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EE 489 TEST #1 February 28, 1989

90/100

I. a) Find the closed loop T.F $\frac{y(s)}{R(s)}$ for the system shown below (Figure 1).

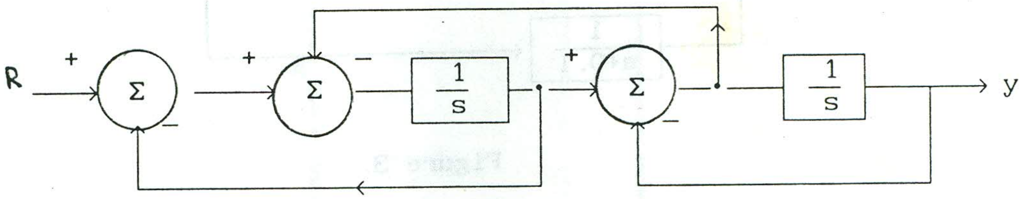


Figure 1.

b) Design the control system given in Figure 2 by choosing appropriate values for k_1 and k_2. The design should satisfy the following.

(i) The settling time ts ≤ 3 sec. (with 2% criterion)

(ii) The percentage over shoot ≤ 5%

Find the type of the system and steady state error to a ramp input.

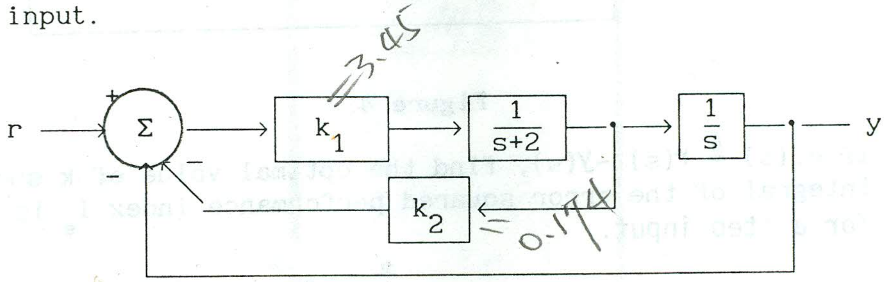


Figure 2.

II. a) Examine whether the system described by the transfer function

$$\frac{y(s)}{R(s)} = \frac{1}{s^2+1}$$

is BIBO stable or not.

b) For the system shown in Figure 3, determine the range of k for which the system is stable. Use the Routh's Criterion.

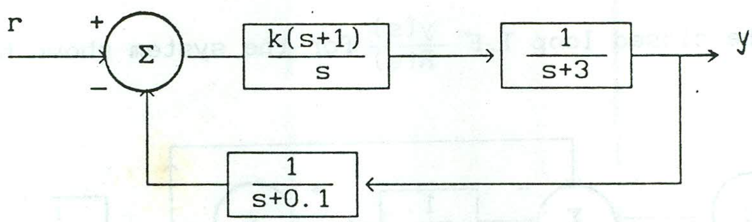


Figure 3.

III. Consider the following system given in Figure 4 and verify that

(a) $$\frac{y(s)}{R(s)} = \frac{2-ks}{s^2+3s+2}$$

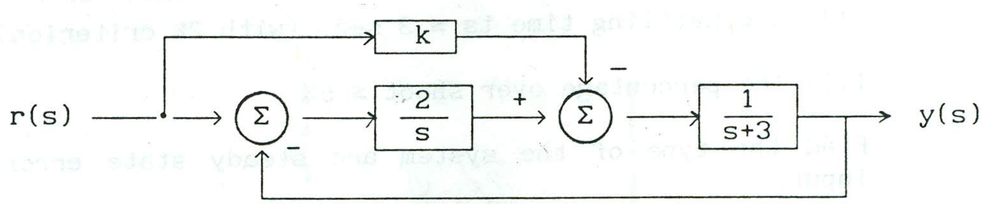


Figure 4.

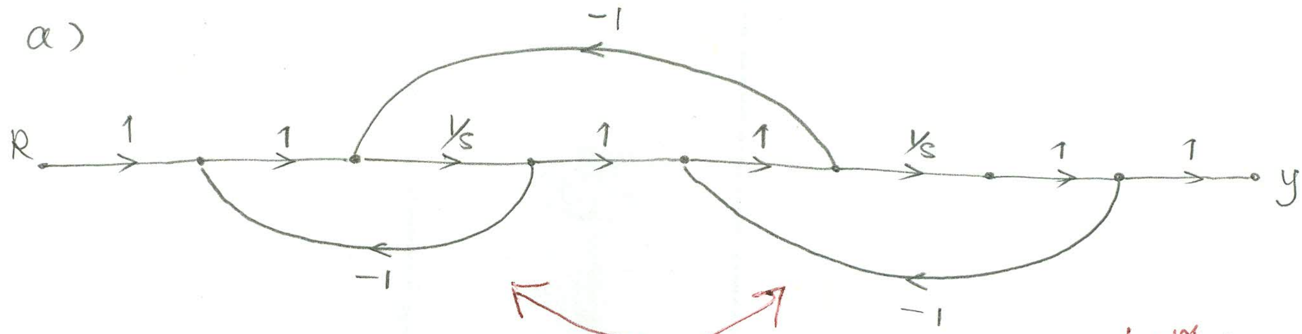
(b) If $e.(s) \triangleq r(s) - y(s)$, find the optimal value of k such that the integral of the error squared performance Index I_s is minimum for a step input.

$$\text{where} \quad I_s = \int_0^{\infty} e^2(t)\,dt = \frac{C_1^2 d_0 + C_0^2 d_2}{2 d_0 d_1 d_2}$$

$$\text{and} \quad E(s) = \frac{\sum_{k=0}^{n-1} C_k s^k}{\sum_{k=0}^{n} d_k s^k}$$

(c) Find the sensitivity of the system with respect to the gain 'k' (S_k^T) over a nominal value of k = -3. Does the sensitivity depend on the input signal frequency?

I. a)



They are non-touching loops

Apply Mason's Rules:

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$$G_1 = \frac{1}{s^2}$$

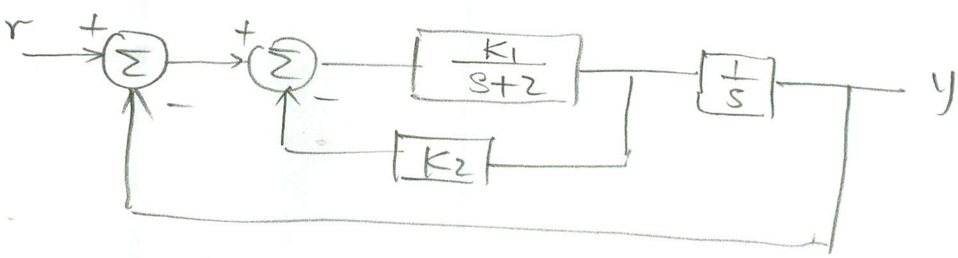
$$\Delta_1 = 1$$

$$\Delta = 1 - \left(-\frac{1}{s} - \frac{1}{s} - \frac{1}{s}\right) + 0$$

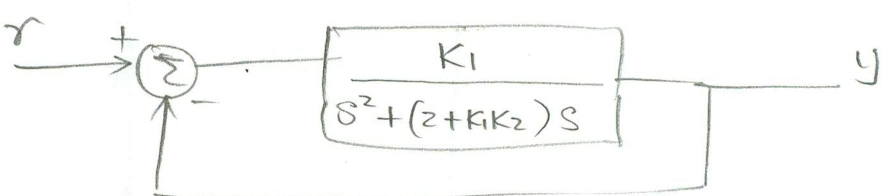
$$= 1 + \frac{3}{s}$$

$$\Rightarrow \text{T.F.} \quad \frac{Y(s)}{R(s)} = \frac{\frac{1}{s^2}}{1+\frac{3}{s}} = \frac{1}{s^2+3s}$$

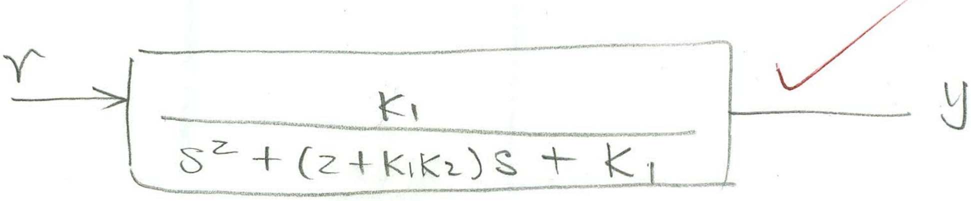
b) $\Rightarrow$



$\Rightarrow$



$\Rightarrow$



$\Rightarrow \quad 2\xi\omega_n = 2 + K_1K_2 \quad$ and $\quad \omega_n^2 = K_1$

(i) for $t_s \leq 3$ sec.

$$t_s \doteq \frac{3.912}{\xi\omega_n} = \frac{7.824}{2\xi\omega_n} = \frac{7.824}{2+K_1K_2} \leq 3$$

$$\Rightarrow K_1K_2 \geq 0.608 \Rightarrow \underline{K_1 \cdot K_2 = 0.7}$$

(ii) $e^{-\pi\xi/\sqrt{1-\xi^2}} \leq 5\% \quad \Rightarrow \quad \xi \doteq 0.7$

$$\xi = \frac{2+K_1K_2}{2\omega_n} = \frac{2+K_1K_2}{2\sqrt{K_1}} \doteq \frac{2+0.7}{2\sqrt{K_1}} = 0.7$$

$$\Rightarrow \underline{K_1 = 3.72}, \quad \underline{K_2 = 0.188}$$

Thus, we have system

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$$\frac{Y(s)}{R(s)} = \frac{3.72}{s^2 + 2.7s + 3.72}$$

$$E(s) = R(s)\left[1 - \frac{Y(s)}{R(s)}\right] = R(s)\cdot\frac{s^2+2.7\cdot s}{s^2+2.7s+3.72}$$

(i) for the unit step input.

$$e_{ss} = \lim_{s\to 0} s\cdot\frac{1}{s}\cdot\frac{s^2+2.7s}{s^2+2.7s+3.72} = 0.$$

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(ii) for — ramp input $\frac{R}{s^2}$:

$$e_{ss} = \lim_{s\to 0} s\cdot\frac{R}{s^2}\cdot\frac{s^2+2.7s}{s^2+2.7s+3.72} = \frac{2.7R}{3.72}$$

$\Rightarrow$ System is Type I and $e_{ss} = \frac{R}{1.38}$

II a) $\Rightarrow$ C.E. $Q(s) = s^2+1$

using Routh table

The test for BIBO stability is
$$\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$$
where $h(t)$ is the impulse response of the system.

$$\begin{array}{c|cc} s^2 & 1 & 1 \\ s^1 & \cancel{0}\ 2 & 0 \\ s^0 & 1 & \end{array} \qquad (s^2+1)' = 2s + 0$$

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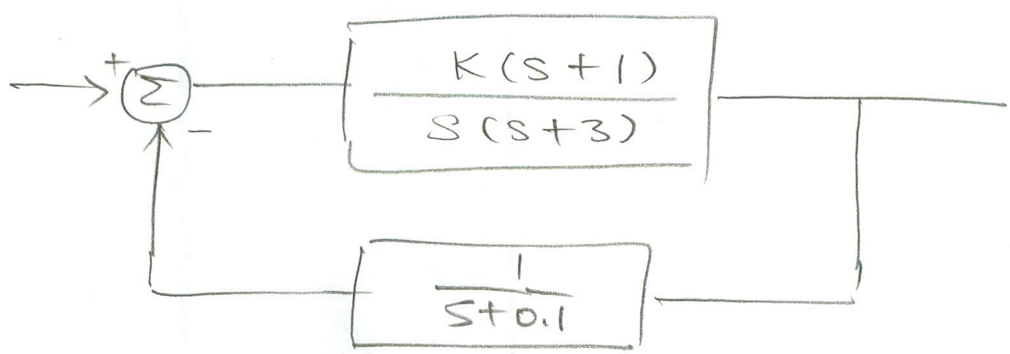
no sign changes in the Routh's table

In this example
$$\int_0^{\infty} |\sin t|\, dt \not< \infty .$$
hence is not BIBO ~~stable~~.

$\Rightarrow$ System is stable, Actually system given above is marginally BIBO stable,

Because it has two simple poles on $j\omega$ axis.

b) $\Rightarrow$



$$\Rightarrow \text{T.F. } T(s) = \frac{K(s+1)(s+0.1)}{s(s+0.1)(s+3) + K(s+1)}$$

$$\Rightarrow \text{C.E. } Q(s) = s(s+0.1)(s+3) + K(s+1)$$

$$= s^3 + 3.1s^2 + (K+0.3)s + K$$

⇒ Routh's Table :

s^3	1	$K+0.3$
s^2	3.1	K
s^1	$0.677K+0.3$	0
s^0	K	

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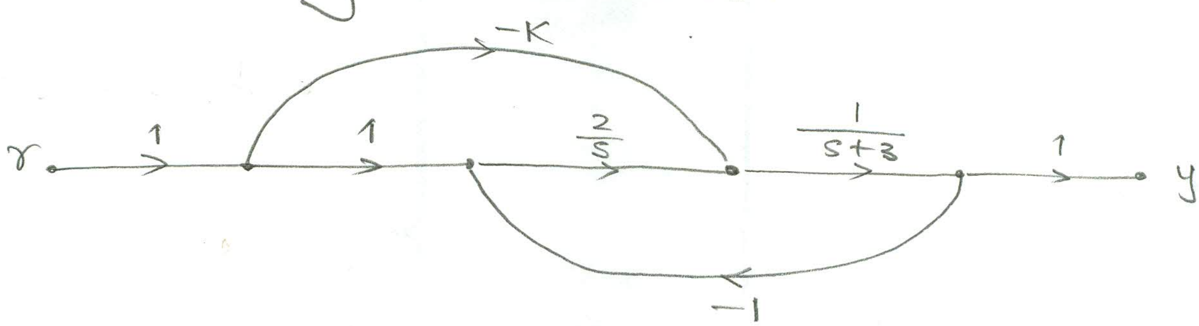
$\Rightarrow \quad 0.677K + 0.3 > 0$ and $K > 0$

Answer : $K > 0$ system is stable

III. (a) using Mason's Rules :



$G_1 = \dfrac{2}{s(s+3)}$, $G_2 = \dfrac{-K}{s+3}$

$\Delta_1 = 1$, $\Delta_2 = 1$

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$$\Delta = 1 - \left(-\frac{2}{s} \cdot \frac{1}{s+3}\right) = \frac{s^2+3s+2}{s(s+3)}$$

$$T(s) = \frac{\dfrac{2}{s(s+3)} - \dfrac{K}{s+3}}{\dfrac{s^2+3s+2}{s(s+3)}} = \frac{2-Ks}{s^2+3s+2}$$

verified

III (b) $e(s) = r(s) - y(s)$

$$= r(s)\left[1 - \frac{y(s)}{r(s)}\right]$$

$$= r(s)\cdot\left[1 - \frac{2 - ks}{s^2+3s+2}\right]$$

$$= \frac{s^2+3s+\cancel{2}-\cancel{2}+ks}{s^2+3s+2}\cdot r(s)$$

$$= \frac{s^2+(3+k)s}{s^2+3s+2}\cdot r(s)$$

for a step input [WLOG assuming $r(s) = \frac{1}{s}$]

$$\Rightarrow e(s) = \frac{s+3+k}{s^2+3s+2} = \frac{s+3+k}{(s+1)(s+2)}$$

$$= \frac{2+k}{s+1} - \frac{1+k}{s+2}$$

$$\Rightarrow e(t) = (2+k)e^{-t} - (1+k)e^{-2t}$$

$$e^2(t) = (2+k)^2 e^{-2t} + (1+k)^2 e^{-4t}$$
$$- 2(2+k)(1+k)e^{-3t}$$

$$\Rightarrow I_s = \int_0^\infty e^2(t)\,dt$$

$$= \int_0^\infty (2+k)^2 e^{-2t}\,dt + \int_0^\infty (1+k^2)e^{-4t}\,dt$$
$$- \int_0^\infty 2(2+k)(1+k)e^{-3t}\,dt$$

$\Rightarrow$ ... 06

$$\Rightarrow I_s = (2+K)^2 \cdot \left[-\tfrac{1}{2} e^{-2t}\Big|_0^\infty\right] + (1+K)^2\left[-\tfrac{1}{4}e^{-4t}\Big|_0^\infty\right]$$

$$- 2(2+K)(1+K)\left[-\tfrac{1}{3}e^{-3t}\Big|_0^\infty\right]$$

$$= \tfrac{1}{2}(2+K)^2 + \tfrac{1}{4}(1+K)^2 - \tfrac{2}{3}(2+K)(1+K)$$

$$= \tfrac{1}{2}(K^2+4K+4) + \tfrac{1}{4}(K^2+2K+1) - \tfrac{2}{3}(K^2+3K+2)$$

$$= \tfrac{1}{12}\left(\underline{6}K^2+24K+24+\underline{3}K^2+6K+3-\underline{8}K^2-24K-16\right)$$

$$= \tfrac{1}{12}\left(K^2+6K+11\right)$$

good

$$\frac{dI_s}{dK} = \frac{1}{12}(2K+6) = 0 \Rightarrow \underline{K=-3}$$

$$\frac{d^2 I_s}{dK} = \frac{1}{6} \geq 0 \Rightarrow I_s \text{ is } \underline{\text{minimum}}$$

Answer $K = -3$ and

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$$\underline{I_s = \frac{1}{6}}$$

III (c) $T(s) = \dfrac{2 - Ks}{s^2+3s+2}$

$$S_K^T = \frac{k}{T} \cdot \frac{dT}{dk}$$

$$= \frac{k}{T} \cdot \frac{-s}{s^2+3s+2}$$

$$= \frac{K}{\dfrac{2-Ks}{s^2+3s+2}} \cdot \frac{-s}{s^2+3s+2}$$

$$= \frac{-Ks}{2-Ks} = \frac{1}{1-\dfrac{2}{Ks}}$$

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For $K = -3$

$$S_K^T \Big|_{K=-3} = \frac{1}{1+0.667/s}$$

The sensitivity is depended on the input signal frequency indeed. for low frequency $(f \to 0)$ $S_K^T \to 0$. for high frequency $(f \to \infty)$ $S_K^T \to 1$.

SPRING 1989 EE 489 TEST # 2.

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1. The root locus for the system depicted by the block diagram in Fig 1a. is shown in Fig 1b.

   a).What value of K will produce roots with zeta=0.707, the damping.
   b).What is the type of the given system ?. find the steady state error to a step of 0.5. let K=6.0.
   c).If the specification on the steady state error $e_{ss} \leq 0.01$ to a step input (at zeta=0.707), Design a compensator and draw the rough sketch of the compensated system root-locus over the Fig 1b.

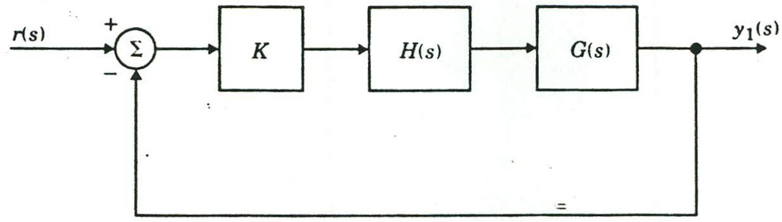

Fig 1a.

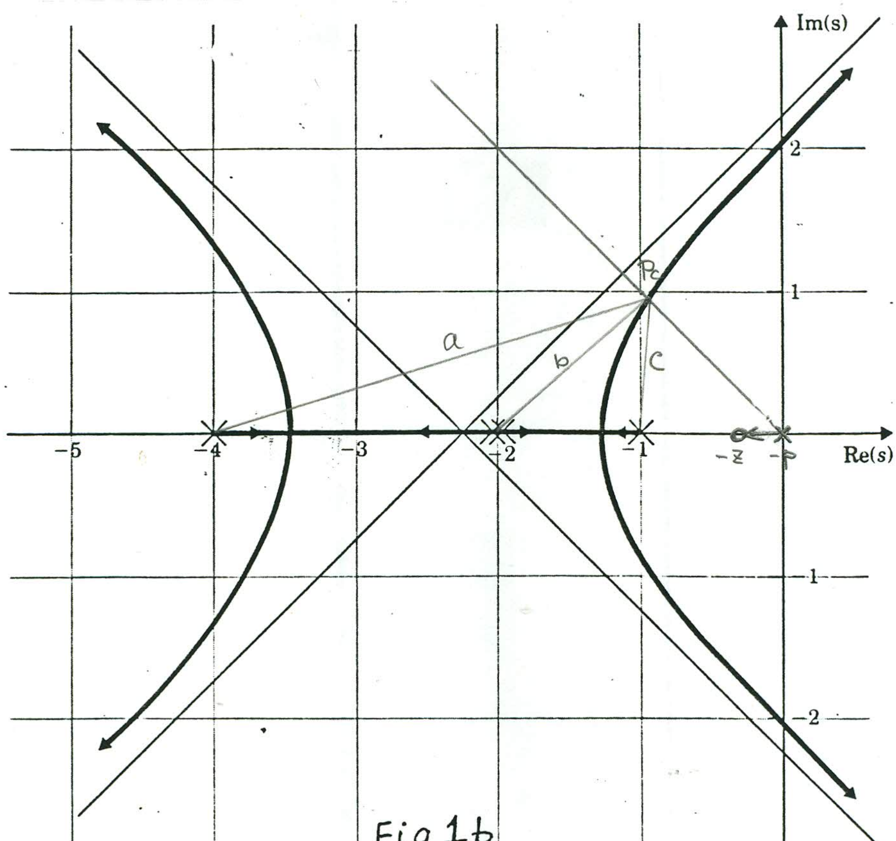

Fig.1b

Solution: a) from the Fig 1 b, we see that the closed loop poles are about at $-0.95 \pm j0.95$.

Then compute

$a \cong 3.18 \qquad b \cong 1.41 \qquad c \cong 0.95$

Then

$$K = ab^2c \cong 6$$

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b) It is type 0 system because G(s) does not has a pole at the origin:

$$G(s) = \frac{6}{(s+1)(s+2)^2(s+4)}, \quad R(s) = \frac{0.5}{s}$$

$$e_{ss} = \lim_{s\to 0} s \cdot R(s) \cdot \frac{1}{1+G(s)} = \lim_{s\to 0} \frac{0.5}{1+\frac{6}{(s+1)(s+2)^2(s+4)}}$$

$$= 0.36364$$

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c) Design for step input $0.5u(t)$:

I think we need a lag compensator to improved it. Let $D(s) = \frac{s+z}{s+p}$.

$$K_{p,comp} = \frac{6}{16} \cdot \frac{z}{p} \simeq 0.375\frac{z}{p}$$

$$e_{ss,comp} = \frac{0.5}{1+0.375\frac{z}{p}} \cong 0.01 \Rightarrow \frac{z}{p} \cong 130$$

15 good

choose $D(s) = \frac{s+0.05}{s+0.000383}$ $(\angle \cdots < 2°)$

2. The bode plot of an open loop Transfer function G(s) is given in Fig 2.

   a). What are the gain cross-over and Phase cross-over frequencies ? Determine the Gain-margin, Phase-margin and the Band Width. Comment on the relative stability of the system.

   b). Design lead compensation that will yield a phase margin of greater than 75deg. Let w_m = 10 rad/sec., be the new gain cross over frequency.

   c). Draw the rough sketch of the Bode plot of the compensated system over the Fig 2.

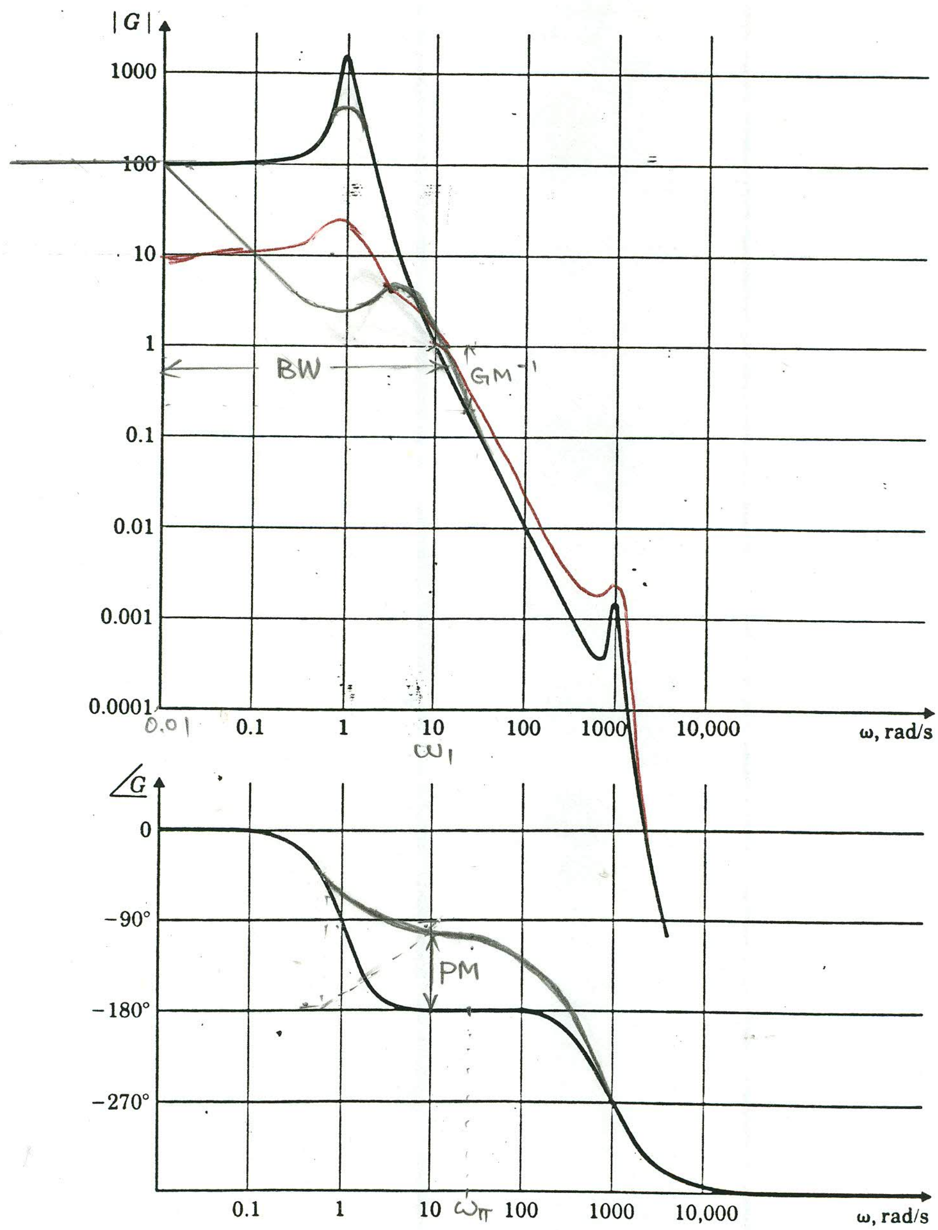

FIG. 2

5+2

Solution : a) gain cross over frequency = 10 rad/sec.

phase cross over frequency = 20 rad/sec.

$GM = 15\ dB \approx 5.6$

$PM \cong 0°$

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$BW \cong 13$ rad/sec. (hard to tell exactly)

This system is almost unstable (poor stability)

b) We try to design a unity dc gain lead compensator

$$D(s) = K \cdot \frac{\tau s+1}{\alpha \tau s+1}, \quad \alpha < 1$$

(i) the additinoal PM $\gtrsim 75°$, select PM = 75°

$$\sin(PM) = 0.9659 = \frac{1-\alpha}{1+\alpha} \Rightarrow \alpha = 0.017332$$

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(ii) $\omega_m = \sqrt{\alpha}\, p \Rightarrow 10 = 0.131652 \cdot p \Rightarrow p \cong 76$

$z = 1.32$

$$\therefore D(s) = 57.7 \cdot \frac{s+1.32}{s+76}$$

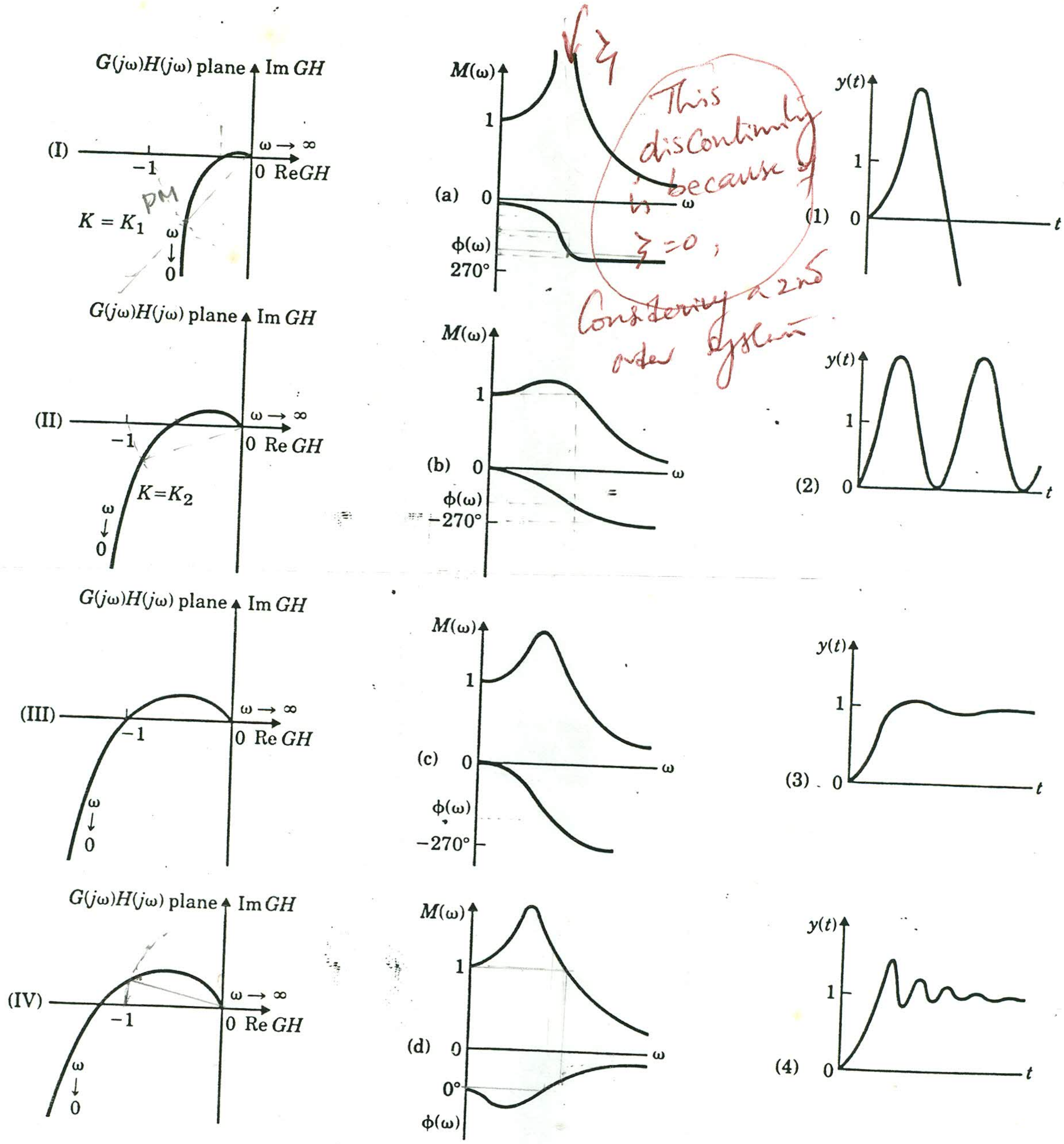
Good

Note: This is only a theorical design. I don't think we can obtained a add. PM = 80° in this design.

c) see the sketch in Fig. 2.

Note:
If you want the new gcf at 10 rad/sec, after Compensation. The gain (by a factor $-10 \log \alpha$ (or) $\frac{1}{\sqrt{\alpha}}$) should be reduced. Hence, the gain of the lead Compensator should be less by about 7 times.

3. Match the step responses given in Fig 3 (1),(2),(3),(4) with the frequency responses (a),(b),(c),(d) and the Nyquist plots I,II,III,IV.



MATCHES = (I) (b) (3) ✓

(II) (c) (4) ✓

(III) (~~d~~ a) (2) ✓

(IV) (~~a~~ d) (1) ✓

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Please Note!

On Closer Examination you will notice the following.

You notice angle of increase due to a pole in RHP!!

EE 489 FINAL TEST Spring 1989. May 3, 1989.

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1). Consider the Transfer function [10]

$$G(s) = \frac{Y(s)}{U(s)} = \frac{s+1}{s^2 + 5s + 6}$$

Write the state space equations for the same.

Let $\dfrac{Y(s)}{U(s)} = \dfrac{Y(s)}{X(s)} \cdot \dfrac{X(s)}{U(s)} = (s+1) \cdot \dfrac{1}{s^2+5s+6}$

$\dfrac{Y(s)}{X(s)} = s+1 \Rightarrow y = \dot{x} + x$

$\dfrac{X(s)}{U(s)} = \dfrac{1}{s^2+5s+6} \Rightarrow \ddot{x} + 5\dot{x} + 6x = u$

Let $x_1 = x$, $x_2 = \dot{x}$ $\Rightarrow$ $\dot{x}_1 = \dot{x} = x_2$

$\dot{x}_2 = \ddot{x} = -5\dot{x} - 6x + u$

$= -5x_2 - 6x_1 + u.$

$y = x_2 + x_1$

$$\Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

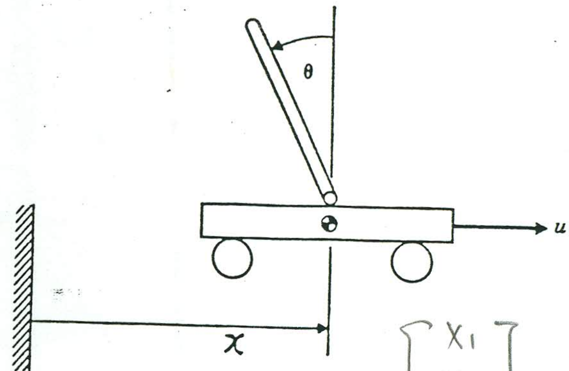
$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

2). The equations of motion for an inverted pendulum on a cart are [10]

$$\ddot{\theta} = \theta + u$$
$$\ddot{x} = -\beta\theta - u$$

Let $\vec{X} = [\theta\ \dot{\theta}\ x\ \dot{x}]$ be the state vector and the output $y = \theta$.
Write the state space equations in the form

$$\dot{X} = FX + GU$$
$$Y = HX + DU$$



Solution:

Let $x_1 = \theta$, $x_2 = \dot{\theta}$
$x_3 = x$, $x_4 = \dot{x}$ or $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix}$

$$\dot{x}_1 = \dot{\theta} = x_2$$
$$\dot{x}_2 = \ddot{\theta} = \theta + u = x_1 + u.$$
$$\dot{x}_3 = \dot{x} = x_4$$
$$\dot{x}_4 = \ddot{x} = -\beta\theta - u = -\beta x_1 - u$$
$$y = \theta = x_1$$

$$\Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -\beta & 0 & 0 & 0 \end{bmatrix}}_{F} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}}_{G} u$$

$$y = \underbrace{[1\ 0\ 0\ 0]}_{H} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \underbrace{0}_{D} \cdot u$$

Ben M. Chen

3). Consider the system [10] -2

$$\dot{X}(t) = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix} X(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} U(t)$$

$$Y(t) = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

a). Is the system stable ?

b). Is it controllable and Observable ?

c). Define what is controllability and observability.

d). Is the above system Linear-time-varying OR Linear-time-invariant system ?. Give your reasons.

a) $F = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix}$

$$|\lambda I - F| = \begin{vmatrix} \lambda+2 & -1 \\ -1 & \lambda \end{vmatrix} = \lambda\cdot(\lambda+2) - 1$$

$$= \lambda^2 + 2\lambda - 1$$

$\Rightarrow$ system is unstable.

b) i) Controllability matrix :, $FG = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

$\mathcal{C} = [G \;\; FG] = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \Rightarrow$ system controllable.

$|\mathcal{C}| \neq 0$

ii) $HF = [1 \;\; 2]\begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix} = [0 \;\; 1]$

Obseverability matrix :

$\mathcal{O} = \begin{bmatrix} H \\ HF \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \Rightarrow |\mathcal{O}| \neq 0$ system observable.

c) Considering a system :

$$\dot{x} = F x + G u$$
$$y = H x + D u$$

X Explanation of what they mean is needed ! Not the tests.

where F is $n \times n$ matrix

Then the controllability matrix given by

$$\mathcal{C} = [G, FG, \cdots, F^{n-1}G]$$

whenever $\mathcal{C}$ is of full rank, system is controllable.

And Observability matrix given by

$$\mathcal{O} = \begin{bmatrix} H \\ HF \\ \vdots \\ HF^{n-1} \end{bmatrix}$$

whenever $\mathcal{O}$ is of full rank (n), system is observable.

Note : There are original definitions for Con. and Obs. but I think they are in EE501

d) System is Linear-time-invariant because all the coft. matrices of the system are independent of time t.

4). Consider the plant described by [20]

$$\dot{X}(t) = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix} X(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} U(t)$$

$$Y(t) = [\,1 \quad 2\,]\, X(t)$$

a). Find an appropriate control law such that the poles of closed loop system are at $-0.5 \pm j\,0.5$

b). Design an observer which is twice faster than the controller.

c). With $U(t) = -K\,\hat{X}(t) + r(t)$, Write the complete state equations of the combined control-law and estimator.

a) The desired pole locations are: $(s+0.5-j0.5)(s+0.5+j0.5)$

$$= s^2 + s + 0.5$$

I would like to design a state feedback controller.

$$u = -Kx + r \quad , \quad K = [K_1 \quad K_2]$$

$$C.E. = |sI - F + G \cdot K|$$

$$= \left| \begin{bmatrix} s+2 & -1 \\ -1 & s \end{bmatrix} + \begin{bmatrix} K_1 & K_2 \\ 0 & 0 \end{bmatrix} \right| = \begin{vmatrix} s+2+K_1 & K_2-1 \\ -1 & s \end{vmatrix}$$

$$= s^2 + (2+K_1)s + (K_2-1) \equiv s^2 + s + 0.5$$

$$\Rightarrow K_1 = -1 \quad , \quad K_2 = 1.5$$

Answer $\quad K = [-1 \quad 1.5]$

b) desired pole locations for observer will be

$$S_{1,2} = -1 \pm j1$$

$$\Rightarrow \quad C.E. = (S+1+j1)(S+1-j1) = S^2 + 2S + 2$$

Assume $L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}$

$$C.E. = |SI - F + LH| = \left| \begin{bmatrix} S+2 & -1 \\ -1 & S \end{bmatrix} + \begin{bmatrix} L_1 & 2L_1 \\ L_2 & 2L_2 \end{bmatrix} \right|$$

$$= \begin{vmatrix} S+2+L_1 & 2L_1-1 \\ L_2-1 & S+2L_2 \end{vmatrix}$$

$$= S^2 + (L_1 + 2L_2 + 2)S + (2L_1 + 5L_2 - 1)$$

$$\equiv S^2 + 2S + 2$$

$$\Rightarrow \begin{cases} L_1 + 2L_2 = 0 \\ 2L_1 + 5L_2 = 3 \end{cases} \Rightarrow \underline{L = \begin{bmatrix} -6 \\ 3 \end{bmatrix}}$$

c) $$\begin{bmatrix} \dot{X}(t) \\ \dot{\hat{X}}(t) \end{bmatrix} = \begin{bmatrix} F & -GK \\ LH & F-LH-GK \end{bmatrix} \begin{bmatrix} X(t) \\ \hat{X}(t) \end{bmatrix} + \begin{bmatrix} G \\ G \end{bmatrix} \cdot r(t)$$

$$y = [H \quad 0] \begin{bmatrix} X(t) \\ \hat{X}(t) \end{bmatrix}$$

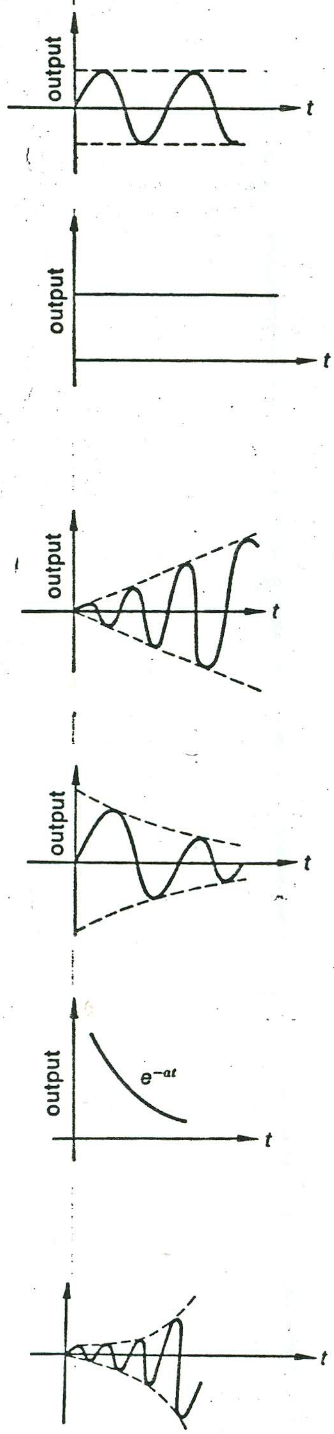
$$\Rightarrow \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{\hat{X}}_1 \\ \dot{\hat{X}}_2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 1 & -1.5 \\ 1 & 0 & 0 & 0 \\ -6 & -12 & 5 & 11.5 \\ 3 & 6 & -2 & -6 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \cdot r$$

$$y = [1 \quad 2 \quad 0 \quad 0] \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

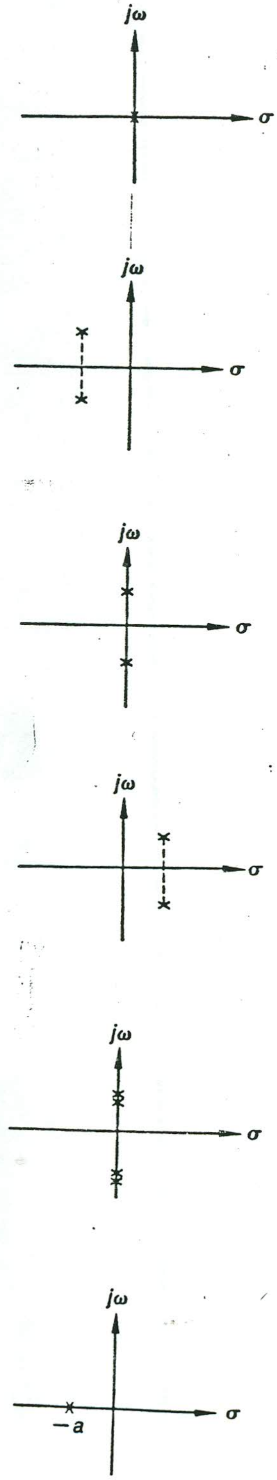
5). Match the location of closed-loop poles on s-plane with the time responses.

[10]

time response



Location of closed loop poles on s-plane



A matches 4 A — 6

B matches 6 B — 5

C matches 2 C — 4

D matches 5 D — 3

E matches 1 E — 2

F matches 3 F — 1

matches :

A ↔ 4 ✓

B ↔ 6 ✓

C ↔ 2 ✓

D ↔ 5 ✓

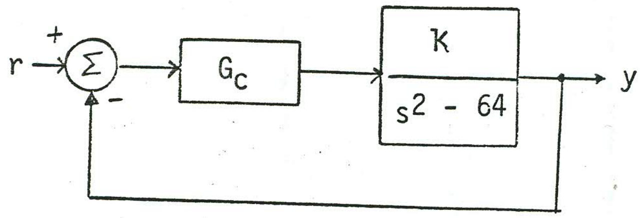
E ↔ 1 ✓

F ↔ 3 ✓

Ben M. Chen

6). Consider

[20]



a). Using root-locus techniques, design a compensation that will give closed loop w_n = 7rad/sec. and a damping ratio of ζ = 0.707. Choose compensator zero to be at -10.

b). Find the operating gain K of the compensated system.

c). What is the steady state error of the compensated system to a unit step.

d). Draw the compensated root - locus.

Solution: a) Refer to page 11 for the sketch.

Compute: $\theta_1 = 158.96°$, $\theta_2 = 59.04°$, $\theta_3 = 45°$

$\theta_1 + \theta_2 + \theta_4 - \theta_3 = \pm 180° \Rightarrow \theta_4 = 7°$

$\tan 7° = 0.12278 = \frac{5}{p-5} \Rightarrow p = 45.72$

Thus

$$G_c = \frac{s+10}{s+45.72}$$ and this is lead compensator

b) Refer to the a, b, c, and d marked on the fig. in page 11.

Compute: $a = \sqrt{5^2 + 42.72^2} \cong 43.01$

$b = \sqrt{5^2 + 5^2} = 7.07$, $c = \sqrt{5^2 + 3^2} = 5.831$

$d = \sqrt{5^2 + 13^2} = 13.928$.

$$\Rightarrow K = \frac{a \cdot c \cdot d}{b} = \frac{43.01 \times 5.831 \times 13.928}{7.07} \cong 494$$

c) $$G_{comp.}(s) = G_s \cdot \frac{K}{s^2 - 64}$$

$$= \frac{494(s+10)}{(s+45.72)(s^2-64)}.$$

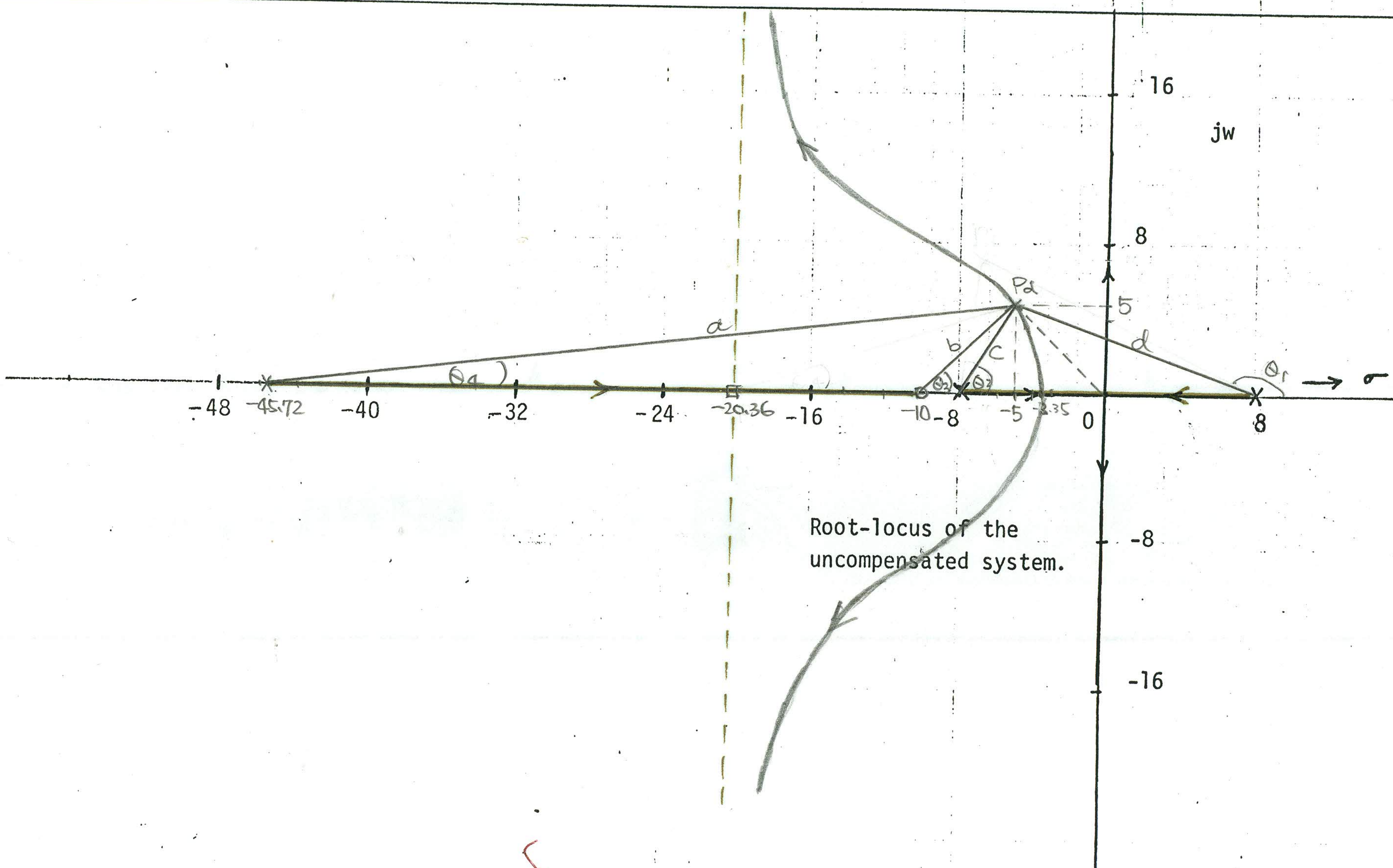
$$K_p = \lim_{s \to 0} G_{comp}(s) = \frac{494 \times 10}{-45.72 \times 64} = -1.688$$

$$e_{ss} = \frac{1}{1+K_p} = -1.45238$$

d) see the sketch on next page.

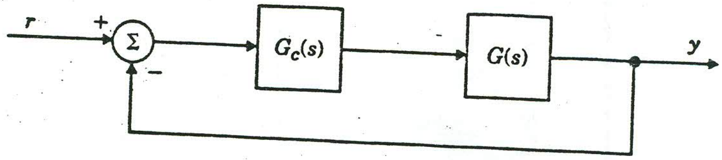
centered point $\alpha = \frac{-45.72+10-8-5+8}{3-1} = -20.36$

Break point $\cong -3.35$



Root-locus of the uncompensated system.

7). Consider [20]



a). Suppose $G(s) = \dfrac{2500\ K}{s(s+25)}$, Design a lead compensator with D.C.gain equal to unity so that the phase-margin $\geq 45^0$, the steady state error due to ramp should be $\leq$ 0.01.

b). Draw the compensated bode-plot. (Use the uncompensated plot given in page 14.)

Solution : a) since G_c is unity D.C. gained

$$\Rightarrow \lim_{s \to 0} G_c = 1$$

$$\therefore K_v = \lim_{s \to 0} s \cdot G_c(s) \cdot G(s)$$

$$= \lim_{s \to 0} \frac{2500\ K}{s+25} = 100K$$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{100K} = 0.01 \quad \Rightarrow \quad K = 1$$

Lead compensator

$$G_c(s) = \frac{\tau s + 1}{\alpha \tau s + 1}, \quad \alpha < 1$$

good

from the uncompensated Bode plot given on page 14, we can find that Phase margin for uncompensated system

$$PM \cong 27°$$

Thus we need to design a compensator which would add $\geq 45° - 27° = 18°$ to the compensated system.

i) Choose

$$PMa = 20° \quad (\text{additional phase margin})$$

ii)

$$\sin(PMa) = \sin 20° = 0.342 = \frac{1-\alpha}{1+\alpha} \Rightarrow \alpha = 0.49$$

iii)

$$10 \log \alpha = -3.09 \text{ dB} \Rightarrow \omega_m \cong 58$$

iv)

$$p = \frac{\omega_m}{\sqrt{\alpha}} \cong 83, \qquad z = \alpha p \cong 40.7$$

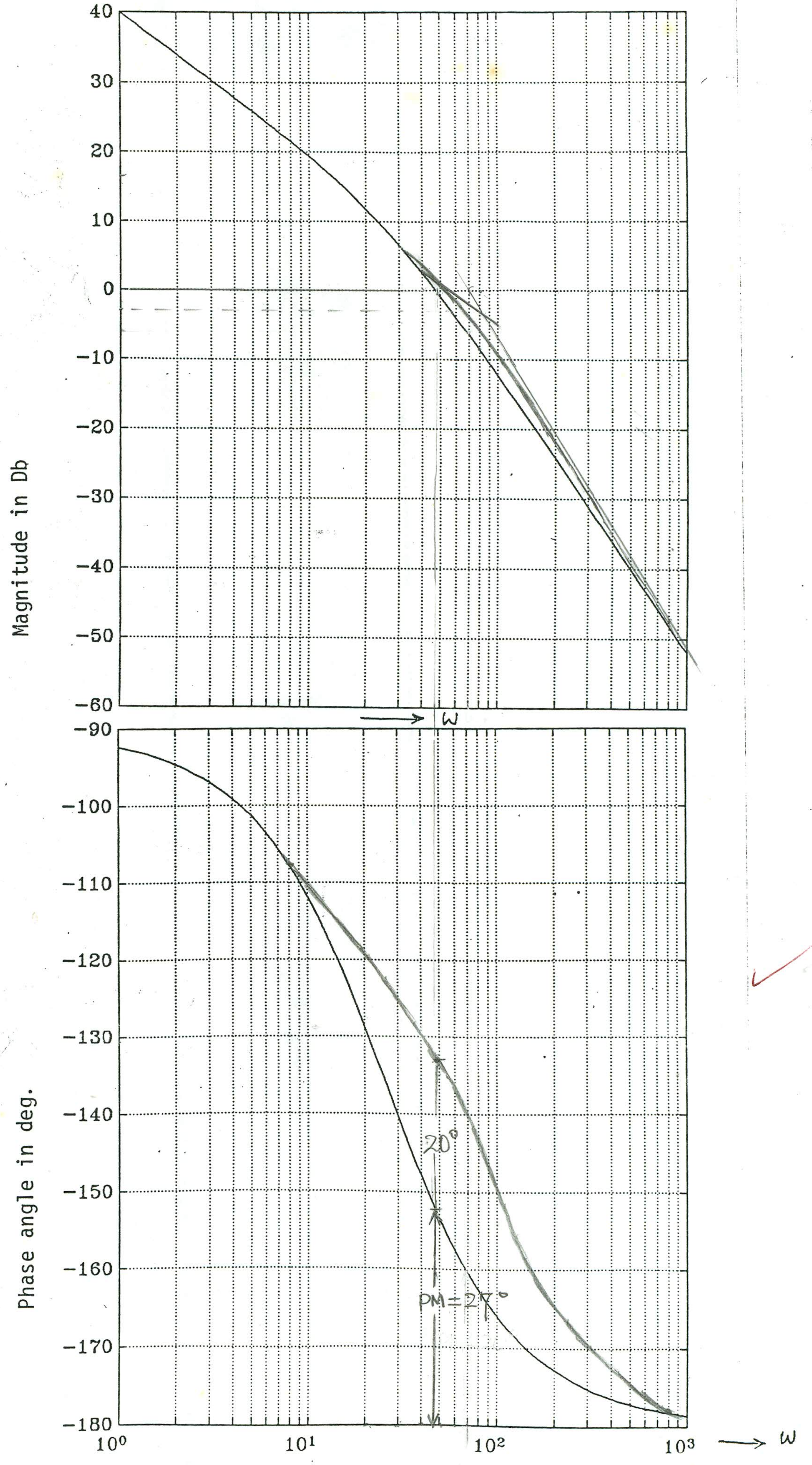
$$\Rightarrow \quad G_c(s) = \beta \cdot \frac{s+40.7}{s+83} \Rightarrow \beta \cong 2.04 \text{ for unity gain.}$$

Thus

$$G_c(s) = 2.04 \cdot \frac{s+40.7}{s+83}$$

b) The Bode plot for compensated system is given on next page.

Problem 7.


Magnitude in Db
40
30
20
10
0
-10
-20
-30
-40
-50
-60
ω
Phase angle in deg.
-90
-100
-110
-120
-130
-140
-150
-160
-170
-180
20°
PM=27°
10^0
10^1
10^2
10^3
ω