Ben M. Chen

Project #1 EE 489 Spring 1989/DUE: April 13, 1989

Refer to the problem of magnetic ball levitator given in the pages ~~45~~ 48 and ~~210~~ 233 of the text book. Also refer to Problem No. ~~4.24~~ 4.26.

a) Write the equations of motion for this setup, define the near ideal specifications of your control system.

b) What is the transfer function from u to e.

c) Of $u = -ke+u_d$, Draw the closed loop control system block-diagram. Where u_d is some constant disturbance of one order of magnitude less than u.

d) What is the characteristic equation? Obtain the root locus for varying values of power amplifier gain k. For what range of k is the system stable?

e) What kind of a Controller Design/Compensator Design do you propose to improve the stability and transient response? Prove your design. Assume linear relationship between i and u, during the transient state.

f) It is further desired that e_{ss}(step) should be very very small (close to .1mm accuracy). If the steady state error is not okay, design an appropriate compensator. Is the Integral-Square-Error-Performance Index (Is) optimal for the selected gain k?

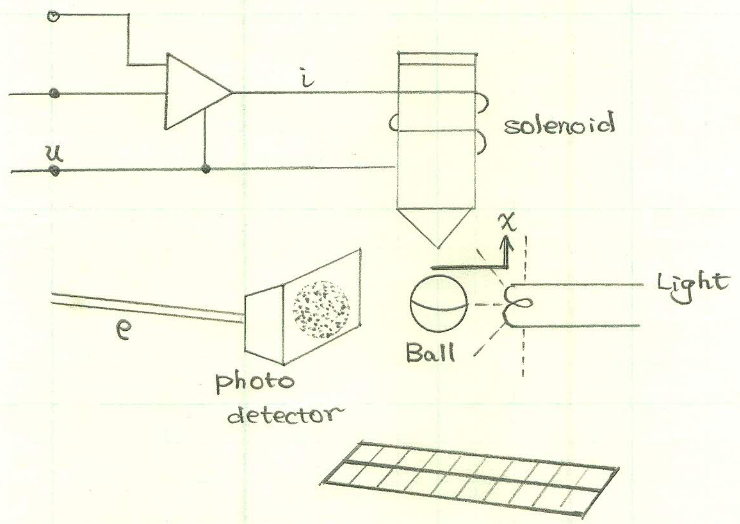
g) Repeat the steps 'e' and 'f' using Bode-Design Techniques, in terms of phase margin, gain margin and frequency response characteristics.

h) If $f = k_2 i + k_1 x$
Study the sensitivity of the <u>overall</u> <u>compensated</u> system with respect to k_1 and k_2. If the $S^T_{k_1}$ or $S^T_{k_2}$ depend on the input signal frequency - plot the same using MATLAB.

i) Use MATLAB, extensively for obtaining the root locus. Bode plots of the uncompensated and compensated systems. Also to obtain the step responses of the final designs to show that your design is meeting all the necessary near ideal specifications.

PROJECT ONE FOR EE 489 — Ben M. Chen

Consider the elementary magnetic suspension system:



Refer to equation (2.48) on page 45, we have

$$m\ddot{x} = f(x, i) - mg \qquad \text{------- (1)}$$

And from the givens in Problem 4.24 on page 211, we know

$$\left.\begin{aligned} m &= 20g = 0.02\ kg \\ e &= 100x \\ i &= u + V_0 \\ f(x, i) &\cong 0.5i + 20x \\ g &= 9.8\ N/Kg \end{aligned}\right\} \cdots (2)$$

a) Write the equations of motion for this setup, define the near ideal specifications of my control system.

Subtitute equation (2) in to (1), we obtain

$$0.02\ddot{x} = 0.5(u + V_o) + 20x - 0.02 \times 9.8$$

or

$$0.02\ddot{x} = 20x + 0.5u + (0.5V_o - 0.196)$$

Select $V_o = 0.392$ V so that the ball is in equilibrium at $x = 0$.

Then we have

$$\left.\begin{aligned} 0.02\ddot{x} &= 20x + 0.5u \\ e &= 100x \end{aligned}\right\} \cdots\cdots (3)$$

And I would like my closed-loop control system having following specifications:

1. Percent Overshot $\leq 20\%$
2. Settling time ($\pm 1\%$) $\leq .02$ sec.

b) What is the transfer function from u to e.

Taking Laplace transform on both sides of the both equations in (3),

$$\begin{cases} 0.02 \cdot S^2 X(S) = 20X(S) + 0.5U(S) \\ E(S) = 100X(S) \end{cases}$$

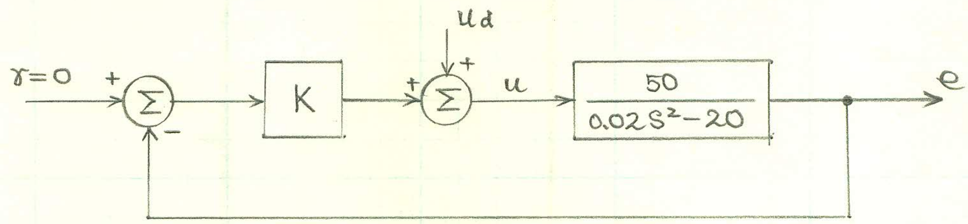
OR

$$\left\{ X(S) = \frac{0.5}{0.02S^2 - 20} U(S) \ ; \quad E(S) = 100X(S) \right.$$

Thus

$$G(S) = \frac{E(S)}{U(S)} = \frac{50}{0.02S^2 - 20}$$

c) Of $u = -ke + u_d$, Draw the closed loop control system block-diagram. Where u_d is some constant disturbance of one order of magnitude less than u.



d) What is the characteristic equation? Obtain the root locus for varying values of power amplifier gain k. For what range of K is the system stable?

Characteristic equation =

$$Q(S) = 1 + K \cdot \frac{50}{0.02S^2 - 20}$$

OR $\quad Q(S) = 0.02S^2 + (50K - 20)$

I would like to use Routh's table to check actual range of K such that the system is stable. And to check with the root locus obtained from MATLAB to verify.

S^2	0.02	$50K - 20$
S^1	0	
S^0	$50K - 20$	

Unfortuanitely, this system alway has two poles on $j\omega$ axis if $K > 0.4$, see root locus on next page.

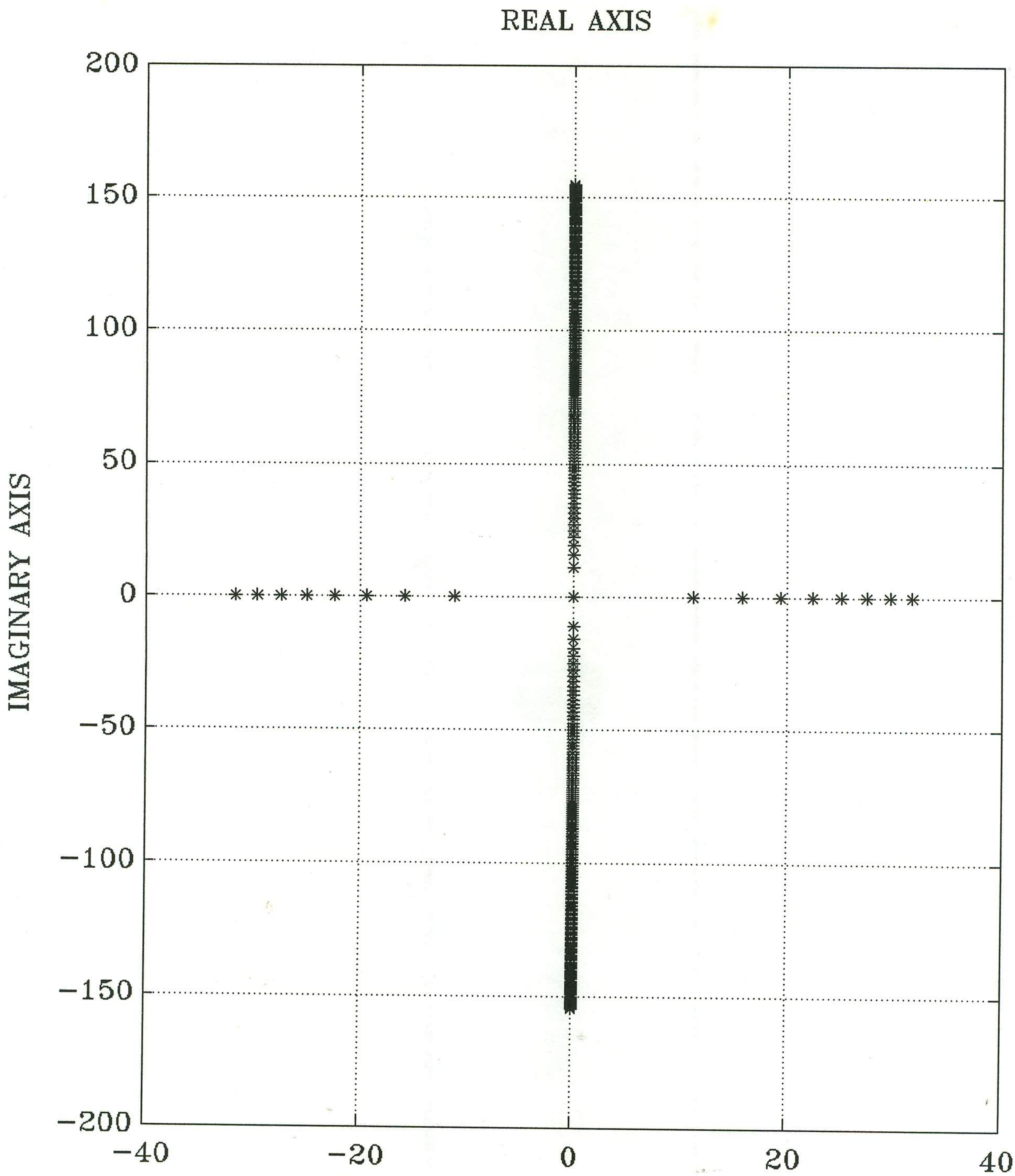


Fig. 1 : ROOT LOCUS FOR THE UNCOMPENSATED SYSTEM

e) <u>what kind of Controller Design / Compensator Design do you propose to improve the stability and transient response? Prove your design.</u>

I would like to design a PD controlle to my system:

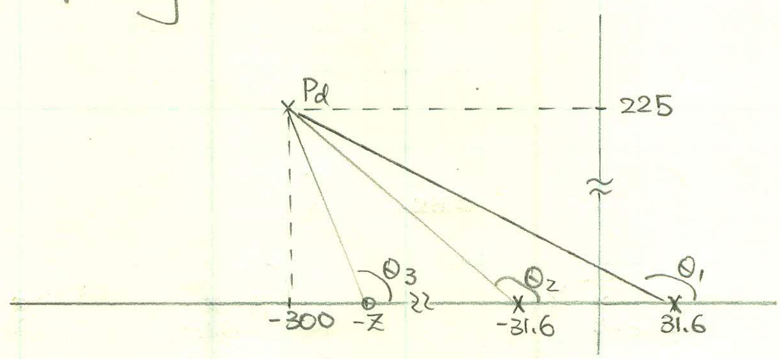
$$D(s) = K(s+z)$$

Select z such that my desired closed-loop poles will meet my specifications:

$$\zeta = 0.8 \quad \text{and} \quad \zeta\omega_n = 300 \Rightarrow \quad \omega_n = 375$$

$$P_{d,1,2} = -300 \pm j225$$

By inspecting



Computed we have

$$\theta_1 = 145.842° \qquad \theta_2 = 140.027°$$

$$\theta_1 + \theta_2 - \theta_3 = 145.842° + 140.027° - \theta_3 = -180°$$

$$\Rightarrow \theta_3 = 105.87°$$

Then, I have

$$\underline{z = 236.03}$$

<u>See Fig. 1 b) for the root locus on next page for proof.</u>

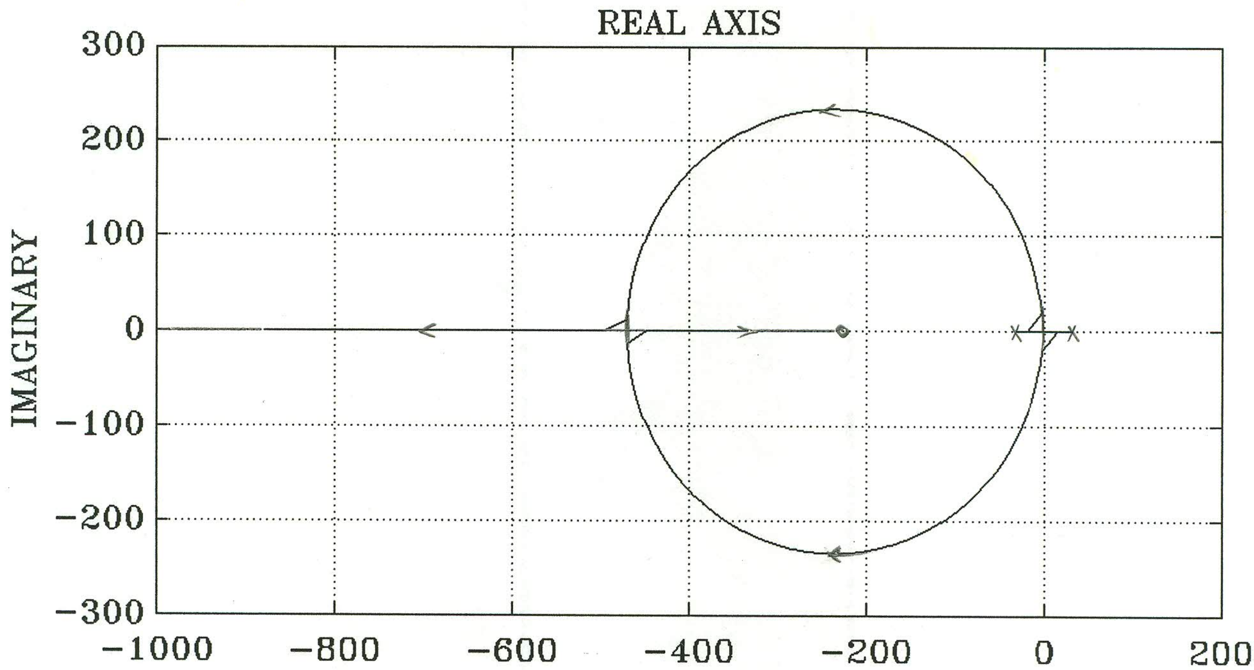


Fig 1 b) : ROOT LOCUS FOR THE PD CONTROLLED SYSTEM

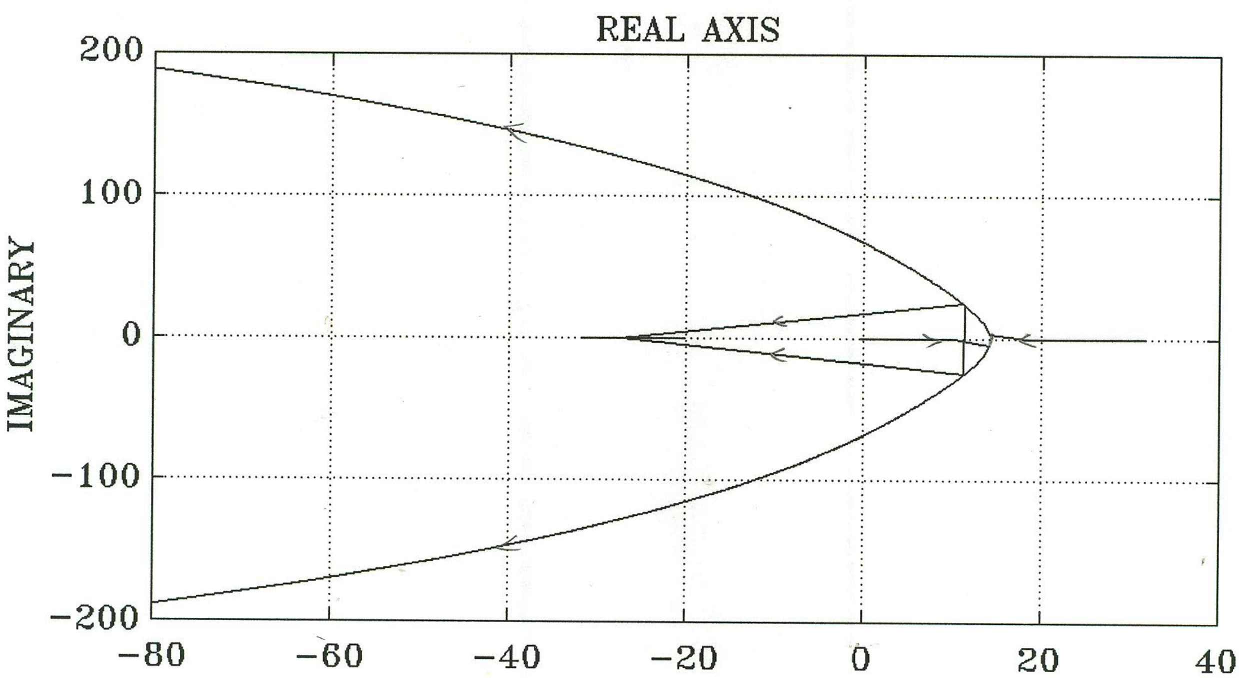
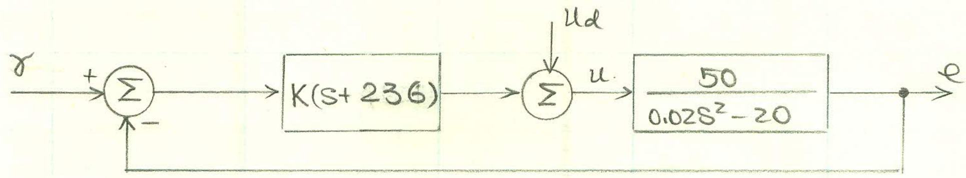


Fig 1 b) : ROOT LOCUS FOR THE COMPENDSATED SYSTEM

e) [CONT.]

Now my control system is



We are going to find value of K such that the closed loop poles at desired locations : $P_{d,1,2} = -300 \pm j\,225$

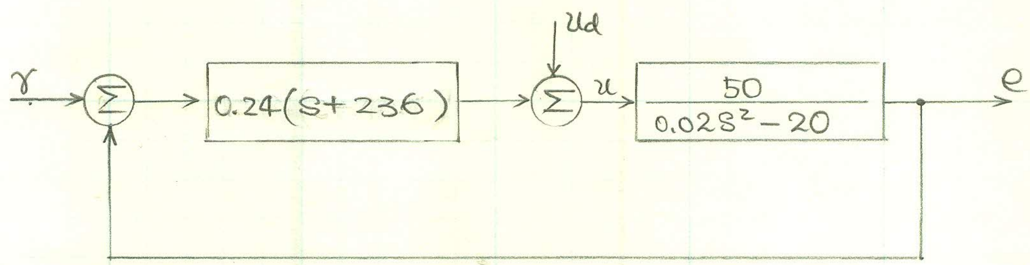
$$Q(s) = 0.02s^2 - 20 + 50Ks + 11800K$$

$$= 0.02\left(s^2 + 2500K \cdot s + 590000 \cdot K - 1000\right)$$

$$= 0.02\left(s^2 + 600s + 140625\right)$$

Thus , we obtain $\underline{K = 0.24}$

Then the final Control system will be



f) It is further desired that e_{ss} (step) should be very very small (close to .1 mm accuracy). If the steady error is not OK, design an appropriate compensator. Is the Integral-Square-Error-Performance-Index (I_s) optimal for the selected K?

Computing e_{ss} due to step input for the system given in e).

$$E(S) = \left[1 - \frac{600(s+236)}{S^2 + 600S + 140600} \right] \cdot \frac{1}{S}$$

$$e_{ss,e} = \lim_{S \to 0} S \cdot E(s) = 1 - 1.007 \doteq -0.007\,m = 7\,mm$$

But our desired steady error is

$$e_{ss,e'} = 0.1\ mm$$

$$\frac{e_{ss,e}}{e_{ss,e'}} = \frac{7}{0.1} = \underline{70}$$

* We see that $e_{ss,x}$ is far from what we desired. So, We need to design a lag compensator to improve the steady error

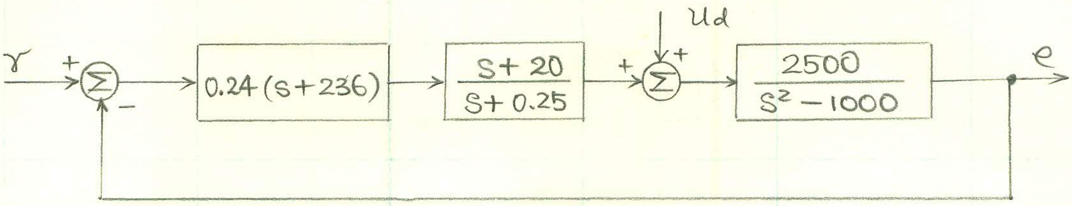
* For Lag Compensator. Let

$$D_L(S) = \frac{S+Z}{S+P} = \frac{S+20}{S+0.25}$$

Then the control system is given. as:

(see Fig 1 c) on page 6 for root locus for compensated system)

f) [CONT.]



$$G(s) = \frac{600\,(s+20)(s+236)}{(s+0.25)(s^2-1000)}$$

$$= \frac{600\,(s^2+256s+4720)}{s^3+0.25s^2-1000s-250}$$

$$T(s) = \frac{600\,(s^2+256s+4720)}{s^3+600.25s^2+152600s+2831750}$$

$$E^*(s) = [1-T(s)]\cdot R(s)$$

$$= \frac{s^3+0.25s^2-1000s-250}{s^3+600.25s^2+152600s+2831750}\cdot\frac{1}{s} \quad \cdots\cdots \quad (3)$$

$$-e_{ss} = +\frac{250}{2831750} = +0.00009\ m = +0.09\ mm < 0.1\ mm$$

for the step input, we see that the $|e_{ss}| = 0.09$ mm. Thus

$I_s = \infty$

So we can conclude that our selection of $K = 0.24$ is not optimal for Integral-Square-Error-Performance-Index.

g) Repeat the steps 'e' and 'f' using Bode Design Tech., in terms of phase margin, gain margin and frequency Response.

- For the original open-loop system we have

$$G(S) = \frac{2500}{S^2 - 1000}$$

Refer to Fig. 3 a) for Bode plots for this system, We see that there are no Gain margin and phase margin for this system. That means this system could not be stable with proportional control, which agrees, of course, with root locus.

- For the PD controlled system, but uncompensated, the

$$G(S) = \frac{50S + 11800}{0.02S^2 - 20}$$

Refer to Fig 3 b) for the plots. we see that the phase plot will never crossover $-180°$. Thus Gain Margin is undefined. But we can measure the $PM = 90°$

- For the compensated - PD controlled system, the

$$G(S) = \frac{2500(S^2 + 256S + 4720)}{S^3 + 0.25S^2 - 1000S - 250}$$

Refer to Fig 3 c) on page 13 for Bode Plots for this system. We have

Phase Margin $= 90°$

Gain Margin $\doteq -40\,dB$.?

Note: This system is unnormal. Because for high gain K. System is always stable. (from root locus)

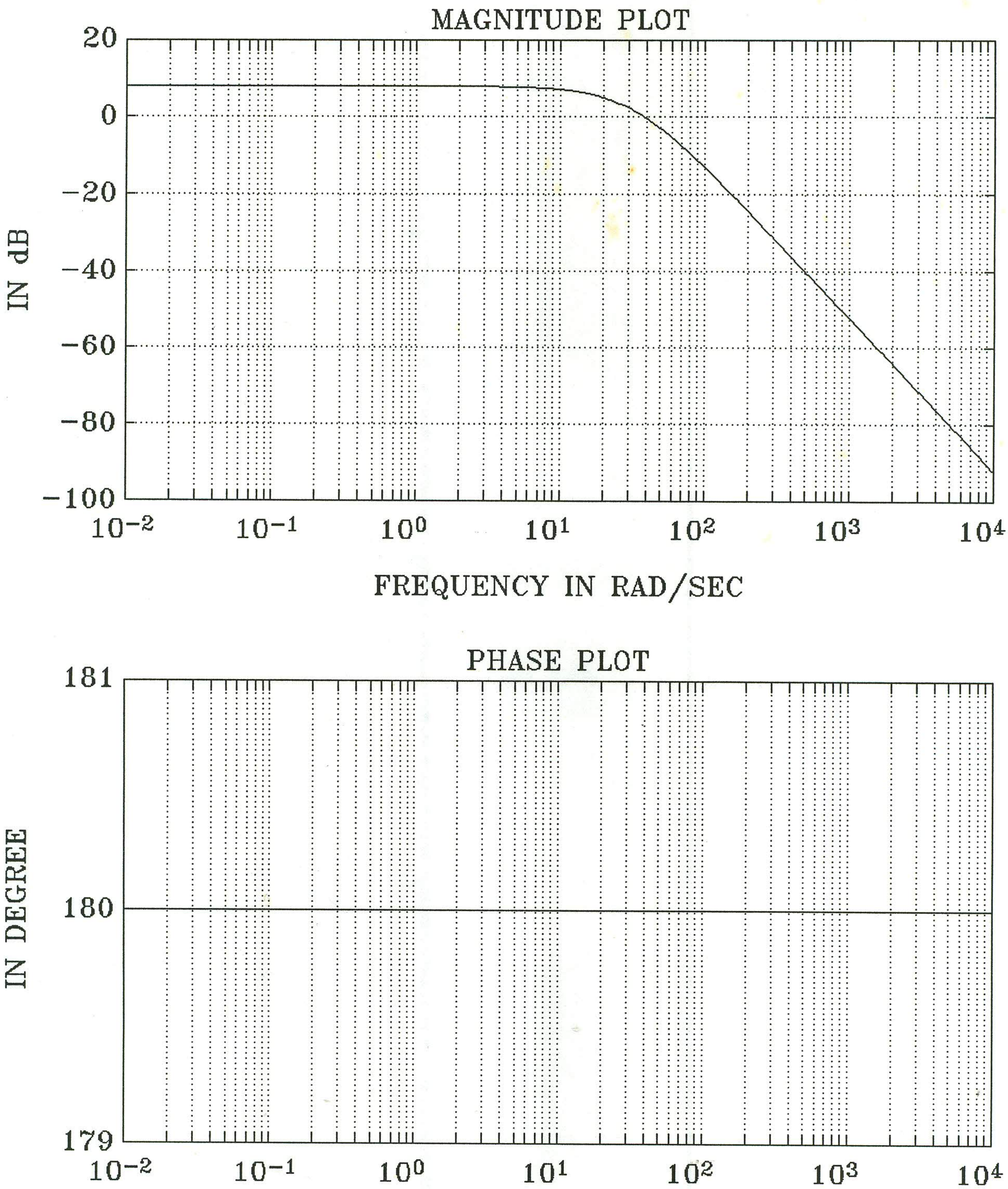


Fig 3 a) : Bode Plots For The Original Open-loop System

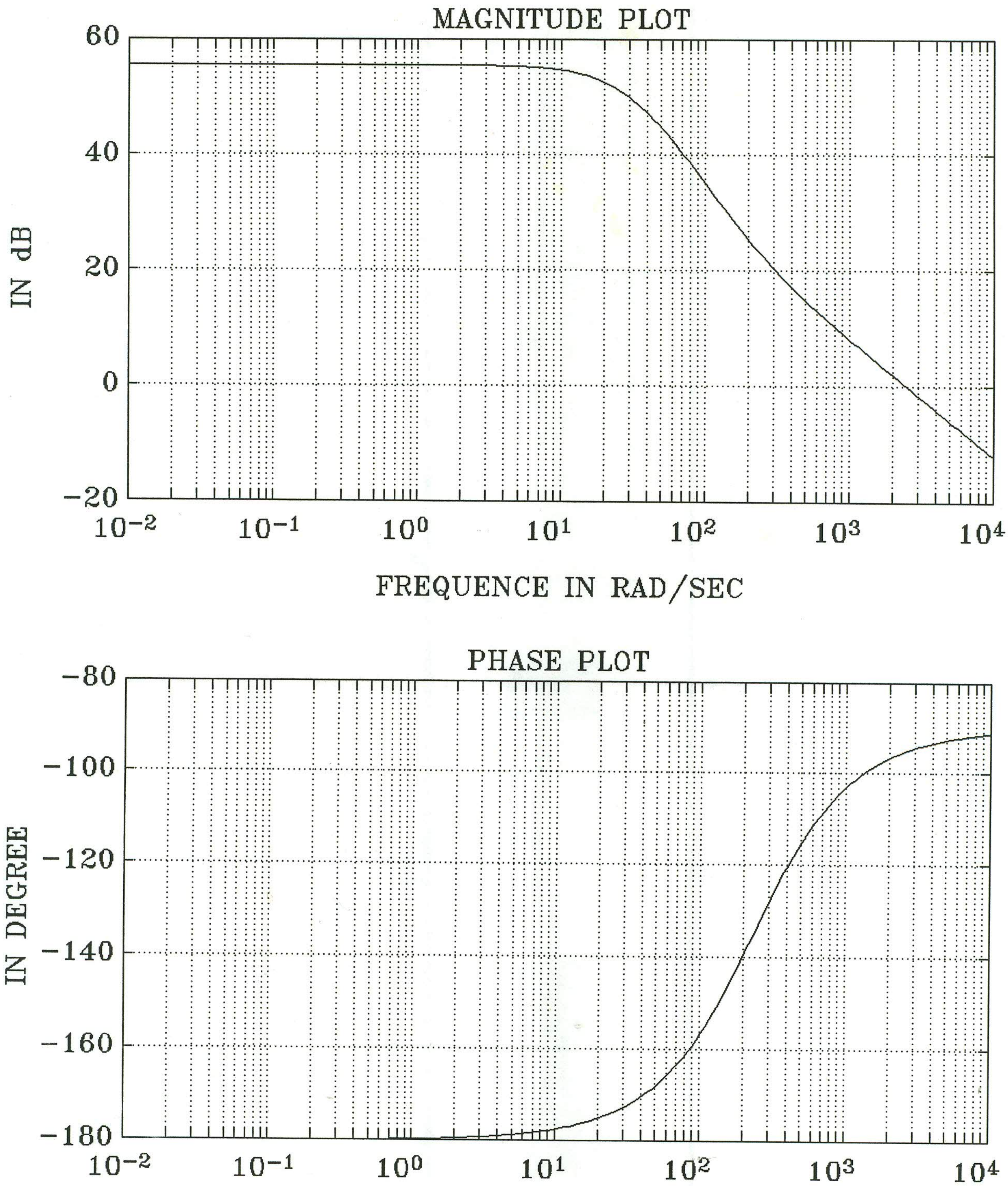


Fig 3 b) : Bode Plot For PD Controlled System

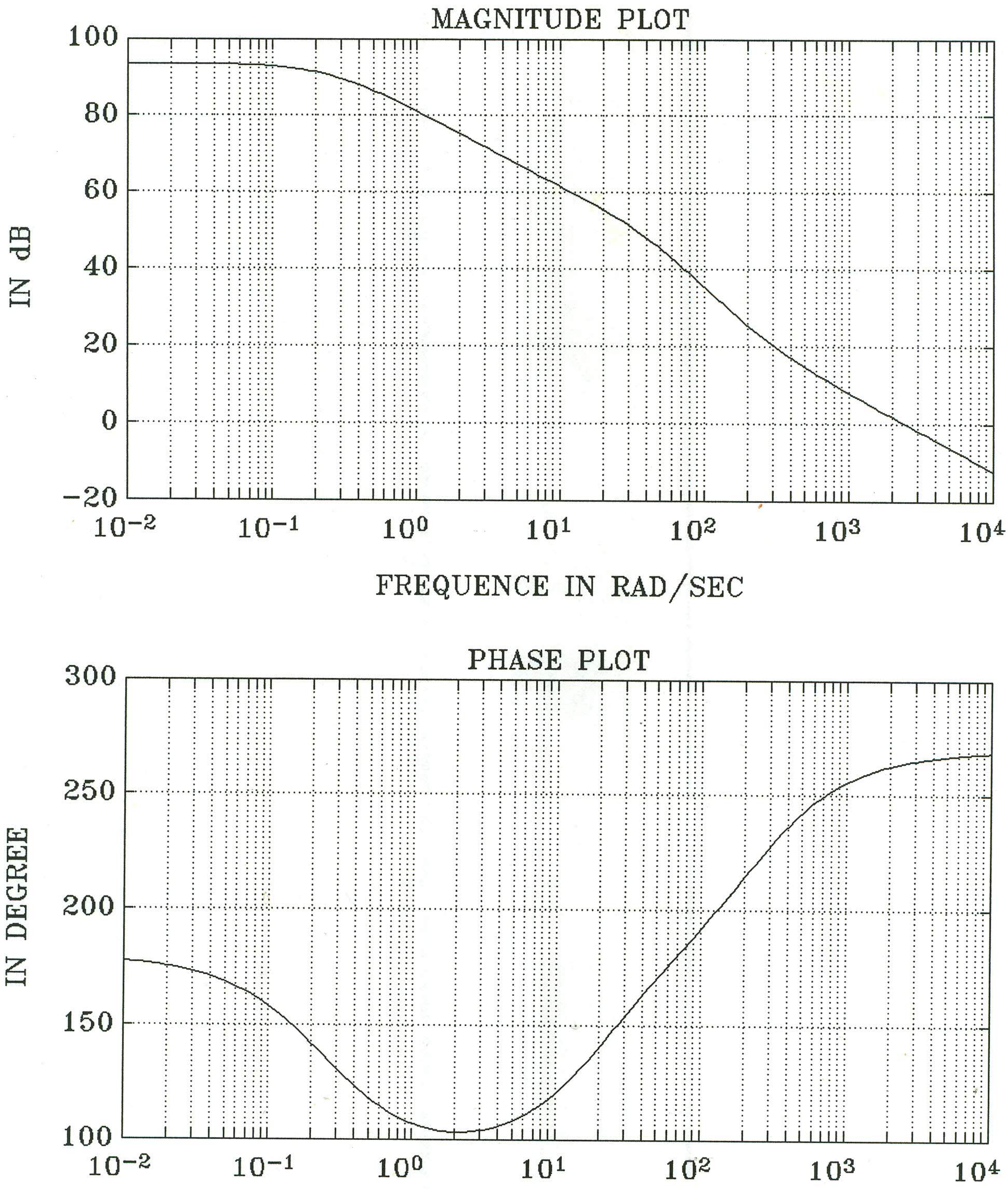


Fig 3 c) : Bode Plot For PD Compensated System

h) If $f = k_2 i + k_1 x$

Study the sensitivity of the overall compensated system with respect to k_1 and k_2. If the $S_{k_1}^T$ or $S_{k_2}^T$ depend on the input signal frequency – plot the same using MATLAB

Refer the begining of this project, we have

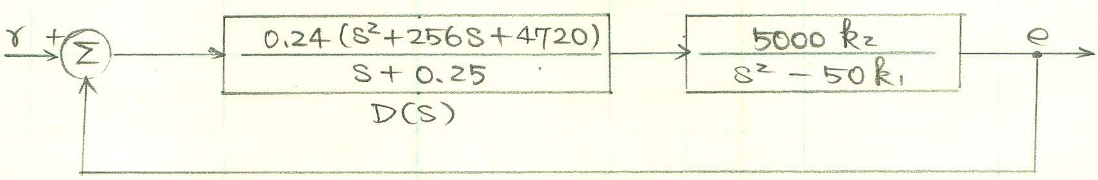
$$0.02\,\ddot{x} = k_2 i + k_1 x - mg$$

$$= k_2 (u + V_0) + k_1 x - mg$$

$$= k_2 u + k_1 x + (k_2 V_0 - mg) \qquad (V_0 \equiv mg/k_2)$$

$$= k_2 u + k_1 x$$

$$\therefore\ G^*(s) = \frac{E(s)}{U(s)} = \frac{100\,k_2}{0.02 s^2 - k_1} = \frac{5000\,k_2}{s^2 - 50 k_1}$$



$$T(s) = \frac{5000 k_2\, D(s)}{s^2 + 5000 k_2 D(s) - 50 k_1}$$

$$S_{k_1}^T = k_1 \cdot \frac{s^2 + 5000 k_2 D(s) - 50 k_1}{5000\, k_2\, D(s)} \cdot \frac{5000 k_2 D(s) \cdot 50}{(s^2 + 5000 k_2 D(s) - 50 k_1)^2}$$

$$= \frac{50 k_1}{s^2 + 5000 k_2 D(s) - 50 k_1}$$

$$S_{k_2}^T = \frac{s^2 - 50 k_1}{s^2 + 5000 k_2 D(s) - 50 k_1}$$

They are depended on input signal frequency.

h) (CONT.)

Subtitude $k_1 = 20$, $k_2 = 0.5$, we have

$$S_{K_1}^{T} = \frac{1000S + 250}{S^3 + 600.25S^2 + 152600S^2 + 2831750}$$

$$S_{K_2}^{T} = \frac{S^3 + 0.25S - 1000S - 250}{S^3 + 600.25S^2 + 152600S^2 + 2831750}$$

Refer to plots on next page. (only magnitude plots are enough).

i) Step Response for design systems.

Fig. 4 on next page is the unit step response for my control system. It is easy to see that

Percent Overshoot $\approx 20\%$

And

Settling time $\cong 0.015$ seconds

Steady error $= 0.09$ mm.

Every thing is perfect to the specifications given at begining of this project.

End of Project One!

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03-26-89

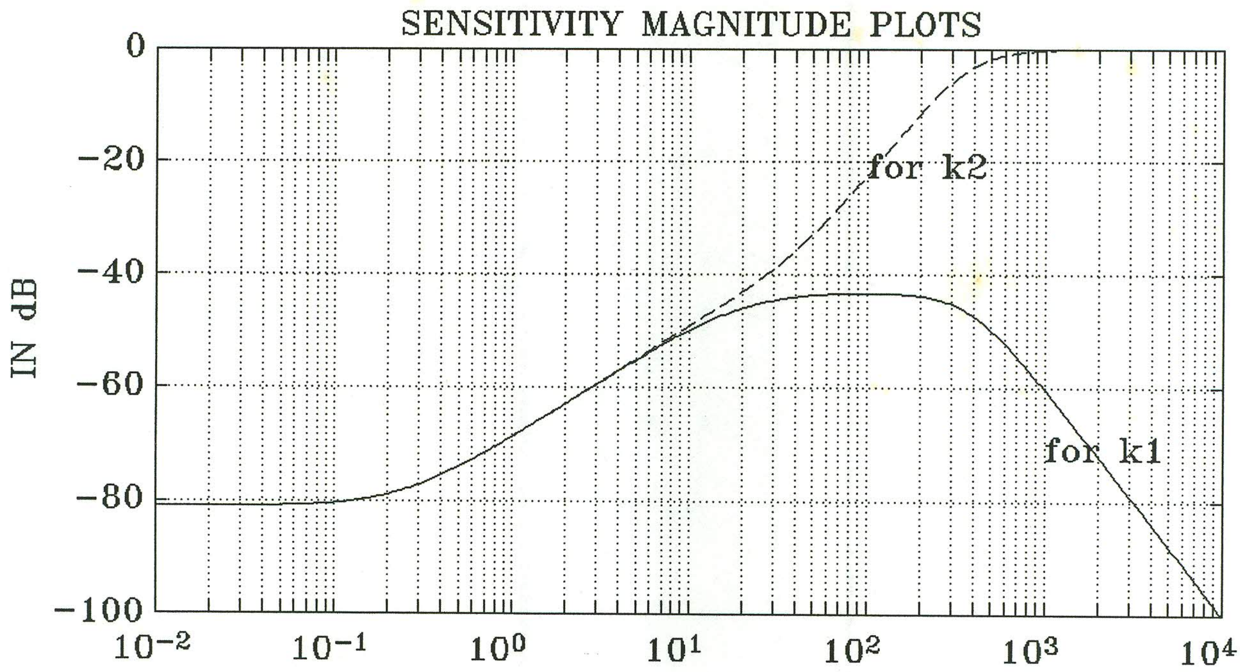

Fig 3:Sensitivities Via Frequency in rad/sec

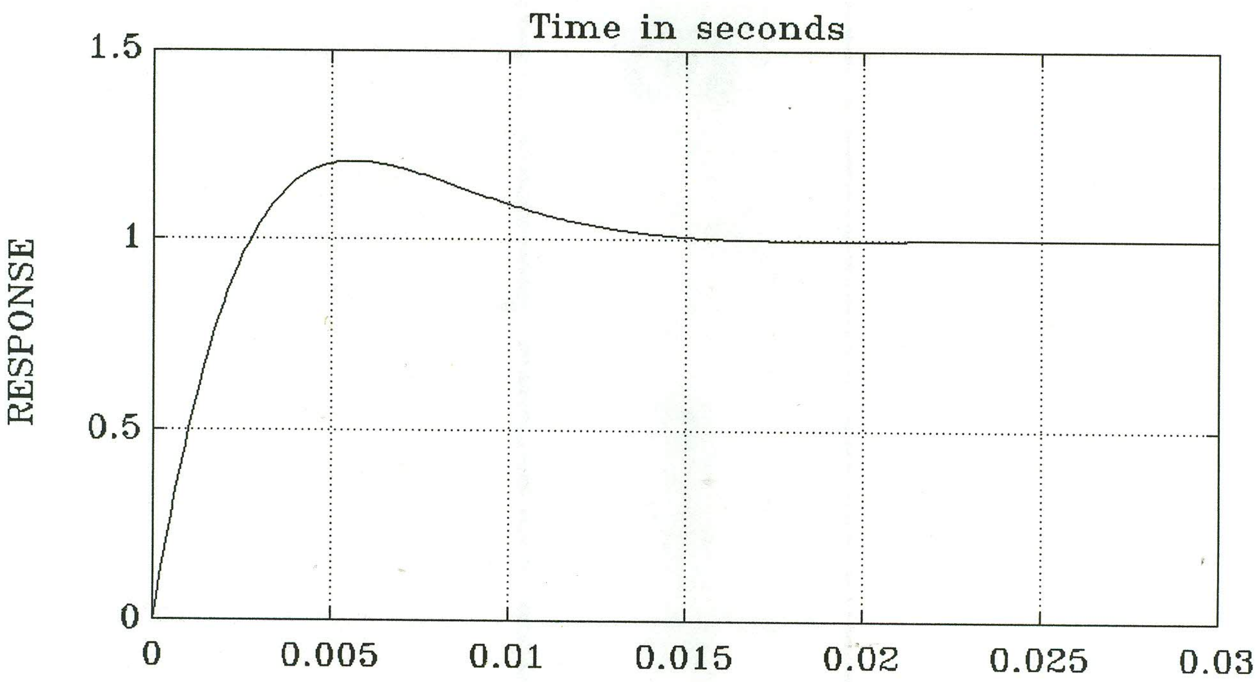

Fig 4 : Step Response For The Complete Designed System

100/100

The Second Project For E-E 489

Introduction To Control System

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April 25, 1989

Project 2 :

Refer to Problem 6.18 on page 428 in text.

a) Check the stability, controlibility, observability of the system. And transform this system to Jordan canonical form.

b) Obtain the transfer function $Y(s)/\Delta(s)$.

c) Design state feedback law, such that the closed loop system has $\xi = 0.707$ and time constant $\tau = 1$ sec.

d) Let $y = u$, design a suitable observer.

e) Obtain a step response of the combined plant and observer. Plot ξ versus $\hat{\xi}$ and θ vs $\hat{\theta}$.

P.S. State equation for Problem 6.18.

$$\begin{bmatrix} \dot{\xi} \\ \dot{\theta} \\ \dot{u} \end{bmatrix} = \begin{bmatrix} -0.4 & 0 & -0.01 \\ 1 & 0 & 0 \\ -1.4 & 9.8 & -0.02 \end{bmatrix} \cdot \begin{bmatrix} \xi \\ \theta \\ u \end{bmatrix} + \begin{bmatrix} 6.3 \\ 0 \\ 9.8 \end{bmatrix} \delta$$

$$y = u = [\,0 \quad 0 \quad 1\,] \cdot \begin{bmatrix} \xi \\ \theta \\ u \end{bmatrix}$$

a) Following are the results I obtained from MATLAB.

The eigenvalues of matrix F are:

$\lambda_1 = -0.6565$; $\lambda_{2,3} = 0.1183 \pm 0.3678\,i$ $\Rightarrow$ System unstable.

Controllability matrix of the system:

$$\mathcal{C} = \begin{bmatrix} 6.3 & -2.618 & 1.1374 \\ 0 & 6.3 & -2.618 \\ 9.8 & -9.016 & 65.5855 \end{bmatrix}$$

rank $(\mathcal{C}) = 3$ implies system is complete controllable.

Obeserability matrix of the system:

$$\mathcal{O} = \begin{bmatrix} 0 & 0 & 1 \\ -1.4 & 9.8 & -0.02 \\ 10.388 & -0.196 & 0.0144 \end{bmatrix}$$

rank $(\mathcal{O}) = 3$ implies system is complete obsevarable.

Note:

$$\begin{bmatrix} q \\ \theta \\ u \end{bmatrix} = T\bar{x}$$

Jordan Canonical Form: (complex form)

Let
$$\begin{bmatrix} q \\ \theta \\ u \end{bmatrix} = \begin{bmatrix} 0.039 & -0.0128 + i\cdot 0.0091 & -0.0128 - i\,0.0091 \\ -0.0594 & 0.0123 + i\cdot 0.0388 & 0.0123 - i\,0.0388 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Subtitute this into original state equation, we have

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -0.6565 & 0 & 0 \\ 0 & 0.1183 + 0.3678i & 0 \\ 0 & 0 & 0.1183 - 0.3678i \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 94.0524 \\ -42.1262 - 85.2245i \\ -42.1262 + 85.2245i \end{bmatrix} \delta$$

$$y = [\,1 \quad 1 \quad 1\,]\,\bar{x}$$

Can you comment on the controllability & observability looking at the matrices

Sorry!

b) Transfer function

$$T(s) = \frac{9.8\,s^2 - 4.9\,s + 61.74}{s^3 + 0.42\,s^2 - 0.006\,s + 0.098}$$

c)

$$u = -\begin{bmatrix} K_1 & K_2 & K_3 \end{bmatrix} \cdot \begin{bmatrix} \delta \\ \theta \\ u \end{bmatrix}$$

$$F' = F - GK = \begin{bmatrix} -0.4 & 0 & -0.01 \\ 1 & 0 & 0 \\ -1.4 & 9.8 & -0.02 \end{bmatrix} - \begin{bmatrix} 6.3 \\ 0 \\ 9.8 \end{bmatrix} \begin{bmatrix} K_1 & K_2 & K_3 \end{bmatrix}$$

$$= \begin{bmatrix} -0.4 - 6.3K_1 & -6.3K_2 & -0.01 - 6.3K_3 \\ 1 & 0 & 0 \\ -1.4 - 9.8K_1 & 9.8 - 9.8K_2 & -0.02 - 9.8K_3 \end{bmatrix}$$

$$|sI - F'| = \begin{vmatrix} s + 0.4 + 6.3K_1 & 6.3K_2 & 0.01 + 6.3K_3 \\ -1 & s & 0 \\ 1.4 + 9.8K_1 & 9.8K_2 - 9.8 & s + 0.02 + 9.8K_3 \end{vmatrix}$$

$$= s(s + 0.4 + 6.3K_1)(s + 0.02 + 9.8K_3) - (9.8K_2 - 9.8)(0.01 + 6.3K_3)$$

$$- (1.4 + 9.8K_1)(0.01 + 6.3K_3)s + 6.3K_2(s + 0.02 + 9.8K_3)$$

$$= s^3 + (0.42 + 6.3K_1 + 9.8K_3)\cdot s^2$$

$$+ (0.028K_1 + 6.3K_2 - 4.9K_3 - 0.006)s + (61.74K_3 - 0.098K_2 + 0.098)$$

desired pole location $\zeta = 0.707$ $\quad \tau = \frac{1}{\zeta\omega_n} = 1 \Rightarrow \omega_n = 1.414$

And I would like to have another pole at $s = -7$

$$C.E. = (s+7)(s^2 + 2s + 2) = s^3 + 9s^2 + 16s + 14$$

Solving linear equation:

$$\begin{cases} 6.3K_1 + 0K_2 + 9.8K_3 = 8.58 \\ 0.028K_1 + 6.3K_2 - 4.9K_3 = 16.006 \\ 0K_1 - 0.098K_2 + 61.74K_3 = 14 \end{cases}$$

Following results are obtained from MATLAB:

$K_1 = 1.0025$ $K_2 = 2.7159$ $K_3 = 0.2311$

Control law: $K = [1.0025 \quad 2.7159 \quad 0.2311]$

d) I would like to have my observer pole twice faster than the controller. $\tau = 0.5$ sec. $\Rightarrow \xi\omega_n = 2 \Rightarrow \omega_n = 2.82885$

Assuming

$$L = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix}, \Rightarrow LH = \begin{bmatrix} 0 & 0 & L_1 \\ 0 & 0 & L_2 \\ 0 & 0 & L_3 \end{bmatrix}$$

$$F - LH = \begin{bmatrix} -0.4 & 0 & -0.01 - L_1 \\ 1 & 0 & -L_2 \\ -1.4 & 9.8 & -0.02 - L_3 \end{bmatrix}$$

$$|S - F + LH| = \begin{vmatrix} S+0.4 & 0 & 0.01 + L_1 \\ -1 & S & L_2 \\ 1.4 & -9.8 & S + 0.02 + L_3 \end{vmatrix}$$

$$= S(S+0.4)(S+0.02+L_3) + 9.8(0.01+L_1)$$

$$- 1.4(0.01+L_1)S + 9.8L_2(S+0.4)$$

$$= S^3 + (L_3 + 0.42)S^2 + (-1.4L_1 + 9.8L_2 + 0.4L_3 - 0.006)S$$

$$+ (9.8L_1 + 3.92L_2 + 0.098)$$

$$= (S+14)(S^2 + 4S + 8) = S^3 + 18S^2 + 64S + 112$$

$$\Rightarrow L = \begin{bmatrix} 8.4353 \\ 7.0187 \\ 17.5800 \end{bmatrix}$$

e) We can combine both controller and obsever as equation 6.81 given on page 348 in the text.

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} F & -GK \\ LH & F-LH-GK \end{bmatrix} \cdot \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} G \\ G \end{bmatrix} \cdot r$$

$$y = [H \quad 0] \cdot \begin{bmatrix} x \\ \hat{x} \end{bmatrix}$$

OR

$$\begin{bmatrix} \dot{q} \\ \dot{\theta} \\ \dot{u} \\ \dot{\hat{q}} \\ \dot{\hat{\theta}} \\ \dot{\hat{u}} \end{bmatrix} = \begin{bmatrix} -0.4 & 0 & -0.01 & -6.31575 & -17.11 & -1.4559 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ -1.4 & 9.8 & -0.02 & -9.8245 & -26.6158 & -2.2647 \\ 0 & 0 & 8.4353 & -6.71575 & -17.11 & -9.9012 \\ 0 & 0 & 7.0187 & 1 & 0 & -7.0187 \\ 0 & 0 & 17.58 & -11.2245 & -16.8158 & -19.8648 \end{bmatrix} \cdot \begin{bmatrix} q \\ \theta \\ u \\ \hat{q} \\ \hat{\theta} \\ \hat{u} \end{bmatrix}$$

$$+ \begin{bmatrix} 6.3 \\ 0 \\ 9.8 \\ 6.3 \\ 0 \\ 9.8 \end{bmatrix} \cdot r$$

$$y = [0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0] \cdot \begin{bmatrix} x \\ \hat{x} \end{bmatrix}$$

From MATLAB, we can obtain the plots (Refer to next page)

Note: Since the step function in MATLAB assuming the initial conditions of state variables to be zero. Thus, we can't really tell the differeces between true and estimated state responses from the plots given on next page.

good

Q.E.D.

Ben m. Chen 04-25-89

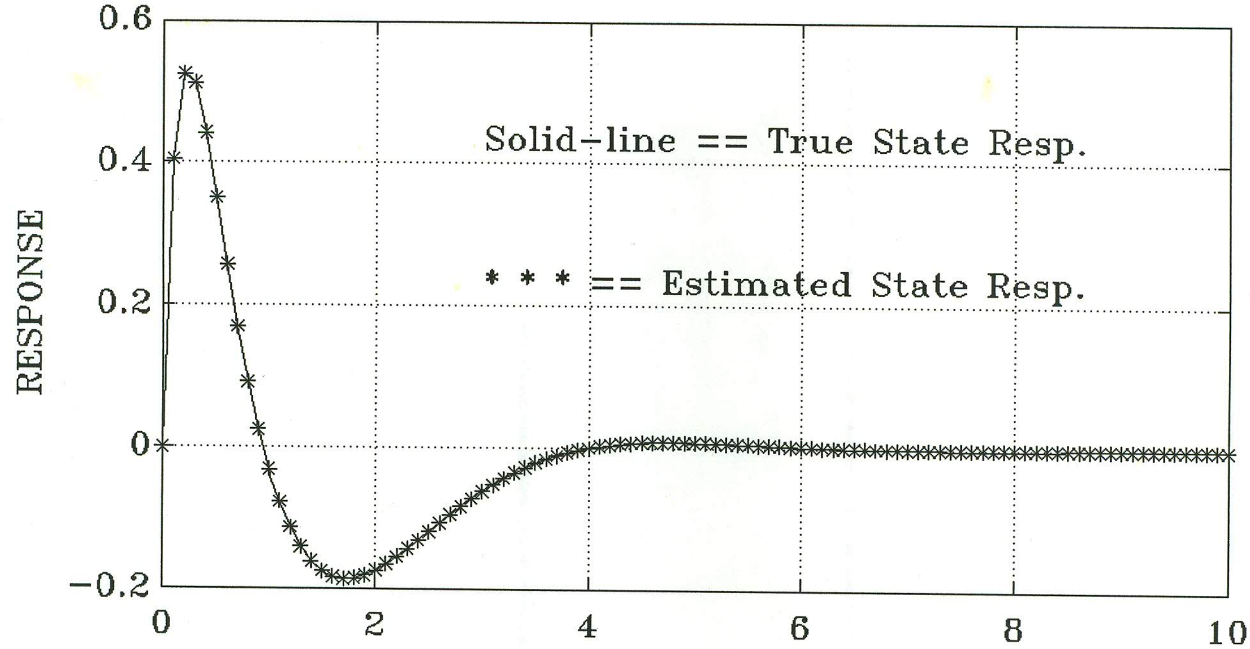

Fig. 1 : True & Estimated Responses of State q.

Initial Conditions for All States Are Zero

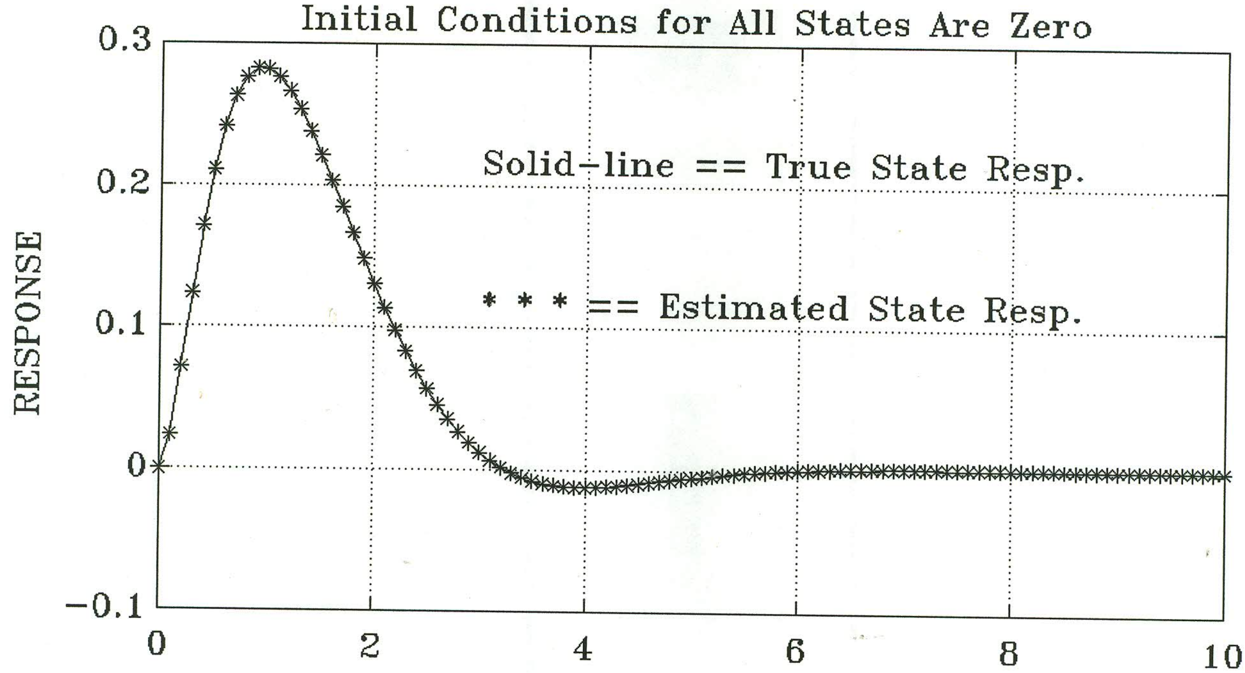

Fig. 2 : True & Estimated Responses of State θ.