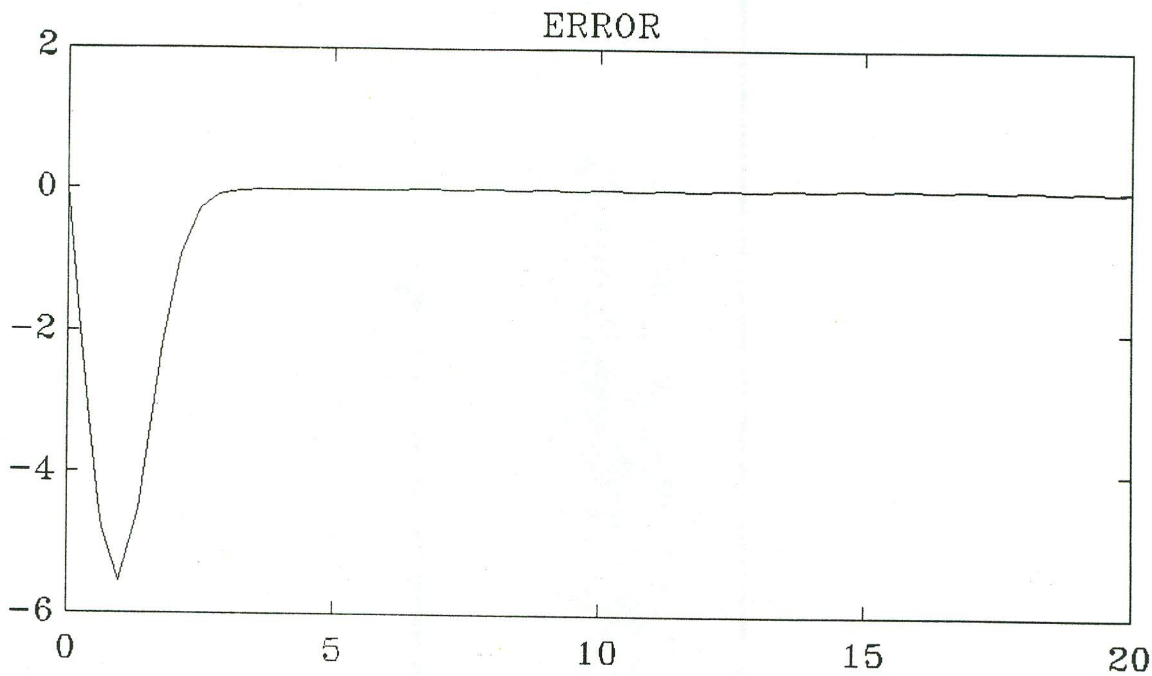
Adaptive law:

$$\dot{c}_1(t) = -e_1\omega_1^1. \qquad x(7)$$

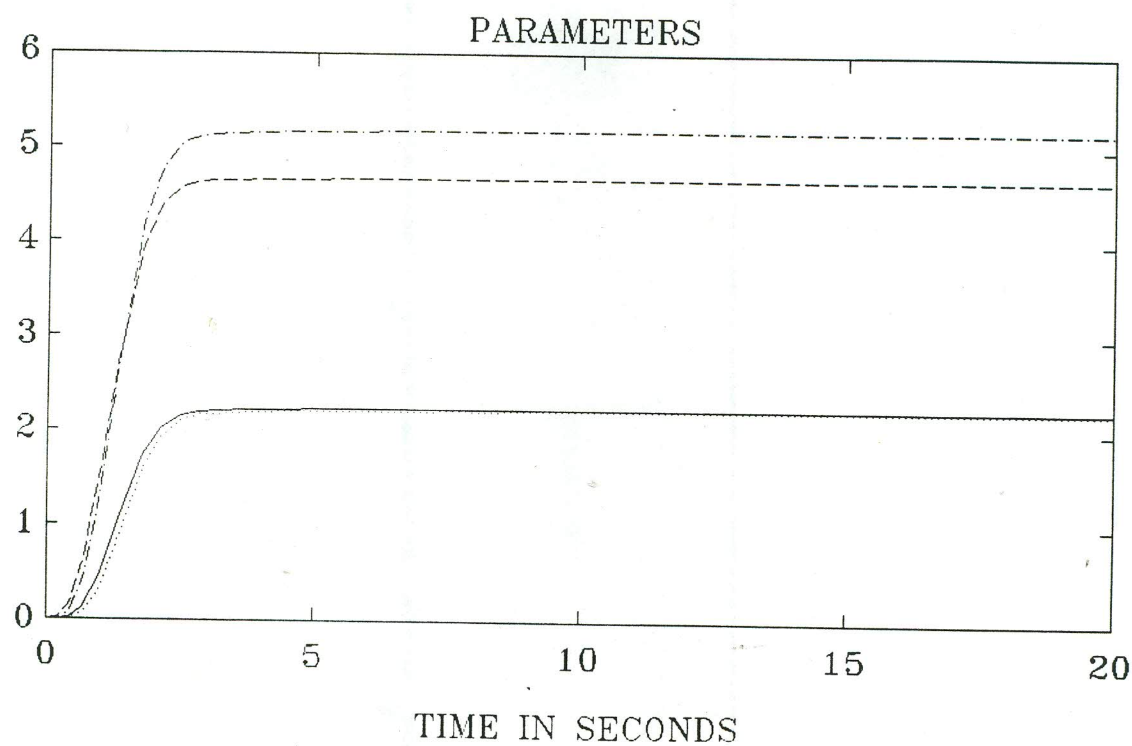
$$\dot{c}_2(t) = -e_1\omega_1^2 \qquad x(8)$$

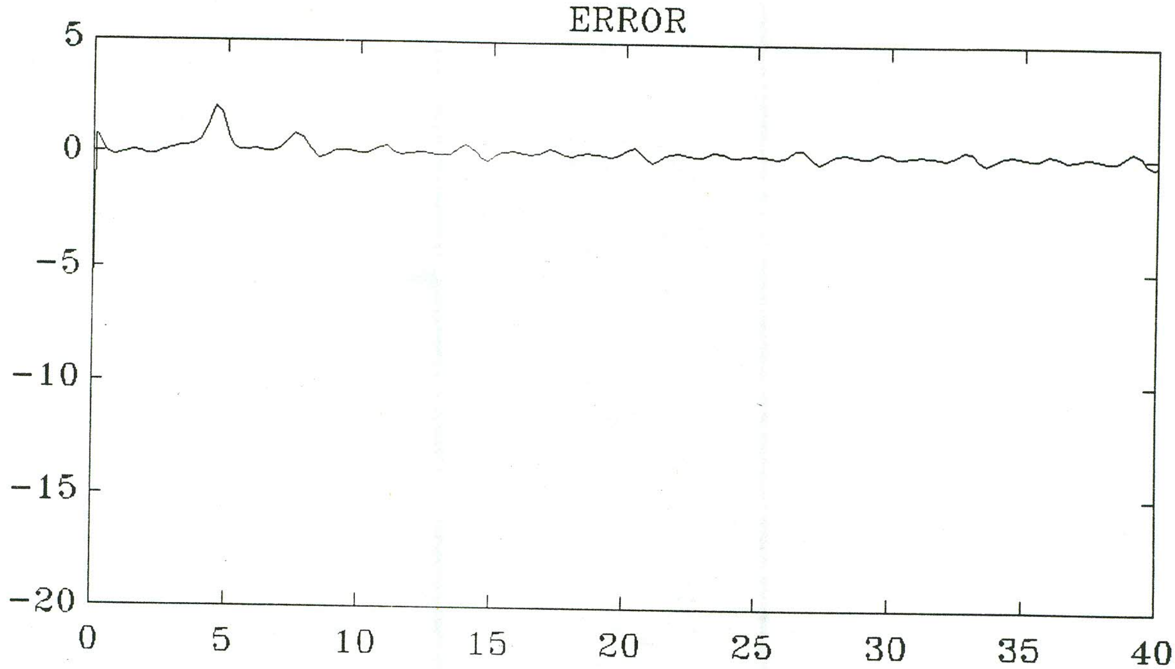
$$\dot{d}_1(t) = -e_1\omega_2^1 \qquad x(9)$$

$$\dot{d}_2(t) = -e_1\omega_2^2 \qquad x(10)$$

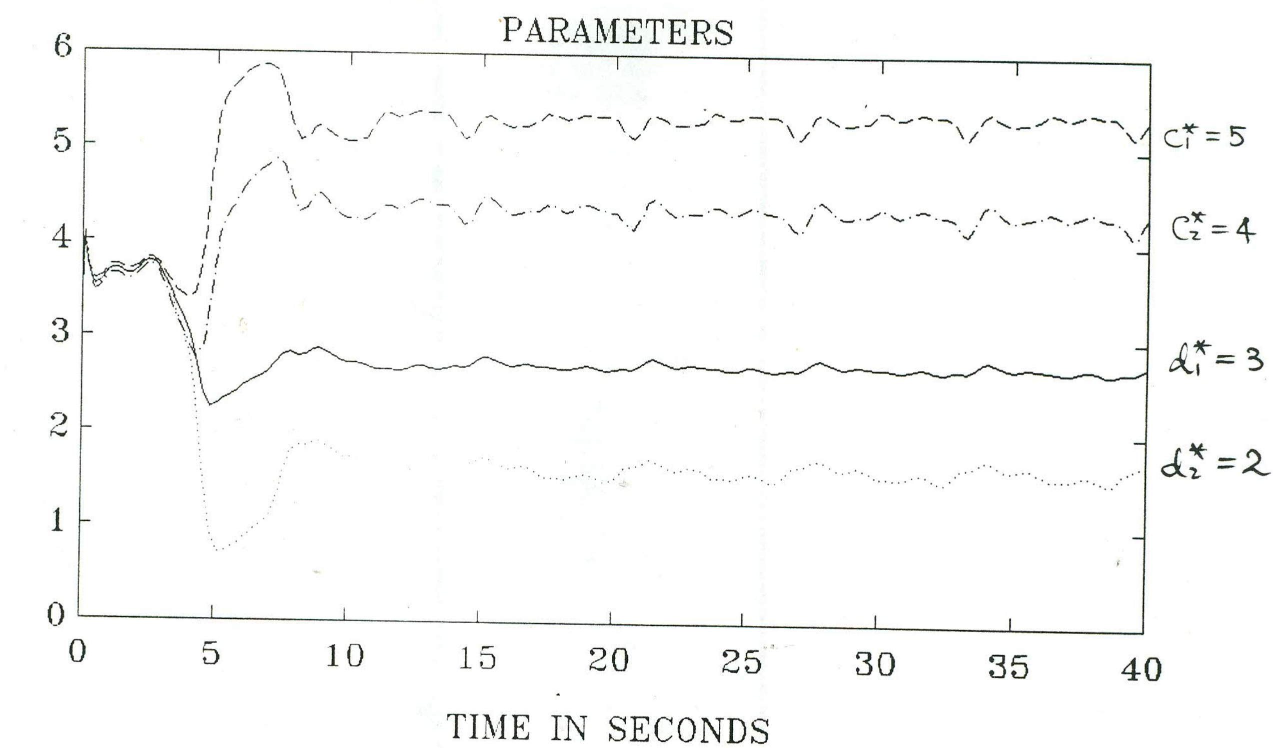

ERROR
2
0
−2
−4
−6
0
5
10
15
20

u=1


PARAMETERS
6
5
4
3
2
1
0
0
5
10
15
20
TIME IN SECONDS



$u=3\sin(t)+.5\cos(2t)+.4\sin(3t)-.7\cos(4t)$



Problem 4.3 Determine the structure of an adaptive observer to estimate the unknown parameters a_1 and a_2 of the transfer function $W(s)$ in Problem 1 when b_1 and b_2 are known.

Answer: We can use the same structure as we did in Problem 4.2 (b). But, since, we know b_1 and b_2, we don't have to adapt the parameter c_1 and c_2.

Simulation Equation:

Plant:

x(1) $\dot{x}_1 = -5x_1 + x_2 + 4u.$

x(2) $\dot{x}_2 = -6x_1 + 9u, \quad y_p = 5\omega_1^1 + 4\omega_1^2 + 2\omega_2^1 + 3\omega_2^2$

Observer:

x(3) $\dot{\omega}_1^1 = -\omega_1^1 + \omega_1^2$

x(4) $\dot{\omega}_1^2 = -\omega_1^2 + u$

x(5) $\dot{\omega}_2^1 = -\omega_2^1 + \omega_2^2$

x(6) $\dot{\omega}_2^2 = -\omega_2^2 + y_p$

Error

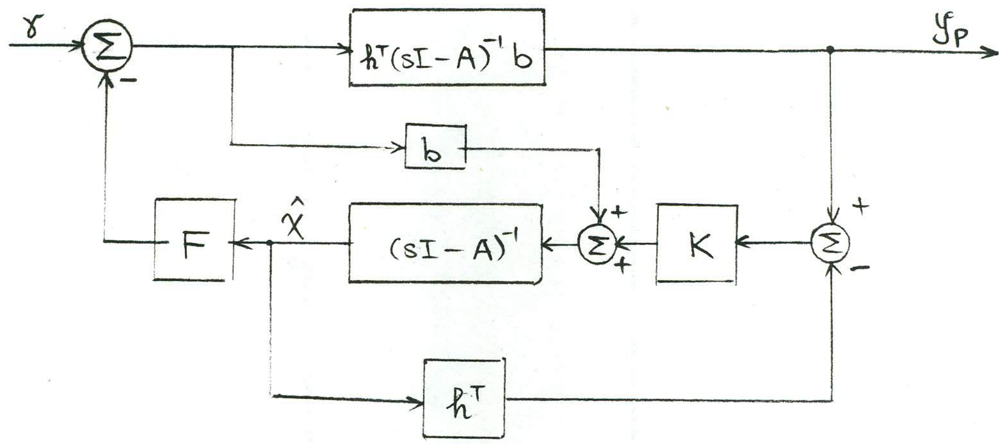
$$e_1 = 5\omega_1^1 + 4\omega_1^2 + d_1(t)\omega_2^1 + d_2(t)\omega_2^2 - y_p$$

Adaptation law

x(7) $\dot{d}_1(t) = -e_1\omega_2^1$

x(8) $\dot{d}_2(t) = -e_1\omega_2^2$

Problem 5.1 Considering the observer based controller



Due to the well-known separation principle, (and in fact it is easy to prove it), the poles of overall system are

$\lambda(A-bF)$ and $\lambda(A-Kh^T)$.

And if (A,b) is complete controllable, $\lambda(A-bF)$ can be arbitrarily assigned in any location.

Proof: (A,b) is controllable $\Rightarrow$ $\exists$ transformation $\ni$ A, b are in controllable canonical form, i.e.,

$$A = \left[\begin{array}{cc} 0 & I \\ \hline \multicolumn{2}{c}{a} \end{array}\right], \quad b = \left[\begin{array}{c} 0 \\ \hline 1 \end{array}\right]$$

$$A - bF = \left[\begin{array}{cc} 0 & I \\ \hline \multicolumn{2}{c}{a - f} \end{array}\right]$$

Hence, one can choose f such that $\lambda(A-bF)$ can be at any desired locations. Now. if (A, h^T) is observable $\Rightarrow$ (A^T, h) is controllable $\Rightarrow$ Q.E.D.

Problem 5.2 (a)

Let $W_p(s) = \frac{Z_p(s)}{R_p(s)}$

with $Z_p(s)$ is of order m, and $R_p(s)$ is of order n

Consider a feedforward compensator with T.F

$$C_{FF}(s) = \frac{Z_{FF}(s)}{R_{FF}(s)} \quad \begin{matrix} \# \; m_f \\ \# \; n_f \end{matrix}$$

Then $W_p(s)\, C_{FF}(s) = \frac{Z_p(s)}{R_p(s)} \cdot \frac{Z_{FF}(s)}{R_{FF}(s)}$

The relative degree of $W_p(s)\, C_{FF}(s)$

$$= n - m + n_f - m_f = n^* + n_f - m_f \geq n^*$$

Then we considers the feedback path compensator

with. T. F. (strictly proper, i.e., $n_F - m_F \geq 1$)

$$C_{FB}(s) = \frac{Z_{FB}(s)}{R_{FB}(s)} \quad \begin{matrix} \# \; m_F \\ \# \; n_F \end{matrix}$$

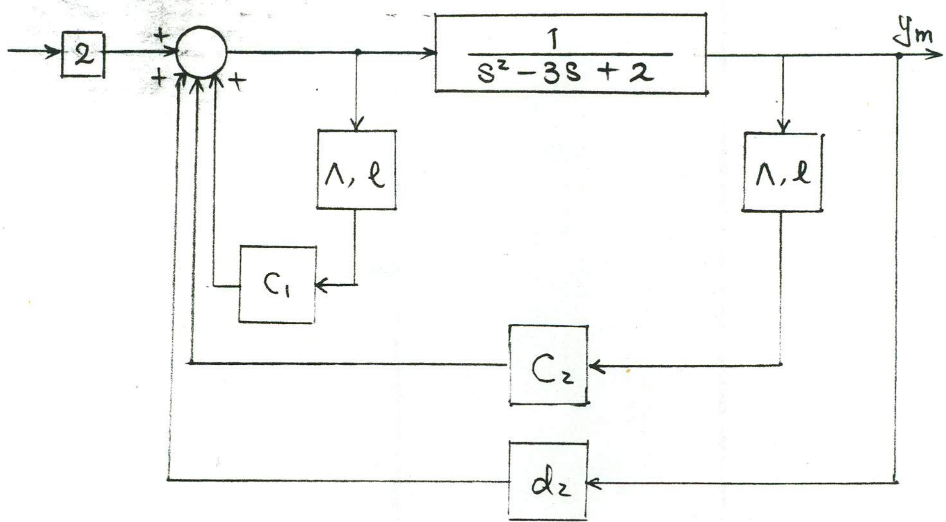
The T.F. of closed-loop system

$$H(s) = \frac{W_p(s)}{1 - W_p(s)\, C_{FB}(s)}$$

$$= \frac{R_{FB}(s) \cdot Z_p(s)}{R_p(s) \cdot R_{FB}(s) - Z_p(s) \cdot Z_{FB}(s)} \quad \begin{matrix} \# \; n_F + m \\ \# \; n + n_F \end{matrix}$$

relative degree $= n + n_F - n_F - m = n^*$. Q.E.D.

Problem 5.2 (b) Consider the configuration following



Let $\Lambda = -1 \Rightarrow \lambda(s) = s+1$, $\ell = 1$

T.F of feedforward controller $= \dfrac{s+1}{s+(1-c_1)}$

T.F. of feedback controller $= \dfrac{c_2 + d_2(s+1)}{s+1}$

$$W_m(s) = \frac{s+1}{(s^2-3s+2)(s+1-c_1) - c_2 - d_2(s+1)}$$

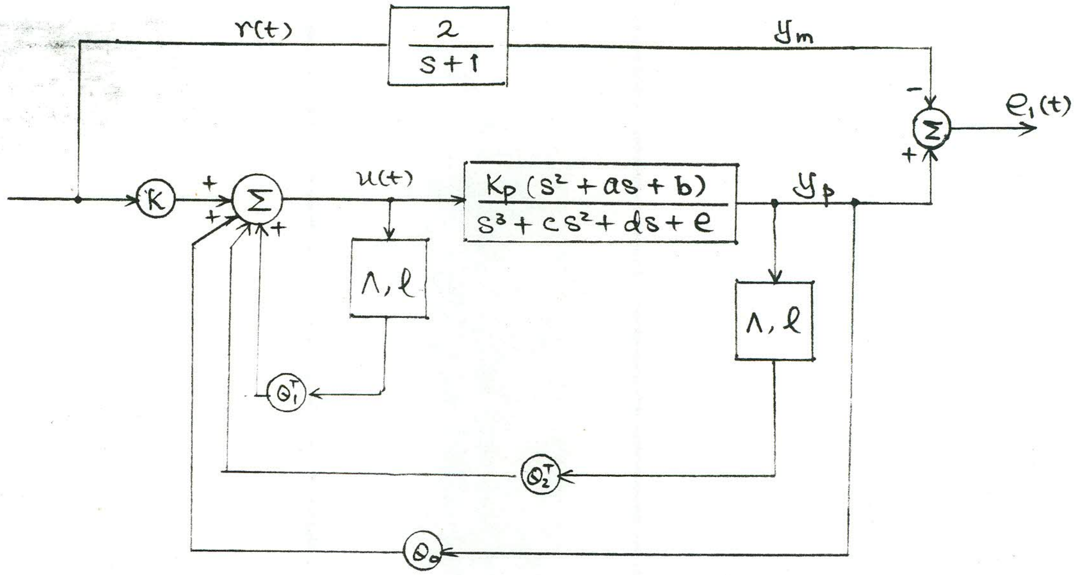
$$= \frac{s+1}{s^3 + (1-c_1-3)s^2 + (2-3+3c_1-d_2)s + (d_2 - c_2 - 2c_1 + 2)}$$

$$\Rightarrow s^3 + (-c_1-2)s^2 + (3c_1 - d_2 - 1)s + (-2c_1 - c_2 - d_2 + 2)$$

$$= s^3 + 4s^2 + 5s + 2$$

$$\Rightarrow \underline{c_1 = -6, \quad d_2 = -24, \quad c_2 = 36.}$$

Problem 5.3 (a)



Adaptive Laws:

$$\dot{k}(t) = -\operatorname{sgn}(k_p)\, e_1(t)\, r(t)$$

$$\dot{\theta}_0(t) = -\operatorname{sgn}(k_p)\, e_1(t)\, y_p(t)$$

$$\dot{\theta}_1(t) = -\operatorname{sgn}(k_p)\, e_1(t)\, \omega_1(t)$$

$$\dot{\theta}_2(t) = -\operatorname{sgn}(k_p)\, e_1(t)\, \omega_2(t).$$

where

$$\dot{\omega}_1(t) = \Lambda \omega_1(t) + l\left(\theta^T(t)\omega(t)\right)$$

$$\dot{\omega}_2(t) = \Lambda \omega_2(t) + l\, y_p$$

Λ is an $(n-1)\times(n-1)$ stable matrix, (Λ, l) controllable.

$$\theta^T(t) = [k(t),\ \theta_1^T(t),\ \theta_0(t),\ \theta_2^T(t)]$$

$$\omega^T(t) = [r(t),\ \omega_1^T(t),\ y_p(t),\ \omega_2^T(t)].$$

Problem 5.3 (b)

(1) The assumption of the sign of k_p is known is used in the adaptive laws. No adaptive laws can be obtained without the sign of k_p. (up to now?).

(2) The assumption of $Z_p(s) = s^2 + as + b$ being Hurwitz is also important. Otherwise, $Z_p(s)$ can not cancled out from $W_o(s)$ in (5.17). No RHP pole-zero cancelations are allowed in control system.

Problem 5.3 (c) Simulation

Let $a=2$, $b=1$, $c=6$, $d=12$, $e=8$, $k_p=2$

$$\Lambda = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}, \quad \ell = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

$$\Rightarrow \lambda(s) = s^2 + 2s + 1, \quad K^* = 1$$

$$C^*(s) = [\theta_1^{1*} \quad \theta_1^{2*}] \cdot \begin{bmatrix} s+1 & -1 \\ 0 & s+1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \theta_1^{2*} s + (\theta_1^{2*} - \theta_1^{1*})$$

$$D(s) = \theta_2^{2*} s + (\theta_2^{2*} - \theta_2^{1*}) + \theta_0^* (s^2 + 2s + 1)$$

$$= \theta_0^* s^2 + (2\theta_0^* + \theta_2^{2*}) s + (\theta_0^* + \theta_2^{2*} - \theta_2^{1*})$$

$$\Rightarrow \lambda(s) Z_p(s) = s^4 + 4s^3 + 6s^2 + 4s + 1$$

$$R_m(s)\lambda(s)Z_p(s) = s^5 + 5s^4 + 10s^3 + 10s^2 + 5s + 1$$

$$R_p(s)[\lambda(s) - C^*(s)] - k_p Z_p(s) D^*(s)$$

$$= (s^3 + 6s^2 + 12s + 8)(s^2 + (2+\theta_1^{2*})s + (1 + \theta_1^{2*} - \theta_1^{1*})) - 2(s^2 + 2s + 1)$$

$$\cdot [\theta_0^* s^2 + (2\theta_0^* + \theta_2^{2*}) s + (\theta_0^* + \theta_2^{2*} - \theta_2^{1*}$$

$$\Rightarrow \theta_0^* =$$

$$\theta_1^{*T} = [\qquad\qquad], \quad \theta_2^{*T} = [\qquad\qquad]$$

Simulation Equations:

Plant:

$$\dot{x}_1 = -6x_1 + x_2 + 2u$$

$$\dot{x}_2 = -12x_1 + x_3 + 4u$$

$$\dot{x}_3 = -8x_1 + 2u \quad , \quad y_p = x_1$$

Model:

$$\dot{x}_4 = -x_4 + r \quad , \quad y_m = x_4$$

Adaptation Law:

x_5 $\quad \dot{k}(t) = -e_1(t)\,r(t)$

x_6 $\quad \dot{\theta}_0(t) = -e_1(t)\,y_p(t)$

x_7 $\quad \dot{\theta}_1^1(t) = -e_1(t)\cdot\omega_1^1(t)$

x_8 $\quad \dot{\theta}_1^2(t) = -e_1(t)\cdot\omega_1^2(t)$

x_9 $\quad \dot{\theta}_2^1(t) = -e_1(t)\,\omega_2^1(t)$

x_{10} $\quad \dot{\theta}_2^2(t) = -e_1(t)\,\omega_2^2(t)$

Filtering:

x_{11} $\quad \dot{\omega}_1^1(t) = -\omega_1^1(t) + \omega_1^2(t)$

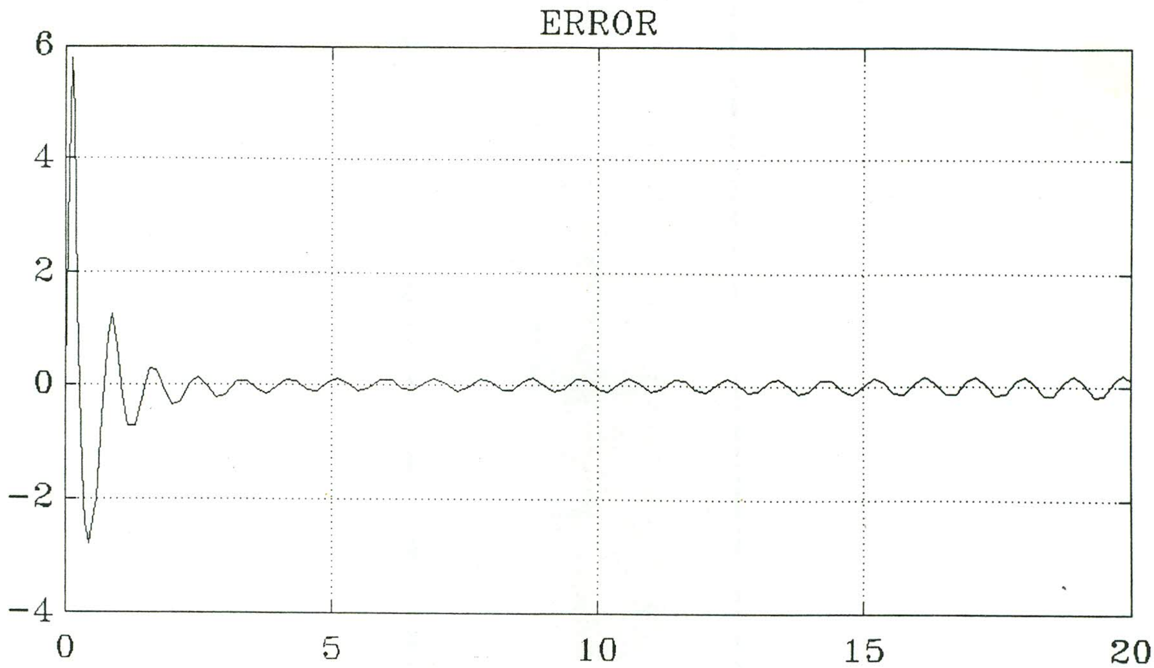
x_{12} $\quad \dot{\omega}_1^2(t) = -\omega_1^2(t) + k\cdot r + \theta_1^1\cdot\omega_1^1 + \theta_1^2\cdot\omega_1^2 + \theta_0\cdot y_p + \theta_2^1\cdot\omega_2^1 + \theta_2^2\cdot\omega_2^2$

x_{13} $\quad \dot{\omega}_2^1(t) = -\omega_2^1(t) + \omega_2^2(t)$

x_{14} $\quad \dot{\omega}_2^2(t) = -\omega_2^2(t) + y_p$

Remark:

$$u(t) = k(t)\cdot r(t) + \theta_1^1(t)\,\omega_1^1(t) + \theta_1^2(t)\,\omega_1^2(t) + \theta_0(t)\,y_p(t) + \theta_2^1(t)\,\omega_2^1(t) + \theta_2^2(t)\cdot\omega_2^2$$



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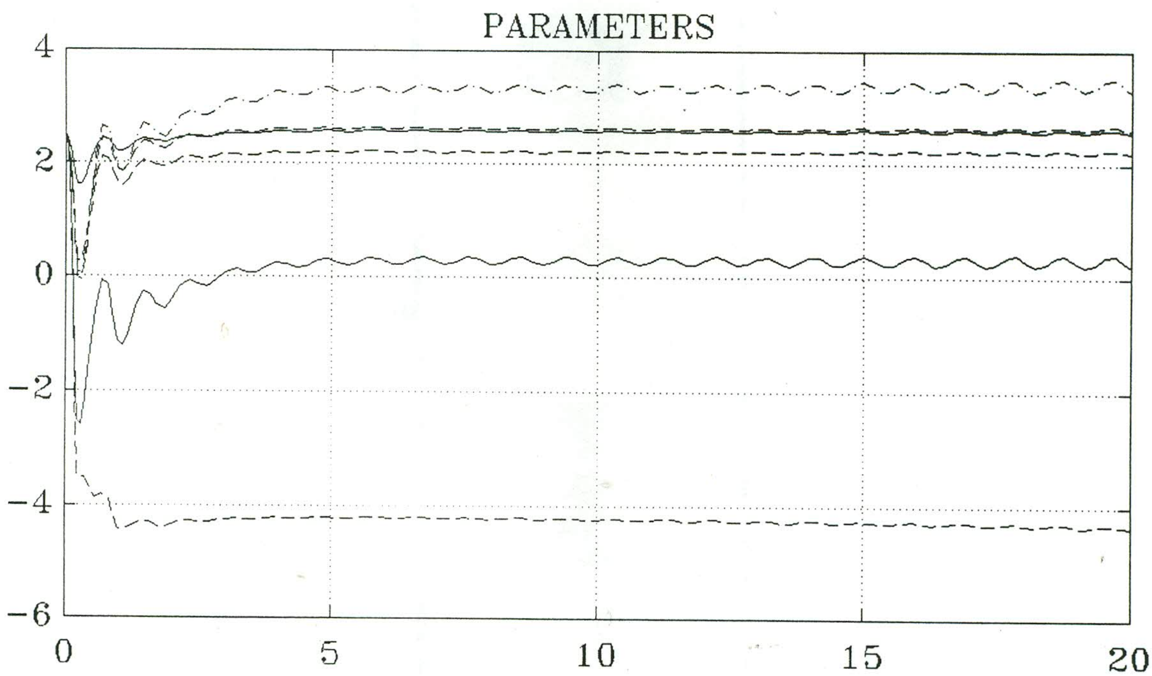
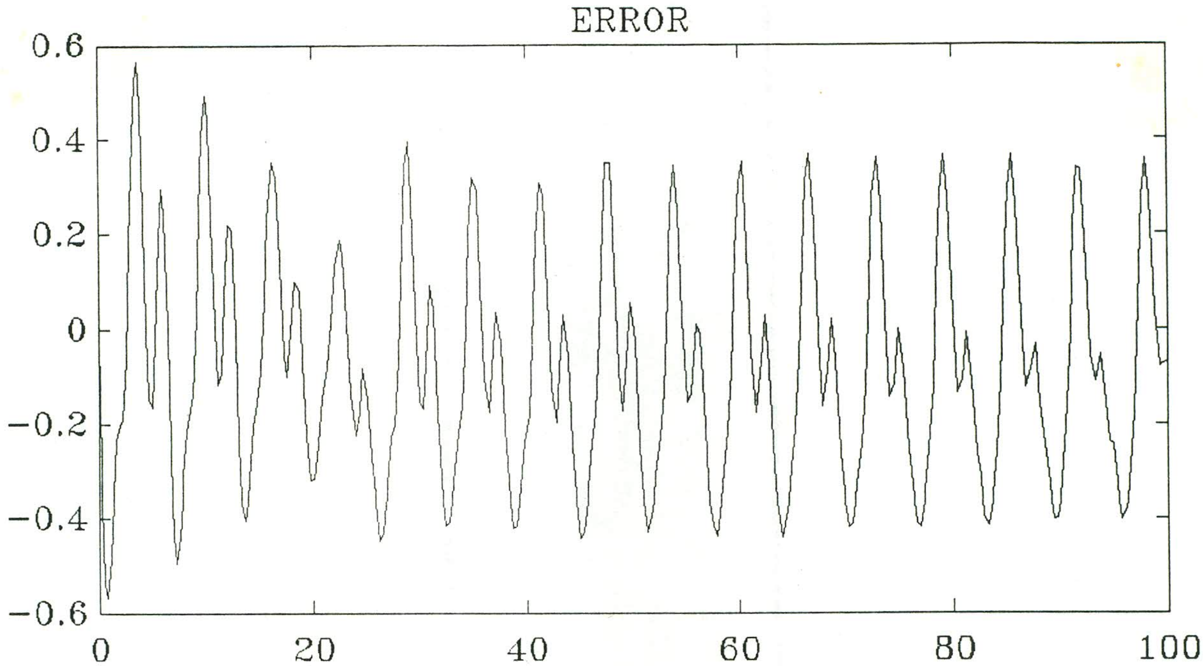


TIME IN SECONDS



r=.5cos(t)+.7sin(2t)+.9cos(3t)+.2sin(4t)