R O B O T I C S

EE/ME 442

KENDALL

WASHINGTON STATE UNVERSITY

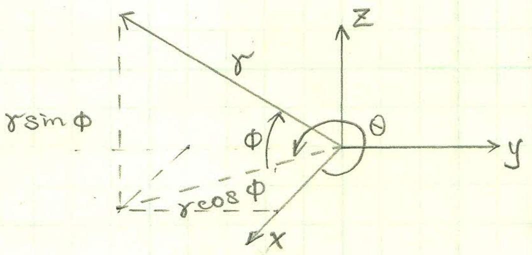
FALL 1988

30/30

CHAPTER 1.

Very Nice!

1. Figure 1.5 shows a three-degrees-of-freedom robot using a spherical coordinate system. Find the kinematic solution for this arm. That is, find expressions for the possition of the hand in space (x, y, and z) as a function of the joint variables γ, ϕ, θ.



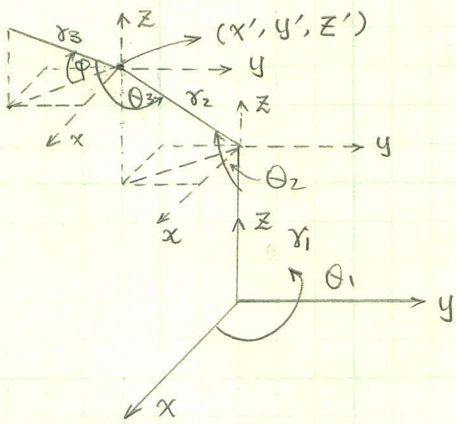
Solution:

$$x = \gamma \cos\phi \cdot \cos\theta$$

$$y = \gamma \cos\phi \cdot \sin\theta$$

$$z = \gamma \cdot \sin\phi$$

2. Solve problem 1 for the articulated arm in Figure 1.8, expressing x, y, and z as functions of θ_1, θ_2, and θ_3.



Solution: Firstmost, we assume that there is no arm 3, that is $\gamma_3 = 0$, Then from prob. 1

$$x' = \gamma_2 \cos\theta_1 \cos(\theta_2 - \tfrac{\pi}{2})$$

$$y' = \gamma_2 \sin\theta_1 \cdot \sin(\theta_2 - \tfrac{\pi}{2}); \quad z' = \gamma_1 + \gamma_2 \sin(\theta_2 - \tfrac{\pi}{2})$$

$$\varphi = \theta_2 + \theta_3 - \tfrac{3\pi}{2}$$

$$\therefore \quad x'' = \gamma_3 \cos\theta_1 \cos(\theta_2 + \theta_3 - \tfrac{3\pi}{2}) = -\gamma_3 \cdot \cos\theta_1 \sin(\theta_2 + \theta_3)$$

$$y'' = \gamma_3 \sin\theta_1 \cos(\theta_2 + \theta_3 - \tfrac{3\pi}{2}) = -\gamma_3 \cdot \sin\theta_1 \cdot \sin(\theta_2 + \theta_3)$$

$$z'' = \gamma_3 \cos(\theta_2 + \theta_3)$$

$$\therefore \begin{cases} x = \gamma_2 \cdot \sin\theta_2 \cdot \cos\theta_1 - \gamma_3 \cos\theta_1 \sin(\theta_2 + \theta_3) \\ y = \gamma_2 \cdot \sin\theta_2 \sin\theta_1 - \gamma_3 \sin\theta_1 \sin(\theta_2 + \theta_3) \\ z = \gamma_1 - \gamma_2 \cdot \cos\theta_2 + \gamma_3 \cos(\theta_2 + \theta_3) \end{cases}$$

4. Solve problem 3 for the articulated arm in Figure 1.8, expressing θ_1, θ_2 and θ_3 as functions of x, y, z.

Solution: From problem 2. we have

$$\frac{y}{x} = \frac{\sin\theta_1[\gamma_2\sin\theta_2 - \gamma_3\sin(\theta_2+\theta_3)]}{\cos\theta_1[\gamma_2\sin\theta_2 - \gamma_3\sin(\theta_2+\theta_3)]} = \tan\theta_1$$

$$\therefore \quad \theta_1 = \tan^{-1}\left(\frac{y}{x}\right) \quad \text{Let} \quad a = \cos\theta_1 \quad b = \sin\theta_1 \quad \cdots \quad (1)$$

$$\frac{x}{a} = \gamma_2\sin\theta_2 - \gamma_3\sin(\theta_2+\theta_3)$$

$$z - \gamma_1 = -\gamma_2\cos\theta_2 + \gamma_3\cos(\theta_2+\theta_3)$$

$$\left(\frac{x}{a}\right)^2 = \gamma_2^2\sin^2\theta_2 + \gamma_3^2\sin^2(\theta_2+\theta_3) - 2\gamma_2\gamma_3\sin\theta_2\sin(\theta_2+\theta_3)$$

$$(z-\gamma_1)^2 = \gamma_2^2\cos^2\theta_2 + \gamma_3^2\cos^2(\theta_2+\theta_3) - 2\gamma_2\gamma_3\cos\theta_2\cos(\theta_2+\theta_3)$$

$$\left(\frac{x}{a}\right)^2 + (z-\gamma_1)^2 = \gamma_2^2 + \gamma_3^2 - 2\gamma_2\gamma_3\cos\theta_3$$

$$\therefore \quad \theta_3 = \cos^{-1}\left(\frac{\gamma_2^2+\gamma_3^2-(x/a)^2+(z-\gamma_1)^2}{2\gamma_2\cdot\gamma_3}\right) \quad \cdots \quad (2)$$

And $\quad x/a = \gamma_2\sin\theta_2 - \gamma_3(\sin\theta_2\cos\theta_3 + \cos\theta_2\sin\theta_3)$

Let $\quad c = \cos\theta_3, \quad d = \sin\theta_3$, then

$$x/a = \gamma_2\sin\theta_2 - \gamma_3\cdot c\cdot\sin\theta_2 - \gamma_3\cdot d\cdot\cos\theta_2$$

$$= (\gamma_2 - \gamma_3\cdot c)\sin\theta_2 - \gamma_3\cdot d\cdot\cos\theta_2$$

$$\frac{x}{a\sqrt{(\gamma_2-\gamma_3\cdot c)^2+\gamma_3^2\cdot d^2}} = \sin\theta_2\cdot\cos\alpha - \cos\theta_2\cdot\sin\alpha = \sin(\theta_2-\alpha)$$

where $\quad \alpha = \tan^{-1}[-\gamma_3\cdot d/(\gamma_2-\gamma_3\cdot c)]$

$$\therefore \theta_2 = \sin^{-1}\left[\frac{x}{a\sqrt{(\gamma_2-\gamma_3\cdot c)^2+\gamma_3^2\cdot d^2}}\right] + \alpha$$

Thus, we have

$$\begin{cases} \theta_1 = \tan^{-1}\left(\frac{y}{x}\right), \quad a \triangleq \cos\theta_1, \quad b \triangleq \sin\theta_1 \\ \theta_3 = \cos^{-1}\left[\frac{\gamma_2^2+\gamma_3^2-(x/a)^2+(z-\gamma_1)^2}{2\gamma_2\cdot\gamma_3}\right], \quad c \triangleq \cos\theta_3, \quad d \triangleq \sin\theta_3 \\ \alpha \triangleq \tan^{-1}[-\gamma_3\cdot d/(\gamma_2-\gamma_3\cdot c)] \\ \theta_2 = \sin^{-1}\left[x/a\cdot\sqrt{(\gamma_2-\gamma_3\cdot c)^2+\gamma_3^2\cdot d^2}\right] + \alpha \end{cases}$$

Q.E.D.

Express $\theta_1, \theta_2, \theta_3$ in terms of x y and z!

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Chapter 6. Write the homogeneous transform matrix that represents

(a) a translation of < 3, 7, 9 >, followed by

(b) a rotation of 90° about the Z axis, followed by

(c) a rotation of −90° about the X axis

Solution :

$$Trans(3,7,9) = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Rot(Z, 90) = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Rot(X, -90) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From MATHLAB

homogeneous transform $= [Rot(X, -90°)] \cdot [Rot(Z, 90°)] \cdot [Trans(3,7,9)]$

$$= \begin{bmatrix} 0 & -1 & 0 & -7 \\ 0 & 0 & 1 & 9 \\ -1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

✓

Chapter 6 : 8. 9. 10

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Good!

For problem 8 to 11, consider the following (refer to Figure 6.6). Let

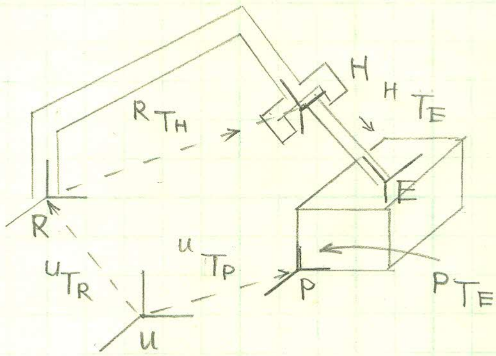

FIGURE 6.6

$$^{U}T_{R} = \begin{bmatrix} 0 & 1 & 0 & 2 \\ -1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{R}T_{H} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

8. (a) Explain how R is oriented relative to U (i.e., what type of rotations or translations have been implemented to derive frame R from frame U).

(b) Repeat the above for frame H relative to R. Assume that only one rotation has been implemented in both case.

Solution: (a) R is obtained from U by

1. A rotation of $-90°$ about z axis, followed by
2. A translation of $[2 \;\; 4 \;\; 5]^T$ ✓

(b) H can be obtained from R by

1. A rotation of $180°$ about x aixs, followed by
2. A traslation of $[1 \;\; 3 \;\; 2]^T$ ✓

9. (a) Given a vector [1, 2, 3] in frame H, find its expression in frame R.

(b) Find the expression of vector [1, 2, 3] (in frame H) in frame U.

Solution: (a)

$$\begin{bmatrix} {}^{R}x \\ {}^{R}y \\ {}^{R}z \\ 1 \end{bmatrix} = {}^{R}T_{H} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

✓

9. (b) I get confused about the statement

(1) if [1, 2, 3] is in frame H, find the expression in U.

$$\begin{bmatrix} {}^{u}x \\ {}^{u}y \\ {}^{u}z \\ 1 \end{bmatrix} = {}^{u}T_{H} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = {}^{u}T_{R} \cdot {}^{R}T_{H} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 2 \\ -1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 4 \\ 1 \end{bmatrix}$$

(2) if [1, 2, 3] is in frame U, Find the expression in H.

$$\begin{bmatrix} {}^{H}x \\ {}^{H}y \\ {}^{H}z \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0 & 1 & 0 & 2 \\ -1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 4 \\ 1 \end{bmatrix}$$

✓

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10. Find ${}^{U}T_{H}$ and verify the result obtained in 8 (b)

<u>Solution:</u> ${}^{U}T_{H} = {}^{u}T_{R} \cdot {}^{R}T_{H}$

$$= \begin{bmatrix} 0 & 1 & 0 & 2 \\ -1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 5 \\ -1 & 0 & 0 & 3 \\ 0 & 0 & -1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

VERIFICATION OF 8 (b):

$$Rot(x, 180^\circ) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad Trans(1, 3, 2) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$TRANS(1, 3, 2) \cdot Rot(x, 180^\circ) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^{R}T_{H}$$

✓

<u>VERIFIED!</u>

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Homework no: 3 EE 442 Robotics Ben M. Chen

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ME 442 Robotics

Forward Kinematic Problem Set One

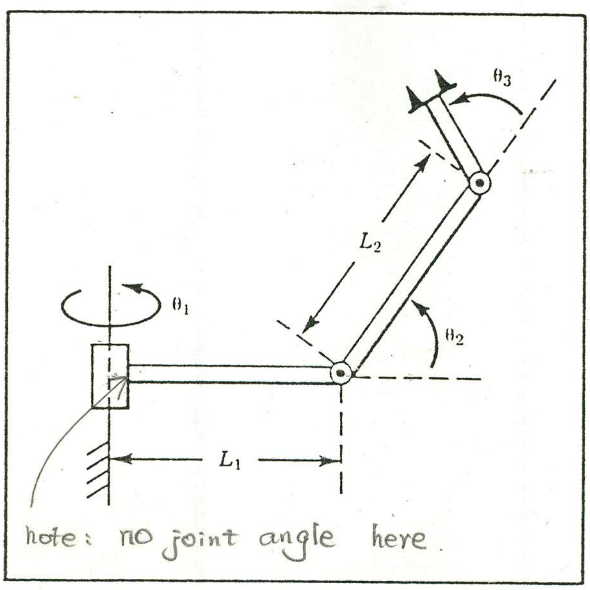
1. The following frame definitions are given as known. Draw a frame diagram that shows the position of these frames with respect to each other. This drawing need not be exact but should provide the relative positions. Then ~~slove~~ solve for ${}^{B}T_{C}$.

$$ {}^{U}T_{A} = \begin{bmatrix} 0.866 & -0.500 & 0.000 & 11.0 \\ 0.500 & 0.866 & 0.000 & -1.0 \\ 0.000 & 0.000 & 1.000 & 8.0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

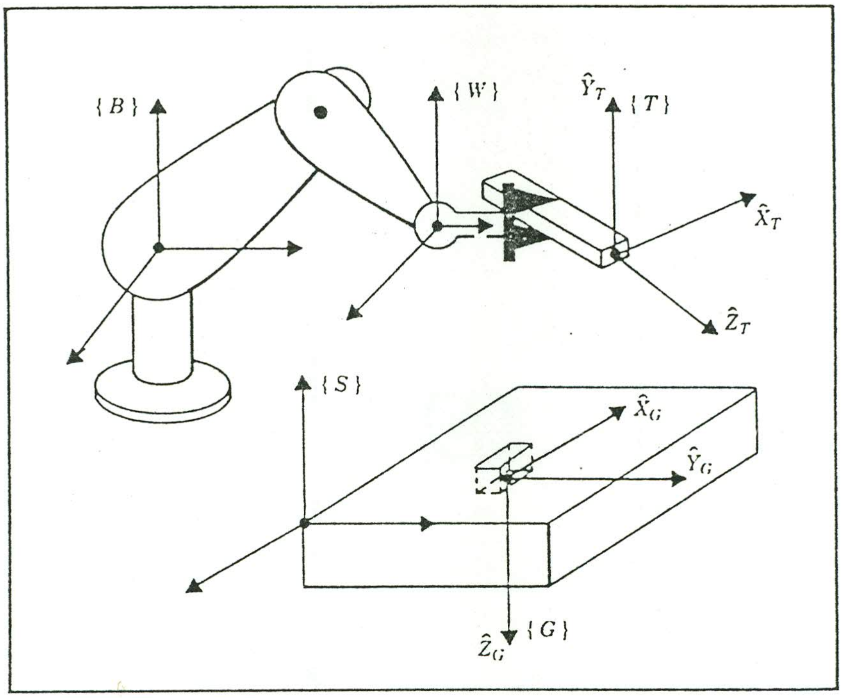
$$ {}^{B}T_{A} = \begin{bmatrix} 1.000 & 0.000 & 0.000 & 0.0 \\ 0.000 & 0.866 & -0.500 & 10.0 \\ 0.000 & 0.500 & 0.866 & -20.0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{C}T_{U} = \begin{bmatrix} 0.866 & -0.500 & 0.000 & -3.0 \\ 0.433 & 0.750 & -0.500 & -3.0 \\ 0.250 & 0.433 & 0.866 & 3.0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

2. For the three degree of freedom arm shown below, derive the link parameters and the kinematic matrix for ${}^{B}T_{W}$.



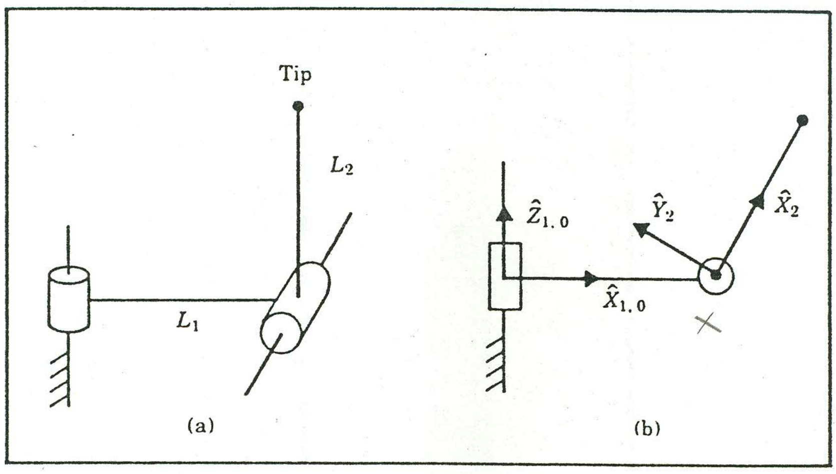
3. For the robotic system shown below, the location of the tool, ${}^{W}T_{T}$, is not known. Using force control, the robot feels around with the tool tip until it inserts into the socket (or goal) at location ${}^{S}T_{G}$. Once in this "calibration" configuration (in which {G} and {T} are coincident), the position of the robot, ${}^{B}T_{W}$, is determined by reading the joint angle sensors and computing the kinematics. Assuming ${}^{B}T_{S}$ and ${}^{S}T_{G}$ are known, give the transform equation to compute the unknown tool frame, ${}^{W}T_{T}$.



4. For the 2-link manipulator shown below, the link transformation matrices, ${}^{0}T_{1}$ and ${}^{1}T_{2}$, were determined. Their product is:

$$
{}^{0}T_{2} = \begin{bmatrix} c_1c_2 & -c_1s_2 & s_1 & l_1c_1 \\ s_1c_2 & -s_1s_2 & -c_1 & l_1s_1 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

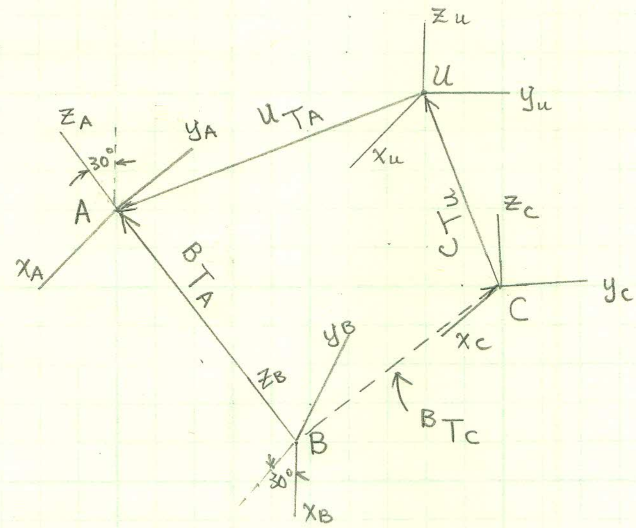
Make the frame assignments and find the expression for the vector ${}^{0}P_{tip}$ which locates the tip of the arm relative to the {0} frame.



Tip
L_2
L_1
(a)
$\hat{Z}_{1,0}$
$\hat{Y}_2$
$\hat{X}_2$
$\hat{X}_{1,0}$
(b)

Forward Kinematic Problem Set One:

1. **Solution:**



From the diagram above, it is easy to see that

$$^BT_C = {}^BT_A \cdot {}^uT_A^{-1} \cdot {}^CT_u^{-1}$$

$$= \begin{bmatrix} 1.000 & 0.000 & 0.000 & 0.0 \\ 0.000 & 0.866 & -0.500 & 10.0 \\ 0.000 & 0.500 & 0.866 & -20.0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0.866 & -0.5 & 0 & 11.0 \\ 0.5 & 0.866 & 0 & -1.0 \\ 0 & 0 & 1 & 8.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0.866 & -0.5 & 0 & -3.0 \\ 0.433 & 0.75 & -0.5 & -3.0 \\ 0.25 & 0.433 & 0.866 & 3.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 0.500 & 0.750 & 0.433 & -6.5754 \\ -0.750 & 0.625 & -0.2165 & 19.7877 \\ -0.433 & -0.2165 & 0.875 & -28.3185 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

✓

2. Link parameters Table

n	α_n	d_n	θ_n
1	90°	0	θ_1
2	0	L_1	θ_2
3	0	L_2	θ_3

Solution:

$$^BT_W = A_1 \cdot A_2 \cdot A_3$$

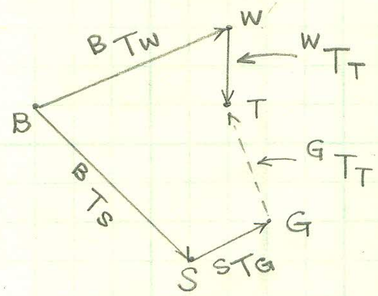
$$= \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} C_2 & -S_2 & 0 & L_1C_2 \\ S_2 & C_2 & 0 & L_1S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} C_3 & -S_3 & 0 & L_2C_3 \\ S_3 & C_3 & 0 & L_2S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_1C_2C_3 - C_1S_2S_3 & -C_1C_2S_3 - C_1S_2C_3 & S_1 & C_1C_2C_3L_2 - C_1S_2C_3L_2 + C_1C_2L_1 \\ S_1C_2C_3 - S_1S_2S_3 & -S_1C_2S_3 - S_1S_2C_3 & -C_1 & S_1C_2C_3L_2 - S_1S_2C_3L_2 + S_1C_2L_1 \\ S_2C_3 - S_3C_2 & -S_2S_3 + C_2C_3 & 0 & S_2C_3L_2 + C_2S_3L_2 + S_2L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Note: This result is identical to the example 7.3 in text. pp. 131–132. ✗

3. <u>Solution:</u>

Once $\{G\}$ and $\{T\}$ are coincident $^{G}T_{T} = I_4$ (I_4 is 4×4 identity matrix)



Then from the diagram on the left, it is easy to see that

$$^{W}T_{T} = {}^{W}T_{B} \cdot {}^{B}T_{S} \cdot {}^{S}T_{G} \cdot {}^{G}T_{T}$$

$$= {}^{B}T_{W}^{-1} \cdot {}^{B}T_{S} \cdot {}^{S}T_{G} \quad \leftarrow \text{answer}$$

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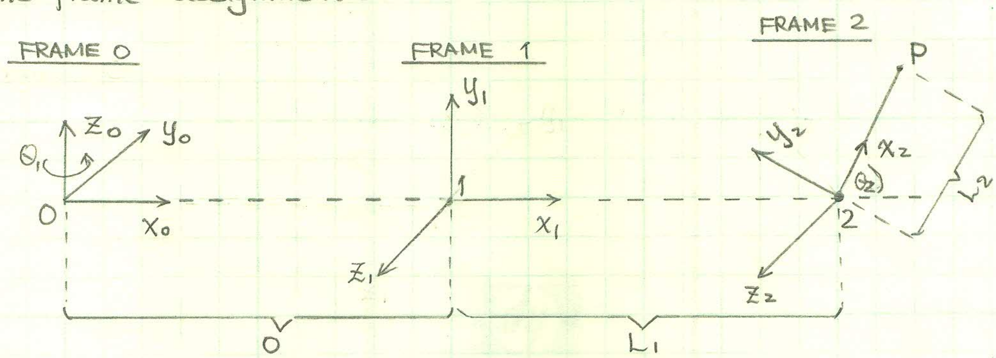
4. The frame assignments

a)



NOTE: FROM THE FIGURE (b) IN THE DIAGRAM, WE SEE THAT FOR

FRAME 0 : X_0, Z_0 ARE KNOWN, THEN y_0 CAN BE ASSIGNED AS ABOVE

FRAME 1 : SAME AS FRAME 0

FRAME 2 : WE KNEW $\hat{X}_2$ and $\hat{Y}_2$, THUS Z_2 IS FIXED.

b) $^{2}P_{tip} = (L_2, 0, 0)$, Thus

$$\begin{bmatrix} {}^{0}P_{tip} \\ 1 \end{bmatrix} = {}^{0}T_{2} \cdot {}^{2}P_{tip} = \begin{bmatrix} C_1C_2 & -C_1S_2 & S_1 & L_1C_1 \\ S_1C_2 & -S_1S_2 & -C_1 & L_1S_1 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} L_2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} L_2C_1C_2 + L_1C_1 \\ L_2S_1C_2 + L_1S_1 \\ L_2S_2 \\ 1 \end{bmatrix}$$

Thus

$$^{0}P_{tip} = (L_2C_1C_2 + L_1C_1,\ L_2S_1C_2 + L_1S_1,\ L_2S_2)$$

Ben M. Chen

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ME 442 Robotics

Inverse Kinematic Problem Set

For problem two in the last homework exercise, you developed the foward kinematic transform ${}^{0}T_{3}$. This homework exercise will focus on the inverse kinematics for this manipulator. Use the following for the questions that follow.

$$
{}^{0}T_{3} = \begin{bmatrix} c_1c_{23} & -c_1s_{23} & s_1 & l_2c_1c_2 + l_1c_1 \\ s_1c_{23} & -s_1s_{23} & -c_1 & l_2s_1c_2 + l_1s_1 \\ s_{23} & c_{23} & 0 & l_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

1. Derive the inverse kinematic relationships for the manipulator joint angles for a specified ${}^{B}T_{W}$ in the manipulators workspace.

2. For a goal orientation specified as Trans(0 0 5)*RPY(0,-45,90), compute the joint angles for a manipulator where $l_1 = 10$ and $l_2 = 11.18$. In a sketch, locate the goal frame and show the manipulator in the configuration computed above. Show the frame axes for joint 3.

3. For a goal orientation specified as Trans(2.486,2.486,8.484)* RPY(45,-45,90), compute the joint angles for a manipulator where $l_1 = 12$ and $l_2 = 12$. In a sketch, locate the goal frame and show the manipulator in the configuration computed above. Show the frame axes for joint 3.

4. Given the following ${}^{B}T_{W}$ goal, compute the joint angles for the same manipulator specified in 3.

$$
{}^{B}T_{W} = \begin{bmatrix} .5 & -.5 & .707 & 4 \\ .5 & -.5 & -.707 & 2 \\ .707 & .707 & 0 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Substitute the computed joint angles and link lengths back into ${}^{0}T_{3}$ and then compare the elements of ${}^{0}T_{3}$ with the elements of ${}^{B}T_{W}$. Discuss this comparison in terms of the manipulator workspace.

Inverse kinematic Problem Set :

1. Solution: Let ${}^0T_3 = {}^BT_W$, we have

$$\begin{bmatrix} C_1C_{23} & -C_1S_{23} & S_1 & l_2C_1C_2+l_1C_1 \\ S_1C_{23} & -S_1S_{23} & -C_1 & l_2S_1S_2+l_1S_1 \\ S_{23} & C_{23} & 0 & l_2S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} n_x & O_x & a_x & P_x \\ n_y & O_y & a_y & P_y \\ n_z & O_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \cdots (1)$$

From the matrix equation above, we see that

① $\begin{cases} S_1 = a_x \\ -C_1 = a_y \end{cases}$

$$\tan\theta_1 = \frac{S_1}{C_1} = \qquad \therefore \quad \theta_1 = \text{ATAN2}(a_x, -a_y)$$

② $\begin{cases} l_2C_1C_2 + l_1C_1 = P_x & \Rightarrow \quad C_2 = (P_x - l_1C_1)/l_2C_1 \\ l_2S_2 = P_z & \Rightarrow \quad S_2 = P_z/l_2 \end{cases}$

$$\tan\theta_2 = \frac{S_2}{C_2} \qquad \theta_2 = \text{ATAN2}\left(\frac{P_z}{l_2}, \frac{P_x - l_1C_1}{l_2C_1}\right) = \text{ATAN2}\left(P_z, \frac{P_x - l_1C_1}{C_1}\right)$$

③ $\begin{cases} S_{23} = n_z \\ C_{23} = O_z \end{cases} \Rightarrow \quad \theta_2 + \theta_3 = \text{ATAN2}(n_z, O_z)$

So, we have the conclusion now:

$$\begin{cases} \theta_1 = \text{ATAN2}(a_x, -a_y) \\ \theta_2 = \text{ATAN2}\left(P_z, \dfrac{P_x - l_1C_1}{C_1}\right) \\ \theta_3 = \text{ATAN2}(n_z, O_z) - \text{ATAN2}\left(P_z, \dfrac{P_x - l_1C_1}{C_1}\right) \end{cases}$$

Note: we see that all the angles are obtained from ATAN2 functions. So, for a given ${}^BT_W$, we can only one set of solution for the joint angles. ✓

2. Solution: From 6.15 on page 115 in text.

$$RPY(\phi_z,\phi_y,\phi_x)=\begin{bmatrix} C\phi_z C\phi_y & C\phi_z S\phi_y S\phi_x - S\phi_z C\phi_x & C\phi_z S\phi_y C\phi_x + S\phi_z S\phi_x & 0 \\ S\phi_z S\phi_y & S\phi_z S\phi_y S\phi_x - C\phi_z C\phi_x & S\phi_z S\phi_y C\phi_x - C\phi_z S\phi_x & 0 \\ -S\phi_y & C\phi_y S\phi_x & C\phi_y C\phi_x & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Trans(a,b,c)=\begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus,

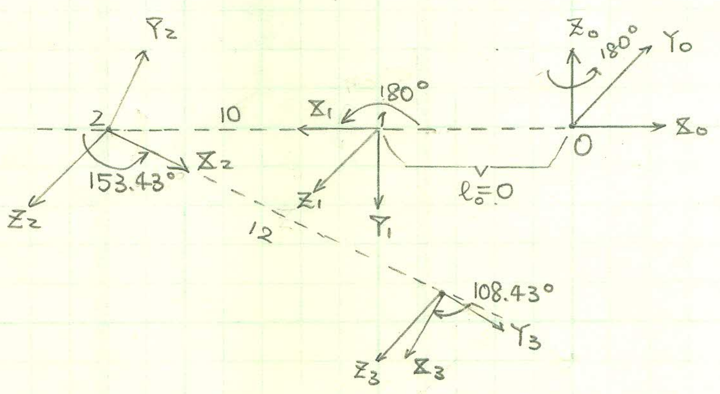
$$RPY(0,-45,90)=\begin{bmatrix} 0.7071 & -0.7071 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0.7071 & 0.7071 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{B}T_W = Trans(a,b,c)\cdot RPY(0,-45,90)=\begin{bmatrix} 0.7071 & -0.7071 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0.7071 & 0.7071 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then Apply the equations in problem 1, we have

$$\begin{cases} \theta_1 = ATAN2(a_x,-a_y) = ATAN2(0,-1) = 180^\circ \\ \theta_2 = ATAN2(P_z,\ P_x/C_1 - l_1) = ATAN2(5,-10) = 153.43^\circ \\ \theta_3 = ATAN2(n_z, O_z) - \theta_2 = ATAN2(0.7071, 0.7071) - 153.43^\circ \end{cases}$$

$$= -108.43^\circ$$



3. Solution:

$$RPY(45,-45,90) = \begin{bmatrix} 0.5000 & -0.5000 & 0.7071 & 0 \\ 0.5000 & -0.5000 & -0.7071 & 0 \\ 0.7071 & 0.7071 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

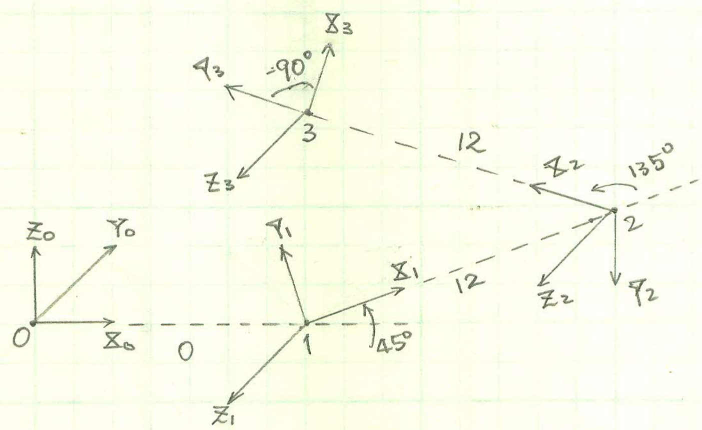
$$Trans(2.486\ ,\ 2.486\ ,\ 8.484) = \begin{bmatrix} 1 & 0 & 0 & 2.486 \\ 0 & 1 & 0 & 2.486 \\ 0 & 0 & 1 & 8.484 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{B}T_W = Trans(2.486, 2.486, 8.484) \cdot RPY(45,-45,90) =$$

$$\begin{bmatrix} 0.5000 & -0.5000 & 0.7071 & 2.486 \\ 0.5000 & -0.5000 & -0.7071 & 2.486 \\ 0.7071 & 0.7071 & 0 & 8.484 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then from the formulars in problem 1, we have

$$\begin{cases} \theta_1 = ATAN2(a_x, -a_y) = ATAN2(0.7071, 0.7071) = 45^\circ \\ \theta_2 = ATAN2(P_z,\ P_x/c_1 - l_1) = ATAN2(8.484, -8.484) = 135^\circ \\ \theta_3 = ATAN2(n_z, O_z) - 135^\circ = ATAN2(0.7071, 0.7071) - 135^\circ = -90^\circ \end{cases}$$



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4. Solution:

$$^{B}T_{W} = \begin{bmatrix} .5 & -.5 & 0.707 & 4 \\ .5 & -.5 & -0.707 & 2 \\ 0.707 & 0.707 & 0 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Once again, applies the equations in problem 1 to this $^{B}T_{w}$, we have

$$\begin{cases} \theta_1 = \text{ATAN2}(0.707, 0.707) = 45° \\ \theta_2 = \text{ATAN2}(8, -6.3431) = 128.41° \\ \theta_3 = \text{ATAN2}(0.707, 0.707) - 128.41° = -83.41° \end{cases}$$

Substitute these angles and $l_1 = l_2 = 12$ to $^{0}T_{3}$, we have

$$^{0}T_{3} = \begin{bmatrix} 0.5 & -0.5 & 0.707 & 3.2135 \\ 0.5 & -0.5 & -0.707 & 15.1342 \\ 0.707 & 0.707 & 0 & 9.4030 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Comparision & discussion:

One can see the differences between the forth columns of $^{B}T_{w}$ and $^{0}T_{3}$. I think the transformation $^{B}T_{w}$ is still within the manipulator workspace NO. The problem is that the equation $^{0}T_{3}$ given at the begining of this homework set is incorrect YES.

For example, recall equation (1) in problem 1, we have

$$l_2 S_2 S_1 + l_1 S_1 = S_1 (l_2 S_2 + l_1) = P_y$$

$$l_2 S_2 = P_z \quad \& \quad S_1 = a_x$$

So, we have $\quad a_x (P_z + l_1) = P_y \quad$ ------ (2)

Then check PROBLEM 2: $a_x(P_z + l_1) = 0 = P_y$. RESULTS IN PROBLEM 2 ARE O.K.

PROBLEM 3: $a_x(P_z + l_1) = 13.07 \neq 2.486 = P_y$, Wrong Results.

PROBLEM 4: $a_x(P_z + l_1) = 14.14 \neq 2 = P_y$, Wrong Results.

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2. Suppose that the hand configuration is given by

$$T_H = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

What is the effect of a differential rotation of 0.15 rad about the Z axis followed by a differential translation of $[0.5\ 0.5\ 1.0]^T$?

Solution: From the givens, we have

$$\delta x = 0, \quad \delta y = 0, \quad \delta z = 0.15$$

$$dx = 0.5, \quad dy = 0.5, \quad dz = 1.0$$

So, we have the differential operator

$$\Delta = \begin{bmatrix} 0 & -0.15 & 0 & 0.5 \\ 0.15 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 1.0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The effect is the differential motion

$$dT_H = \Delta T_H$$

$$= \begin{bmatrix} 0 & -0.15 & 0 & 0.5 \\ 0.15 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 1.0 \\ 0 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.15 & 0 & 0 & -0.4 \\ 0 & 0 & 0.15 & 0.65 \\ 0 & 0 & 0 & 1.0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

✓ #

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8.7 6. Using the strategy described in section 8.2.3, find ${}^H J_{21}$. That is find $\partial y / \partial \theta_1$ for the articulated manipulator.

Solution:

$$ {}^H J_{21} = \frac{{}^H \partial y}{\partial \theta_1} = {}^0 O_y \, {}^0 P_x - {}^0 O_x \, {}^0 P_y $$

$$ = [-S_1 (C_{234} C_5 S_6 + S_{234} C_6) - C_1 S_5 S_6] $$
$$ \cdot C_1 \cdot [C_{234} a_4 + C_{23} a_3 + C_2 a_2] $$
$$ - [-C_1 (C_{234} C_5 S_6 + S_{234} C_6) + S_1 S_5 S_6] $$
$$ \cdot S_1 \cdot [C_{234} a_4 + C_{23} a_3 + C_2 a_2] $$

$$ = \{ C_{234} a_4 + C_{23} a_3 + C_2 a_2 \} \cdot $$
$$ \cdot \{ -S_1 C_1 C_{234} C_5 S_6 - S_1 C_1 S_{234} C_6 - C_1^2 S_5 S_6 $$
$$ + S_1 C_1 C_{234} C_5 S_6 + S_1 C_1 S_{234} C_6 - S_1^2 S_5 S_6 \} $$

$$ = S_5 S_6 \cdot [C_{234} \cdot a_4 + C_{23} \cdot a_3 + C_2 \cdot a_2] \quad \# $$

✓

7. (a) The articulated manipular is in the configuration given in Example 8.2. Given a differential motion of $[0.1, 0, 0, 0.1, 0.2, 0]^T$, find the differential displacement of the hand frame.

Solution: From Eq. (8.69), we have

$$^{H}\begin{bmatrix} dx \\ dy \\ dz \\ \delta x \\ \delta y \\ \delta z \end{bmatrix} = \begin{bmatrix} 0 & 0.2 & 0.2 & 0.2 & 0 & 0 \\ 0 & -2 & -1 & 0 & 0 & 0 \\ 0.2 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0.1 \\ 0 \\ 0 \\ 0.1 \\ 0.2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.02 \\ 0.00 \\ 0.02 \\ +0.10 \\ 0.00 \\ 0.10 \end{bmatrix}$$

(b) Find $^{H}\Delta$ for this configuration.

$$^{H}\Delta = \begin{bmatrix} 0 & -0.10 & 0 & 0.02 \\ 0.10 & 0 & 0.10 & 0.00 \\ 0 & -0.10 & 0 & 0.02 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(c) Find the motion of the hand, as measured in base coordinates; that is, find dT_H

$$dT_H = T_H\,{}^{H}\Delta$$

$$= \begin{bmatrix} 0 & 1 & 0 & -0.20 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & -0.10 & 0 & 0.02 \\ 0.10 & 0 & 0.10 & 0.00 \\ 0 & -0.10 & 0 & 0.02 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.10 & 0 & 0.10 & 0 \\ 0 & 0.10 & 0 & -0.02 \\ 0 & 0.10 & 0 & -0.02 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

#

8. Given that the articulated manipular is in the following state

$$ {}^{R}T_H = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & -1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

find the joint velocities corresponding to a Cartesian velocity of

$$ \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\phi}_x \\ \dot{\phi}_y \\ \dot{\phi}_z \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.2 \\ 0.7 \\ 0 \\ 0 \\ 0.1 \end{bmatrix} $$

Solution: From givens, we have

$$ {}^{H}\Delta = \begin{bmatrix} 0 & -0.1 & 0 & 0.2 \\ 0.1 & 0 & 0 & 0.2 \\ 0 & 0 & 0 & 0.7 \\ 0 & 0 & 0 & 0 \end{bmatrix} $$

And then

$$ dT = {}^{R}T_H \cdot {}^{H}\Delta = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & -1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & -0.1 & 0 & 0.2 \\ 0.1 & 0 & 0 & 0.2 \\ 0 & 0 & 0 & 0.7 \\ 0 & 0 & 0 & 0 \end{bmatrix} $$

$$ = \begin{bmatrix} 0.1 & 0 & 0 & 0.2 \\ 0 & -0.1 & 0 & 0.2 \\ 0 & 0 & 0 & -0.7 \\ 0 & 0 & 0 & 0 \end{bmatrix} $$

We have $\dot{P}_x = 0.2$, $\dot{P}_y = 0.2$

$$ \theta_1 = \tan^{-1}\frac{P_y}{P_x} = \tan^{-1}\frac{2}{1} = 1.107 \text{ rad} $$

$$ \dot{\theta}_1 = \frac{\cos\theta_1 \dot{P}_y - \sin\theta_1 \dot{P}_x}{P_x\cos\theta_1 + P_y \sin\theta_1} = -0.04 \text{ rad} / \Delta t \text{ samp} $$

#

From Eq. (7.53), we have

$$C_3 = \cos\theta_3 = \frac{P_1^2 + P_2^2 - a_3^2 - a_2^2}{2a_2a_3} = -0.8333$$

$$2a_2 \cdot a_3 \cdot \cos\theta_3 = P_1^2 + P_2^2 - a_3^2 - a_2^2$$

$$-2a_2 \cdot a_3 \cdot \sin\theta_3 \cdot \dot\theta_3 = 2 \cdot P_1 \cdot \dot P_1 + 2P_2\dot P_2$$

$$\dot\theta_3 = \frac{P_1 \cdot \dot P_1 + P_2 \cdot \dot P_2}{a_2 a_3 \sin\theta_3} \quad \cdots \quad (1)$$

where $P_1 = P_x \cdot C_1 + P_y \cdot S_1 - a_4 C_{234} = P_x \cdot C_1 + P_y \cdot S_1$

$$\dot P_1 = \dot P_x C_1 - P_x \cdot S_1 \cdot \dot\theta_1 + \dot P_y S_1 + P_y \cdot C_1 \cdot \dot\theta_1 + a_4 S_{234}\overset{0}{\cancel{(\dot\theta_2 + \dot\theta_3 + \dot\theta_4)}}$$

$\cdots$ (2)

$$P_2 = P_z - a_4 S_{234} = P_z - a_4$$

$$\dot P_2 = \dot P_z - a_4 C_{234} \cdot \overset{0}{\cancel{(\dot\theta_2 + \dot\theta_3 + \dot\theta_4)}}$$

From (7.45)

$$a_z C_{234} = 0 \qquad (a_x = a_y = 0)$$

$$\dot a_z C_{234} - a_z S_{234} (\dot\theta_2 + \dot\theta_3 + \dot\theta_4) = 0.$$

$$\theta_2 + \theta_3 + \theta_4 = 90^\circ$$

$$\therefore \quad \dot\theta_2 + \dot\theta_3 + \dot\theta_4 = 0$$

$\therefore$ $P_1 = P_x \cdot C_1 + P_y \cdot S_1 = 0.44735 + 2 \times 0.89436 = 2.23607$

$P_2 = P_z - a_4 = 5 - .2 = 4.8$

$\dot P_1 = .2 \times 0.44735 + 1 \times 0.89436 \times 0.04 + 0.2 \times 0.89436 - 2 \times 0.44735 \times 0.04$

$= 0.26833$

$\dot P_2 = \dot P_z = -0.7$

$\theta_3 = 146.44^\circ \qquad \sin\theta_3 = 0.55277$

<u>Joint Velocities:</u>

$$\dot\theta_3 = \frac{2.23607 \times 0.26833 + 4.8 \times (-.7)}{3 \times 4 \times 0.55277}$$

$$= -0.41609 \text{ rad}/\Delta t \text{ samp.}$$

OK

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Good!

ME 442 Robotics

Path Planning Problem Set

The following questions concern path planning using the two link planor manipulator used in class for examples. Link lengths are $l_1 = 10$ and $l_2 = 8$. The hand frame does not rotate with respect to the base. The questions will use the following four hand frame positions and the elbow up option for the links:

P_i	P_x	P_y	time, sec
1	10	4	0
2	8	6	1
3	8	8.8	2
4	10	10.8	3

1. Using a constant velocity trajectory with parabolic blends, compute blend times for the joint motions for a movement between P_1 and P_4 (do not pass through P_2 and P_3). Use a constant acceleration of 120% of the minimum permissible value. What will be ${}^0T_3$ at 1.0 second? At this instant, what is the roll angle of the hand frame? For these last two questions, maintain four decimal places of accuracy.

2. For the joint angle, θ_1, and using all four points, plan a path using a 3rd order polynomial for P_2 to P_3 and 4th order polynomials for P_1 to P_2 and P_3 to P_4. Match velocity at P_2 and P_3 and use zero velocity and acceleration at P_1 and P_4. For the velocities at P_2 and P_3, use the heuristic that was presented in class.

3. Repeat 2 using P_2 and P_3 as via points. Instead of polynomial paths, use constant velocities between the points and constant acceleration blends at the points. Path must start at P_1 and end at P_4. Use ±100 degrees per sec^2 for the blend acceleration with the sign selected using the algorithm for blend paths. Provide answer as a table that indicates the times for blends with the acceleration values and times for constant velocity with velocity values.

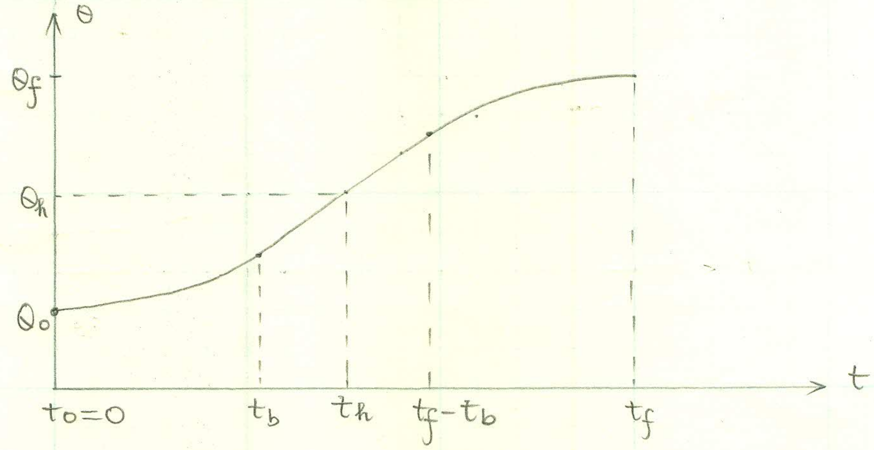
Solution: From the lecture on September 14, we have the inverse kinematis for the "elbow up" planor manipulator. And since hand frame does not rotate with respect to the base, this implies $\theta_1+\theta_2+\theta_3=2k\pi$. Then we have following equations:

$$
\begin{cases}
C_2 = (P_x^2+P_y^2-l_1^2-l_2^2)/2l_1 l_2 \\
S_2 = -\sqrt{1-C_2^2} \\
K_1 = l_1 + l_2 C_2 \\
K_2 = l_2 \cdot S_2 \\
\theta_1 = \mathrm{ATAN2}(P_y, P_x) - \mathrm{ATAN2}(K_2, K_1) \\
\theta_2 = \mathrm{ATAN2}(S_2, C_2), \qquad \theta_3 = -\theta_1 - \theta_2
\end{cases}
$$

Using these equations, we are able to compute

P_i	P_x	P_y	θ_1	θ_2	θ_3	t(sec)
1	10	4	66.91994°	−107.4576°	40.53766°	0
2	8	6	84.02625°	−113.5782°	29.55195°	1
3	8	8.8	89.48176°	−98.1057°	8.62394°	2
4	10	10.8	78.08437°	−70.7919°	−7.29247°	3

(1)



From notes, we have following formulas:

$$\begin{cases} \ddot{\theta}_{min} = 4(\theta_f - \theta_0)/t_f^2 & \text{---- (1)} \\ \ddot{\theta} = 1.2 \times \ddot{\theta}_{min} & \cdots \text{ (2)} \\ t_b = \frac{t_f}{2} - \frac{\sqrt{\ddot{\theta}^2 t_f^2 - 4\ddot{\theta}(\theta_f - \theta_0)}}{2\ddot{\theta}} & \cdots \text{ (3)} \end{cases}$$

And
$$\begin{cases} \theta(t) = \theta_0 + \frac{1}{2}\ddot{\theta} t^2 \ ; & 0 \le t \le t_b \\ \theta(t) = \theta_0 + \frac{1}{2}\ddot{\theta} t_b^2 + (t - t_b)\cdot\ddot{\theta}\cdot t_b & t_b \le t \le t_f - t_b \\ \theta(t) = \theta_0 + \frac{1}{2}\ddot{\theta} t_b^2 + (t_f - 2t_b)\cdot\ddot{\theta}\cdot t_b & ; \ t_f - t_b \le t \le t_f \\ \quad + \ddot{\theta} t_b\cdot(t - t_f + t_b) - \frac{1}{2}\ddot{\theta}(t - t_f + t_b)^2 & \end{cases}$$

Then for θ_1:

$\theta_{1f} = 78.08437^\circ \qquad \theta_{10} = 66.91994^\circ \qquad t_f = 3$ sec.

$\ddot{\theta}_1 \min = 4.96197 \text{ deg}/s^2 \quad \Rightarrow \quad \ddot{\theta}_1 = 5.95436 \text{ deg}/s^2$

$t_{b1} = 1.5 - 0.6124 = 0.8876$ second.

$\theta_1(1 \text{ sec}) = 66.91994 + \frac{1}{2} \times 5.95436 \times (.8876)^2 +$

$(1 - 0.8876) \times 5.95436 \times 0.8876 = 69.8594^\circ$

for θ_2:

$\theta_{2f} = -70.7919^\circ \qquad \theta_{20} = -107.4576^\circ \qquad t_f = 3$ sec.

$\ddot{\theta}_2 \min = 16.2959 \quad \Rightarrow \quad \ddot{\theta}_2 = 19.5550 \text{ deg}/\sec^2$

$t_{b2} = 1.5 - 0.6124 = 0.8876$ second.

(Note: it is easy to prove that t_b is independent of θ_f and θ_0)

$\theta_2(1 \text{ sec.}) = -97.8035^\circ$

for θ_3 :

$$\theta_{30} = 40.53766^\circ \qquad \theta_{3f} = -7.29247^\circ$$

$$\ddot{\theta}_{3\,min} = -21.25784 \text{ deg/sec}^2 \Rightarrow \ddot{\theta}_3 = -25.5094 \text{ deg/sec}^2$$

$$t_{3b} = t_{1b} = t_{2b} = 0.8876 \text{ sec.}$$

$$\theta_3(1\text{ sec.}) = 27.9441^\circ$$

Check : $\theta_1(1\text{ sec.}) + \theta_2(1\text{ sec.}) + \theta_3(1\text{ sec.}) =$

$$69.8598 - 97.8035 + 27.9441 = 0^\circ \qquad \text{(agreed)}$$

Answer : Blend time :
(i) from 0 to 0.8876 seconds ; $\ddot{\theta}$
(ii) from 0.8876 to 2.1124 seconds ; $\dot{\theta}$
(iii) from 2.1124 to 3 seconds ; $-\ddot{\theta}$

Subtitute $\theta_1(1\text{ sec.})$, $\theta_2(1\text{ sec.})$ and $\theta_3(1\text{ sec.})$ into the form in notes , we have

$$^0T_3 = \begin{bmatrix} C_{123} & -S_{123} & 0 & l_1C_1 + l_2C_{12} \\ S_{123} & C_{123} & 0 & l_1S_1 + l_2S_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

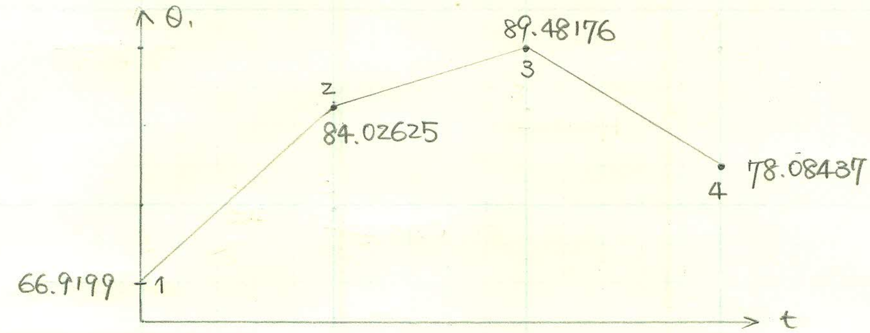
$$= \begin{bmatrix} 1 & 0 & 0 & 10.51052 \\ 0 & 1 & 0 & 5.6397 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Roll angle of the hand frame at $t = 1.0$ second

$$\theta_3(1\text{ sec.}) = 27.9441^\circ$$

From now on, all $\theta_1, \theta_2, \cdots$ represent θ_1 in detterent segments:

2.



From the lecture, we have (heuristic)

$$\dot{\theta}_K = \frac{\theta_{K+1} - \theta_K}{2(t_{K+1} - t_K)} + \frac{\theta_K - \theta_{K-1}}{2(t_K - t_{K-1})} \qquad \text{for } k = 2, 3.$$

$$\dot{\theta}_{12} = (84.02625 - 66.9199) = 17.10635$$

$$\dot{\theta}_{23} = (89.48176 - 84.02625) = 5.45551$$

$$\dot{\theta}_{34} = 78.08437 - 89.48176 = -11.39739$$

$$\therefore \quad \dot{\theta}_2 = 11.28093 \ \text{deg/sec}.$$

$$\dot{\theta}_3 = 0 \ \text{deg/second}.$$

Path constraints:

(66.9199, 0)	(84.02625, 11.28093)	(89.48176, 0)	(78.08437, 0)
P_1	P_2	P_3	P_4

For Path From P_1 to P_2:

$$\theta_1(t) = a_{10} + a_{11} t_1 + a_{12} \cdot t_1^2 + a_{13} \cdot t_1^3 + a_{14} \cdot t_1^4$$

$$\theta_1(0) = a_{10} = 66.9199$$

$$\dot{\theta}_1(0) = a_{11} = 0.$$

$$\ddot{\theta}_1(0) = 2a_{12} = 0.$$

$$\theta_1(1) = a_{10} + a_{13} + a_{14} = 84.02625$$

$$\dot{\theta}_1(1) = 3a_{13} + 4a_{14} = 11.28093$$

$$\Rightarrow \begin{cases} a_{13} + a_{14} = 17.10635 \\ 3a_{13} + 4a_{14} = 11.28093 \end{cases}$$

$$\Rightarrow a_{13} = 57.14447 \ ; \quad a_{14} = -40.03812$$

$$\therefore \quad \theta_1(t_1) = 66.9199 + 57.14447 t_1^3 - 40.03812 t_1^4$$

2nd segment : from P_2 to P_3

$$\theta_2(t_2) = a_{20} + a_{21} \cdot t_2 + a_{22} \cdot t_2^2 + a_{23} t_2^3$$

$$\theta_2(0) = a_{20} = 84.02625$$

$$\dot{\theta}_2(0) = a_{21} = 11.28093$$

$$\theta_2(1) = a_{20} + a_{21} + a_{22} + a_{23} = 89.48176$$

$$\dot{\theta}_2(1) = a_{21} + 2a_{22} + 3a_{23} = 0$$

$$\Rightarrow \begin{cases} a_{22} + a_{23} = -5.82542 \\ 2a_{22} + 3a_{23} = -11.28093 \end{cases} \Rightarrow \begin{cases} a_{22} = -6.19533 \\ a_{23} = 0.36991 \end{cases}$$

$$\therefore \quad \underline{\theta_2(t_2) = 84.02625 + 11.28093 \cdot t_2 - 6.19533\, t_2^2 + 0.36991\, t_2^3}$$

Last segment :

$$\theta_3(t_3) = a_{30} + a_{31} \cdot t_3 + a_{32} \cdot t_3^2 + a_{33} \cdot t_3^3 + a_{34} t_3^4$$

$$\theta_3(0) = a_{30} = 89.48176$$

$$\dot{\theta}_3(0) = a_{31} = 0$$

$$\theta_3(1) = 89.48176 + 0 + a_{32} + a_{33} + a_{34} = 78.08437$$

$$\dot{\theta}_3(1) = 2a_{32} + 3a_{33} + 4a_{34} = 0$$

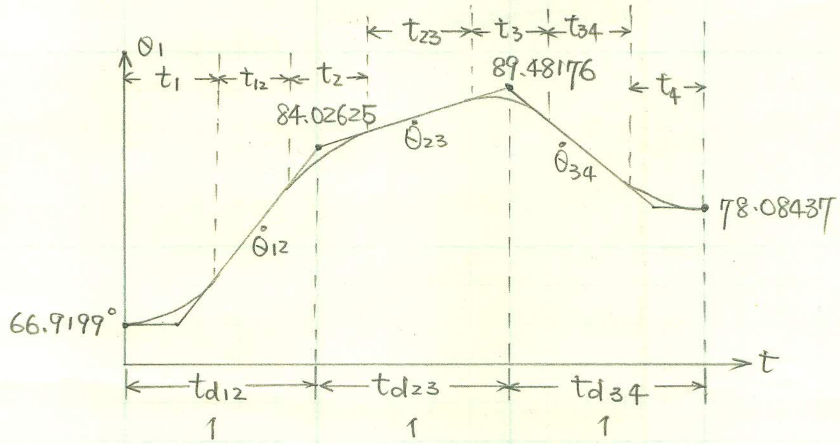
$$\ddot{\theta}_3(1) = 2a_{32} + 6a_{33} + 12a_{34} = 0$$

Solve $\begin{cases} a_{32} + a_{33} + a_{34} = -11.39739 \\ 2a_{32} + 3a_{33} + 4a_{34} = 0 \\ 2a_{32} + 6a_{33} + 12a_{34} = 0 \end{cases}$

We have $\begin{cases} a_{32} = -68.384 \\ a_{33} = 91.179 \\ a_{34} = -34.192 \end{cases}$

$$\therefore \quad \underline{\theta_3(t_3) = 89.48176 - 68.384 \cdot t_3^2 + 91.179\, t_3^3 - 34.192\, t_3^4}$$

3.



Given: $|\ddot\theta_1| = |\ddot\theta_2| = |\ddot\theta_3| = |\ddot\theta_4| = 100$ degrees/sec²

First segment:

$$\ddot\theta_1 = \mathrm{SGN}(84.02625 - 66.9199) \cdot 100 = 100 \text{ degrees/sec}^2$$

$$t_1 = 1 - \left[1^2 - \frac{2(84.02625 - 66.9199)}{100} \right]^{1/2}$$

$$= 0.1889 \text{ seconds}$$

$$\dot\theta_{12} = (\theta_2 - \theta_1)/(td_{12} - 0.5\, t_1)$$

$$= (84.02625 - 66.9199)/(1 - 0.5 \times 0.1889)$$

$$= 18.8906 \text{ degrees/sec.}$$

$$t_{12} = 1 - 0.1889 - \tfrac{1}{2} t_2 = 0.7439 \text{ seconds}$$

— — — — — — — —

Point 2:

$$\dot\theta_{23} = \frac{\theta_3 - \theta_2}{td_{23}} = \frac{89.48176 - 84.02625}{1} = 5.4555 \text{ deg/sec.}$$

$$\ddot\theta_2 = \mathrm{SGN}(\dot\theta_{23} - \dot\theta_{12})|\ddot\theta_2| = \mathrm{SGN}(5.4555 - 18.8906) \cdot 100$$

$$= -100 \text{ deg/sec}^2$$

$$t_2 = (5.4555 - 18.8906)/(-100) = 0.1344 \text{ seconds}$$

$$t_{23} = 1 - \tfrac{1}{2} \times 0.1344 - \tfrac{1}{2} t_3 = 0.8449 \text{ sec.}$$

Last segment:

$$\ddot{\theta}_4 = SGN(89.48176 - 78.08437) \cdot |\ddot{\theta}_4|$$

$$= 100 \text{ degrees/sec}^2$$

$$t_4 = t_{d34} - \left(t_{d34}^2 + \frac{2(\theta_4 - \theta_3)}{\ddot{\theta}_4}\right)^{1/2}$$

$$= 1 - \left(1 + \frac{2(78.08437 - 89.48176)}{100}\right)^{1/2}$$

$$= 0.1213 \text{ seconds.}$$

$$\dot{\theta}_{34} = \frac{78.08437 - 89.48176}{1 - 0.5 \times 0.1213} = -12.1335 \text{ deg/sec.}$$

$$t_{34} = 1 - 0.1213 - \tfrac{1}{2} t_3 = 0.7907 \text{ second.}$$

Point 3:

$$\ddot{\theta}_3 = SGN(\dot{\theta}_{34} - \dot{\theta}_{23}) \cdot |\ddot{\theta}_3|$$

$$= SGN(-12.1335 - 5.4555) \cdot 100 = -100 \text{ degrees/sec}^2$$

$$t_3 = (\dot{\theta}_{34} - \dot{\theta}_{23}) / \ddot{\theta}_3 = (-12.1335 - 5.4555)/-100 = 0.1759 \text{ sec.}$$

P_i	θ_i	t_i	t_{ij}	$\dot{\theta}_{ij}$	$\ddot{\theta}_i$
1	66.9199	0.1889			100
			0.7439	18.8906	
2	84.0263	0.1344			-100
			0.8449	5.4555	
3	89.4818	0.1759			-100
			0.7907	-12.1335	
4	78.0844	0.1213			100

Total time

= 3.0000 seconds.

FILE: ROBOT HOMEWORK A - 10/20/88 15:55:15 PAGE: 001

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HOMEWORK NO:7 FOR EE/ME 442 ROBOTICS BEN M CHEN 10-20-88

Ben M. Chen

1. RETURN ON INVESTMENT CALCULATION

TOOL: PROJECT: SCREW ROBOT

TOTAL PROJECT COST SAVED PER PRODUCT(SEE MAKEBUY WORKSHEET)	$0.35 — .35 - .11 ⇒ .24 −2
QUALITY SAVINGS PER UNIT:	$0.13
INITIAL EXPENSED INVESTMENT:	$20000
CAPITAL INVESTMENT:	$25000
ENGINEERING MAN MONTHS:	3
ENG MAN MONTHS EXPENSE RATE:	$5000
SALVAGE VALUE:	N/A
TOTAL INITIAL TOOL COST:	5 $60000 −1

TAX RATE 50.00%

		YEAR 1	YEAR 2	YEAR 3	YEAR 4	YEAR 5
VOLUME(UNIT PER YEAR)		120000	120000	120000	120000	120000
ANNUAL SAVINGS		$57600	$57600	$57600	$57600	$57600
ANNUAL MAINTENANCE COST		$1000	$1000	$1000	$1000	$1000
CAPITAL DEPRECIATION		$5250	$5250	$5250	$5250	$5250
TAXABLE INCOME		$51350	$51350	$51350	$51350	$51350
NET CASH FLOW	($60000)	$25675	$25675	$25675	$25675	$25675

PAYBACK (MONTHS) 28.04

COST OF CAPITAL 20.00%

	YEAR 1	YEAR 2	YEAR 3	YEAR 4	YEAR 5
CASH FLOW PV	$21396	$17830	$14858	$12382	$10318
CUM PV CASH FLOW	($38604)	($20774)	($5916)	$6466	$16784

IRR 32.19%

2. I think the labor savings is under stated.

3. Increase the labor savings, cut down the initial expensed investment and captial investment.
OR, increase the product volume.

4. Labor savings or product volume.

5. No, if I am the boss. Yes, if I'm hunting an job.

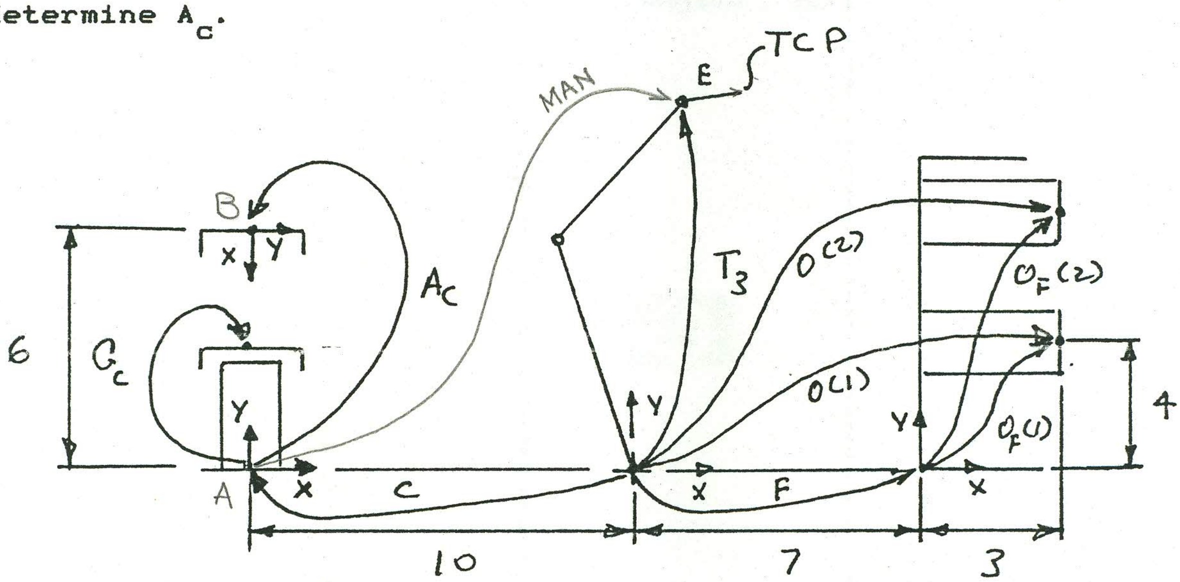
5. (CONT.) if we put 60000 dollars in the bank and assume that the interest is 8.5% per year for the 5-year term, we will ~~have~~ be more profitable and no risk.

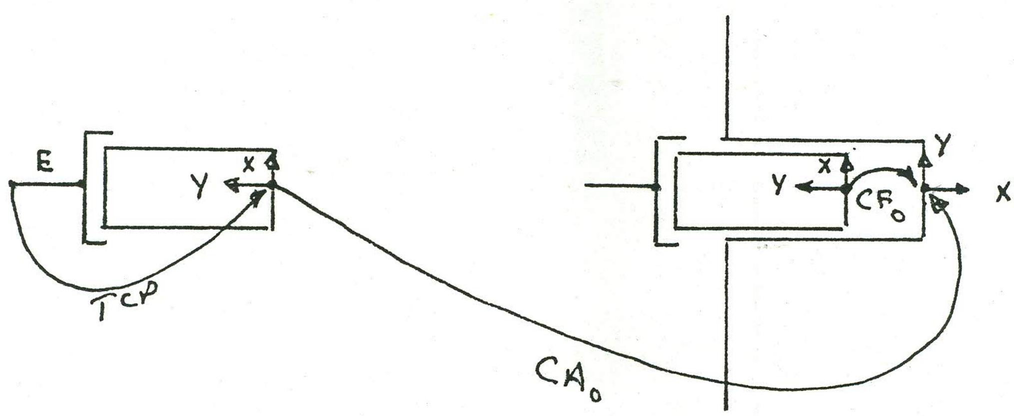
Benmei Chen

ME 442 Robotics

Programming Practice Problem

The two link planner robot is used for a pick and place task. Pins are picked up at a single location and then moved and inserted into a receiver assembly. The assembly recieves two pins and then is removed to be replaced by a new assembly that will recieve pins. When the assembly is in position, it is rigidly held by a fixture so that the insertion of pins will not change its location. The situation is shown below and the arrows and labels identify frame transformations. Use the pseudo code style to prepare a program that accomplishes this task. Using the dimensions and orientations shown, determine A_c.





Programming Practice Problem

Program:

```
TCP = E
k = 1
(WHILE
MAN = C^-1 * T3
MOVE  Ac
MOVE  Gc
GRASP
TCP = E * Gc^-1
O[k] = F * OF[k]
MAN = O[k]^-1 * T3
MOVE  CAo
MOVE  CFo
O[k] = T3 * TCP * CFo^-1
F = O[k] * OF[k]^-1
RELEASE
MAN = CFo^-1 * O[k]^-1 * T3
TCP = E
MOVE  Ac
k = k + 1
WHILE ) k < 2
:
END
```

From the diagram, we see that, if we define the coordinate systems A, B as those in the diagram, then B can be obtained by rotation of $-90°$ about z axis, followed by a displacement of $[0, 6, 0]^T$.

$$A_c = \text{Tran}[0,6,0] * \text{Rot}(z, -90°)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

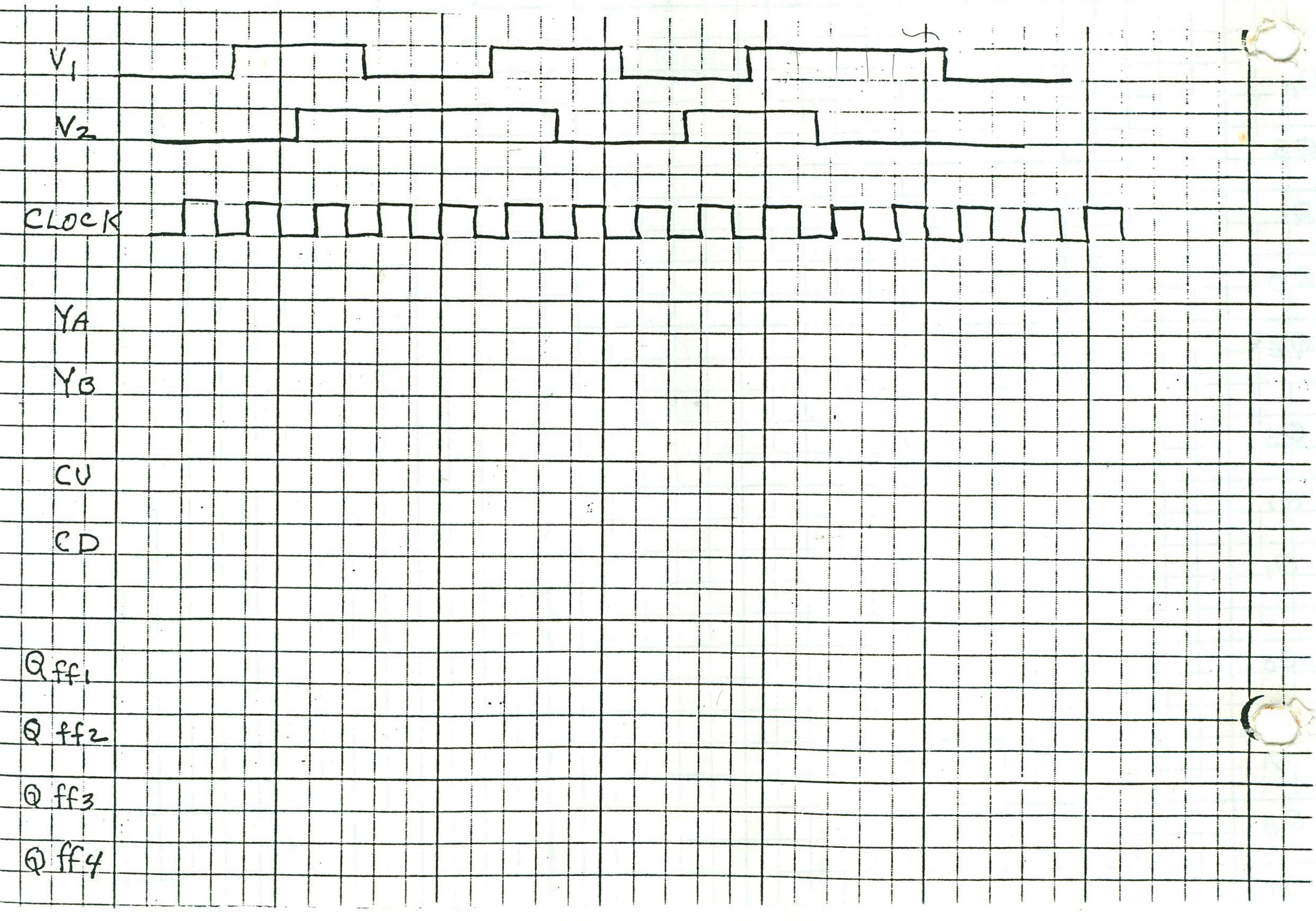
$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

EE/ME 442 H.W. DUE 11/9/88

① THE FOLLOWING PROBLEMS FROM YOUR TEXT
2.3, 2.6, 2.7, 2.8, 2.9, 2.10, 2.11

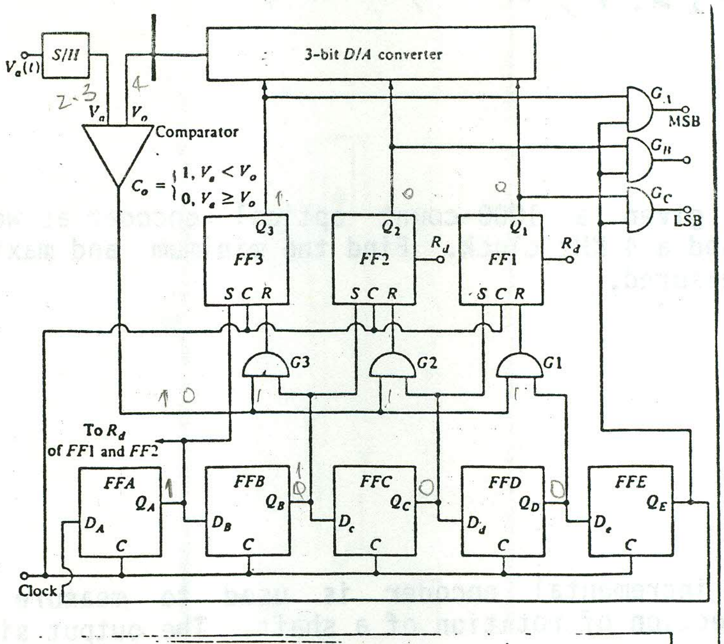
② You are given a 1000-count optical encoder as well as one 8-bit counter and a 4 KHz clock. Find the minimum and maximum speeds that can be measured.

③ An incremental encoder is used to measure the position and the direction of rotation of a shaft. The output signals (V_1 and V_2) of the encoder are decoded by the sequential machine of Figure 2.12 (p. 29) and the synchronizing circuit of Figure 2.14 (p. 30) respectively. Complete the following timing diagrams and compare the outputs of the two decoders.

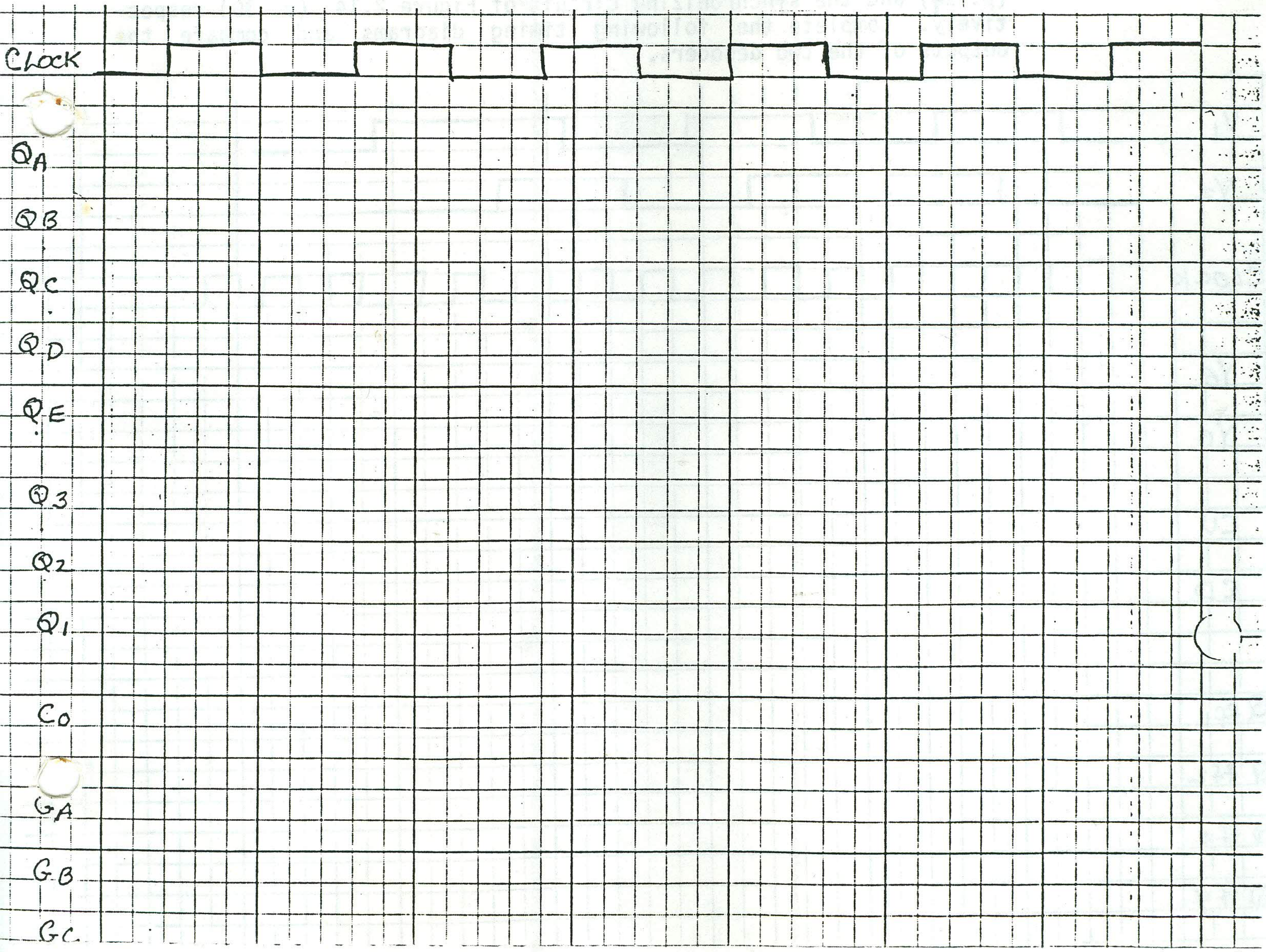


4

For the 3-bit successive approximation A/D converter shown complete the timing diagram. Make sure that you start the process properly. Assume that $V_a = 2.3$ V. What is the purpose of FFE?



A 3-bit successive-approximation A/D converter.



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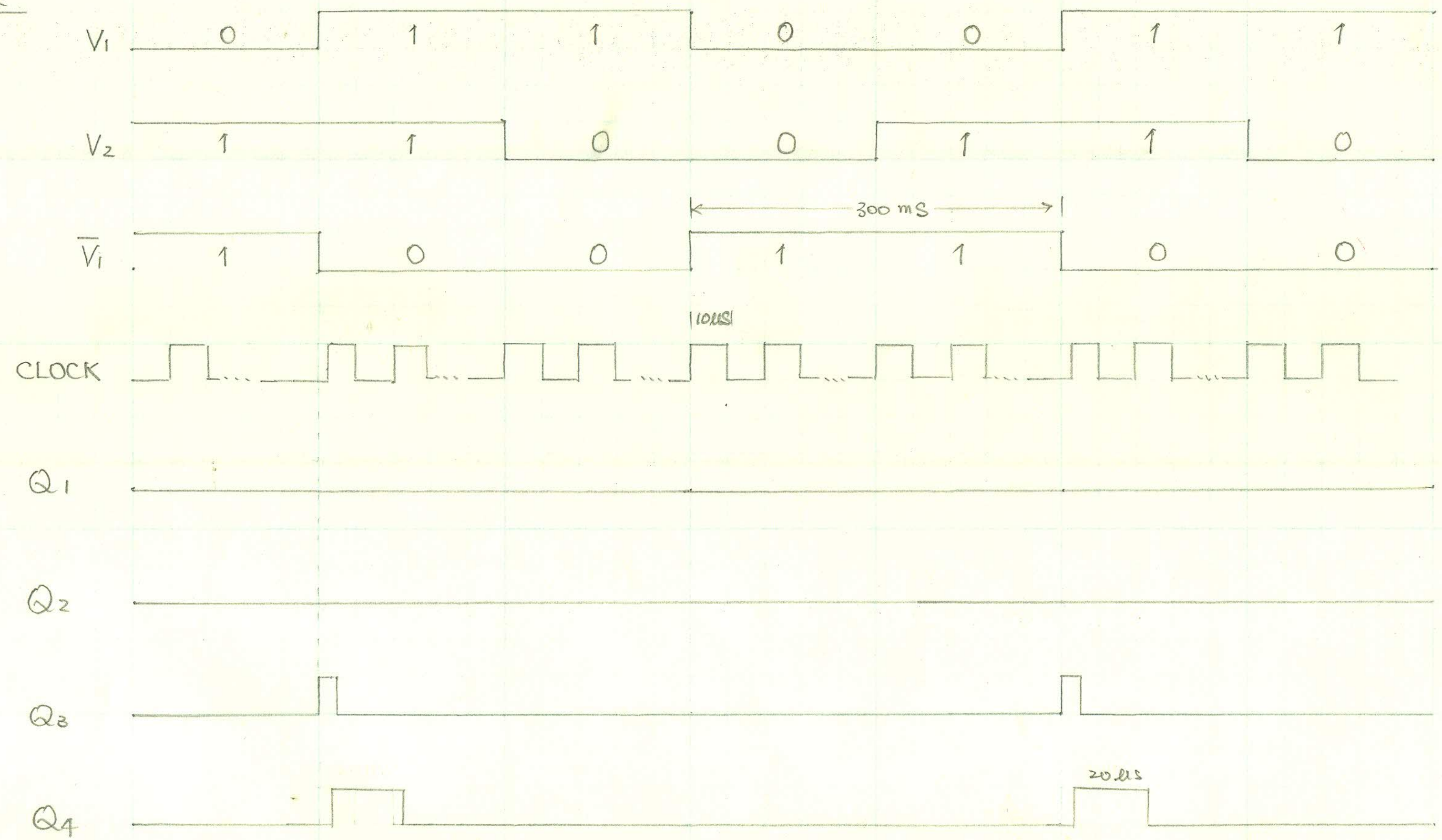
2.3 Assume that a sequence of pulses exactly like those shown in Figure 2.11 (count up case) are input to the circuit of Fig. 2.14. Assume that the pulses are 500 ms wide. Furthermore, assume that the clock input is is a 50-KHz, Construct a detailed timing diagram showing the signals on the Q outputs of all four flip-flops.

Solution:



2.6 You have some optical shaft encoders with 1000 cycles of the V_1 output for one revolution of shaft, Assuming max. speed of the shaft is on revolution per second, dicuss the required clock speeds.

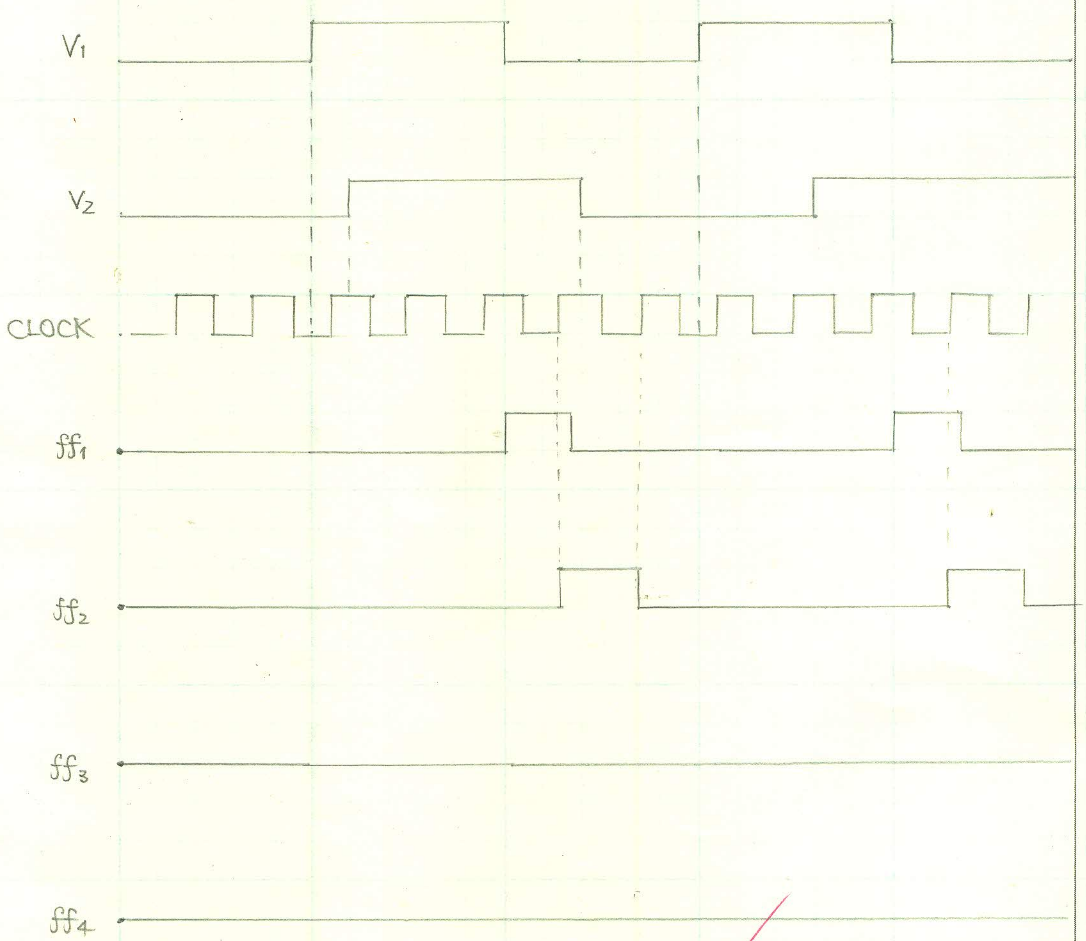
Solution: the pulse wide for $V_1 = 1/1000 = 1$ ms

Then A 1-KHz clock will give 1 count at this rate. But, it gives us more measurement errors.

We may choose the clock rate much higher than 1 K.

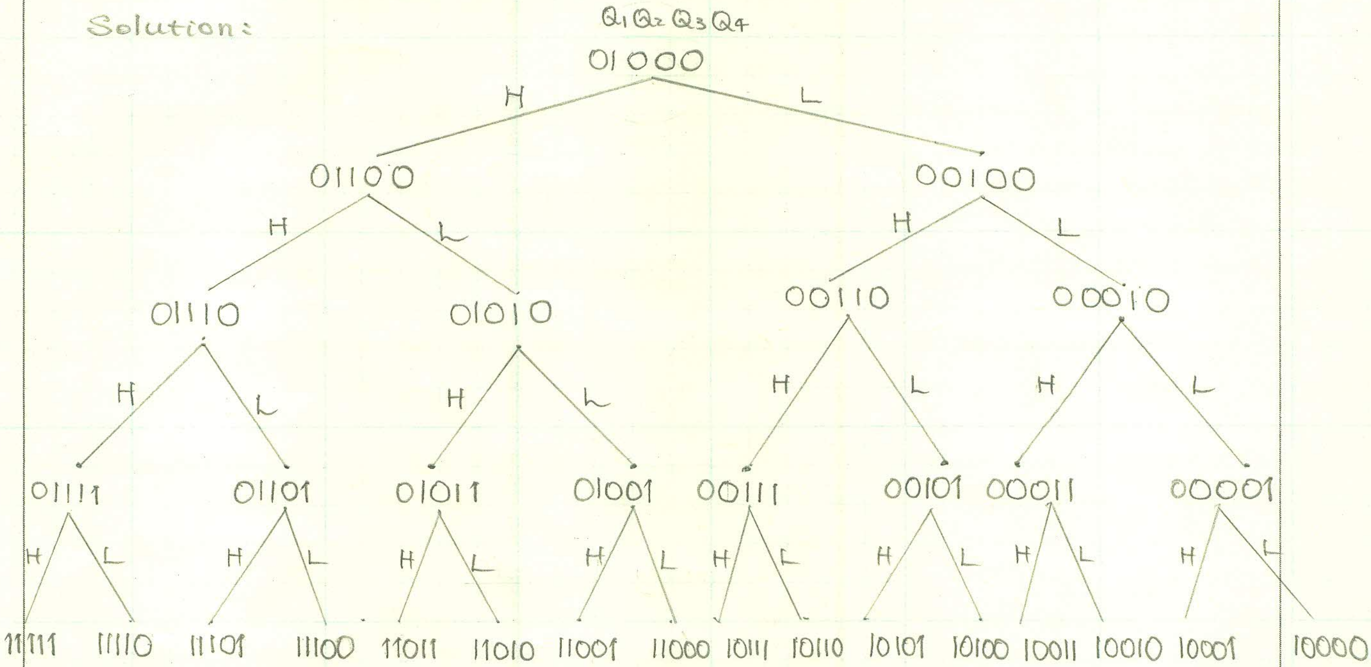
2.7 On the timing diagram in Figure 2.P.1, show the outputs of the four flip-flops in Fig. 2.14.

Solution:



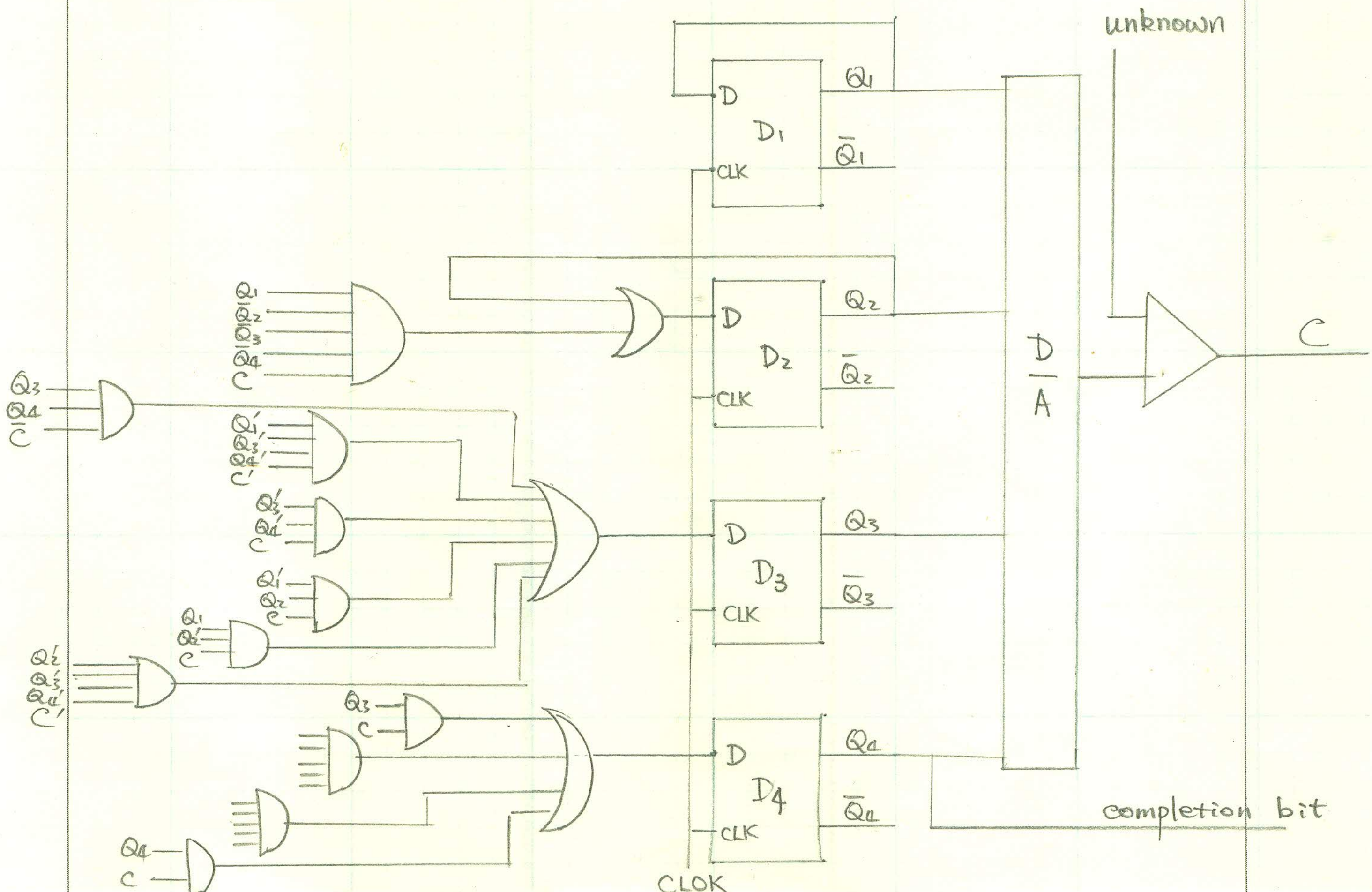
2.8 Design a succesive approximation register (sequential machine) for a 4-bit A/D converter. A state diagram is all that is required.

Solution:



State Table (see next page)

Circuit:



State Table:

C Output of Comp.		Q_1	Q_2	Q_3	Q_4	Q_1^+	Q_2^+	Q_3^+	Q_4^+	Completion bit
(L)	0	0	0	0	0	X	X	X	X	X
	0	0	0	0	1	0	0	0	0	1
	0	0	0	1	0	0	0	0	1	0
	0	0	0	1	1	0	0	1	0	1
	0	0	1	0	0	0	0	1	0	0
	0	0	1	0	1	0	1	0	0	1
	0	0	1	1	0	0	1	0	1	0
	0	0	1	1	1	0	1	1	0	1
	0	1	0	0	0	0	1	0	0	0
	0	1	0	0	1	1	0	0	0	1
	0	1	0	1	0	1	0	0	1	0
	0	1	0	1	1	1	0	1	0	1
	0	1	1	0	0	1	0	1	0	0
	0	1	1	0	1	1	1	0	0	1
	0	1	1	1	0	1	1	0	1	0
	0	1	1	1	1	1	1	1	0	1
(H)	1	0	0	0	0	X	X	X	X	X
	1	0	0	0	1	0	0	0	1	1
	1	0	0	1	0	0	0	1	1	0
	1	0	0	1	1	0	0	1	1	1
	1	0	1	0	0	0	1	1	0	0
	1	0	1	0	1	0	1	0	1	1
	1	0	1	1	0	0	1	1	1	0
	1	0	1	1	1	0	1	1	1	1
	1	1	0	0	0	1	1	0	0	0
	1	1	0	0	1	1	0	0	1	1
	1	1	0	1	0	1	0	1	1	0
	1	1	0	1	1	1	0	1	1	1
	1	1	1	0	0	1	1	1	0	0
	1	1	1	0	1	1	1	0	1	1
	1	1	1	1	0	1	1	1	1	0
	1	1	1	1	1	1	1	1	1	1

From This Table, we see that:

$$\text{Completion} = Q_4$$

$$Q_1^+ = Q_1 \quad ; \quad Q_2^+ = Q_2 + Q_1 \cdot \bar{Q}_2 \cdot \bar{Q}_3 \cdot \bar{Q}_4 \cdot C$$

$$Q_3^+ = Q_3 \cdot Q_4 \cdot \bar{C} + \bar{Q}_1 \bar{Q}_3 \bar{Q}_4 \cdot \bar{C} + \bar{Q}_2 \cdot \bar{Q}_3 \cdot \bar{Q}_4 \cdot \bar{C}$$
$$+ \bar{Q}_3 \cdot \bar{Q}_4 \cdot C + \bar{Q}_1 \cdot Q_2 \cdot C + Q_1 \bar{Q}_2 \cdot C$$

$$Q_4^+ = \bar{Q}_1 Q_3 \bar{Q}_4 \bar{C} + \bar{Q}_2 Q_3 \bar{Q}_4 \bar{C} + \bar{Q}_1 \bar{Q}_2 \bar{Q}_3 \bar{Q}_4 \bar{C}$$
$$+ Q_3 \cdot C + Q_4 \cdot C$$

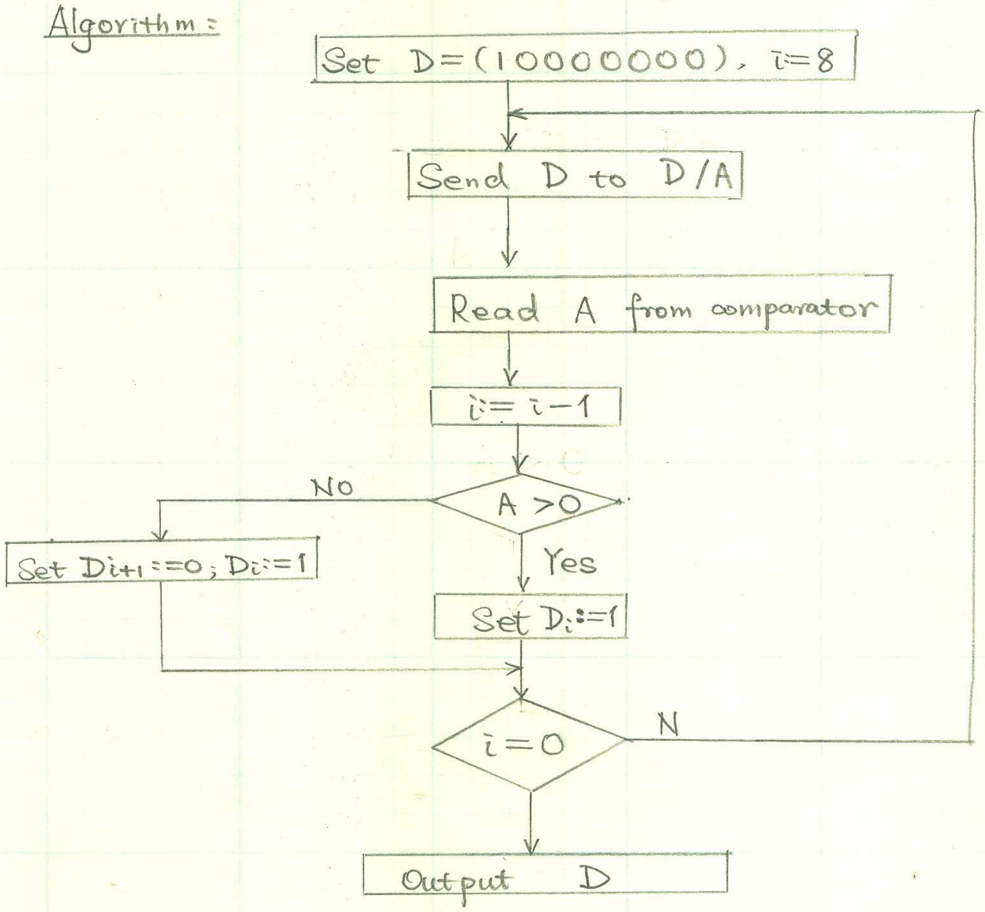
2.9 Flow chart a program to perform A/D conversion using the succesive approximation algorithm. Assume that the computer has one input, from analog comparator, and eight outputs, to the D/A.

Solution: Assume that the eight outputs, to D/A are

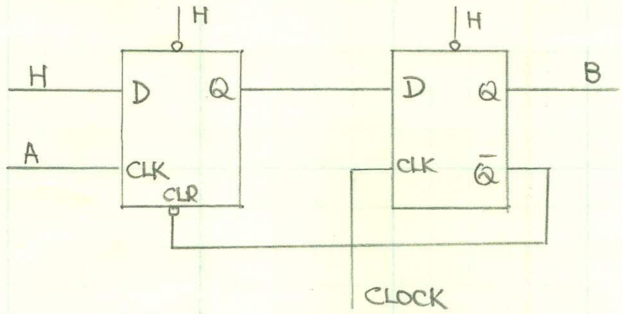
$$D = (D_1 \ D_2 \ D_3 \ D_4 \ D_5 \ D_6 \ D_7 \ D_8)$$

And the analog unknown is A (A=1 (H) A=0 (L))

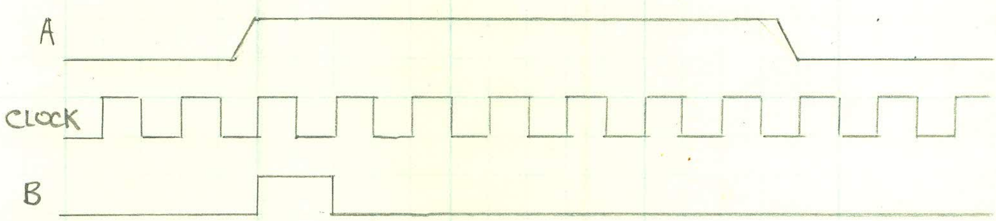
Algorithm:



2.10. Solution: Circuit



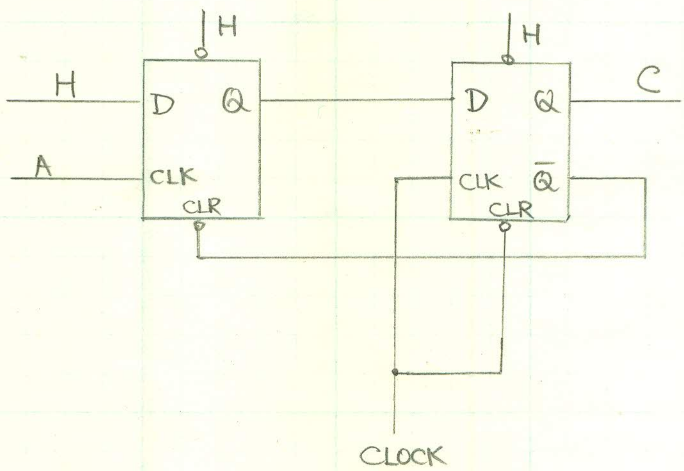
Timing diagram:



Checked!

2.11 Solution: If all the devices in my design is ideally, I think I can have the output C by AND B to clock. That means if all the devices do not have time delays.

The other way to have C is:



Handout 2. You are given a 1000-count optical encoder as well as one 8-bit counter and a 4 KHz clock. Find the min. & max. speeds that can be measured.

Solution: If we choose to use a 4 KHz clock, 255 counts (8 bits) will require 63.75 ms.

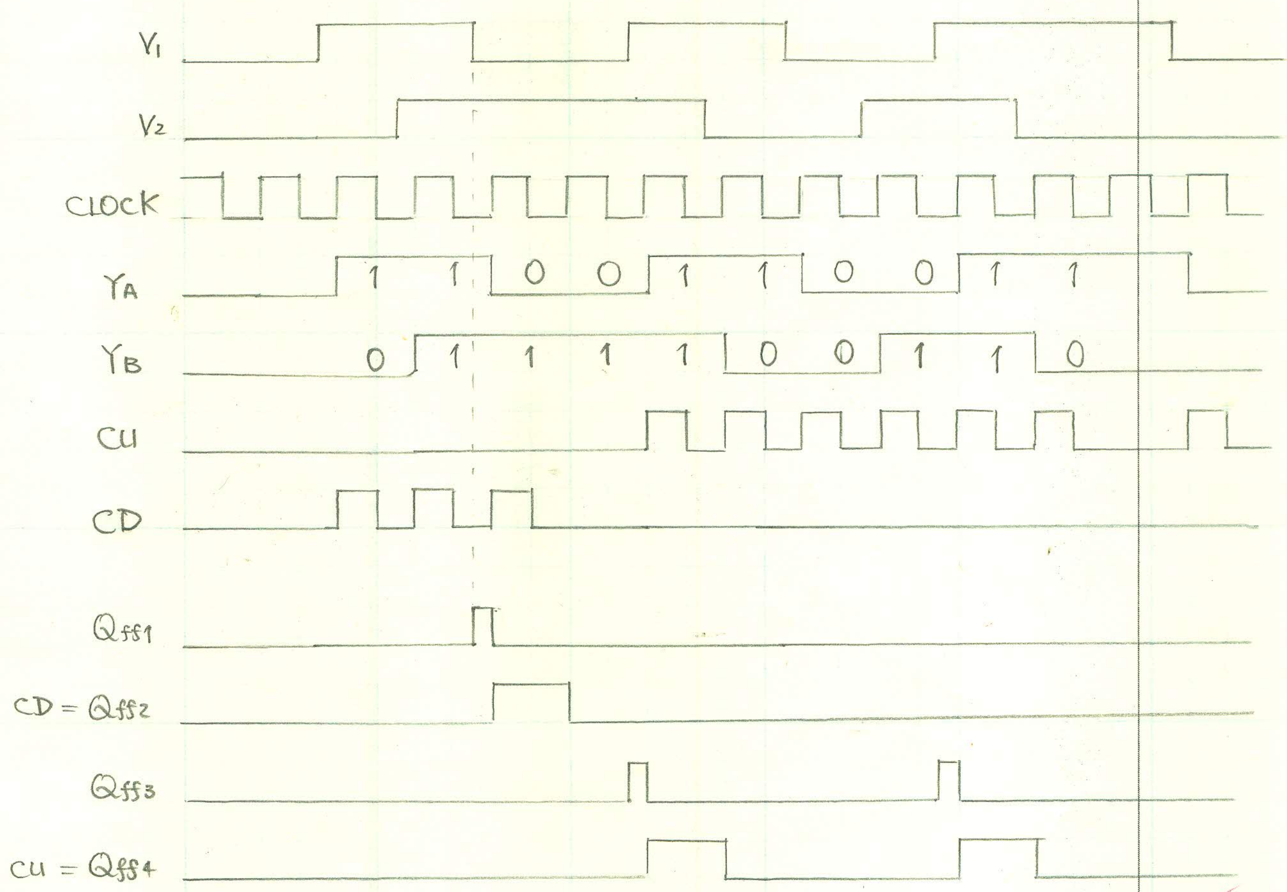
$\Rightarrow V_1$ pulse $\leq$ 63.75 ms

$1000 \times 63.75\text{ ms} = 63.75$ second.

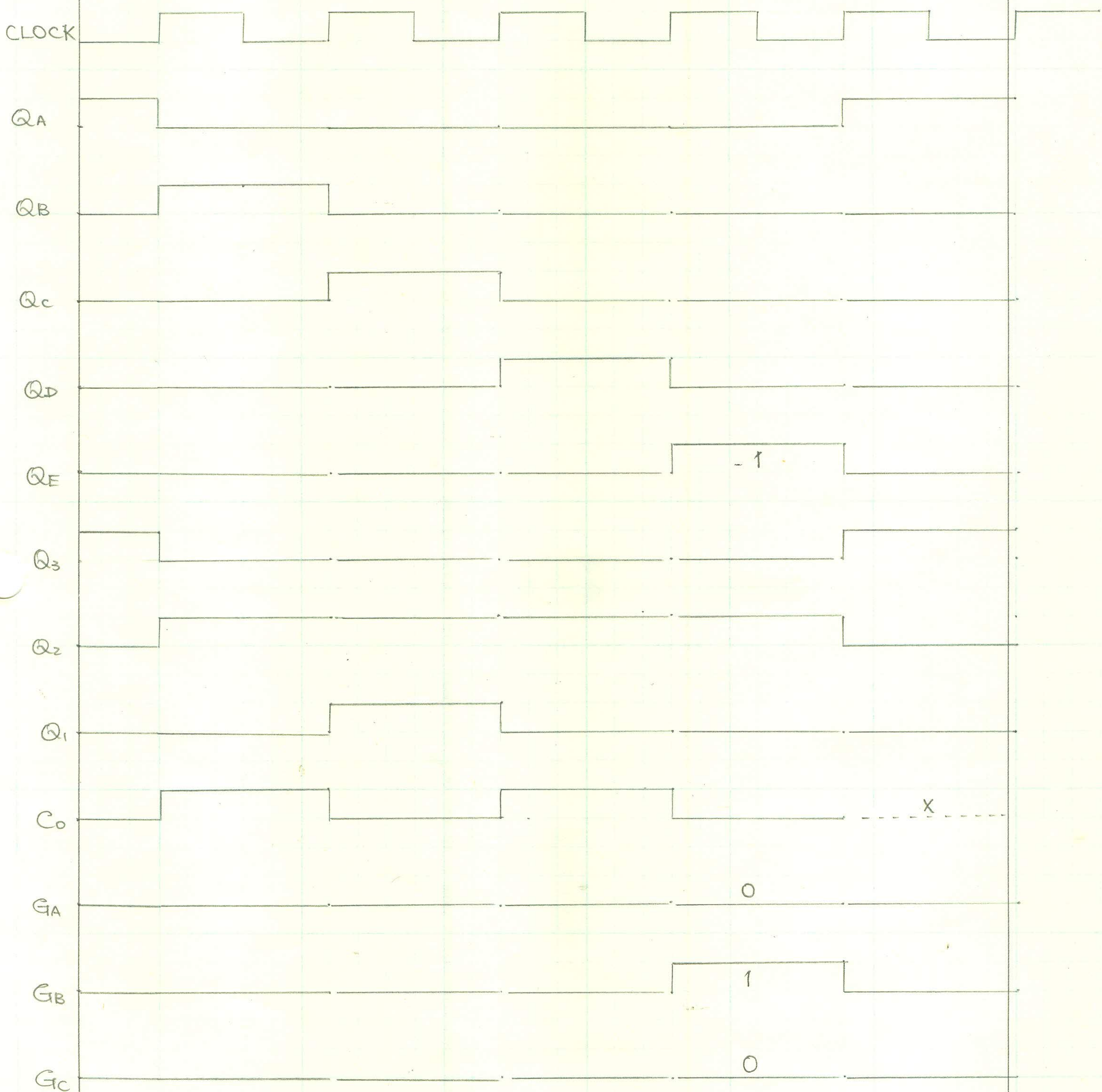
Minmum velocity = 0.015686 revolution/second.

maximum velocity = 4 revolutions/second.

Handout 3: Solution



Handout 4.



The usage of FFE (1) comparision is finished, when $Q_E=1$

(2) Read the results from G_A, G_B, G_C, when $Q_E=1$

(3) Reset the converter, when $Q_E=1$.