135/150

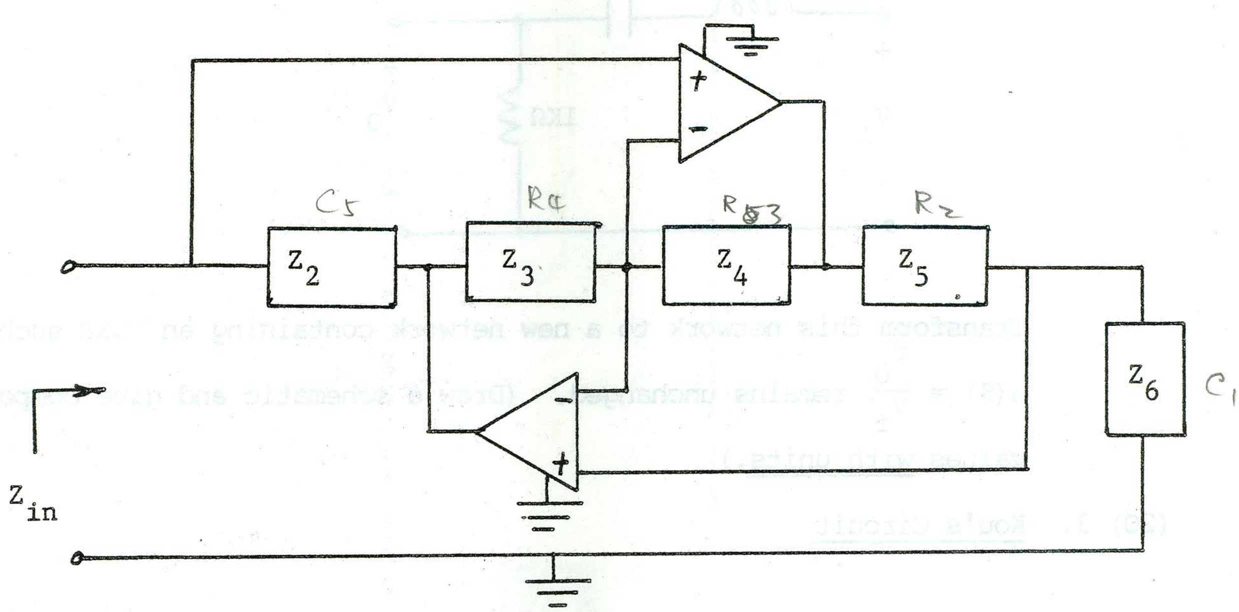
E-E 449 ACTIVE FILTER DESIGN
Two Hour Examination
April 28, 1987
Dr. R. A. Birgenheier

Benmei Chen

OPEN BOOK

points

(25) 1. Antoniou's GIC Circuit



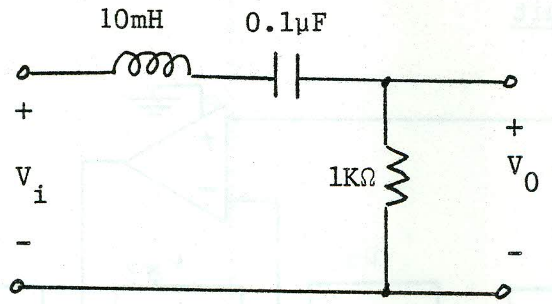
a) Z_6 is the impedance of a 1K ohm resistor. Find component values (not impedances) such that Z_{in} is the impedance of a 1μF capacitor.

b) Z_6 is the impedance of a 1μF capacitor. Find component values (not impedances) such that Z_{in} is the impedance of a $1nF^2$ FDNR.

c) Find component values to simulate a 1mH inductor.

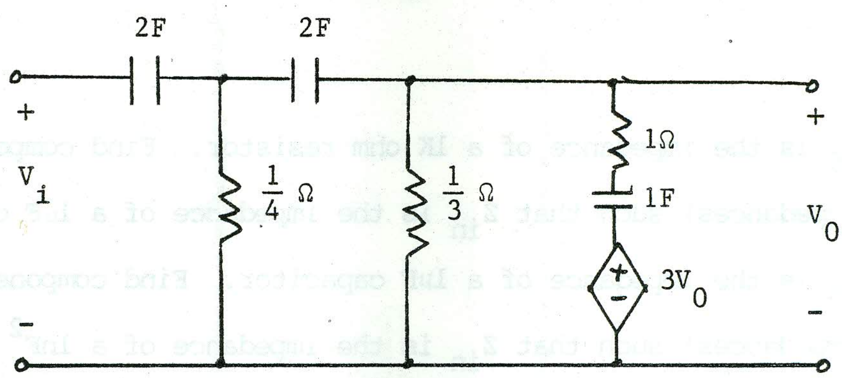
(points)

(20) 2. Frequency-dependent Negative Resistor



Transform this network to a new network containing an FDNR such that $H(S) = \frac{V_0}{V_i}$ remains unchanged. (Draw a schematic and give component values with units.)

(20) 3. Kou's Circuit



For the above network

$$H(S) = \frac{V_0}{V_i} = \frac{S^2}{S^2 + 3S + 3}$$

Perform appropriate frequency and impedance scaling to realize the following transfer function using practical component values.

$$H(S) = \frac{S^2}{S^2 + 3 \times 10^{-3} S + 3 \times 10^{-6}}$$

(Draw a schematic with component values.)

Points

(20) 4. RC Decomposition

$$Z(S) = \frac{S - 1}{(S + 1)(S + 2)}$$

Find the RC decomposition (i.e., find realizations for $Z_{RC}^{(i)}$ and $Z_{RC}^{(ii)}$)

(20) 5. ω_0 and Q Sensitivities

$$H(S) = \frac{(R+1)}{(R_1+1)} \left[\frac{S^2}{S^2 + \frac{R_1(R+1)}{(R_1+1)} S + R} \right]$$

Find $S_R^{\omega_0}$, $S_{R_1}^{\omega_0}$, S_R^Q and $S_{R_1}^Q$

(20) 6. Multiple-amplifier Biquad

Suppose

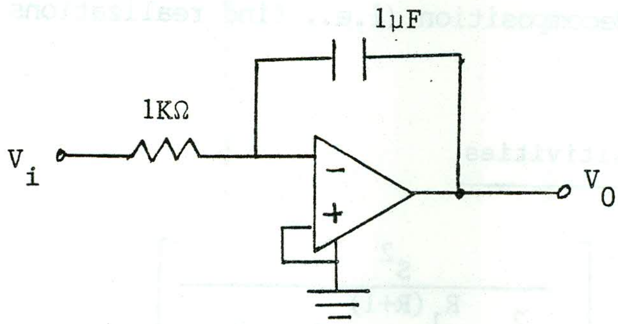
$$\frac{V_1}{V_i} = \frac{-k_1 S}{S^2 + b_1 S + b_0}$$

$$\frac{V_3}{V_i} = \frac{-k_2}{S^2 + b_1 S + b_0}$$

Show how V_i, V_1 and V_3 can be summed to realize the transfer function of a HPF. Find k_1 and k_2 in terms of b_0 and b_1.

Points

(25) 7. Non-ideal Integrator



$$A(s) = \frac{10^7}{S + 100}$$

The transfer function of the non-ideal integrator is

$$H(s) = -\frac{1}{RC}\,\frac{A_0\omega_a}{S^2 + A_0\omega_a S + \frac{\omega_a}{RC}}$$

a) determine the pole locations

b) suppose we have a magnitude-response tolerance of $\varepsilon_p = 2\%$ and a phase response tolerance of $\varepsilon_\phi = 3^0$. What is the useful frequency range of the integrator?

1. (a) $Z_6 = R_6 = 1000\Omega$

LET $Z_2 = R_2 = Z_3 = R_3 = Z_5 = R_5 = 1000\Omega$

$$Z_4 = \frac{1}{C_4 S}, \qquad C = \frac{C_4 \cdot R_3 \cdot R_5}{R_2 R_L}$$

$$C_4 = \frac{C R_2 R_L}{R_3 \cdot R_5} = C = 1\,\mu F$$

(b) $$D = \frac{R_2 R_4 C_1 C_5}{R_3} = 1\,nF^2$$

LET $R_3 = R_4 = R_5 = 1000\Omega$ $C_2 = 1\,\mu F$

$$C_6 = \frac{D R_4}{R_5 R_3 C_2} = \frac{1\times10^{-9}\times10^{3}}{10^3\times10^3\times10^{-6}} = 1\,\mu F$$

(c) $$\frac{R_2 R_4 R_6 C_5}{R_3} = 1\,mH$$

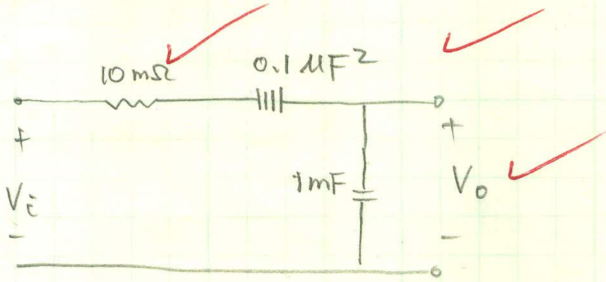
LET $R_2 = R_4 = R_6 = R_3 = 1K\Omega$.

$$C_5 = \frac{10^3\times10^{-3}}{10^3\times10^3\times10^3} = 1\,nF$$

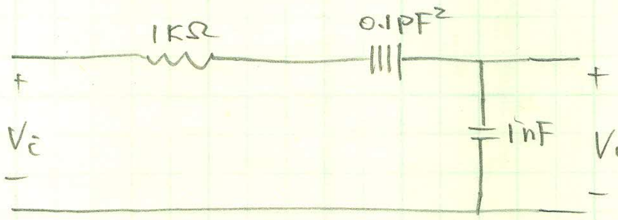
* FOR THE RESISTORS $Z_X = R_X$

FOR capacitors $Z_X = \frac{1}{C_X S}$ $X = 1, 2, 3, 4, 5, 6$

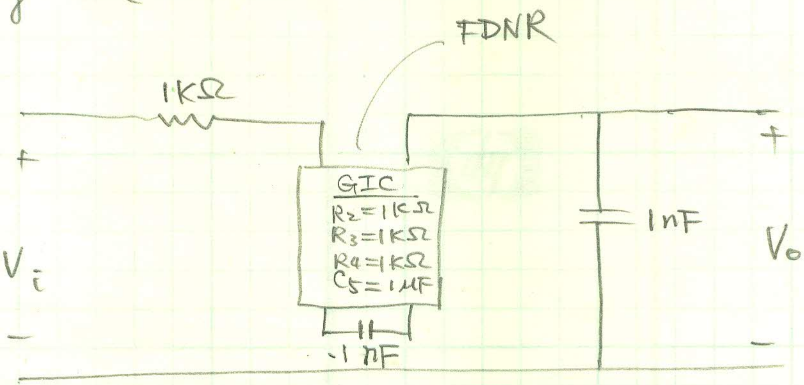
2. Devided each element in circuit by s , we have



Impedance scale by 10^6 , we have



using the FDNR



3.

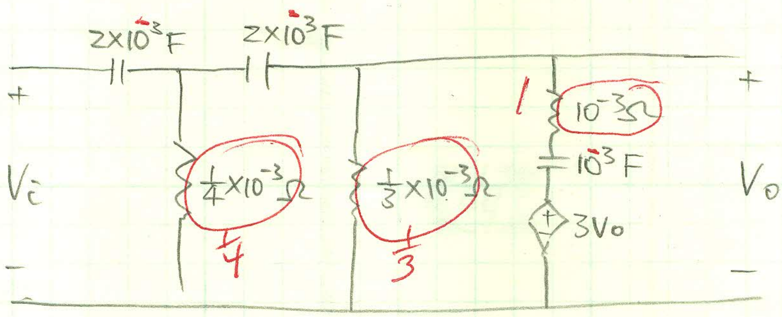
$$H(s) = \frac{s^2}{s^2 + 3\times10^{-3}s + 3\times10^{-6}}$$

$$= \frac{s^2\times10^6}{s^2\times10^6 + 3\times10^3 s + 3}$$

$$= \frac{(1000s)^2}{(1000s)^2 + 3(1000s) + 3}$$

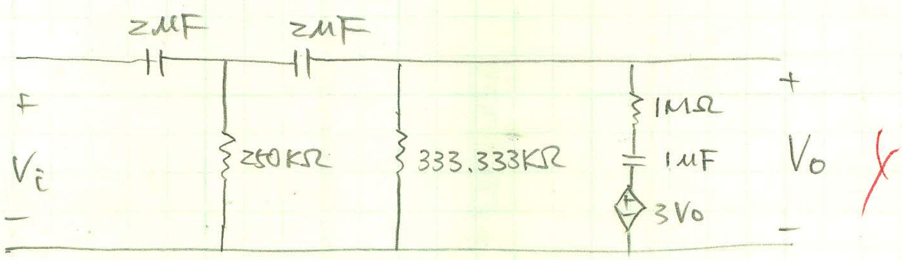
$$= \frac{\hat{s}^2}{\hat{s}^2 + 3\hat{s} + 3} \qquad \hat{s} = 1000s$$

So, we need ~~to~~ scale ~~to~~ the freguey in original circuit by a factor 10^{-3}



−15

Impedance scale by 10^9



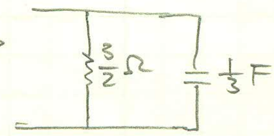
4.

$$Z(s) = \frac{s-1}{(s+1)(s+2)}$$

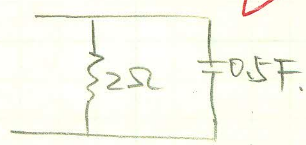
$$= \frac{3}{s+2} - \frac{2}{s+1}$$

$$\triangleq Z_{RC}^{(i)} - Z_{RC}^{(ii)}$$

$$Z_{RC}^{(i)} = \frac{3}{s+2} = \frac{1}{\frac{2}{3} + \frac{1}{3}s} \Rightarrow$$



$$Z_{RC}^{(ii)} = \frac{2}{s+1} = \frac{1}{0.5 + 0.5s} \Rightarrow$$



5. $\omega_0 = \sqrt{b_0} = \sqrt{R}$

$$\frac{\partial \omega_0}{\partial R} = \frac{1}{2}\frac{1}{\sqrt{R}} = \frac{1}{2}\cdot\frac{\sqrt{R}}{R} = \frac{\omega_0}{2R}$$

$$S_R^{\omega_0} \triangleq \frac{R}{\omega_0}\cdot\frac{\partial \omega_0}{\partial R} = \frac{R}{\omega_0}\cdot\frac{\omega_0}{2R} = 0.5$$

$$S_{R_1}^{\omega_0} \triangleq \frac{R_1}{\omega_0}\cdot\frac{\partial \omega_0}{\partial R_1} = 0$$

$$Q = \sqrt{R}\cdot\frac{R_1+1}{R_1(R+1)}$$

$$\frac{\partial Q}{\partial R} = \frac{1}{2}\cdot\frac{1}{R}\sqrt{R}\cdot\frac{R_1+1}{R_1(R+1)} - \sqrt{R}\cdot\frac{R_1+1}{R_1}\cdot\frac{1}{(R+1)^2}$$

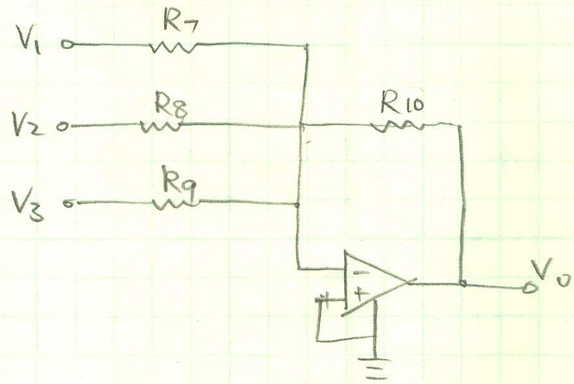
$$= \frac{1}{2}\frac{1}{R}\cdot Q - Q\cdot\frac{1}{R+1} = -\frac{1\cdot(R-1)}{2R(R+1)}\cdot Q$$

$$S_R^Q \triangleq \frac{R}{Q}\cdot\frac{\partial Q}{\partial R} = -\frac{1}{2}\cdot\frac{(R-1)}{R+1} \approx -\frac{1}{2} \quad \text{IF } R \gg 1$$

$$\frac{\partial Q}{\partial R_1} = \frac{\sqrt{R}}{R+1}\cdot\frac{\partial\frac{(R_1+1)}{R_1}}{\partial R_1} = -\frac{\sqrt{R}}{R+1}\cdot\frac{1}{R_1^2} = -Q\cdot\frac{1}{R_1(R_1+1)}$$

$$S_{R_1}^{Q} \triangleq \frac{R_1}{Q}\cdot\frac{\partial Q}{\partial R_1} = \frac{R_1}{Q}\cdot\left(-Q\cdot\frac{1}{R_1(R_1+1)}\right) = -\frac{1}{R_1+1} = 0 \quad \text{IF } R_1 \gg 1$$

6. Using the summer below.



the output of the summer is

$$\frac{V_0}{V_i} = -\frac{R_{10}}{R_7}\frac{V_1}{V_i} - \frac{R_{10}}{R_8}\frac{V_3}{V_i} - \frac{R_{10}}{R_9}\frac{V_i}{V_i}$$

$$= -\frac{R_{10}}{R_7}\cdot\frac{-k_1 S}{S^2+b_1S+b_0} - \frac{R_{10}}{R_8}\cdot\frac{-k_2}{S^2+b_1S+b_0} - \frac{R_{10}}{R_9}$$

$$= -\frac{\frac{R_{10}}{R_9}S^2+\left(\frac{R_{10}}{R_9}b_1-\frac{R_{10}}{R_7}k_1\right)S+\left(\frac{R_{10}}{R_9}b_0-\frac{R_{10}}{R_8}k_2\right)}{S^2+b_1S+b_0}$$

IF WE PICK $R_7 = R_8 = R_9 = 1000\Omega$, we have

$$\frac{V_0}{V_i} = -\frac{\frac{R_{10}}{1000}S^2+(b_1-k_1)S+(b_0-k_2)}{S^2+b_1S+b_0}$$ ✓

IN ORDER TO OBTAIN A HPF,

WE have to let $k_1 = b_1$ ✓, $k_2 = b_0$ ✓

However, in general,

$$k_1 = \frac{R_7}{R_9}b_1$$

$$k_2 = \frac{R_8}{R_9}\cdot b_0$$ where R_8, R_9 can be any reasonable values.

7. From givens, we have

$A_o \omega_a = 10^7 \qquad \omega_a = 100$

so, $A_o = 10^7/100 = 10^5$

a) The pole locations

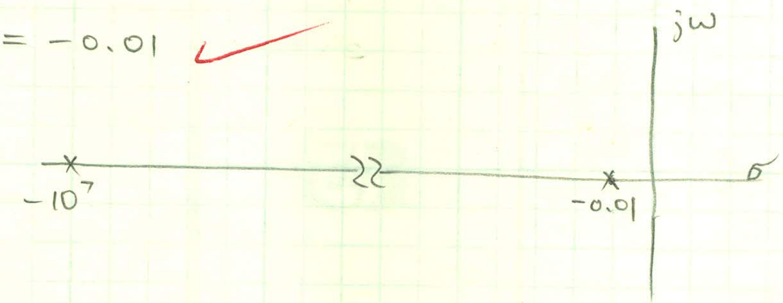
$$S_{1,2} = -\frac{A_o \omega_a}{2} \pm \frac{1}{2}\sqrt{A_o^2 \omega_a^2 - \frac{4\omega_a}{RC}}$$

$$= -\frac{10^5 \times 100}{2} \pm \frac{1}{2}\sqrt{(10^7)^2 - \frac{4\times 100}{10^3\times 10^{-6}}}$$

$$= -5\times 10^6 \pm 4999999.99$$

$S_1 = -9999999.99 \approx -10^7$

$S_2 = -0.01$



(b) $$\omega_{m1} = \frac{1}{RCA_o} \cdot \frac{1}{\sqrt{0.02\,\varepsilon_p}}$$

$$= \frac{1}{10^3\times 10^{-6}\times 10^5} \cdot \frac{1}{\sqrt{0.02\times 2}} = 0.05 \text{ rad/sec.}$$

$$\omega_{m2} = \frac{1}{A_o RC}\tan(90^\circ - 3^\circ) = 0.1908 \text{ rad/sec.}$$

$$\omega_{M1} = A_o \omega_a \sqrt{0.02\,\varepsilon_p} = 10^5 \times 100\sqrt{0.02\times 2} = 2\times 10^6 \text{ rad/sec.}$$

$$\omega_{M2} = A_o \omega_a \tan 3^\circ = 524077.7928 \text{ rad/sec.}$$

So, useful freq. range

.1908 rad/sec. to 524077.7928 rad/sec.