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Benmei Chen

EE/ME 442

Exam 1 — Robotics — **Fall, 1988**

1. Using the following hand frame matrix, determine the roll, pitch, and yaw angles plus origin translation values. On the graph paper provided below, draw this frame. Label the axes and show the angles.

$$\begin{bmatrix} 0 & -1 & 0 & 5 \\ .866 & -.5\ 0 & -.5 & 0 \\ .5 & 0 & .866 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

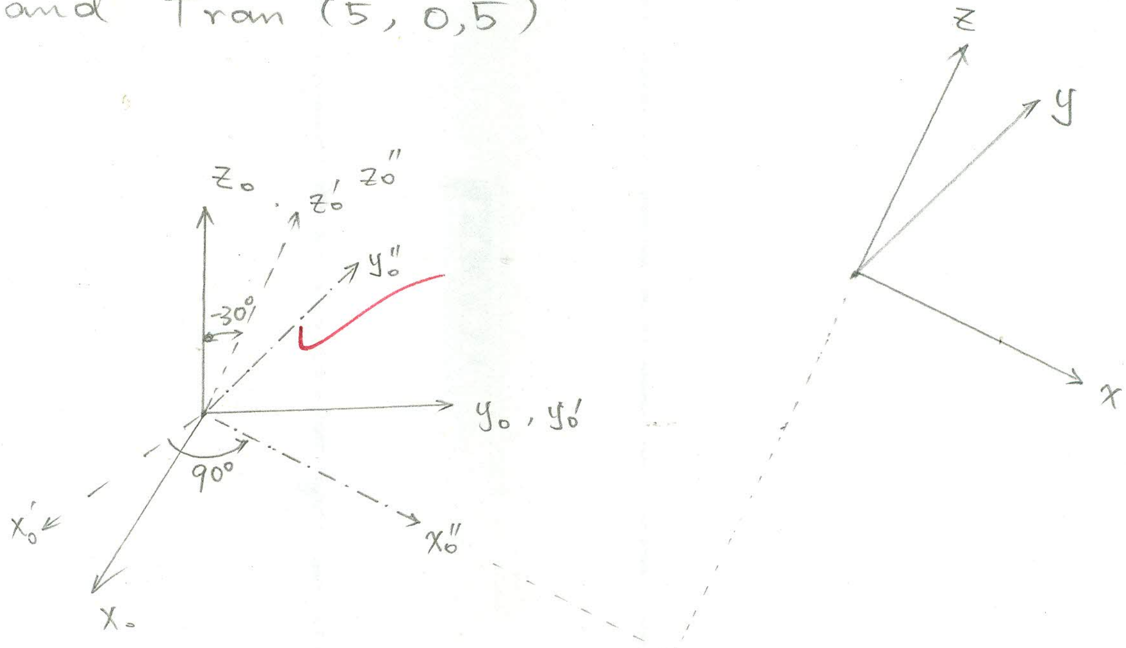
From the formulars of $RPY(\phi_z, \phi_y, \phi_x)$, we have

$-\sin\phi_y = 0.5 \quad \Rightarrow \cos\phi_y \neq 0 \quad \Rightarrow \cos\phi_x = 0$

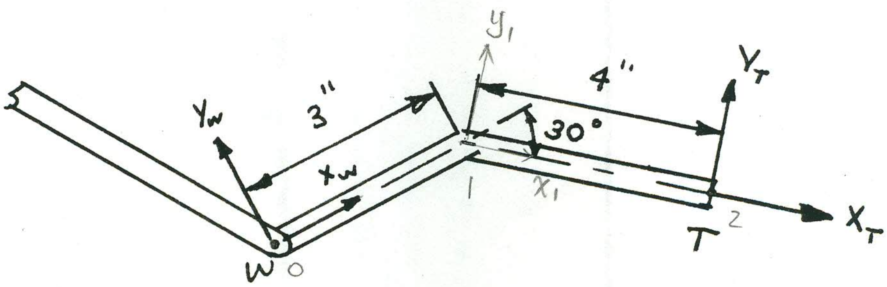
10/10

Select $\underline{\phi_x = 0°}$, $\underline{\phi_y = -30°}$, $\underline{\phi_z = 90°}$

and Tran (5, 0, 5)



2. Shown below is the wrist joint of a robot. The tool that is shown is an insertion device that must be positioned with respect to the goal frame before the insertion operation can be started. What is the transformation matrix for this tool frame with respect to the wrist.



Link parameter Table

n	α_n	d_n	θ_n
1	0°	3	−30°
2	0°	4	0°

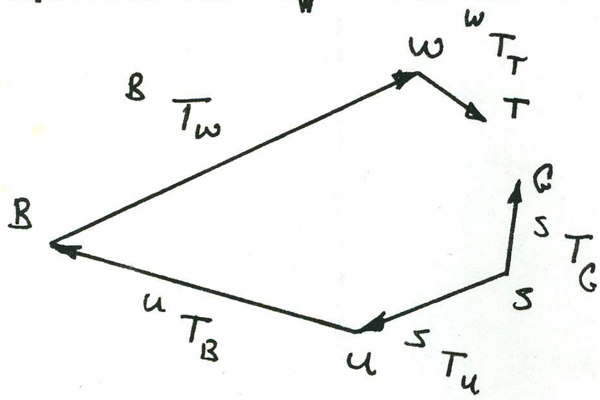
$$
{}^{W}T_T = \begin{bmatrix} 0.866 & 0.5 & 0 & \cancel{2.598}\ 3 \\ -.5 & 0.866 & 0 & \cancel{-1.500}\ 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

−2

8/10

$$
= \begin{bmatrix} 0.866 & 0.500 & 0.000 & 3.464 \\ 0.000 & 0.866 & 0.000 & -1.500 \\ 0.000 & 0.000 & 1.000 & 0.000 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

3. When the tool frame is aligned with the goal frame, what is the transform equation for ${}^{B}T_{W}$. Transforms ${}^{U}T_{B}$, ${}^{S}T_{U}$, ${}^{S}T_{G}$, and ${}^{W}T_{T}$ are known.



Solution:

$${}^{B}T_{W} = {}^{B}T_{U} \cdot {}^{U}T_{S} \cdot {}^{S}T_{G} \cdot {}^{G}T_{T} \cdot {}^{T}T_{W}$$

when the tool frame is aligned with the goal frame.

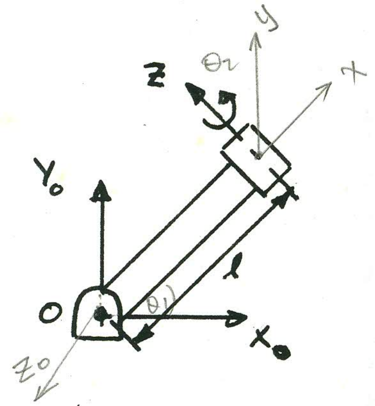
$${}^{G}T_{T} = I_4$$

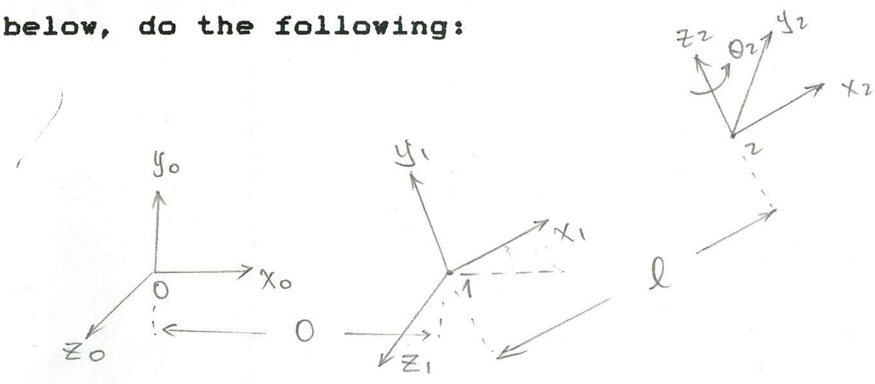
$${}^{B}T_{W} = {}^{B}T_{U} \cdot {}^{U}T_{S} \cdot {}^{S}T_{G} \cdot {}^{T}T_{W}$$

$$= ({}^{U}T_{B})^{-1} \cdot ({}^{S}T_{U})^{-1} \cdot {}^{S}T_{G} \cdot ({}^{W}T_{T})^{-1}$$

5/5

4. For the manipulator shown below, do the following:





a. Prepare a link parameter table.

b. Develop the forward kinematic transformation ${}^{0}T_{2}$.

c. Derive the inverse kinematic equations for the joint angles.

d. For the following hand frame and l = ~~6~~ 5, compute the joint angles, draw the position of the robot and show the hand frame orientation.

$$\begin{bmatrix} .867 & -.867 & -.5 & 4.33 \\ .5 & -.5 & .867 & 2.5 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solution: a) Link Parameter Table

n	α_n	d_n	θ_n
~~1~~	~~0°~~	~~0~~	~~θ_1~~
2	0. ~~90°~~	l	θ_1
3	$-90°$	0	θ_2

5 ~~2~~/5

I see the problem here but I don't have time.

b) $${}^{0}T_{2} = \begin{bmatrix} C1 & -S1 & 0 & 0 \\ S1 & C1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} C2 & 0 & -S2 & lC2 \\ S2 & 0 & C2 & lS2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \underbrace{\begin{bmatrix} C12 & 0 & -S12 & lC12 \\ S12 & 0 & C12 & lS12 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{algebra OK}}$$

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c) $${}^{0}T_{2} = \begin{bmatrix} C12 & 0 & -S12 & lC12 \\ S12 & 0 & C12 & lS12 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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$C_{12} = n_x \qquad S_{12} = n_y \qquad \Rightarrow \qquad \theta_1 + \theta_2 = ATAN2(n_y, n_x)$

I can only determine $\theta_1 + \theta_2$ in this case.

-3

d) $\theta_1 + \theta_2 = 29.97°$ 2/5

① From the Table you gave in Class:

n	α_n	d_n	θ_n
1	$-90°$	ℓ	θ_1
2	0	0	θ_2

Then we have

$$A_1 = \begin{bmatrix} C_1 & 0 & -S_1 & \ell C_1 \\ S_1 & 0 & C_1 & \ell S_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & 0 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then,

$$^0T_2 = A_1 A_2 = \begin{bmatrix} C_1 C_2 & -C_1 S_2 & -S_1 & \ell C_1 \\ S_1 C_2 & -S_1 S_2 & C_1 & \ell S_1 \\ -S_2 & -C_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

② From my Link Parameter Table

n	α_n	d_n	θ_n
1	$0°$	0	θ_1
2	$-90°$	ℓ	$0°$
3	$0°$	0	θ_2

Then we have

$$A_1 = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 0 & 0 & \ell \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} C_2 & -S_2 & 0 & 0 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then in my case

$${}^0T_3 = A_1 A_2 \cdot A_3 = \begin{bmatrix} C_1 & 0 & -S_1 & \ell C_1 \\ S_1 & 0 & C_1 & \ell S_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} C_2 & -S_2 & 0 & 0 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_1C_2 & -C_1S_2 & -S_1 & \ell C_1 \\ S_1C_2 & -S_1S_2 & C_1 & \ell S_1 \\ -S_2 & -C_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$= {}^0T_2$ from your table

This shows that my link table is O.K, isn't it ?

3.9
4.0

81
~~76~~
100

Ben M. Chen

EE/ME 442

Exam 2 Robotics Fall, 1988

1. Prepare a link parameter table for the following manipulator. (10%)

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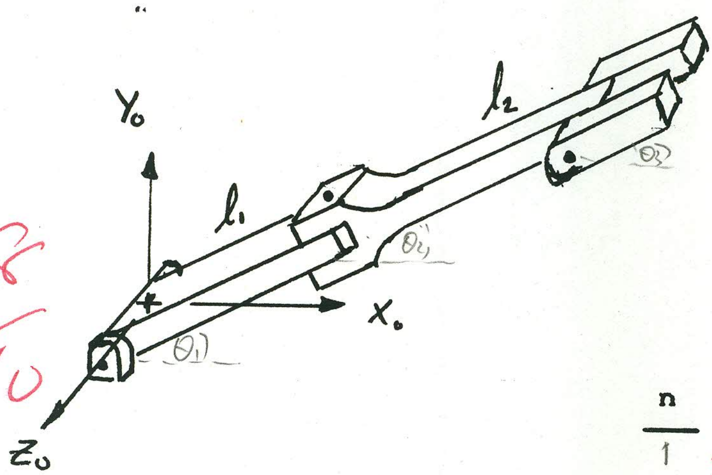
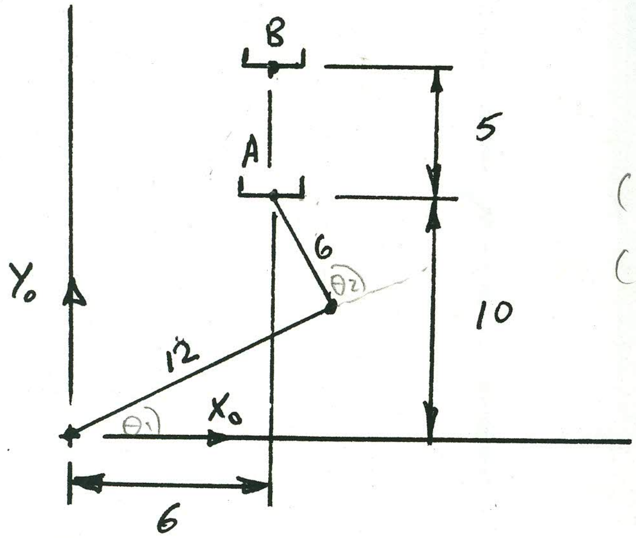


n	α_n	a_n	θ_n
1	-90	l_1	θ_1
2	90	l_2	θ_2
3	0	0	θ_3

2. The two link planor robot used in class examples moves the object from position A to position B. Use parabolic blends and constant velocity to move the object. Object starts at rest and ends at rest. The required time for the motion is 4 seconds and the blend time should be 10% of this time. What acceleration should be used for the blend on the joint path for θ_2? (20%)

19/20



$(P_x^A, P_y^A, P_z^A) = (6, 10, 0)$

$(P_x^B, P_y^B, P_z^B) = (6, 15, 0)$

Solution:

For point A: $c_2 = (6^2 + 10^2 - 12^2 - 6^2)/(2\times 12\times 6)$

$= -0.3056$

$s_2 = \pm 0.9522$

From the diagram, we have "elbow down case"

$\theta_2^A = 107.79°$

For point B: $c_2 = (6^2 + 15^2 - 12^2 - 6^2)/(2\times 12\times 6)$

$= 0.5625$

$s_2 = \pm 0.8268$

$\theta_2^B = 55.77°$

$$\ddot{\theta}^2_{min} = \frac{4\,(+55.77° - 107.79°)}{16} = -13.005 \text{ deg/sec}^2$$

$$t_b = \frac{4}{2} - \frac{\sqrt{\ddot{\theta}^2 t_f^2 - t_f^2\cdot\ddot{\theta}\cdot\ddot{\theta}_{min}}}{2\cdot|\ddot{\theta}|}$$

assume $\ddot{\theta} = \alpha\cdot\ddot{\theta}_{min}$

$$t_b = 2 - \frac{t_f\cdot\sqrt{1-1/\alpha}}{2} \Rightarrow t_b = 2(1-\sqrt{1-1/\alpha})$$

$t_b = 10\%\cdot t_f = 0.4 = 2(1-\sqrt{1-1/\alpha})$

$\Rightarrow \alpha = 2.7778$

$\ddot{\theta}_2 = \alpha\,\ddot{\theta}_{min} = 2.7778\times(-13.005) = -36.1244 \text{ deg/sec}^2$

+36.1 at 2nd blnk

3. Use the one link manipulator from exam 2. The transformations are

$$A_1 = \begin{bmatrix} c_1 & 0 & -s_1 & lc_1 \\ s_1 & 0 & c_1 & ls_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_2 = \begin{bmatrix} c_1c_2 & -c_1s_2 & -s_1 & lc_1 \\ s_1c_2 & -s_1s_2 & c_1 & ls_1 \\ -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The link length is 10 inches and using a position of θ_1 = 45 and θ_2 = 30, what will be the cartesian velocities in frame 2 for a θ_1 velocity of 3 degrees/sec and a θ_2 velocity of 2 degrees/sec? (30%)

$\dot{\theta}_1 = 3$ deg/sec $\qquad \dot{\theta}_2 = 2$ deg/sec.

$\theta_1 = 45°$ $\qquad \theta_2 = 30°$

First, we compute Jacobian in Frame 2. ${}^2J$

$$\begin{bmatrix} {}^2\dot{x} \\ {}^2\dot{y} \end{bmatrix} = {}^2J \cdot \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$${}^2J = \begin{bmatrix} \frac{{}^2\partial x}{\partial \theta_1} & \frac{{}^2\partial x}{\partial \theta_2} \\ \frac{{}^2\partial y}{\partial \theta_1} & \frac{{}^2\partial y}{\partial \theta_2} \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

Subtitude the formular in text, we have.

$$\frac{{}^2\partial z}{\partial \theta_1} = a_y \cdot p_x - a_x \cdot p_y = C_1^2 \cdot l + S_1^2 \cdot l = \underline{10}$$

$$\frac{{}^2\partial z}{\partial \theta_2} = 0.$$

+4

$$\frac{{}^2\partial x}{\partial \theta_1} = {}^0n_y\,{}^0p_x - {}^0n_x\,{}^0p_y$$

$$= S_1 C_2 \cdot l C_1 - S_1 C_2\, l \cdot S_1$$

$$= \sin 45^\circ \cdot \cos 30^\circ \cdot 10 \cdot \cos 45^\circ - \cos 45^\circ \cdot \cos 30^\circ \cdot 10 \cdot \sin 45^\circ$$

$$= 0$$

$$\frac{{}^2\partial y}{\partial \theta_1} = {}^0O_y\,{}^0p_x - {}^0O_x\,{}^0p_y$$

$$= -S_1 S_2 \cdot l C_1 + C_1 S_2 \cdot l \cdot S_1$$

$$= -\sin 45^\circ \cdot \sin 30^\circ \cdot 10 \cdot \cos 45^\circ + \cos 45^\circ \cdot \sin 30^\circ \cdot 10 \cdot \sin 45^\circ$$

$$= 0$$

15
$\cancel{10}$
/ 30

$$\frac{{}^2\partial x}{\partial \theta_2} = S_2 \cdot 0 - C_2 \cdot 0 = 0$$

$$\frac{{}^2\partial y}{\partial \theta_2} = C_2 \cdot 0 + S_2 \cdot 0 = 0$$

${}^2J_{31} - {}^2J_{62}$ −16

We have ${}^2J = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\begin{bmatrix} {}^2v_x \\ \vdots \\ {}^2\omega_z \end{bmatrix} = J \begin{bmatrix} \dot\theta_1 \\ \dot\theta_2 \end{bmatrix}$

$$\begin{bmatrix} {}^2\dot x \\ {}^2\dot y \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-\cancel{4}3$

~~Normally, we don't have cartesian velocities. But we do have the rotation velocities, I think~~

$${}^2\dot z = [10, 0] \cdot \begin{bmatrix} 3 \\ 2 \end{bmatrix} = 30$$

4. **For a starting point of θ_o = 100 degrees, a via point of θ_v = 120 degrees, and a final point of θ_f = 80 degrees, determine the blend time needed at the via point when using a blend period acceleration of ±50 degrees per second. (20%)**

$t_{do\cancel{a}v} = 3$ sec.

$t_{dvf} = 3$ sec.

$\theta_o = 100^\circ$, $\theta_f = 80^\circ$ $\theta_v = 120^\circ$

First segment:

20/20

$$\ddot{\theta}_1 = \text{sign}(\theta_v - \theta_o)\cdot|\ddot{\theta}_1| = \text{sig}(120-100)\cdot 50 = 50 \text{ deg/sec}^2$$

$$t_o = 3 - \left[9 - \frac{2(120-100)}{50}\right]^{1/2} = 0.1364 \text{ sec}$$

$$\dot{\theta}_{ov} = (120-100)/(3 - 0.5\times 0.1364) = 6.8218 \text{ deg/sec.}$$

$$t_{ov} = 3 - 0.1364 - 0.5 * t_v = 2.6555 \text{ sec.}$$

Last segment:

$$\ddot{\theta}_f = \text{sign}(120-80)\cdot 50 = 50 \text{ deg/sec}^2$$

$$t_f = 3 - \left[3^2 + \frac{2(80-120)}{50}\right]^{1/2} = 0.2797 \text{ sec.}$$

$$\dot{\theta}_{vf} = (80^\circ - 120^\circ)/(3 - 0.5\times 0.2797) = -13.9853 \text{ deg/sec.}$$

$$t_{vf} = 3 - 0.2797 - 0.5 * t_v = 2.5122 \text{ sec.}$$

Via point:

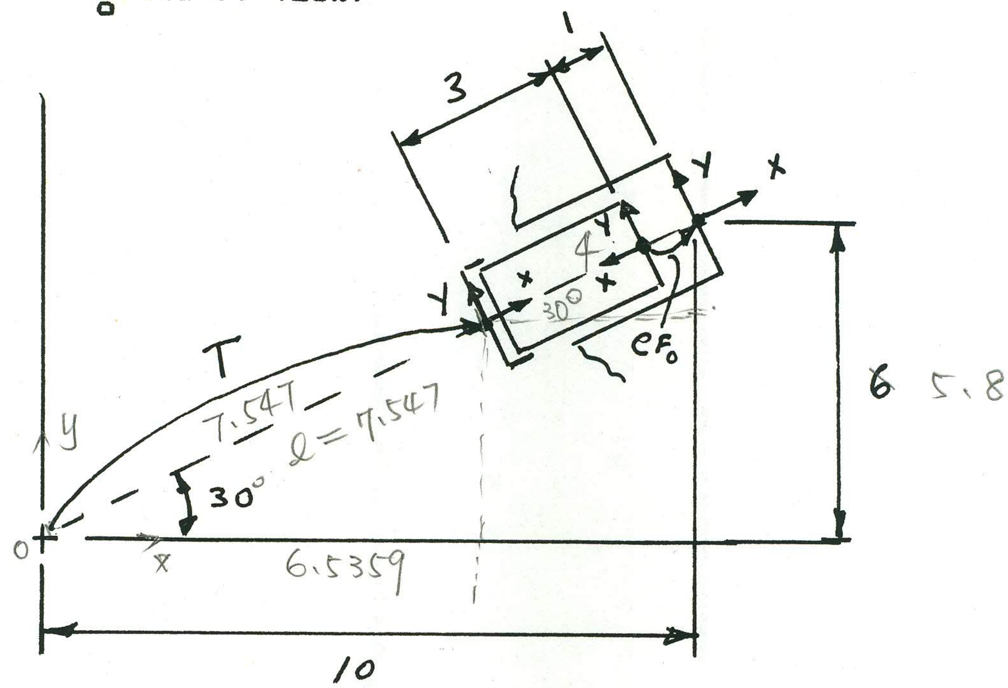
$$\ddot{\theta}_v = \text{sig}(\dot{\theta}_{vf} - \dot{\theta}_{ov})\cdot 50 = \text{Sign}(-13.9853 - 6.8218)\cdot 50 = -50 \text{ d}$$

$$t_v = (-13.9853 - 6.8218)/(-50) = 0.4161 \text{ sec.}$$

Table =

point	t_{ij}	t_i
o		0.1364
	2.6555	
v		0.4161
	2.5122	
f		0.2797

5. The pin must be inserted into the hole as shown below.
Determine CF_o and T? (20%)



$\frac{x}{4} = \cos 30°$.

$\frac{6.5359}{T} = \cos$

19/20

$$T = \begin{bmatrix} \cos 30° & -\sin 30° & 0 & 0 \\ \sin 30° & \cos 30° & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 7.547 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.866 & -0.5 & 0 & 0 \\ 0.5 & 0.866 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 7.547 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.866 & -0.5 & 0 & 6.5357 \\ 0.5 & 0.866 & 0 & 3.7735 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$CF_o = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

-1