Analog and Digital Filter

E-E 448

Instructor:

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Homework and Tests

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3-5C.

$\frac{40}{50}$

$R(\omega) = (4\omega^2 + 160)/(\omega^2 + 16)$

$M(S) = R(\omega)|_{\omega=\frac{s}{j}} = (-4S^2+160)/(-s^2+16)$

$C(S) \triangleq -4S^2 + 160$

$D(S) \triangleq -S^2 + 16$

1) $D(S) = -S^2 + 16 = (S+4)(-S+4)$

$B(S) = S+4$

$M_2(S) = 4$

$N_2(S) = S$

2) $A(S) = a_0 + a_1 S + a_2 S^2 + a_3 S^3$

$M_1(S) = a_0 + a_2 S^2$

$N_1(S) = a_1 S + a_3 S^3$

3) $M_1(S)M_2(S) - N_1(S)N_2(S) = 4a_0 + 4a_2 S^2 - a_1 S^2 - a_3 S^4 = C(S)$

$4a_0 + (4a_2 - a_1)S^2 - a_3 S^4 = -4S^2 + 160$

$a_0 = 40, \quad a_3 = 0, \quad 4a_2 - a_1 = -4$

In this problem, we are not able to determine the extract values of a_1 and a_2. Assume $a_1 = 4$, $a_2 = 0$

$F(S) = (4S + 40)/(S+4)$ (minimum phase function)

CHECK: $R(\omega) = \mathrm{Re}\{F(S)|_{s=j\omega}\} = (4\omega^2 + 160)/(\omega^2+16)$

Let $a_1 = 35.36$, $a_2 = 7.84$, we obtain

$F(S) = (7.84S^2 + 35.36S + 40)/(S+4)$ (mini-phase, also)

CHECK: $F(\omega) = F(S)|_{s=j\omega} = (-7.84\omega^2 + 40 + j35.36\omega)/(4+j\omega)$

$= (-7.84\omega^2 + 40 + j35.36\omega)(4 - j\omega)/(\omega^2+16)$

$= [(4\omega^2 + 160) + j(141.44\omega + 7.84\omega^2 - 40\omega)]/(\omega^2+16)$

CHECKED

3-6e

$$X(\omega) = (-3\omega^3 - \omega)/(\omega^6 + 2\omega^4 + \omega^2 + 36)$$

$$N(S) = jX(\omega)\big|_{\omega=\frac{s}{j}} = (3S^3 - S)/(-S^6 + 2S^4 - S^2 + 36)$$

$$C(S) \triangleq 3S^3 - S$$

$$D(S) \triangleq -S^6 + 2S^4 - S^2 + 36$$

1) $D(S) = -S^6 + 2S^4 - S^2 + 36 = (-S+2)(S+2)(S^2-2S+3)(S^2+2S+3)$

$$= (-S+2)(S^2-2S+3)(S+2)(S^2+2S+3)$$

$$B(S) = (S+2)(S^2+2S+3) = S^3 + 4S^2 + 7S + 6$$

$$M_2(S) = 4S^2 + 6$$

$$N_2(S) = S^3 + 7S$$

2) $A(S) = a_1 S + a_0$

$M_1(S) = a_0$, $N_1(S) = a_1 S$

$$N_1(S)M_2(S) - N_2(S)M_1(S) = 4a_1S^3 + 6a_1S - a_0S^3 - 7a_0S$$

$$= (4a_1 - a_0)S^3 + (6a_1 - 7a_0)S = C(S) = 3S^3 - S$$

$$\begin{cases} 4a_1 - a_0 = 3 \\ 6a_1 - 7a_0 = -1 \end{cases} \Rightarrow \begin{cases} a_1 = 1 \\ a_0 = 1 \end{cases}$$

✓

3) $F(S) = (S+1)/(S^3 + 4S^2 + 7S + 6)$

CHECK: $F(\omega) = F(S)\big|_{s=j\omega} = (j\omega+1)/(-j\omega^3 - 4\omega^2 + j7\omega + 6)$

$$= (j\omega+1)[-4\omega^2 + 6 + j(\omega^3 - 7\omega)]/(\omega^6 + 2\omega^4 + \omega^2 + 36)$$

$$= [-\omega^4 + 3\omega^2 + 6 + j(-3\omega^3 - \omega)]/(\omega^6 + 2\omega^4 + \omega^2 + 36)$$

$$X(\omega) = (-3\omega^3 - \omega)/(\omega^6 + 2\omega^4 + \omega^2 + 36)$$

✓

CHECKED!

3-8 d

$$\Phi(\omega) = -\tan^{-1}(-3\omega^3-\omega)/(-\omega^4+3\omega^2+6)$$

$$\Phi(S) \triangleq \Phi(\omega)\big|_{\omega=\frac{S}{j}} = -\tan^{-1}(3S^3-S)/j(-S^4-3S^2+6)$$

$$\Phi_o(S) = 3S^3-S$$

$$\Phi_e(S) = -S^4-3S^2+6$$

1) $P(S) = \Phi_o(S)+\Phi_e(S) = -S^4+3S^3-3S^2-S+6 = (-S+2)(S+1)(S^2-2S+3)$

2) $B(-S) = (-S+2)(S^2-2S+3)$

$B(S) = (S+2)(S^2+2S+3) = S^3+4S^2+7S+6$

$A(S) = S+1$

3) $F(S) = (S+1)/(S^3+4S^2+7S+6)$

CHECK = $F(\omega) = F(S)\big|_{s=j\omega}$ (See 3-6e)

$$= [(-\omega^4+3\omega^2+6)+j(-3\omega^3-\omega)]/(\omega^6+2\omega^4+\omega^2+36)$$

$$\Phi(\omega) = -\tan^{-1}(-3\omega^3-\omega)/(-\omega^4+3\omega^2+6)$$

CHECKED!

You did wrong prob. −5

3-9C

$$|H(j\omega)|^2 = (\omega^4 - 9\omega^2 + 25)/(\omega^6 + 3\omega^4 - 3\omega^2 + 4)$$

1) $\dfrac{C(S)}{D(S)} \triangleq |H(j\omega)|^2 \Big|_{\omega = \frac{s}{j}}$

$$= (S^4 + 9S^2 + 25)/(-S^6 + 3S^4 + 3S^2 + 4)$$

2) $C(S) = S^4 + 9S^2 + 25 = (S^2 + S + 5)(S^2 - S + 5)$

$A(S) = S^2 + S + 5$

3) $D(S) = -S^6 + 3S^4 + 3S^2 + 4 = (-S+2)(S+2)(S^2+S+1)(S^2-S+1)$

$$= (-S+2)(S^2-S+1)\cdot(S+2)(S^2+S+1)$$

$B(S) = S^3 + 3S^2 + 3S + 2$

4) $H(S) = (S^2 + S + 5)/(S^3 + 3S^2 + 3S + 2)$

CHECK : $H(j\omega) = (-\omega^2 + j\omega + 5)/(-j\omega^3 - 3\omega^2 + j3\omega + 2)$

$$|H(j\omega)|^2 = \frac{(-\omega^2 + 5 + j\omega)(-\omega^2 + 5 - j\omega)}{(-3\omega^2 + 2 + j3\omega - j\omega^3)(-3\omega^2 + 2 - j3\omega + j\omega^3)}$$

$$= (\omega^4 - 9\omega^2 + 25)/(\omega^6 + 3\omega^4 - 3\omega^2 + 4)$$

CHECKED !

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3-9e

$$|H(j\omega)|^2 = (\omega^4+2\omega^2+1)/(\omega^8-5\omega^6+11\omega^4-11\omega^2+4)$$

1) $\frac{C(s)}{D(s)} \triangleq |H(j\omega)|^2\Big|_{\omega=\frac{s}{j}} = (s^4-2s^2+1)/(s^8+5s^6+11s^4+11s^2+4)$

$C(s) = s^4-2s^2+1$

$D(s) = s^8+5s^6+11s^4+11s^2+4$

2) $C(s) = s^4-2s^2+1 = (s-1)^2(s+1)^2 = (s^2-2s+1)(s^2+2s+1)$

$A(s) = s^2+2s+1$

3) $D(s) = s^8+5s^6+11s^4+11s^2+4 = (s^4+2s^2+1)(s^4+3s^2+4)$

$= (s-j)(s+j)(s-j)(s+j)(s-0.5+j1.323)(s-0.5+j1.323)(s+0.5+j1.323)\cdot(s+0.5-j1.323)$

$B(s) = (s^2+1)\cdot(s+0.5+j1.323)(s+0.5-j1.323)$

$= (s^2+1)(s^2+s+2) = s^4+s^3+3s^2+s+2$

4) $H(s) = (s^2+2s+1)/(s^4+s^3+3s^2+s+2)$

CHECK: $H(j\omega) = H(s)\Big|_{s=j\omega} = (-\omega^2+1+j2\omega)/(2+\omega^4-3\omega^2+j\omega-j\omega^3)$

$|H(j\omega)|^2 = (-\omega^2+1+j2\omega)(-\omega^2+1-j2\omega)/(2+\omega^4-3\omega^2+j\omega-j\omega^3)(2+\omega^4-3\omega^2-j\omega+j\omega^3)$

$= (\omega^4+2\omega^2+1)/(\omega^8-5\omega^6+11\omega^4-11\omega^2+4)$

CHECKED !

Wrong problem

−5

$\frac{57}{60}$

4.1

(d) $p(s) = 3s^4 + 2s^3 + 3s^2 + 2s + 3$

$M(s) = 3s^4 + 3s^2 + 3$

$N(s) = 2s^3 + 2s$

$$\begin{array}{r|l} & \frac{3}{2}s \\ 2s^3+2s & 3s^4+3s^2+3 \\ & 3s^4+3s^2 \\ \hline & 3 \end{array}$$

$$\begin{array}{r|l} & ? \\ 3 & 2s^3+2s \end{array}$$

SO, $p(s)$ is neither Hurwitz nor modified Hurwitz polynomial.

* IF $p(s)$ is a Hurwitz polynomial, $T(s) \triangleq \frac{M(s)}{N(s)}$ or $\frac{N(s)}{M(s)}$ could be writen as this form $T(s) = q_1 s + \cfrac{1}{q_2 s + \cfrac{1}{q_3 s + \cfrac{1}{\ddots \cfrac{1}{q_n s}}}}$. In this problem, $T(s)$ can't be writen in this form, SO, $p(s)$ is not a Hurwitz or modified polynomial.

(e) $p(s) = s^4 + 2s^3 + 3s^2 + 4s + 2$

$M(s) = s^4 + 3s^2 + 2$

$N(s) = 2s^3 + 4s$

$$\begin{array}{r|l} & \frac{1}{2}s \\ 2s^3+4s & s^4+3s^2+2 \\ & s^4+2s^2 \\ \hline & s^2+2 \end{array}$$

$$\begin{array}{r|l} & 2s \\ s^2+2 & 2s^3+4s \\ & 2s^3+4s \\ \hline & 0 \end{array}$$

$k(s) = s^2 + 2$

$\hat{T}(s) = \frac{s^2+2}{2s} = \frac{1}{2}s + \frac{1}{s}$

SO, $p(s)$ is a modified Hurwitz polynomial.

(f) $p(s) = s^4 + 5s^3 + 11s^2 + 11s + 4$

$M(s) = s^4 + 11s^2 + 4$, $N(s) = 5s^3 + 11s$

$$\begin{array}{r|l} & \frac{1}{5}s \\ 5s^3+11s & s^4+11s^2+4 \\ & s^4+\frac{11}{5}s^2 \\ \hline & \frac{44}{5}s^2+4 \end{array}$$

$$\begin{array}{r|l} & 25/44\, s \\ \frac{44}{5}s^2+4 & 5s^3+11s \\ & 5s^3+25/11\, s \\ \hline & 96/11 \cdot s \end{array}$$

$$\begin{array}{r|l} & \frac{121}{120}s \\ \frac{96}{11}\cdot s & \frac{44}{5}s^2+4 \\ & \frac{44}{5}s^2 \\ \hline & 4 \end{array}$$

$$\begin{array}{r|l} & \frac{24}{11}s \\ 4 & \frac{96}{11}\cdot s \\ & \frac{96}{11}\cdot s \\ \hline & 0 \end{array}$$

SO, $p(s)$ is a Hurwitz polynomial.

(i) $p(s) = s^5+s^4+3s^3+2s^2+2s+2$

$N(s) = s^5+3s^3+2s$

$M(s) = s^4+2s^2+2$

$$s^4+2s^2+2 \,\big)\, s^5+3s^3+2s \quad \text{quotient } s$$
$$s^5+2s^3+2s$$
$$s^3$$

$$s^3 \,\big)\, s^4+2s^2+2 \quad \text{quotient } s$$
$$s^4$$
$$2s^2+2$$

$$2s^2+2 \,\big)\, s^3 \quad \text{quotient } \tfrac{1}{2}s$$
$$s^3+s$$
$$-s$$

$$-s \,\big)\, 2s^2+2 \quad \text{quotient } -2s$$
$$2s^2$$
$$2$$

So, $p(s)$ is neither Hurwitz nor modified Hurwitz polynomial.

(p) $p(s) = s^6+2s^4+s^2+3$

$M(s) = s^6+2s^4+s^2+3$

$N(s) = 0$

$k(s) = s^6+2s^4+s^2+3$ ($p(s)$ is not a Hurwitz.)

$$\psi(s) = \frac{s^6+2s^4+s^2+3}{6s^5+8s^3+2s}$$

$$6s^5+8s^3+2s \,\big)\, s^6+2s^4+s^2+3 \quad \text{quotient } 1/6\cdot s$$
$$s^6+\tfrac{4}{3}s^4+\tfrac{1}{3}s^2$$
$$\tfrac{2}{3}s^4+\tfrac{2}{3}s^2+3$$

$$\tfrac{2}{3}s^4+\tfrac{2}{3}s^2+3 \,\big)\, 6s^5+8s^3+2s \quad \text{quotient } 9s$$
$$6s^5+6s^3+27s$$
$$2s^3-25s$$

$$2s^3-25s \,\big)\, \tfrac{2}{3}s^4+\tfrac{2}{3}s^2+3 \quad \text{quotient } \tfrac{1}{3}s$$
$$\tfrac{2}{3}s^4-\tfrac{25}{3}s^2$$
$$\tfrac{27}{3}s^2+3$$

$$\tfrac{27}{3}s^2+3 \,\big)\, 2s^3-25s \quad \text{quotient } \tfrac{6}{27}s$$
$$2s^3+\tfrac{18}{27}s$$
$$-\tfrac{683}{27}s$$

$$\vdots$$

$$-\tfrac{683}{27}s \,\big)\, \tfrac{27}{3}s^2+3 \quad \text{quotient } -\tfrac{243}{683}s$$
$$9s^2$$
$$3$$

So, $p(s)$ is not a modified Hurwitz polynomial.

4-7

(b) $F(s) = \frac{1}{s}$

$F(s)$ is PR.

Let $s = \sigma + j\omega$, $F(s) = \frac{1}{s} = \frac{1}{\sigma+j\omega} = \frac{1}{\sigma^2+\omega^2}(\sigma - j\omega)$

$\omega = 0 \longrightarrow F(s) = \frac{\sigma}{\sigma^2+\omega^2}$ real.

$\sigma \geq 0 \longrightarrow \mathrm{Re}\{F(s)\} = \sigma/\sigma^2+\omega^2 \geq 0$.

(d) $F(s) = \dfrac{s+4}{s^2+s+15}$

(1) $s=\sigma$, $F(s)=F(\sigma)=\dfrac{\sigma+4}{\sigma^2+\sigma+15}$ is real

(2) $B(s) = s^2+s+15$ is Hurwitz polynomial.

(3) $F(s)=\dfrac{s+4}{s^2+s+15} = \dfrac{0.5-j0.456}{(s+0.5-j3.84)} + \dfrac{0.5+j0.456}{(s+0.5+j3.84)}$?

SO, F(S) is not PR ! ✓ ?

-3

Need to check $R(j\omega)$!

(f) $F(s)= \dfrac{s^2+9}{s^3+4s}$

(1) $F(\sigma) = (s^2+9)/(s^3+4s)\big|_{s=\sigma} = (\sigma^2+9)/(\sigma^3+4\sigma)$ real .

(2) $B(s) = s^3+4s = s(s+j2)(s-j2)$ modified Hurwitz .

(3) $F(s) = \dfrac{9/4}{s} + \dfrac{-\frac{5}{8}}{s+j2} + \dfrac{-\frac{5}{8}}{s-j2}$ negative residues ✓

SO, F(S) is not PR. ✓

(h) $F(s) = \dfrac{s^3+6s^2+2s+1}{(s+1)^2}$

(1) F(S) is real when σ is real

(2) $B(s) = (s+1)^2$ is Hurwitz

(3) $A(s) = s^3+6s^2+2s+1 = (s+5.68)(s+0.16-j0.388)(s+0.16+j0.388)$

A(s) is Hurwitz.

$G(s) \triangleq 1/F(s) = B(s)/A(s) = (s+1)^2/(s+5.68)(s+0.16-j0.388)(s+0.16+j0.388)$

$= \dfrac{0.715}{s+5.68} + \dfrac{0.076-j0.201}{s+0.16-j0.388} + \dfrac{0.076+j0.201}{s+0.16-j0.388}$

∴ G(s) is not a PR fuction

F(s) is also not PR. ✓

(j) $F(s) = (s^3+6s^2+2s+1)/(s^3+3s+1)$

$B(s) = s^3+3s+1$

$N_B(s) = s^3+3s$

$M_B(s) = 1$

$$\begin{array}{r|l} & \quad ? \\ 1. & \overline{s^3+3s} \end{array}$$

So, B(s) is neither Hurwitz nor modified Hurwitz polynomial.

F(s) isn't PR. ✓

(1) $F(s) = (s^4+s^3+2s^2+5)/(4s^2+2s+1)$

F(s) isn't PR. Because the difference between degrees A(s) and B(s) is 2. ✓

(n) $F(s) = (10s^4+8s^2+1)/(4s^5+10s^3+4s)$

(1) F(s) is real when s is real

(2) $B(s) = 4s^5+10s^3+4s$

$N_B(s) = 4s^5+10s^3+4s$, $M_B(s) = 0$

$k(s) = 4s^5+10s^3+4s$

$$\hat{T}(s) = \frac{4s^5+10s^3+4s}{20s^4+30s^2+4}$$

$$\begin{array}{r|l} & \frac{1}{5}s \\ 20s^4+30s^2+4 & \overline{4s^5+10s^3+4s} \\ & 4s^5+6s^3 \\ \hline & 4s^3+4s \end{array}$$

$$\begin{array}{r|l} & 5s \\ 4s^3+4s & \overline{20s^4+30s^2+4} \\ & 20s^4+20s^2 \\ \hline & 10s^2+4 \end{array}$$

$$\begin{array}{r|l} & \frac{2}{5}s \\ 10s^2+4 & \overline{4s^3+4s} \\ & 4s^3+\frac{8}{5}s \\ \hline & \frac{12}{5}s \end{array}$$

$$\begin{array}{r|l} & \frac{25}{6}s \\ \frac{12}{5}s & \overline{10s^2+4} \\ & 10s^2 \\ \hline & 4 \end{array}$$

$$\begin{array}{r|l} & \frac{12}{20}s \\ 4 & \overline{\frac{12}{5}s} \\ & \frac{12}{5}s \\ \hline & 0 \end{array}$$

B(s) is modified Hurwitz. ✓

(3) $F(s) = (10s^4+8s^2+1)/(4s^5+10s^3+4s)$

$$= (10s^4+8s^2+1)/4s\cdot(s-j\tfrac{\sqrt{2}}{2})(s+j\tfrac{\sqrt{2}}{2})(s+j\sqrt{2})(s-j\sqrt{2})$$

$$= \frac{2.5}{s} + \frac{25/24}{s-j\sqrt{2}} + \frac{25/24}{s+j\sqrt{2}} + \frac{1/12}{s+j\sqrt{2}/2} + \frac{1/12}{s-j\sqrt{2}/2}$$

(4) $F(j\omega)=(10\omega^4-8\omega^2+1)/(j4\omega^5-j10\omega^3+j4\omega)$

$$= -j\ (10\omega^4-8\omega^2+1)/(4\omega^5-10\omega^3+4\omega)$$

$\therefore \mathrm{Re}\{F(j\omega)\}=0$

SO, F(s) is PR. ✓

5-1. DETERMINE which of the following F(S) are realizable as lossless DP Functions. Justify your answers.

$\frac{109}{120}$

Good job!

(a) $F(S) = (S^4+5S^2+6)/(S^4+3S^2+2)$

Answer: F(S) is unrealizable.

Because the difference between the degrees of (S^4+5S^2+6) and (S^4+3S^2+2) is zero. This doesn't satisfy Property 3. ✓

(c) $F(S) = (S^3+5S)/(S^4+3S^2+2)$

$= S(S+j\sqrt{5})(S-j\sqrt{5})/[(S-j)(S+j)(S-j\sqrt{2})(S+j\sqrt{2})]$

Answer = F(S) is unrealizable. X −5

Because, F(S) does not satisfy Proper 6. The poles and zeros are along the imaginary axis like:

∞ (zero), $j\sqrt{5}$ (zero), $j\sqrt{2}$ (pole), j (pole), 0 (zero), …

(e) $F(S) = (S^3+1.5S)/(S^2+1)$

Answer: F(S) is realizable. ✓

It is easy to check that F(S) satisfies Property 1, 2, 3, 4 and 6. And

$$F(j\omega)/j = (-j\omega^3+j1.5\omega)/j(-\omega^2+1) = (-\omega^3+1.5\omega)/(-\omega^2+1)$$

$$\frac{d\,F(j\omega)/j}{d\omega} = [(-3\omega^2+1.5)(-\omega^2+1)-(-2\omega)\cdot(-\omega^3+1.5\omega)]/(1-\omega^2)^2$$

$$= (\omega^4-1.5\omega^2+1.5)/(1-\omega^2)^2$$

$$= [(\omega^2-0.75)^2+0.9375]/(1-\omega^2)^2$$

> 0 (except the poles of F(S)). ✓

(g) $F(S) = (S^5+3S^3+2S)/(S^4+5.5S^2+6)$

$= S(S+j)(S-j)(S+j\sqrt{2})(S-j\sqrt{2})/[(S-j1.225)(S+j1.225)(S+j2)(S-j2)]$

Answer: F(S) is unrealizable.

Because the poles and zeros of F(S) don't alternate with each other along the imaginary axis.

∞ (pole), $j2$ (pole), $j1.225$ (pole), $j\sqrt{2}$ (zero), j (zero), 0 (zero), $-j$ (zero), $\cdots$

(i) $F(S) = (S^4+3S^2+2)/(S^5+5.5S^3+6)$

Answer: F(S) is unrealizable.

Because $F(-S) = (S^4+3S^2+2)/(-S^5-5.5S^3+6) \neq -F(S)$.

5-7 Realize the following DP admittance functions by two Foster's forms and two Cauer's forms. If a given Y(S) is not realizable, state your reasons.

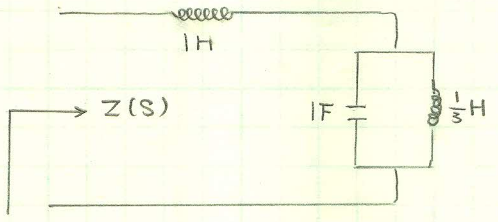
(b) $Y(S) = (S^2+3)/[S(S^2+4)]$

FIRST FOSTER'S FORM:

$Z(S) = S(S^2+4)/(S^2+3)$

$$\left.\frac{Z(S)}{S}\right|_{S^2=p} = (p+4)/(p+3) = 1 + \frac{1}{p+3}$$

$$Z(S) = S + \frac{S}{S^2+3} = S + \frac{1}{S + \frac{1}{\frac{1}{3}S}}$$



CHECK: $$Z(S) = S + \frac{\frac{1}{3}S\cdot(\frac{1}{S})}{\frac{1}{3}S + \frac{1}{S}} = S + \frac{S}{S^2+3} = \frac{S(S^2+4)}{S^2+3}$$

CHECKED!

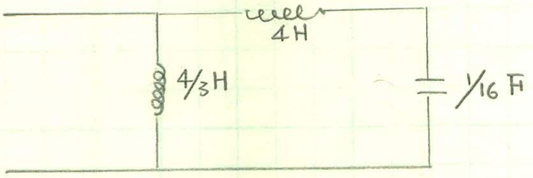
5-7(b) CONT.

SECOND FOSTER'S FORM :

$$Y(s) = (s^2+3)/[s(s^2+4)]$$

$$\frac{Y(s)}{s}\Big|_{s^2=p} = (p+3)/[p(p+4)] = \frac{\frac{3}{4}}{p} + \frac{\frac{1}{4}}{p+4}$$

$$Y(s) = \frac{3/4}{s} + \frac{1/4\, s}{s^2+4} = \frac{1}{4/3\, s} + \frac{1}{4s + \frac{1}{1/16\, s}}$$



CHECK :

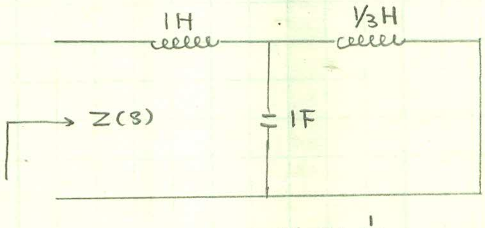
$$Y(s) = \frac{1}{4/3 s} + \frac{1}{4s + \frac{16}{s}} = (s^2+3)/[s(s^2+4)]$$

CHECKED !!

FIRST CAUER'S FORM :

$$Z(s) = s(s^2+4)/(s^2+3) = (s^3+4s)/(s^2+3)$$

$$s^2+3 \overline{\big)\, s^3+4s} \quad s$$
$$s^2+3s$$
$$s \overline{\big)\, s^2+3} \quad s$$
$$s^2$$
$$3 \overline{\big)\, s} \quad \tfrac{1}{3}s$$
$$s$$
$$0$$



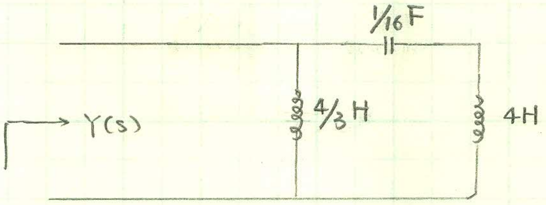
CHECK : $Z(s) = s + \dfrac{\frac{1}{3}s \cdot \frac{1}{s}}{\frac{1}{3}s + \frac{1}{s}} = (s^3+4s)/(s^2+3)$ CHECKED!!

SECOND CAUER'S FORM :

$$Y(s) = (s^2+3)/(s^3+4s)$$

$$4s+s^3 \overline{\big)\, 3+s^2} \quad \frac{3}{4s}$$
$$3 + \tfrac{3}{4}s^2$$
$$\tfrac{1}{4}s \overline{\big)\, 4 + s^2} \quad \frac{16}{s}$$
$$4 + 0$$
$$s \overline{\big)\, \tfrac{1}{4}} \quad \tfrac{1}{4s}$$
$$\tfrac{1}{4}$$
$$0$$

5-7(b) CONT.



✓

CHECK: $Y(S) = \frac{3}{4S} + \frac{1}{4S + \frac{16}{S}} = (S^2+3)/(S^3+4S)$ CHECKED!! ✓

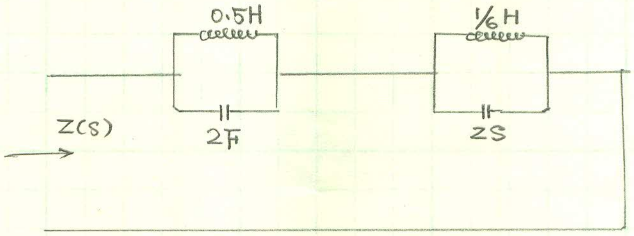
(c) $Y(S) = (S^2+1)(S^2+3)/[S(S^2+2)]$

FIRST FOSTER'S FORM

$Z(S) = S(S^2+2)/[(S^2+1)(S^2+3)]$

$$\frac{Z(S)}{S}\Big|_{S^2=p} = (p+2)/[(p+1)(p+3)] = \frac{0.5}{p+1} + \frac{0.5}{p+3}$$

$$Z(S) = \frac{0.5S}{S^2+1} + \frac{0.5S}{S^2+3} = \frac{1}{2S+\frac{1}{0.5S}} + \frac{1}{2S+\frac{1}{\frac{1}{6}S}}$$



✓

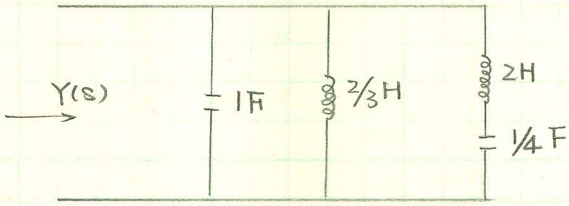
CHECK: $Z(S) = \frac{1}{2S+\frac{2}{S}} + \frac{1}{2S+\frac{6}{S}} = S(S^2+2)/[(S^2+1)(S^2+3)]$ CHECKED ✓

SECOND FOSTER'S FORM

$Y(S) = (S^2+1)(S^2+3)/[S(S^2+2)]$

$$\frac{Y(S)}{S}\Big|_{S^2=p} = (p+1)(p+3)/[p(p+2)] = 1 + \frac{\frac{3}{2}}{p} + \frac{\frac{1}{2}}{p+2}$$

$$Y(S) = S + \frac{1}{\frac{2}{3}S} + \frac{1}{2S + \frac{1}{\frac{1}{4}S}}$$



✓

CHECK: $Y(S) = S + \frac{3}{2S} + \frac{1}{2S+\frac{4}{S}} = (S^2+1)(S^2+3)/[S(S^2+2)]$

CHECKED !! ✓

5-7 (c) CONT.

FIRST CAUER'S FORM

$$Y(S) = (S^2+1)(S^2+3)/[S(S^2+2)] = (S^4+4S^2+3)/(S^3+2S)$$

$S^3+2S \overline{)\,S^4+4S^2+3\,}$ quotient S

S^4+2S^2

$2S^2+3 \overline{)\,S^3+2S\,}$ quotient $\frac{1}{2}\cdot S$

$S^3+\frac{3}{2}\cdot S$

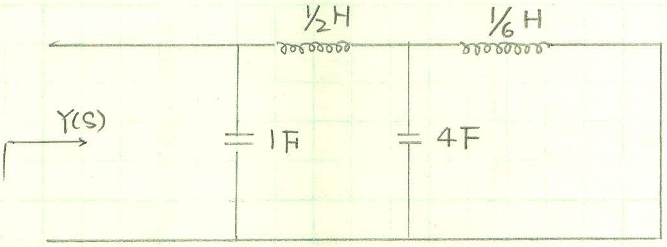
$\frac{1}{2}\cdot S \overline{)\,2S^2+3\,}$ quotient $4S$

$2S^2$

$3 \overline{)\,\frac{1}{2}\cdot S\,}$ quotient $\frac{1}{6}\cdot S$

$\frac{1}{2}\cdot S$

0



$$\text{CHECK:}\quad Y(S) = S + \cfrac{1}{0.5S + \cfrac{1}{4S+\frac{6}{S}}} = \frac{(S^2+1)(S^2+3)}{S(S^2+2)}$$ CHECKED!!

SECOND CAUER'S FORM

$$Y(S) = (S^4+4S^2+3)/(S^3+2S)$$

$2S+S^3 \overline{)\,3+4S^2+S^4\,}$ quotient $\frac{3}{2S}$

$3+\frac{3}{2}S^2$

$\frac{5}{2}S+S^3 \overline{)\,2+S^2\,}$ quotient $\frac{4}{5S}$

$2+\frac{4}{5}S^2$

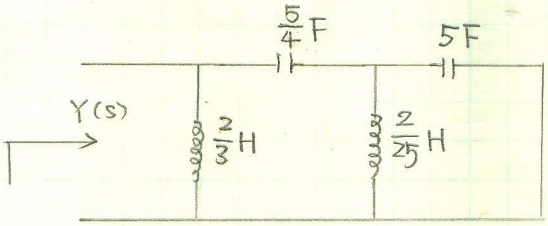
$\frac{1}{5}S \overline{)\,\frac{5}{2}+S^2\,}$ quotient $\frac{25}{2S}$

$\frac{5}{2}$

$S \overline{)\,\frac{1}{5}\,}$ quotient $\frac{1}{5S}$

$\frac{1}{5}$

0



$$\text{CHECK:}\quad Y(S) = \frac{3}{2S} + \cfrac{1}{\frac{4}{5S} + \cfrac{1}{\frac{25}{2S}+5S}}$$

$$= \frac{3}{2S} + \frac{25S(2S^2+5)}{50S^2+100} = \frac{3}{2S} + \frac{S(2S^2+5)}{2S^2+4}$$

$$= \frac{S^4+4S^2+3}{S^3+2S}$$ CHECKED!

5-7 (d) $Y(S) = S(S^2+2)(S^2+4)/(S^2+1)(S^2+3)$

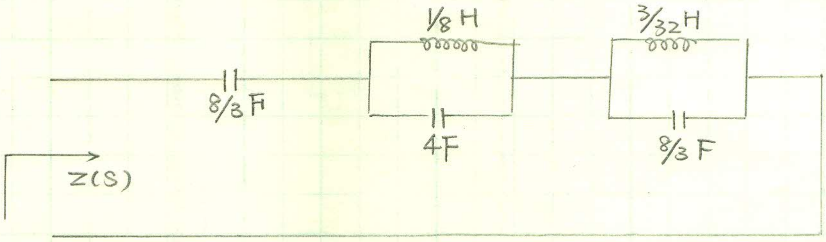
FOSTER'S FIRST FORM:

$$Z(S) = (S^2+1)(S^2+3)/S(S^2+2)(S^2+4)$$

$$\frac{Z(S)}{S}\Big|_{s^2=p} = (p+1)(p+3)/p(p+2)(p+4)$$

$$= \frac{\frac{3}{8}}{p} + \frac{\frac{1}{4}}{p+2} + \frac{\frac{3}{8}}{p+4}$$

$$Z(S) = \frac{1}{8/3\,S} + \frac{1}{4S + \frac{1}{1/8\,S}} + \frac{1}{8/3\,S + \frac{1}{3/32\,S}}$$



CHECK: $Z(S) = \frac{1}{8/3\,S} + \frac{1}{4S + \frac{8}{S}} + \frac{1}{8/3\,S + \frac{32}{3S}}$

$$= \frac{3/8}{S} + \frac{0.25S}{S^2+2} + \frac{3/8\,S}{S^2+4}$$

$$= \frac{3/8\,(S^2+2)(S^2+4) + 0.25S^2(S^2+4) + 3/8\,S^2(S^2+2)}{S(S^2+2)(S^2+4)}$$

$$= \frac{S^4+4S^2+3}{S(S^2+2)(S^2+4)} = \frac{(S^2+1)(S^2+3)}{S(S^2+2)(S^2+4)}$$ CHECKED!

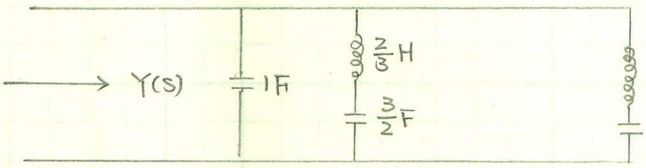
FOSTER'S SECOND FORM:

$$Y(S) = S(S^2+2)(S^2+4)/(S^2+1)(S^2+3)$$

$$\frac{Y(S)}{S}\Big|_{s^2=p} = (p+2)(p+4)/(p+1)(p+3)$$

$$= \frac{\frac{3}{2}}{p+1} + \frac{\frac{1}{2}}{p+3} + 1$$

$$Y(S) = \frac{1}{2/3\,S + \frac{1}{\frac{3}{2}S}} + \frac{1}{2S + \frac{1}{1/6\,S}} + S$$



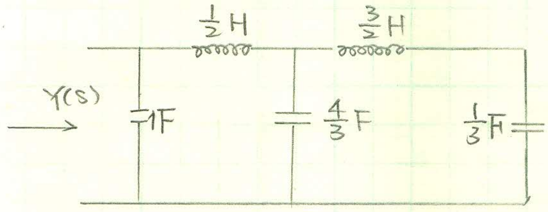
CHECK: $Y(S) = \frac{1}{2/3\,S + \frac{2}{3S}} + \frac{1}{2S + \frac{6}{S}} + S = \frac{S(S^4+6S^2+8)}{(S^2+1)(S^2+3)}$ CHECKED!

5-7 (d) CONT. **CAUER'S FIRST FORM:**

$$Y(S) = S(S^2+2)(S^2+4)/(S^2+1)(S^2+3)$$

$$= (S^5+6S^3+8S)/(S^4+4S^2+3)$$

$$S$$
$$S^4+4S^2+3 \overline{)\ S^5+6S^3+8S}$$
$$S^5+4S^3+3S$$
$$\tfrac{1}{2}S$$
$$2S^3+5S \overline{)\ S^4+4S^2+3}$$
$$S^4+\tfrac{5}{2}S^2$$
$$\tfrac{4}{3}S$$
$$\tfrac{3}{2}S^2+3 \overline{)\ 2S^3+5S}$$
$$2S^3+4S$$
$$\tfrac{3}{2}S$$
$$S \overline{)\ \tfrac{3}{2}S^2+3}$$
$$\tfrac{3}{2}S^2$$
$$\tfrac{1}{3}S$$
$$3 \overline{)\ S}$$
$$S$$
$$0$$



✓

CHECK: $$Y(S) = S + \cfrac{1}{\frac{1}{2}S + \cfrac{1}{\frac{4}{3}S + \cfrac{1}{\frac{3}{2}S + \frac{3}{S}}}}$$

$$= S + \cfrac{1}{0.5S + \cfrac{1}{\frac{4}{3}S + \frac{2S}{3S^2+6}}} = S + \cfrac{1}{0.5S + \frac{3S^2+6}{4S(S^2+2)}}$$

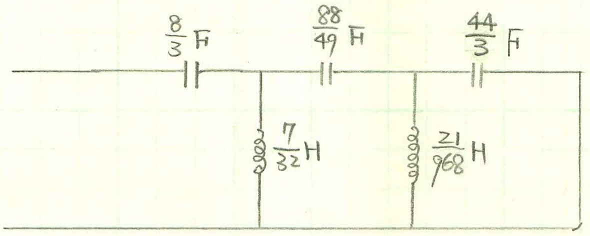
$$= (S^5+6S^3+8S)/(S^4+4S^2+3)$$ CHECKED!

✓

CAUER'S SECOND FORM:

$$Z(S) = (S^4+4S^2+3)/(S^5+6S^3+8S)$$

$$\frac{3}{8S}$$
$$8S+6S^3+S^5 \overline{)\ 3+4S^2+S^4}$$
$$3+\tfrac{9}{4}S^2+\tfrac{3}{8}S^4$$
$$\frac{32}{7S}$$
$$\tfrac{7}{4}S+\tfrac{5}{8}S^3 \overline{)\ 8+6S^2+S^4}$$
$$8+\tfrac{20}{7}S^2$$
$$\frac{49}{88S}$$
$$\tfrac{22}{7}S+S^3 \overline{)\ \tfrac{7}{4}+\tfrac{5}{8}S^2}$$
$$\tfrac{7}{4}+\tfrac{49}{88}S^2$$
$$\frac{968}{21S}$$
$$\tfrac{6}{88}S \overline{)\ \tfrac{22}{7}+S^2}$$
$$\tfrac{22}{7}$$
$$\frac{3}{44s}$$
$$S \overline{)\ \tfrac{3}{44}}$$
$$\tfrac{3}{44}$$
$$0$$



✓

Check -3

5-7 (e)

$$Y(S) = (S^2+2)(S^2+4)/S(S^2+1)(S^2+3)$$

Y(S) is not realizable, because the zeros and poles of Y(S) do not alternate with each other along the imaginary axis.

∞ (zero), $j4$ (zero), ...

5-8

$$Z(S) = \frac{(S^2+1)(S^2+4)}{S(S^2+3)(S^2+5)}$$

(a) START WITH CAUER'S FIRST FORM (FOR TWO ELEMENTS), AND COMPLETE THE REALIZATION WITH CAUER'S SECOND FORM.

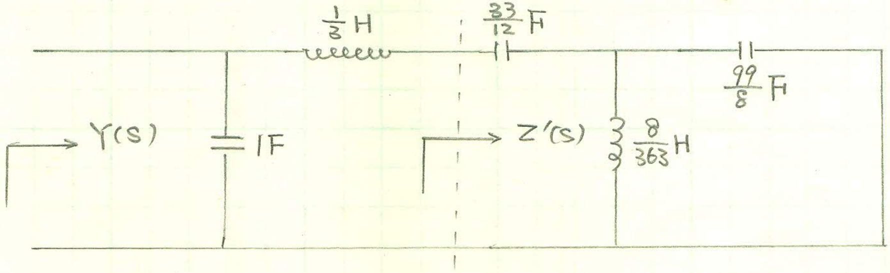
FIRST FORM: $Y(S) = (S^5+8S^3+15S)/(S^4+5S^2+4)$

$$\begin{array}{r} S \\ S^4+5S^2+4 \overline{\big)\, S^5+8S^3+15S} \\ \underline{S^5+5S^3+4S} \\ 3S^3+11S \end{array} \qquad \begin{array}{r} \frac{1}{3}S \\ \overline{\big)\, S^4+5S^2+4} \\ \underline{S^4+\frac{11}{3}S^2} \\ \frac{4}{3}S^2+4 \end{array}$$

$$Y(S) = S + \frac{1}{\frac{1}{3}S + Z'(S)}$$

SECOND FORM: $Z'(S) = \frac{1}{Y(S)-S} - \frac{1}{3}S = \frac{4S^2+12}{9S^3+33S}$

$$\begin{array}{r} \frac{12}{33S} \\ 33S+9S^3 \overline{\big)\, 12+4S^2} \\ \underline{12+\frac{36}{11}S^2} \\ \frac{8}{11}S \end{array} \qquad \begin{array}{r} \frac{363}{8S} \\ \overline{\big)\, 33+9S^2} \\ \underline{33} \\ 9S \end{array} \qquad \begin{array}{r} \frac{8}{99S} \\ \overline{\big)\, \frac{8}{11}} \\ \underline{\frac{8}{11}} \\ 0 \end{array}$$



5-8 (a) CONT.

CHECK: $$Z'(s) = \frac{12}{33s} + \frac{1}{\frac{363}{8s} + \frac{99}{8}s}$$

$$= \frac{12}{33s} + \frac{8s}{33(3s^2+11)} = \frac{4s^2+12}{9s^3+33s}$$

$$Y(s) = s + \frac{1}{\frac{1}{3}s + \frac{4s^2+12}{9s^3+33s}} = \frac{s^5+8s^3+15s}{(s^4+5s^2+4)}$$

CHECKED!

(b) START WITH CAUER'S SECOND FORM (FOR 3 elements); COMPLETE THE REALIZATION WITH CAUER'S FIRST FORM.

Cauer's sec. F: $$Z(s) = \frac{s^4+5s^2+4}{s^5+8s^3+15s}$$

$$\begin{array}{r} \frac{4}{15s} \\ 15s+8s^3+s^5 \overline{\big)\,4+5s^2+s^4} \\ 4+\frac{32}{15}s^2+\frac{4}{15}s^4 \\ \hline \frac{43}{15}s+\frac{11}{15}s^3 \end{array}$$

$$\begin{array}{r} \frac{225}{43s} \\ \frac{43}{15}s+\frac{11}{15}s^3 \overline{\big)\,15+8s^2+s^4} \\ 15+\frac{165}{43}s^2 \\ \hline \frac{179}{43}s+s^3 \end{array}$$

$$\begin{array}{r} \frac{1849}{2685s} \\ \frac{179}{43}s+s^3 \overline{\big)\,\frac{43}{15}+\frac{11}{15}s^2} \\ \frac{43}{15}+\frac{1849}{2685}s^2 \\ \hline \frac{24}{537}s \end{array}$$

$$\begin{array}{r} \frac{32041}{344s} \\ \frac{24}{537}s \overline{\big)\,\frac{179}{43}+s^2} \\ \frac{179}{43} \\ \hline s \end{array}$$

$$\begin{array}{r} \frac{24}{537s} \\ s \overline{\big)\,\frac{24}{537}} \\ \frac{24}{537} \\ \hline 0 \end{array}$$

$$Z(s) = \frac{1}{15s} + \frac{1}{\frac{225}{43s} + \frac{1}{\frac{1849}{2685s} + Z'(s)}}$$

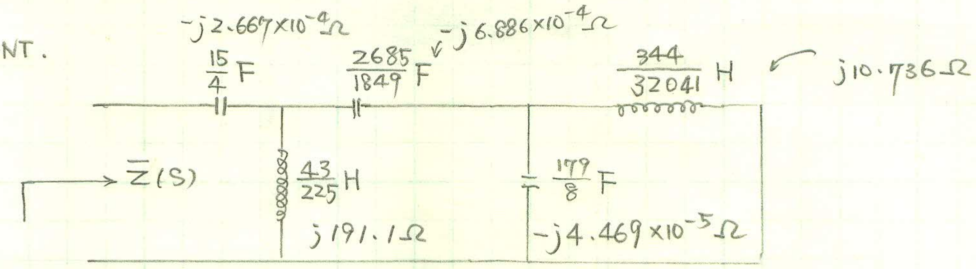
$$Z'(s) = \frac{1}{\frac{32041}{344s} + \frac{537s}{24}} = \frac{344s}{7697s^2+32041}$$

Cauer's 1st: $$Y'(s) = \frac{7697s^2+32041}{344s}$$

$$\begin{array}{r} \frac{179}{8}s \\ 344s \overline{\big)\,7697s^2+32041} \\ 7697s^2 \\ \hline 32041 \end{array}$$

$$\begin{array}{r} \frac{344}{32041}s \\ 32041 \overline{\big)\,344s} \\ 344s \\ \hline 0 \end{array}$$

5-8 (b) CONT.



IN ORDER TO CHECK THE REALIZATION WITH A EASY WAY, WE LET

$$s = j\omega \quad \text{AND} \quad \omega = 1000 \text{ rad/s}$$

$$\overline{Z}'(j1000) = -j0.001000003\ \Omega$$

$$Z(S)\big|_{S=j1000} = \frac{1000^4 - 5\times1000^2 + 4}{j(1000^5 - 8\times1000^3 + 15\times1000)} = -j0.001000003\ \Omega$$

<u>80% CHECKED!!</u>

-2

5-8 (c) Cauer's first form for 1 element. Cauer's second form for 1 element. Complete the realization with Foster's first form

Cauer's 1st: $Y(S) = \frac{S^5 + 8S^3 + 15S}{S^4 + 5S^2 + 4}$

$$Y(S) \triangleq S + Y_1(S)$$

$$Y_1(S) = Y(S) - S = \frac{S^5+8S^3+15S}{S^4+5S^2+4} - S = \frac{3S^3+11S}{S^4+5S^2+4}$$

Cauer's 2nd: $Z_1(S) \triangleq 1/Y_1(S) = (S^4+5S^2+4)/(3S^3+11S)$

$$\triangleq \frac{4}{11S} + Z_2(S)$$

$$Z_2(S) = Z_1(S) - \frac{4}{11S} = \frac{S^4+5S^2+4}{3S^3+11S} - \frac{4}{11\cdot S}$$

$$= \frac{11S^3+43S}{33S^2+121}$$

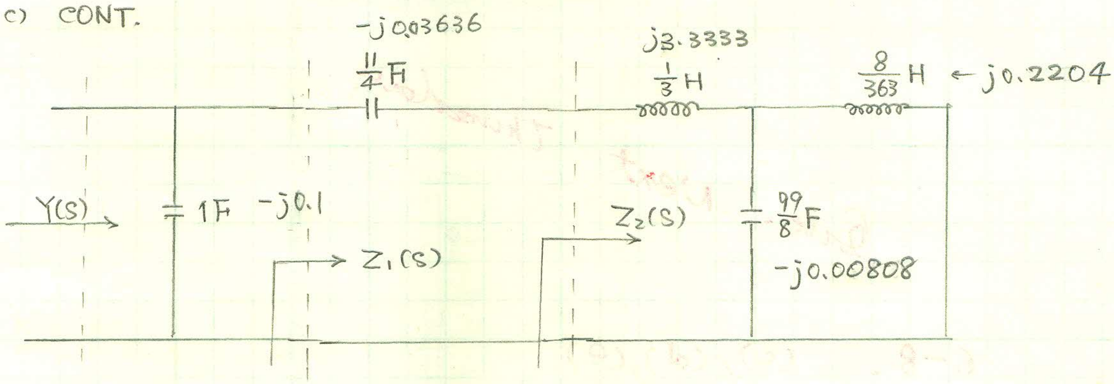
Foster's 1st:

$$\frac{Z_2(S)}{S}\bigg|_{S^2=P} = \frac{11P+43}{33P+121} = \frac{1}{3} + \frac{\frac{8}{3}}{33P+121}$$

$$Z_2(S) = \frac{1}{3}S + \frac{\frac{8}{3}S}{33S^2+121}$$

$$= \frac{1}{3}S + \cfrac{1}{\frac{99}{8}S + \cfrac{1}{\frac{8}{363}S}}$$

5-8 (c) CONT.



CHECK : SAME AS part (b). we check the answer in easy way. Let $s = j \cdot 10$

$Y(j \cdot 1) = j\,9.69589$ ℧

$Y(S)\,|_{s=j\cdot 1} = (j10^5 - j8\times10^3 + j150)/(10^4 - 5\times10^2 + 4)$

$= j\,9.69592$ ℧

90% CHECKED!!

−1

6-1. DETERMINE WHICH OF THE FOLLOWING Z(S) ARE REALIZABLE AS RC DP IMPEDANCE FUNCTIONS:

$\frac{50}{50}$

(b)
$$Z(S) = \frac{S^2+5S}{S^2+3S+2}$$
$$= \frac{S(S+5)}{(S+1)(S+2)}$$

$S=0$ (zero); $S=-1$ (pole); $S=-2$ (pole); $S=-5$ (zero) ✓

SO, Z(S) IS NOT REALIZABLE. ✓

(d)
$$Z(S) = (S+1.5)/(S^2+2S+1)$$
$$= (S+1.5)/(S+1)(S+1)$$

$S=-1$ is not a simple pole ✓

SO, Z(S) IS NOT REALIZABLE. ✓

(f)
$$Z(S) = (S^2+7S+12)/(S^3+3S^2+3S+1)$$
$$= (S+3)(S+4)/(S+1)(S+1)(S+1)$$

$S=-1$ IS NOT A SIMPLE POLE. ✓

SO, Z(S) IS NOT REALIZABLE. ✓

(h)
$$Z(S) = (S^3+8S^2+17S+10)/(S^3+11.5S^2+39S+36)$$
$$= (S+5)(S+1)(S+2)/(S+6)(S+1.5)(S+4)$$

Z(S) Does not satisfy PROPERTY ZRC 5 ✓

$1.5 > 1$, SO, Z(S) IS NOT REALIZABLE. ✓

(j)
$$Z(S) = (S^3+6S^2+8.75S+3)/(S^2+3S+2) \triangleq A(S)/B(S)$$

Because degree of $A(S) = 3 >$ degree of $B(S) = 2$ ✓

SO, Z(S) IS NOT REALIZABLE. ✓

6-2. DETERMINE WHICH OF THE FOLLOWING Y(S) ARE REALIZABLE AS RC DP ADMITTANCE FUNCTIONS:

(a) $Y(s) = (s^2+7s+12)/(s^2+3s+2)$

$= (s+3)(s+4)/(s+1)(s+2)$

$s=-1$ (pole) ; $s=-2$ (pole) ; $s=-3$ (zero) ; $Z=-4$ (zero)

SO, Y(S) IS NOT REALIZABLE. ✓

(c) $Y(s) = (s^2+3s+2)/(s+5) = (s+1)(s+2)/(s+5)$

$s=-1$ (zero) ; $s=-2$ (zero) ; $s=-5$ (pole)

SO, Y(S) IS NOT REALIZABLE. ✓

(e) $Y(s) = (s+1.5)/(s^2+2s+1) \triangleq A(s)/B(s)$

Because degree of $A(s) = 1$ < degree of $B(s) = 2$.

SO, Y(S) IS NOT REALIZABLE. ✓

(g) $Y(s) = (s^3+6s^2+11s+6)/(s^2+4s+3.75)$

$= (s+3)(s+1)(s+2)/(s+1.5)(s+2.5)$

Y(s) satisfies ALL PROPERTIES FOR 1−5.

$Y(s) = [(s^3+4s^2+3.75s)+(2s^2+7.25s+6)]/[(s+1.5)(s+2.5)]$

$= s + (2s^2+7.25s+6)/[(s+1.5)(s+2.5)]$

$= s + [(2s^2+8s+7.5)-(0.75s+1.5)]/[(s+1.5)(s+2.5)]$

$$= s + 2 - \frac{0.375}{s+1.5} - \frac{0.375}{s+2.5}$$

SATISFY PROPERTY YRC 6.

Y(S) DOES'NT HAVE A POLE AT $s=0$. PRO. YRC 7 HOLDS.

$$Y(\sigma) = \sigma + 2 - \frac{0.375}{\sigma+1.5} - \frac{0.375}{\sigma+2.5}$$

$$\frac{dY(\sigma)}{d\sigma} = 1 + \frac{0.375}{(\sigma+1.5)^2} + \frac{0.375}{(\sigma+2.5)^2} > 0$$, PRO. YRC 8 HOLDS.

SO, Y(S) IS REALIZABLE. ✓

6-2 CONT.

(i) $Y(S) = (S^2+3S+2)/(S^3+6S^2+8.75S+3) \triangleq A(S)/B(S)$

Because degree of $A(S) = 2 <$ degree of $B(S) = 3$

SO, $Y(S)$ IS NOT REALIZABLE. ✓

EE 448 . H.W. #5 FILTER 71/100 Benmei Chen

6-8. *Realize the following RC DP admittance functions by Foster's two forms and Cauer's two forms.*

(c) $Y(S) = (S+1)(S+5)/(S+4)(S+6)$

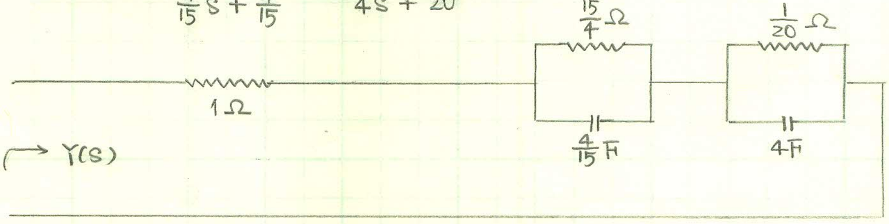
$= (S^2+6S+5)/(S^2+10S+24)$

FOSTER'S FIRST FORM:

$Z(S) = (S^2+10S+24)/(S^2+6S+5) = [(S^2+6S+5)+(4S+19)]/(S^2+6S+5)$

$= 1 + (4S+19)/(S+1)(S+5) = 1 + \frac{15/4}{S+1} + \frac{1/4}{S+5}$

$= 1 + \frac{1}{\frac{4}{15}S + \frac{4}{15}} + \frac{1}{4S+20}$



CHECK: $1 + \frac{15/4}{S+1} + \frac{1/4}{S+5} = (S^2+6S+5+\frac{15}{4}S+\frac{75}{4}+\frac{S}{4}+\frac{1}{4})/(S+1)(S+5)$

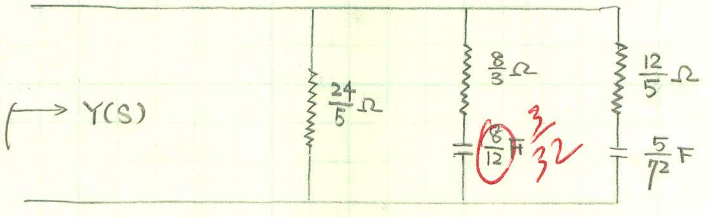
$= (S^2+10S+24)/(S+1)(S+5) = Z(S)$ CHECKED!

FOSTER'S SECOND FORM:

$Y(S) = (S+1)(S+5)/(S+4)(S+6)$

$Y(S)/S = (S+1)(S+5)/S(S+4)(S+6) = \frac{5/24}{S} + \frac{3/8}{S+4} + \frac{5/12}{S+6}$

$Y(S) = 5/24 + \frac{1}{\frac{8}{3} + \frac{1}{8/12 \cdot S}} + \frac{1}{\frac{12}{5} + (\frac{5}{72}S)^{-1}}$



−3

CHECK: $5/24 + 3S/8(S+4) + 5S/12(S+6)$

$= [5(S+4)(S+6)+9S(S+6)+10S(S+4)]/24(S+4)(S+6)$

$= (S^2+6S+5)/(S+4)(S+6) = Y(S)$ CHECKED!!

CAUER'S FIRST FORM:

$$Y(S) = (S^2+6S+5)/(S^2+10S+24)$$

$$Z(S) = (S^2+10S+24)/(S^2+6S+5)$$

$$\begin{array}{r} 1 \\ S^2+6S+5 \,\big)\, S^2+10S+24 \\ S^2+6S+5 \\ \hline 4S+19 \end{array}$$

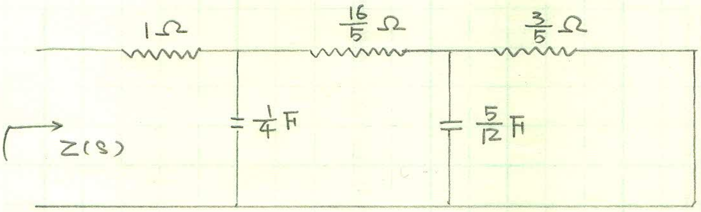
$$\begin{array}{r} \frac{1}{4}S \\ \big)\, S^2+6S+5 \\ S^2+19/4\,S \\ \hline 5/4\,S+5 \end{array}$$

$$\begin{array}{r} \frac{16}{5} \\ \big)\, 4S+19 \\ 4S+16 \\ \hline 3 \end{array}$$

$$\begin{array}{r} \frac{5}{12}S \\ \big)\, \frac{5}{4}S+5 \\ \frac{5}{4}S \\ \hline 5 \end{array}$$

$$\begin{array}{r} \frac{3}{5} \\ \big)\, 3 \\ 3 \\ \hline 0 \end{array}$$

✓



CHECK: IT'S MORE DIFFICALT TO CHECK THE RESULT THAN TO RE-DO IT.

IN ORDER TO CHECK IN EASY WAY, WE LET $s = j\omega$. CHOOSE

$\omega = 1$ rad/s, $Z_{GIVEN}(j1) = 2.923 - j1.885\ \Omega$

$Z_{realiz.} = 2.92308 - j1.88462\ \Omega$.

✓

Now, I DARE TO say 95% CHECKED!

Cauer's second Form: $Y(S) = (S^2+6S+5)/(S^2+10S+24)$

$$Z(S) = (S^2+10S+24)/(S^2+6S+5)$$

$$\begin{array}{r} \frac{5}{24} \\ 24+10S+S^2 \,\big)\, 5+6S+S^2 \\ 5+\frac{25}{12}S+\frac{5}{24}S^2 \\ \hline \frac{47}{12}S+\frac{19}{24}S^2 \end{array}$$

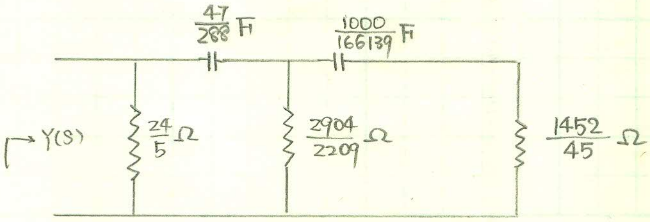
$$\begin{array}{r} \frac{288}{47S} \\ \big)\, 24+10S+S^2 \\ 24+\frac{228}{47}S \\ \hline \frac{242}{47}+S \end{array}$$

$$\begin{array}{r} 2209/2904 \\ \big)\, \frac{47}{12}+\frac{19}{24}S \\ \frac{47}{12}+2209/2904\,S \\ \hline \frac{45}{1452}S \end{array}$$

$$\begin{array}{r} 166.139/S \\ \big)\, \frac{242}{47}+S \\ \frac{242}{47} \\ \hline 1 \end{array}$$

$$\begin{array}{r} \frac{45}{1452} \\ \big)\, \frac{45}{1452} \\ \frac{45}{1452} \\ \hline 0 \end{array}$$

Check -2



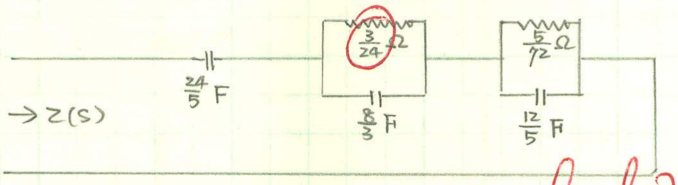
✓

d) $Y(S) = S(S+4)(S+6)/(S+1)(S+5)$

FOSTER'S FIRST FORM :

$$Z(S) = (S+1)(S+5)/S(S+4)(S+6)$$

$$= \frac{5/24}{S} + \frac{3/8}{S+4} + \frac{5/12}{S+6} = \frac{1}{\frac{24}{5}S} + \frac{1}{\frac{8}{3}S+\frac{24}{3}} + \frac{1}{\frac{12}{5}S+\frac{72}{5}}$$



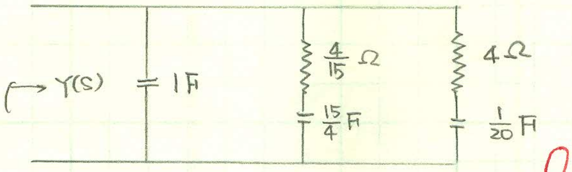
check?

−4

FOSTER'S SECOND FORM :

$$Y(S)/S = (S+4)(S+6)/(S+1)(S+5) = 1 + \frac{15/4}{S+1} + \frac{1/4}{S+5}$$

$$Y(S) = S + \frac{1}{\frac{4}{15} + \frac{1}{\frac{15}{4}S}} + \frac{1}{4 + \frac{1}{\frac{1}{20}S}}$$



✓

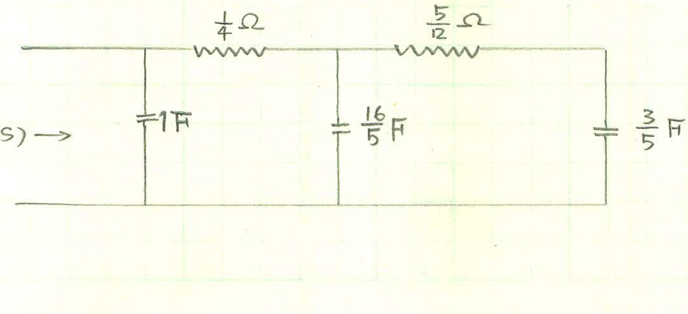
check?

−2

CAUER'S FIRST FORM :

$$Y(S) = (S^3+10S^2+24S)/(S^2+6S+5)$$

$$S^2+6S+5 \,\overline{)\, S^3+10S^2+24S} \quad S$$
$$S^3+6S^2+5S$$
$$4S^2+19S \,\overline{)\, S^2+6S+5} \quad \frac{1}{4}$$
$$S^2+\frac{19}{4}S$$
$$\frac{5}{4}S+5 \,\overline{)\, 4S^2+19S} \quad \frac{16}{5}S$$
$$4S^2+16S$$
$$3S \,\overline{)\, \frac{5}{4}S+5} \quad \frac{5}{12}$$
$$\frac{5}{4}S$$
$$5 \,\overline{)\, 3S} \quad \frac{3}{5}S$$
$$3S$$
$$0$$



check?

−2

6-8 d) (CONT.)

Cauer's second form : $Z(S) = (S^2+6S+5)/(S^3+10S^2+24S)$

$\frac{5}{24S}$

$24S+10S^2+S^3 \overline{)\;5+6S+S^2}$

$5+\frac{25}{12}S+\frac{5}{24}S^2$

$\frac{47}{12}+\frac{19}{24}S$ $\quad \frac{288}{47}$

$\overline{)\;24+10S+S^2}$

$24+\frac{288}{47}S$

$\frac{242}{47}S+S^2$ $\quad 2209/2904S$

$\overline{)\;\frac{47}{12}+\frac{19}{24}S}$

$\frac{47}{12}+2209/2904S$

$\frac{45}{1452}$ $\quad 166139/1000$

$\overline{)\;\frac{242}{47}+S}$

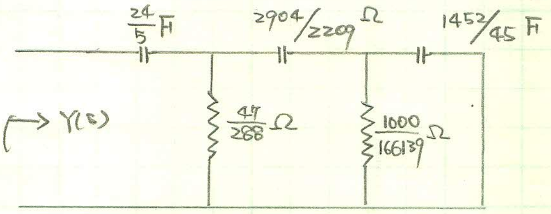
$\frac{242}{47}$

S $\quad \frac{45}{1452S}$

$\overline{)\;\frac{45}{1452}}$

$\frac{45}{1452}$

0



Check -2

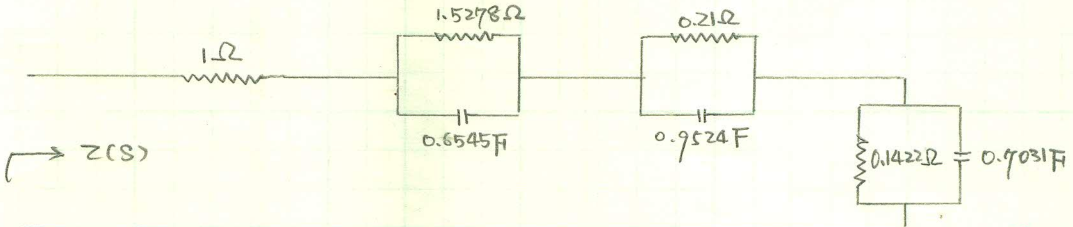
6-8 e) $Y(S) = (S+1)(S+5)(S+10)/(S+2)(S+6)(S+12)$

$Z(S) = (S+2)(S+6)(S+12)/(S+1)(S+5)(S+10)$

FOSTER'S FIRST FORM :

$$Z(S) = 1 + \frac{1.5278}{S+1} + \frac{1.05}{S+5} + \frac{1.4222}{S+10}$$

$$= 1 + \frac{1}{0.6545S+\frac{1}{1.5278}} + \frac{1}{0.9524S+\frac{1}{0.21}} + \frac{1}{0.7031S+\frac{1}{0.1422}}$$



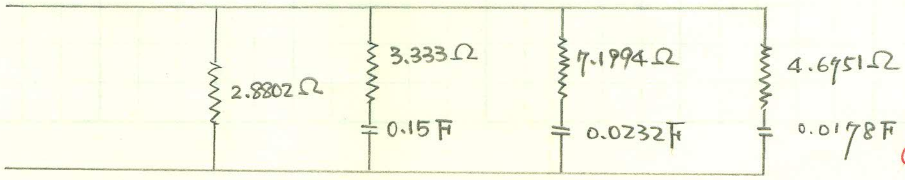
Check -2

FOSTER'S SECOND FORM:

$$Y(S)/S = (S+1)(S+5)(S+10)/S(S+2)(S+6)(S+12)$$

$$= \frac{0.3472}{S} + \frac{0.3}{S+2} + \frac{0.1389}{S+6} + \frac{0.2139}{S+12}$$

$$= \frac{1}{2.8802S} + \frac{1}{3.333S+\frac{1}{0.15}} + \frac{1}{7.1994S+\frac{1}{0.0232}} + \frac{1}{4.6751S+\frac{1}{0.0178}}$$



Check -2

6-8 e) (CONT.) Cauer's first FORM:

$$Y(S) = (S+1)(S+5)(S+10)/(S+2)(S+6)(S+12)$$

$$= (S^3+16S^2+65S+50)/(S^3+20S^2+108S+142)$$

$$Z(S) = (S^3+20S^2+108S+142)/(S^3+16S^2+65S+50)$$

$$1$$
$$S^3+16S^2+65S+50 \,\big)\, S^3+20S^2+108S+142$$
$$S^3+16S^2+65S+50$$
$$4S^2+43S+92$$

$$\tfrac{1}{4}S$$
$$4S^2+43S+92 \,\big)\, S^3+16S^2+65S+50$$
$$S^3+\tfrac{43}{4}S^2+23S$$
$$\tfrac{21}{4}S^2+42S+50$$

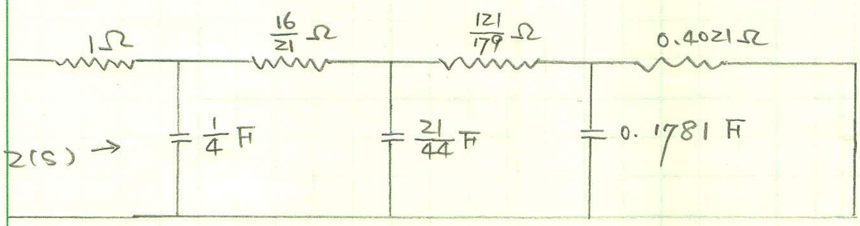
$$\tfrac{16}{21}$$
$$\tfrac{21}{4}S^2+42S+50 \,\big)\, 4S^2+43S+92$$
$$4S^2+32S+\tfrac{800}{21}$$
$$11S+\tfrac{1132}{21}$$

$$\tfrac{21}{44}S$$
$$11S+\tfrac{1132}{21} \,\big)\, \tfrac{21}{4}S^2+42S+50$$
$$\tfrac{21}{4}S^2+\tfrac{283}{11}S$$
$$\tfrac{179}{11}S+50$$

$$\tfrac{121}{179}$$
$$\tfrac{179}{11}S+50 \,\big)\, 11S+\tfrac{1132}{21}$$
$$11S+33.799$$
$$20.106$$

$$0.1781S$$
$$20.106 \,\big)\, 3.58S+50$$
$$3.58S$$
$$50$$

$$0.4021$$
$$50 \,\big)\, 20.106$$
$$20.106$$
$$0$$



check -2

Cauer's second FORM:

$$0.3521$$
$$142+108S+20S^2+S^3 \,\big)\, 50+65S+16S^2+S^3$$
$$50+38.03S+7.04S^2+0.3521S^3$$
$$26.97S+8.96S^2+0.64796^3$$

$$5.265/S$$
$$\big)\, 142+108S+20S^2+S^3$$
$$142+47.18S+3.41S^2$$
$$60.82+16.59S+S^2$$

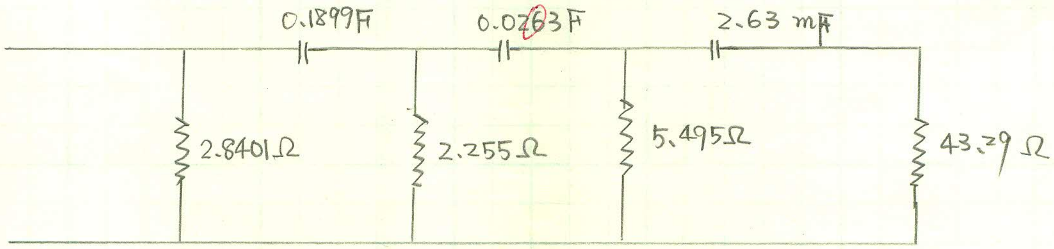
$$0.4434$$
$$\big)\, 26.97+8.96S+0.648S^2$$
$$26.97+7.36S+0.4434S^2$$
$$1.6S+0.205S^2$$

$$38.01/S$$
$$\big)\, 60.82+16.59S+S^2$$
$$60.82+7.79S$$
$$8.8+S$$

$$0.182$$
$$\big)\, 1.6+0.205S$$
$$1.6+0.182S$$
$$0.0231S$$

$$380.48/S$$
$$\big)\, 8.8+S$$
$$8.8$$
$$1$$

$$0.0231$$
$$1 \,\big)\, 0.0231$$
$$0.0231$$
$$0$$



check -2

6-10 (b) $Z(S) = (S+2)(S+4)/(S+1)(S+3)(S+5)$

$$= (S^2+6S+8)/(S^3+9S^2+23S+15)$$

Cauer's second form = (FOR TWO CAPACITORS)

$$\frac{15}{8}$$

$$8+6S+S^2 \overline{\big) 15+23S+9S^2+S^3}$$

$$15+\frac{45}{4}S+\frac{15}{8}S^2$$

$$\frac{32}{47S}$$

$$\frac{47}{4}S+\frac{57}{8}S^2+S^3 \overline{\big) 8+6S+S^2}$$

$$8+\frac{228}{47}S+\frac{32}{47}S^2$$

$$\frac{2209}{216}$$

$$\frac{54}{47}+\frac{15}{47}S \overline{\big) \frac{47}{4}+\frac{57}{8}S+S^2}$$

$$\frac{47}{4}+\frac{705}{216}S$$

$$0.2976/S$$

$$\frac{834}{216}S+S^2 \overline{\big) \frac{54}{47}+\frac{15}{47}S}$$

$$\frac{54}{47}+0.29768$$

$$178.76$$

$$0.0216 \overline{\big) 3.86+S}$$

$$3.86$$

$$0.0216/S$$

$$S \overline{\big) 0.0216}$$

$$0.0216$$

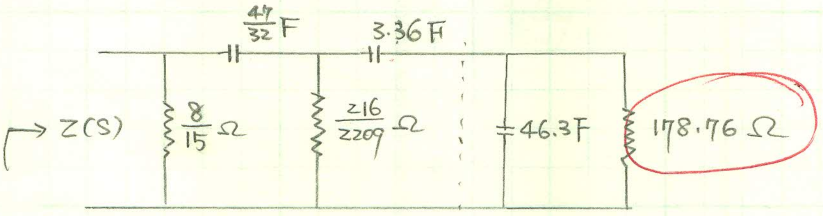
$$0$$

$$Y_1(S) = \frac{1}{178.76} + \frac{S}{0.0216}$$

$$= \frac{0.0216+178.76S}{3.86} = 46.3S + 0.0056$$

Cauer's First Form : (completion)

$$Y_1(S) = 46.3S + 0.0056 = 46.3S + \frac{1}{178.76} \qquad Z_1(S) = \frac{1}{46.3S+\frac{1}{178.76}}$$



-3

check

(c) 1. Cauer's FIRST FORM FOR one capasitor:

$$Y(S) = (S^3+9S^2+23S+15)/(S^2+6S+8) = (S^3+6S^2+8S+3S^2+15S+15)/(S^2+6S+8)$$

$$= S + (3S^2+15S+15)/(S^2+6S+8)$$

2. Cauer's second FORM FOR one CAPASITOR:

$$15/8$$

$$8+6S+S^2 \overline{\big) 15+15S+3S^2}$$

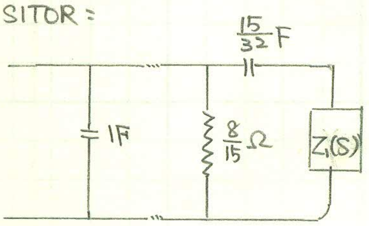
$$15+\frac{45}{4}S+\frac{15}{8}S^2$$

$$\frac{32}{15S}$$

$$\frac{15}{4}S+\frac{9}{8}S^2 \overline{\big) 8+6S+S^2}$$

$$8+\frac{12}{5}S$$

$$\frac{18}{5}+S$$

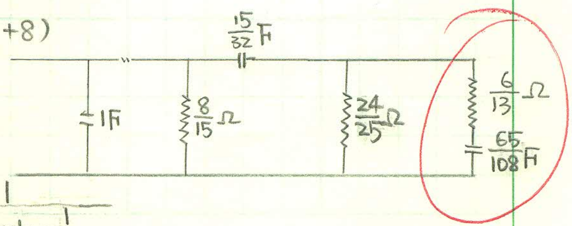


-3

check

$$\frac{15}{8} + \frac{1}{Z_1(S)+\frac{32}{15S}} = (3S^2+15S+15)/(S^2+6S+8)$$

$$\therefore Z_1(S) = \frac{40S+144}{45S+150}$$



FOSTER'S SECOND FORM FOR COMPLETION = $Y_1(S) = \frac{25}{24} + \frac{1}{\frac{6}{13}+\frac{1}{\frac{65}{108}S}}$

6-12 d) $Z(S) = (S+2)(S+4)/S(S+3) + (S^2+3)(S^2+6)/S(S^2+4)$

$$Z_1(S) = (S+2)(S+4)/S(S+3)$$

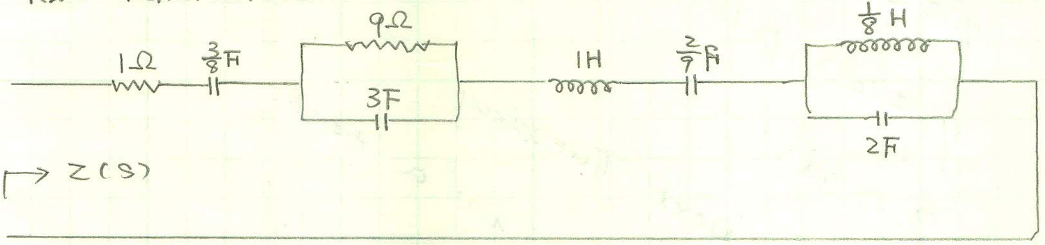
$$= 1 + \frac{\frac{8}{3}}{S} + \frac{\frac{1}{3}}{S+3} = 1 + \frac{1}{\frac{3}{8}S} + \frac{1}{3S+\frac{1}{9}}$$

$$Z_2(S) = (S^2+3)(S^2+6)/S(S^2+4)$$

$$\frac{Z_2(S)}{S}\Big|_{S^2=P} = (P+3)(P+6)/P(P+4) = 1 + \frac{\frac{9}{2}}{P} + \frac{\frac{1}{2}}{P+4}$$

$$Z_2(S) = S + \frac{1}{\frac{2}{9}S} + \frac{1}{2S+\frac{1}{\frac{1}{8}S}}$$

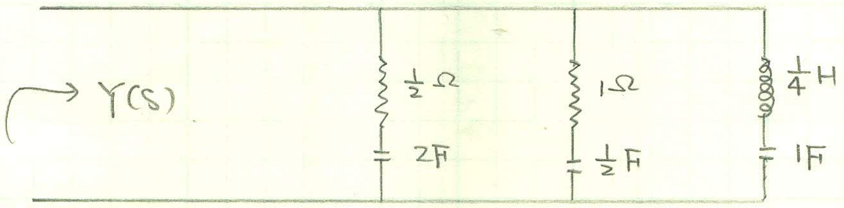
RLC REALIZATION :



6-13 f) $Y(S) = (3S^4+9S^3+24S^2+28S)/(S+1)(S+2)(S^2+4)$

$$\frac{Y(S)}{S} = \frac{2}{S+1} + \frac{1}{S+2} + \frac{4}{S^2+4}$$

$$Y(S) = \frac{1}{\frac{1}{2}+\frac{1}{2S}} + \frac{1}{1+\frac{1}{\frac{1}{2}S}} + \frac{1}{\frac{1}{4}S+\frac{1}{S}}$$



CHECK : $\frac{1}{4}S + \frac{1}{S} = \frac{S^2+4}{4S} \Rightarrow \frac{4S}{S^2+4}$

$$1 + \frac{2}{S} = \frac{S+2}{S} \Rightarrow \frac{S}{S+2}$$

$$\frac{1}{2} + \frac{1}{2S} = \frac{S+1}{2S} \Rightarrow \frac{2S}{S+1}$$

$$Y(S) = \frac{2S}{S+1} + \frac{S}{S+2} + \frac{4S}{S^2+4}$$

$$= [2S(S+2)(S^2+4) + S(S+1)(S^2+4) + 4S(S+1)(S+2)]/(S+1)(S+2)(S^2+4)$$

$$= (3S^4+9S^3+24S^2+28S)/(S+1)(S+2)(S^2+4)$$ CHECKED!

4 18

6.13R

EE 448 FILTER | HOMEWORK #6 | Benmei Chen

30/30

PROBLEM 1. FROM $\xi_i = a^2\xi_{11} + 2ab\,\xi_{12} + b^2\cdot\xi_{22}^2 \geqslant 0 \Rightarrow \xi_{11}\cdot\xi_{22} - \xi_{12}^2 \geqslant 0$

SOLUTION: $\xi_i = a^2\xi_{11} + 2ab\,\xi_{12} + b\,\xi_{22}^2 \geqslant 0$ ... (19)

IF $ab > 0$, WE GET

$$\frac{1}{2}\left[\frac{a}{b}\xi_{11} + \frac{b}{a}\xi_{22}\right] + \xi_{12} \geqslant 0 \quad \cdots \quad (a)$$

IF $ab < 0$, THEN $\frac{a}{b} < 0$ and $\frac{b}{a} < 0$

LET $x = -\frac{a}{b} > 0$, WE GET

$$\frac{1}{2}\left[\frac{a}{b}\xi_{11} + \frac{b}{a}\xi_{22}\right] + \xi_{12} \leqslant 0$$

$$\Rightarrow -\frac{1}{2}\left[x\,\xi_{11} + \frac{1}{x}\xi_{22}\right] + \xi_{12} \leqslant 0$$

$$\frac{1}{2}\left[x\,\xi_{11} + \frac{1}{x}\xi_{22}\right] - \xi_{12} \geqslant 0 \quad \cdots \quad (b)$$

FROM (a) AND (b), WE OBTAIN

$$\frac{1}{2}\left[\frac{\alpha}{b}\xi_{11} + \frac{b}{\alpha}\xi_{22}\right] \pm \xi_{12} \geqslant 0 \quad \cdots \quad (c)$$

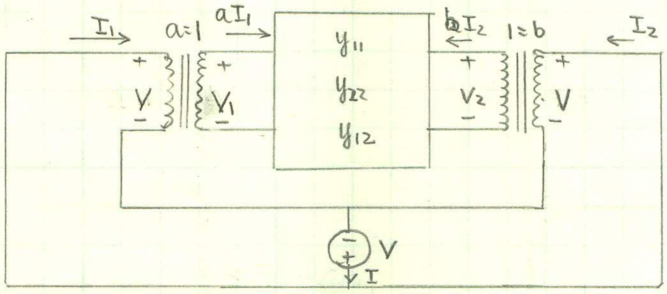
WHERE $\alpha = \begin{cases} a & \text{IF } ab > 0 \\ -a & \text{IF } ab < 0 \end{cases}$

INEQUALTION (c) is just same as the INEQUALTION (21) on PAGE 261 IN notes. HENCE, WITH THE same MOTHED IN THE NOTES, WE HAVE

$$\xi_{11}\cdot\xi_{22} - \xi_{12}^2 \geqslant 0.$$

✓

PROBLEM 2: Develop residue and real part conditions for the y-parameters of a 2-port RLC network.



$$\begin{cases} aI_1 = y_{11}V_1 + y_{12}V_2 \\ bI_2 = y_{12}V_1 + y_{22}V_2 \end{cases} \quad \text{and} \quad \begin{cases} V_1 = V/a \\ V_2 = V/b \end{cases}$$

$$I = I_1 + I_2 = \frac{1}{a}(y_{11}V_1 + y_{12}V_2) + \frac{1}{b}(y_{12}V_1 + y_{22}V_2)$$

$$= \left(\frac{1}{a^2}y_{11} + \frac{2}{ab}y_{12} + \frac{1}{b^2}y_{22}\right) \cdot V$$

$$\therefore Y_{in}(s) = \frac{1}{a^2}y_{11} + \frac{2}{ab}y_{12} + \frac{1}{b^2}y_{22}$$

Suppose $Y_{in}(s)$ has a pole at $s = P_i$

$$\xi_{11}^{(i)} \triangleq y_{11}(s-P_i)\big|_{s=P_i}$$

$$\xi_{12}^{(i)} \triangleq y_{12}(s-P_i)\big|_{s=P_i}$$

$$\xi_{22}^{(i)} \triangleq y_{22}(s-P_i)\big|_{s=P_i}$$

$$\therefore \xi^{(i)} = \frac{1}{a^2}\xi_{11}^{(i)} + \frac{2}{ab}\xi_{12}^{(i)} + \frac{1}{b^2}\xi_{22}^{(i)}$$

FOR AN RC NETWORK, WE KNOW THAT THE POLE OF Y_{RC} ARE SIMPLE, AND HAVE NEGATIVE RESIDUES AT FINITY POLES, ALSO HAVE A POSITIVE RESIDUE AT INFINITY POLE. SO, $\xi^{(i)} \leq 0$ P_i is FINITY.

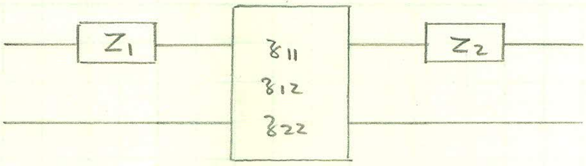
$\xi^{(i)} \geq 0$ P_i is INFINITY.

FOR AN LC NETWORK, THE POLES OF Y_{LC} ARE SIMPLE AND HAVE POSITIVE RESIDUES. SO, $\xi^{(i)} \geq 0$.

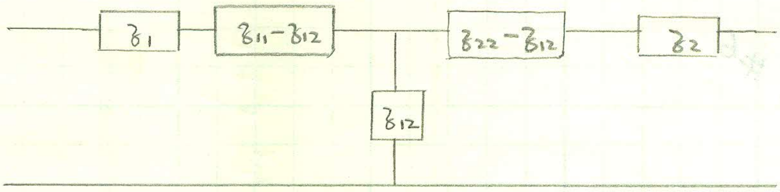
Now, how can I derive equations like (26) and (29) in the notes' book?

PROBLEM 3 : USE T AND π NETWORKS TO VERIFY (30) AND (31)

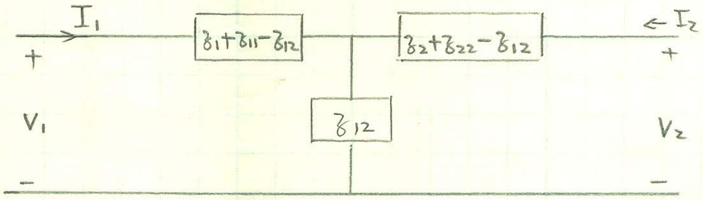
SOLUTION: FOR NETWORK :



EQUALE TO : (T NETWORK)



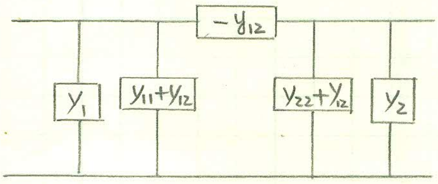
$\Rightarrow$



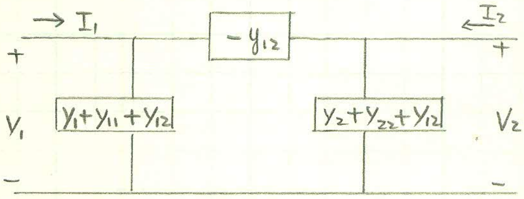
$$\begin{cases} V_1 = I_1(z_1+z_{11}-z_{12}) + (I_1+I_2)\,z_{12} \\ V_2 = I_2(z_2+z_{22}-z_{12}) + (I_1+I_2)\,z_{12} \end{cases}$$

$$\Rightarrow \begin{cases} V_1 = I_1\cdot(z_1+z_{11}) + I_2\cdot z_{12} \\ V_2 = I_1\cdot z_{12} + I_1\,(z_2+z_{22}) \end{cases} \Rightarrow \begin{cases} z_{11}' = z_1 + z_{11} \\ z_{22}' = z_2 + z_{22} \end{cases} \text{and } z_{12}' = z_{12}$$

FOR NETWORK :



$\Rightarrow$



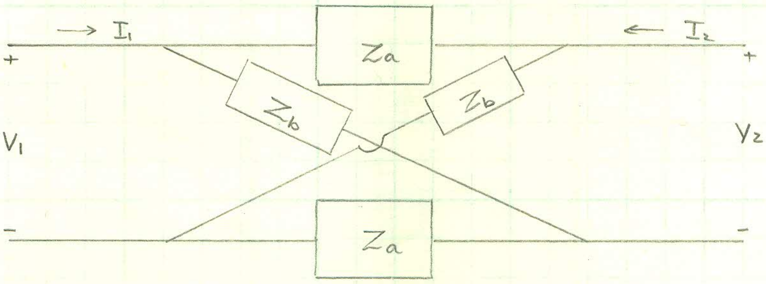
$$\begin{cases} I_1 = V_1(y_1+y_{11}+y_{12}) + (V_1-V_2)\,(-y_{12}) \\ I_2 = V_2(y_2+y_{22}+y_{12}) + (V_2-V_1)\,(-y_{12}) \end{cases}$$

$$\Rightarrow \begin{cases} I_1 = V_1\cdot(y_1+y_{11}) + y_{12}\cdot V_2 \\ I_2 = V_1\,y_{12} + V_2\cdot(y_2+y_{22}) \end{cases} \Rightarrow \begin{cases} y_{11}' = y_1 + y_{11} \\ y_{22}' = y_2 + y_{22} \end{cases} \text{and } y_{12}' = y_{12}$$

HOMEWORK PROBLEM 3.28 ON PAGE 280 IN NOTES :

$\frac{30}{30}$

A LATTICE NETWORK :



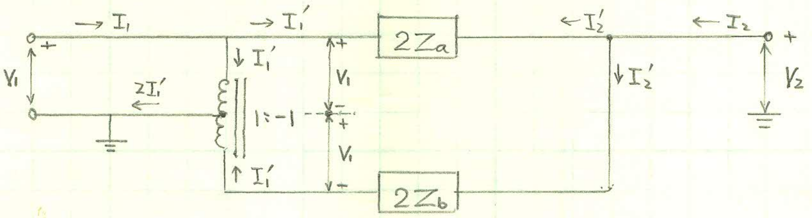
LET $\begin{cases} V_1 = z_{11} I_1 + z_{12} I_2 \\ V_2 = z_{12} I_1 + z_{22} I_2 \end{cases}$

$$z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} = \frac{1}{2}(Z_a + Z_b)$$

$$z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} = \frac{1}{2}(Z_a + Z_b)$$

$$z_{12} = \frac{V_2}{I_1}\Big|_{I_2=0} = \frac{1}{2}(Z_b - Z_a)$$

(a) SHUNT-TYPE Cauer equivalent :



$I_2 = 0$: $2V_1 = I_1' \cdot (2Z_a + 2Z_b) = 2I_1'(Z_a + Z_b) = I_1(Z_a + Z_b)$

$$z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} = \frac{1}{2}(Z_a + Z_b)$$

$$V_2 = I_1' \cdot 2Z_b - V_1 = -I_1' \cdot 2Z_a + V_1 \Rightarrow \quad V_1 = (Z_a + Z_b) I_1'$$

$$\Rightarrow V_2 = I_1' \cdot (Z_b - Z_a)$$

$$z_{12} = \frac{V_2}{I_1}\Big|_{I_2=0} = \frac{V_2}{2I_1'} = \frac{1}{2}(Z_b - Z_a)$$

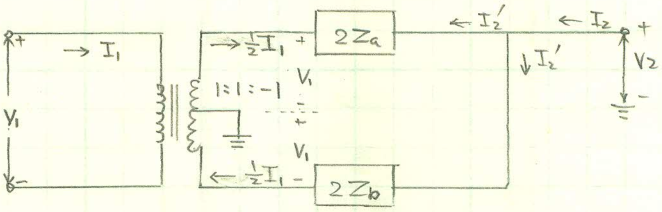
$I_1 = 0$: $V_2 = 2Z_a \cdot I_2' + V_1 = 2Z_b \cdot I_2' - V_1 \Rightarrow V_1 = (Z_b - Z_a) I_2'$

$$V_2 = (Z_a + Z_b) I_2'$$

$$z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} = \frac{1}{2}(Z_a + Z_b)$$

CHECKED !

(b) Hybrid lattice equivalent :



$I_2=0$: $2V_1 = \frac{1}{2} I_1 \cdot (2Z_a + 2Z_b) = (Z_a+Z_b)\cdot I_1$

$z_{11} = \frac{V_1}{I_1} = \frac{1}{2}(Z_a+Z_b)$

$V_2 = -\frac{1}{2} I_1 \cdot 2Z_a + V_1 = \frac{1}{2} I_1 \cdot 2Z_b - V_1 \Rightarrow \frac{1}{2}(Z_a+Z_b)\cdot I_1$

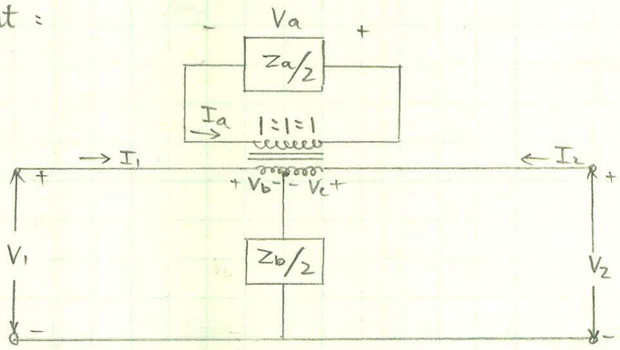
$\Rightarrow V_2 = \frac{1}{2}(Z_b - Z_a)\cdot I_1$

$z_{12} = \frac{V_2}{V_1} = \frac{1}{2}(Z_b - Z_a)$

$I_1=0$: same as z_{22} in part (a)

$z_{22} = \frac{1}{2}(Z_a+Z_b)$ CHECKED!

(c) Bridged-T equivalent :



$I_2=0$: $I_a = I_1$, $V_b = V_a = I_a \cdot Z_a/2 = I_1 \cdot \frac{Z_a}{2}$

$V_1 = V_b + I_1 \cdot Z_b/2 = \frac{1}{2}(Z_a+Z_b)\cdot I_1$

$z_{11} = \frac{V_1}{I_1} = \frac{1}{2}(Z_a+Z_b)$

$V_c = -V_b = -\frac{Z_a}{2}\cdot I_1$

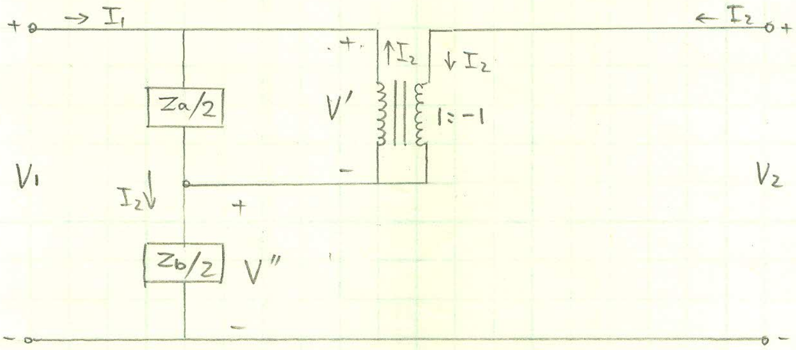
$V_2 = -\frac{Z_a}{2} I_1 + I_1 \cdot Z_b/2 = \frac{1}{2}(Z_b-Z_a)\cdot I_1$

$I_1=0$: $I_a = -I_2$ $V_c = -V_a = \frac{1}{2}\cdot Z_a \cdot I_2$

$V_2 = \frac{1}{2}(Z_a+Z_b) I_2$

$z_{22} = \frac{1}{2}(Z_a+Z_b)$ CHECKED !

(d) Series-type Cauer equivalent



Let $I_2 = 0$, $V_1 = (\frac{1}{2} Z_a + \frac{1}{2} \cdot Z_b) I_1$

$$z_{11} = \frac{V_1}{I_1} = \frac{1}{2}(Z_a + Z_b)$$

$$V_2 = -V' + V'' = \frac{1}{2}(Z_b - Z_a) \cdot I_1$$

$$z_{12} = \frac{V_2}{I_1} = \frac{1}{2}(Z_b - Z_a)$$

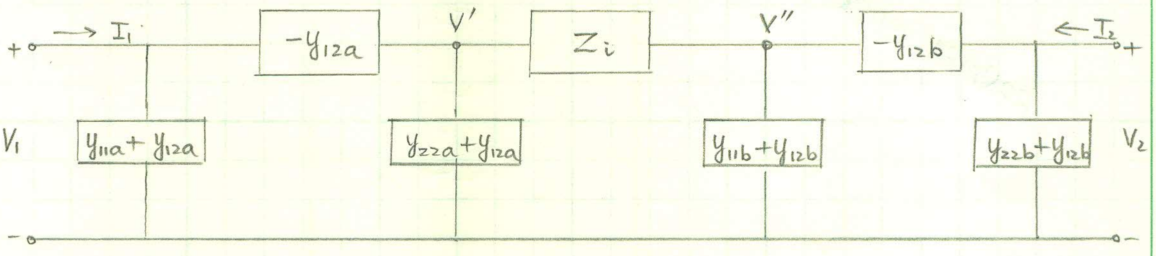
Let $I_1 = 0$, $V_2 = (Z_a/2 + Z_b/2) \cdot I_2$

$z_{22} = \frac{1}{2}(Z_a + Z_b)$. SHOWN !

EE 448 . H.W. #7 FILTER Bonmei Chen

FOR HOMEWORK

Repeat the analysis on pages 281–283 by considering a shunt element of a ladder between the ends of the ladder represented by π networks and y – parameters. Show that zero impedance of the shunt element may not necessarily imply zero transmission.



$$\frac{V'}{V_1} = H_a(S) = -\frac{y_{12a}}{y_{22a}}$$

$$\frac{V_2}{V''} = H_b(S) = -\frac{y_{12b}}{y_{22b}}$$

$$\frac{V''}{V'} = H'(S) = \frac{1}{(y_{11b}+y_{12b})Z_i+1}$$

$$\therefore H(S) = \frac{V_2}{V_1} = \frac{V_2}{V''}\cdot\frac{V''}{V'}\cdot\frac{V'}{V_1} = H_b(S)\cdot H'(S)\cdot H_a(S)$$

$$= \frac{y_{12a}\,y_{12b}}{y_{22a}\,y_{22b}\left[(y_{11b}+y_{12b})Z_i+1\right]}$$

Suppose Z_i has a zero at $S = S_0$.

$$H(S_0) = y_{12a}(S_0)\cdot y_{12b}(S_0) \,/\, (y_{22a}(S_0)\cdot y_{22b}(S_0))$$

is not necessarry to be zero.

So, zero impedance of the shunt element may not necessarily imply zero transmission.

$\frac{120}{120}$

ANALOG AND DIGITAL FILTER

HOMEWORK #8

NOV. 3, 1986

BENMEI CHEN

CHAPTER SEVEN: PROBLEM 7.3 (c) (e) (h)

7.6(e) (g)

7-3 SYNTHESIZE EACH OF THE FOLLOWING TRANSFER FUNCTIONS BY TWO RC LADDER NETWORKS [VIA $Z_{11}(S)$ AND $y_{22}(S)$]

(c) $H(S) = \dfrac{k}{(S+1)(S+4)}$

1. $Z_{11}(S) = \dfrac{(S+1)(S+4)}{(S+0.5)(S+2)} = \dfrac{S^2+5S+4}{S^2+2.5S+1}$

$Z_{21}(S) = \dfrac{k}{(S+0.5)(S+2)}$

$$S^2+2.5S+1 \,\overline{\big)\, S^2+5S+4}\quad 1$$

$$\underline{S^2+2.5S+1}$$

$$2.5S+3 \,\overline{\big)\, S^2+2.5S+1}\quad \tfrac{2}{5}S$$

$$\underline{S^2+\tfrac{6}{5}S}$$

$$1.3S+1 \,\overline{\big)\, 2.5S+3}\quad \tfrac{25}{13}$$

$$\underline{2.5S+\tfrac{25}{13}}$$

$$\tfrac{14}{13} \,\overline{\big)\, 1.3S+1}\quad \tfrac{169}{140}S$$

$$\underline{1.3S}$$

$$1 \,\overline{\big)\, 14/13}\quad 14/13$$

$$\underline{14/13}$$

$$0$$

2. $y_{22}(S) = \dfrac{(S+1)(S+4)}{S+2} = \dfrac{S^2+5S+4}{S+2}$; $\quad y_{12}(S) = -\dfrac{k}{S+2}$

$$S+2 \,\overline{\big)\, S^2+5S+4}\quad S$$

$$\underline{S+2S}$$

$$3S+4 \,\overline{\big)\, S+2}\quad \tfrac{1}{3}$$

$$\underline{S+\tfrac{4}{3}}$$

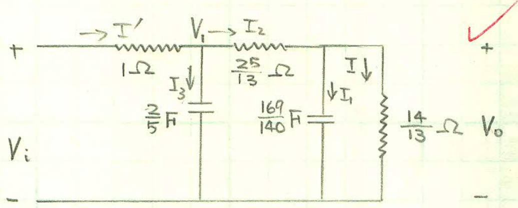
$$\tfrac{2}{3} \,\overline{\big)\, 3S+4}\quad \tfrac{9}{2}S$$

$$\underline{3S}$$

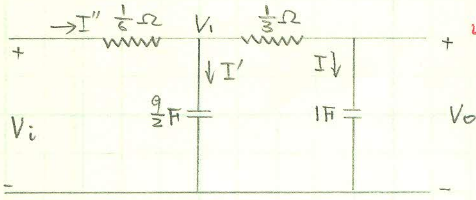
$$4 \,\overline{\big)\, \tfrac{2}{3}}\quad \tfrac{1}{6}$$

$$\underline{2/3}$$

$$0$$



(VIA $Z_{11}(S)$)



(VIA $y_{22}(S)$)

CHECK: LET $I = 1A$ $\quad V_o = \frac{14}{13}$ V

$I_1 = \frac{14}{13} / \frac{140}{169S} = \frac{13S}{10}$ $\Rightarrow I_2 = 1 + 1.3S \Rightarrow$

$V_1 = 3 + 2.5S \Rightarrow I_3 = 1.2S + S^2$

$I' = S^2 + 2.5S + 1 \Rightarrow V_i = S^2 + 5S + 4$

$H(S) = V_o/V_i = \frac{14}{13} / (S+1)(S+4)$ CHECKED!

CHECK: LET $I = 1$ A $\Rightarrow V_o = 1/S$

$\Rightarrow V_1 = \frac{1}{3} + \frac{1}{S} \Rightarrow I' = \frac{3}{2}S + \frac{9}{2}$

$\Rightarrow I'' = \frac{3}{2}S + \frac{11}{2} \Rightarrow V_i = \frac{1}{4}S + \frac{11}{12} + \frac{1}{3} + \frac{1}{S} = \frac{1}{4}S + \frac{5}{4} + \frac{1}{S}$

$H(S) = \frac{1}{S} / (\frac{1}{4}S + \frac{5}{4} + \frac{1}{S})$

$= \frac{1}{4} / (S^2 + 5S + 4) = \frac{1}{4} / (S+1)(S+4)$ CHECKED!

7.3 (e) $H(S) = kS^2/[(S+1)(S+4)]$

1. $Z_{11}(S) = \frac{(S+1)(S+4)}{(S+0.5)(S+2)} = \frac{S^2+5S+4}{S^2+2.5S+1}$; $Z_{12}(S) = \frac{kS^2}{(S+0.5)(S+2)}$

$4+5S+S^2 \overline{)\,1+2.5S+S^2}$ $\quad \frac{1}{4}$

$1+1.25S+\frac{1}{4}S^2$

$1.25S+\frac{3}{4}S^2 \overline{)\,4+5S+S^2}$ $\quad \frac{16}{5S}$

$4+\frac{12}{5}S$

$\frac{13}{5}+S \overline{)\,1.25+\frac{3}{4}S}$ $\quad \frac{25}{52}$

$1.25+25/52\,S$

$\frac{7}{26}S \overline{)\,\frac{13}{5}+S}$ $\quad \frac{338}{35S}$

$\frac{13}{5}$

$1 \overline{)\,\frac{7}{26}}$ $\quad \frac{7}{26}$

$\frac{7}{26}$

0

2. $Y_{22}(S) = \frac{(S+1)(S+4)}{S+2} = \frac{S^2+5S+4}{S+2}$; $Y_{12}(S) = -\frac{kS^2}{S+2}$

$2+S \overline{)\,4+5S+S^2}$ $\quad 2$

$4+2S$

$3S+S^2 \overline{)\,2+S}$ $\quad \frac{2}{3S}$

$2+\frac{2}{3}S$

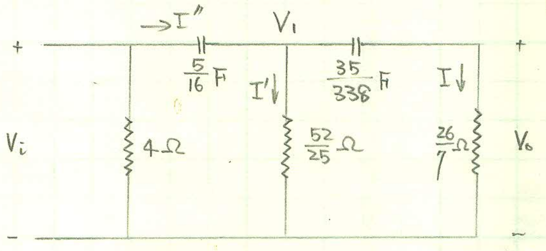
$\frac{1}{3} \overline{)\,3+S}$ $\quad 9$

3

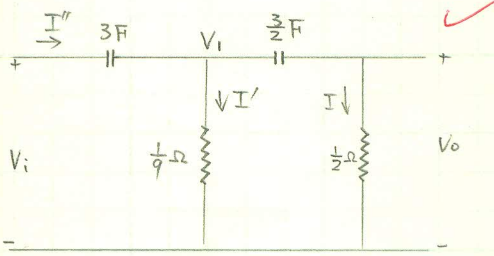
$S \overline{)\,\frac{1}{3}}$ $\quad \frac{1}{3S}$

$\frac{1}{3}$

0



(VIA $Z_{11}(S)$)



(VIA $Y_{22}(S)$)

CHECK: LET $I = 1\,A$, $V_o = \frac{26}{7}$ $\Rightarrow$

$V_1 = 3.7143 + 9.6571/S \Rightarrow I' = 1.7857 + 4.6429/S$

$\Rightarrow I'' = 2.7857 + 4.6429/S$

$\Rightarrow V_i = (2.7857 + 4.6429/S) \times 3.2/S + 3.7143 + 9.6571/S$

$= 14.8571/S^2 + 18.5713/S + 3.7143$

$= (S^2+5S+4)/0.2692S^2$

$H(S) = V_o/V_i = S^2/(S^2+5S+4)$ CHECKED!

CHECK: $I = 1A \Rightarrow V_o = \frac{1}{2}\,V$

$\Rightarrow V_1 = \frac{1}{2} + \frac{2}{3S} \Rightarrow I' = \frac{9}{2} + \frac{6}{S}$

$\Rightarrow I'' = \frac{11}{2} + \frac{6}{S} \Rightarrow$

$V_i = (\frac{11}{2} + \frac{6}{S}) \cdot \frac{1}{3S} + \frac{1}{2} + \frac{2}{3S}$

$= \frac{2}{S^2} + \frac{5}{2S} + \frac{1}{2}$

$= \frac{S^2+5S+4}{2S^2}$

$H(S) = S^2/(S^2+5S+4)$ CHECKED!

7.3 (h) $H(S) = kS^2 / [(S+1)(S+2)(S+3)]$

1. $$Z_{11}(S) = \frac{(S+1)(S+2)(S+3)}{(S+0.5)(S+1.5)(S+2.5)} = \frac{S^3+6S^2+11S+6}{S^3+4.5S^2+5.75S+1.875}$$

USING CAUER'S FIRST FORM FOR ONE CAPACITOR.

$$Z_{11}(S) = 1 + \frac{1.5S^2+5.25S+4.125}{S^3+4.5S^2+5.75S+1.875}$$

$$= 1 + \cfrac{1}{\frac{2}{3}S + \cfrac{S^2+3S+1.875}{1.5S^2+5.25S+4.125}}$$

$$y'(S) = \frac{S^2+3S+1.875}{1.5S^2+5.25S+4.125}$$

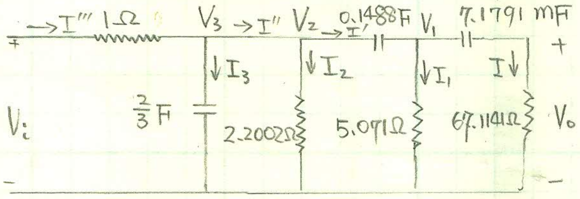
$$= 0.4545 + \cfrac{1}{6.7226/S + \cfrac{1}{0.1972 + \cfrac{1}{139.2932/S + \cfrac{1}{0.0149}}}}$$

2. $$y_{22}(S) = \frac{(S+1)(S+2)(S+3)}{(S+1.5)(S+2.5)} = \frac{S^3+6S^2+11S+6}{S^2+4S+3.75}$$

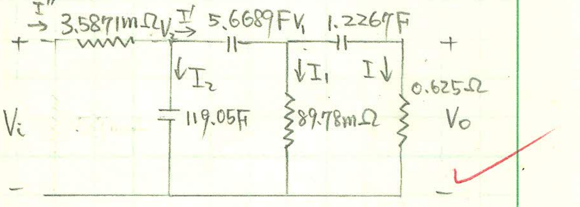
USING CAUER'S SECOND FORM FOR TWO CAPACITORS

$$y_{22}(S) = 1.6 + \cfrac{1}{0.8152/S + \cfrac{1}{11.138 + \cfrac{1}{0.1764/S + Z'(S)}}}$$

$$Z'(S) = \cfrac{1}{278.7738 + \cfrac{1}{0.0084/S}} = \frac{1}{278.7738 + 119.0476S}$$



(VIA $Z_{11}(S)$)



(VIA $y_{22}(S)$).

CHECK: LET $I = 1A$, $V_o = 67.1141\,V \Rightarrow V_1 = 67.1141 + 139.2932/S$

$\Rightarrow I_1 = 13.2349 + 27.4606/S \Rightarrow I' = 14.2349 + 27.4606/S$

$\Rightarrow V_2 = 67.1141 + 234.9579/S + 184.547/S^2 \Rightarrow I_2 = 30.5036 + 106.789/S + 83.8774/S^2$

$\Rightarrow I'' = 44.7385 + 134.2499/S + 83.8774/S^2$

$\Rightarrow I_3 = 44.7427S + 156.6386 + 123.0313/S$

$\Rightarrow I''' = 44.7427S + 201.3976 + 267.8689/S + 83.8774/S^2$

$\Rightarrow V_i = 44.7427S + 268.5117 + 502.8268/S + 268.4244/S^2$

$\Rightarrow H(S) = 1.5\,S^2 \;/ (S^3 + 6.0012S^2 + 11.2S + 5.9993)$

CHECKED.

CHECK: $I = 1 \Rightarrow V_o = 0.625 \Rightarrow V_1 = 0.625 + 0.8152/S$

$\Rightarrow I_1' = 9.0797/S + 6.9613 \Rightarrow I' = 7.9613 + 9.0797/S$

$\Rightarrow V_2 = 2.2196/S + 1.6017/S^2 + 0.625$

$\Rightarrow I_2 = 264.2434 + 190.6824/S + 74.4063S$

$\Rightarrow I'' = 74.4063S + 272.2047 + 199.7621/S$

$\Rightarrow V_i = 0.2669S + 0.9764 + 0.7165/S$

$+ 2.2196/S + 1.6017/S^2 + 0.625$

$= 0.2669S + 1.6014 + 2.9361/S + 1.6017/S^2$

$\Rightarrow H(S) = 234S^2 / (S^3 + 6S^2 + 11.0007S + 6.0011)$

7.6 (e) $H(S)=\frac{kS^4}{(S^2+1)(S^2+4)}$

1. $Z_{11}(S)=\frac{(S^2+1)(S^2+4)}{S(S^2+2)}=\frac{S^4+5S^2+4}{S^3+2S}$

$$2S+S^3 \overline{\big)\,4+5S^2+S^4}\quad \frac{2}{S}$$

$$\frac{4+2S^2}{3S+S^3}\ \overline{\big)\,2+S^2}\quad \frac{2}{3S}$$

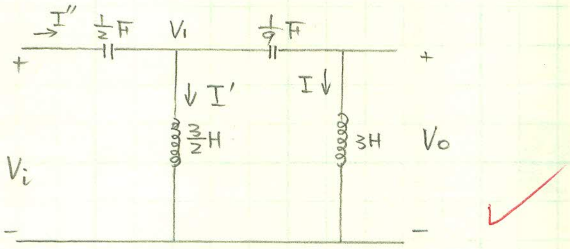
$$\frac{2+\frac{2}{3}S^2}{\frac{1}{3}S}\ \overline{\big)\,3+S^2}\quad \frac{9}{S}$$

$$\frac{3}{S}\ \overline{\big)\,\frac{1}{3}}\quad \frac{1}{3S}$$

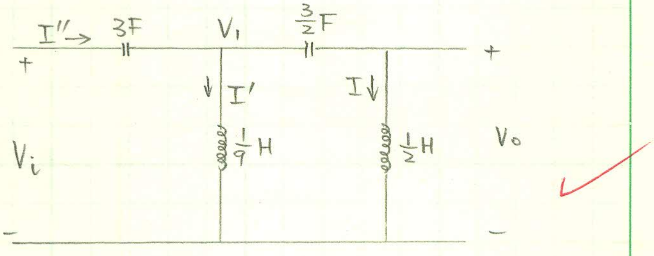
$$\frac{1/3}{0}$$

2. $y_{22}(S)=\frac{(S^2+1)(S^2+4)}{S(S^2+2)}$

$$=\frac{2}{S}+\cfrac{1}{\frac{2}{3S}+\cfrac{1}{\frac{9}{S}+\cfrac{1}{\frac{1}{3S}}}}$$



(VIA $Z_{11}(S)$)



(VIA $Y_{22}(S)$)

CHECK: LET I = 1 A

$V_o = 3S$

$V_1 = 3S+\frac{9}{S}$

$I' = 2+\frac{6}{S^2}$

$I'' = 3+\frac{6}{S^2}$

$V_i = \frac{6}{S}+\frac{12}{S^3}+3S+\frac{9}{S}$

$= (3S^4+15S^2+12)/S^3$

$H(S) = V_o/V_i$

$= 3S^4/(3S^4+15S^2+12)$

$= S^4/(S^2+1)(S^2+4)$ CHECKED!

CHECK: LET I = 1 A

$V_o = \frac{1}{2}S$

$V_1 = \frac{1}{2}S+\frac{2}{3S}$

$I' = \frac{9}{2}+\frac{6}{S^2}$

$I'' = \frac{11}{2}+\frac{6}{S^2}$

$V_i = \frac{11}{6S}+\frac{2}{S^3}+\frac{1}{2}S+\frac{2}{3S}$

$= (S^4+5S^2+4)/2S^3$

$H(S) = V_o/V_i$

$= S^4/(S^4+5S^2+4)$

$= S^4/(S^2+1)(S^2+4)$ CHECKED!

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7.6 (g) $H(S) = kS^2/[(S^2+2)(S^2+5)(S^2+6)]$

1. $Z_{11}(S) = (S^2+2)(S^2+5)(S^2+6)/[S(S^2+4)(S^2+5.5)]$

$$= (S^6 + 13S^4 + 52S^2 + 60)/(S^5 + 9.5S^3 + 22S)$$

$$= S + \cfrac{1}{0.2857S + \cfrac{1}{3.7692S + \cfrac{1}{0.07942S + \cfrac{1}{126.947S + \cfrac{1}{0.001535S}}}}}$$

$$Y'(S) = \cfrac{1}{126.947S + \cfrac{1}{0.001535S}}$$

$$= \cfrac{1}{\cfrac{1}{0.001535S} + \cfrac{1}{\cfrac{1}{126.947S}}}$$

2. $y_{22}(S) = \dfrac{(S^2+2)(S^2+5)(S^2+6)}{S(S^2+4)(S^2+5.5)}$

$$= \frac{S^6 + 13S^4 + 52S^2 + 60}{S^5 + 9.5S^3 + 22S}$$

$$= 2.7273/S + \frac{1}{0.8432/S + Z'(S)}$$

$$Z'(S) = \frac{1}{y_{22}(S) - 2.7273/S} - 0.8432/S$$

$$= \frac{0.1568S^3 + 0.8381S}{S^4 + 10.27727S^2 + 26.0907}$$

$$Y'(S) = \frac{S^4 + 10.27727S^2 + 26.0907}{0.1568S^3 + 0.8381S}$$

$$= 6.37755S + \cfrac{1}{0.03182S + \cfrac{1}{624.817S + \cfrac{1}{0.0003023S}}}$$

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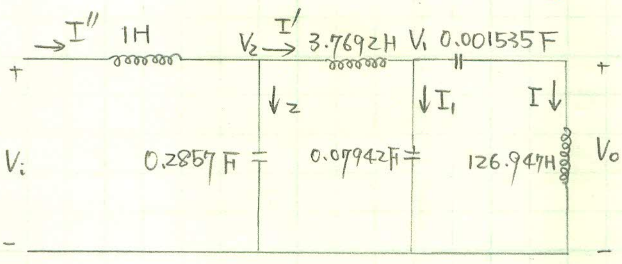

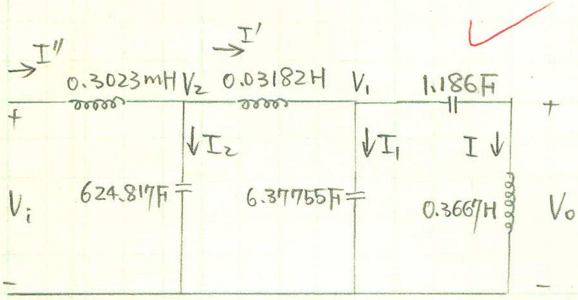

CHECK: $I = 1$ A

$V_0 = 126.947S$

$V_1 = 126.947S + 651.4658/S$

$I_1 = 10.0821S^2 + 51.7394$

$I' = 10.0821S^2 + 52.7394$

$V_2 = 38.0015S^3 + 325.7323S + 651.4658/S$

$I_2 = 10.857S^4 + 93.0617S^2 + 186.1238$

$I'' = 10.857S^4 + 103.1438S^2 + 238.8632$

$V_i = 10.857S^5 + 141.1453S^3 + 564.5955S$

$+ 651.4658/S$

$= (10.857S^6 + 141.1453S^4 + 564.5955S^2$

$+ 651.4658)/S$

$= 10.857(S+2)(S+5)(S+6)/S$

$\therefore H(S) = V_0/V_i = 11.69264S^2/[(S+2)(S+5)(S+6)]$

CHECKED!

CHECK: $I = 1$A

$V_0 = 0.3667S$

$V_1 = 0.3667S + 0.8432/S$

$I_1 = 2.3386S^2 + 5.378$

$I' = 2.3386S^2 + 6.378$

$V_2 = 0.0744S^3 + 0.5696S + 0.8432/S$

$I_2 = 46.4864S^4 + 355.8958S^2 + 526.846$

$I'' = 46.4864S^4 + 358.2344S^2 + 533.224$

$V_i = 0.0141S^5 + 0.1827S^3 + 0.7308S + 0.8432/S$

$= 0.0141(S^5 + 12.9574S^3 + 51.8298S + 59.8014)/S$

$= (S+1.9976)(S+5.17)(S+5.78)/70.92199S$

$H(S) = 26S^2/(S+1.9976)(S+5.17)(S+5.78)$

CHECKED!

Could I let the computer do check later on?

Yes you can.

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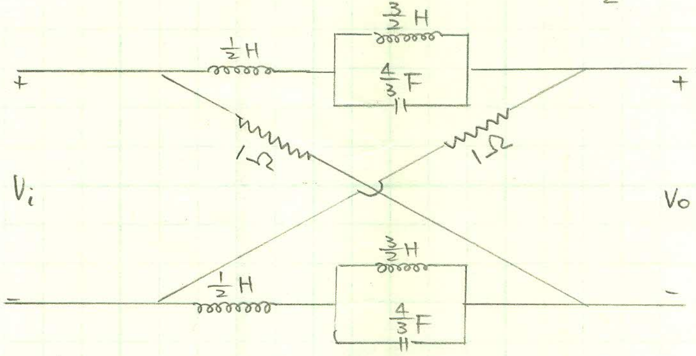
7.9 (C) $H(S) = \dfrac{-S^3 + 2S^2 - 2S + 1}{S^3 + 2S^2 + 2S + 1}$

$$= \frac{1 - \dfrac{S^3 + 2S}{2S^2 + 1}}{1 + \dfrac{S^3 + 2S}{2S^2 + 1}} \qquad Z_a = \frac{S^3 + 2S}{2S^2 + 1}, \quad Z_b = 1$$

$$= \frac{\dfrac{2S^2 + 1}{S^3 + 2S} - 1}{\dfrac{2S^2 + 1}{S^3 + 2S} + 1} \qquad Z_b = \frac{2S^2 + 1}{S^3 + 2S}, \quad Z_a = 1$$

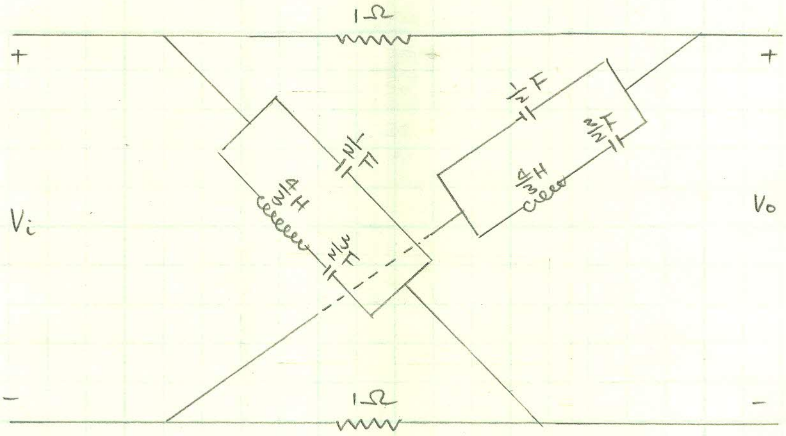
1. $Z_a = \dfrac{S^3 + 2S}{2S^2 + 1} = \dfrac{1}{2}S + \dfrac{1}{\dfrac{4}{3}S + \dfrac{1}{\frac{3}{2}S}}$



CHECK: $Z_a = \dfrac{1}{2}S + \dfrac{\frac{3}{2}S \cdot \frac{3}{4S}}{\frac{3}{2}S + \frac{3}{4S}} = \dfrac{S^3 + 2S}{2S^2 + 1}$, $Z_b = 1$

$H(S) = \dfrac{Z_b - Z_a}{Z_b + Z_a} = \dfrac{1 - \frac{S^3 + 2S}{2S^2 + 1}}{1 + \frac{S^3 + 2S}{2S^2 + 1}} = \dfrac{-S^3 + 2S^2 - 2S + 1}{S^3 + 2S^2 + 2S + 1}$ CHECKED!

2. $Z_b = \dfrac{2S^2 + 1}{S^3 + 2S}$, $Y_b = \dfrac{S^3 + 2S}{2S^2 + 1} = \dfrac{1}{2}S + \dfrac{1}{\dfrac{4}{3}S + \dfrac{1}{\frac{3}{2}S}}$



CHECK: $Z_b = (\frac{4}{3}S + \frac{2}{3S}) \cdot \frac{2}{S} \Big/ (\frac{4}{3}S + \frac{2}{3S} + \frac{2}{S}) = (\frac{4}{3}S^2 + \frac{2}{3}) \cdot 2 / (\frac{4}{3}S^3 + \frac{2}{3}S + 2S)$

$= (2S^2 + 1)/(S^3 + 2S)$, $Z_a = 1$

$H(S) = \dfrac{Z_b - Z_a}{Z_b + Z_a} = \dfrac{-S^3 + 2S^2 - 2S + 1}{S^3 + 2S^2 + 2S + 1}$ CHECKED!

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7.10 $H(s)=p(-s)/p(s)$, $p(s)$ is a polynomial.

SHOW $\Phi_H(\omega) = 2\,\Phi_p(\omega) + \alpha$

PROOF: $H(s)=p(-s)/p(s)$

LET $p(s)=m(s)+n(s)$, WHERE . $m(s)$ is even and $n(s)$ is odd.

$$p(-s)=m(s)-n(s)$$

$$p(j\omega)=m(\omega)+j\,n(\omega)$$

$$p(-j\omega)=m(\omega)-j\,n(\omega)$$, WHERE , $m(\omega)$ and $n(\omega)$ are real,

$$H(j\omega)=P(-j\omega)/p(j\omega)$$

$$= [m(\omega)-j\,n(\omega)]/[m(\omega)+j\,n(\omega)]$$

$$= \frac{1-j\cdot\frac{n(\omega)}{m(\omega)}}{1+j\cdot\frac{n(\omega)}{m(\omega)}}$$

$$= \frac{(1-j\cdot\frac{n(\omega)}{m(\omega)})(1-j\,\frac{n(\omega)}{m(\omega)})}{1+n^2(\omega)/m^2(\omega)}$$

$$\Phi_p(\omega) = -\angle p(j\omega) = -\tan^{-1}\frac{n(\omega)}{m(\omega)}$$

$$\Phi_H(\omega) = -\angle H(\omega) = \begin{cases} 2\Phi_p(\omega) & -45^\circ<\Phi_p(\omega)<45^\circ \\ 2\Phi_p(\omega)+180^\circ & -90^\circ<\Phi_p(\omega)<-45^\circ \\ 2\Phi_p(\omega)-180^\circ & 45^\circ<\Phi_p(\omega)<90^\circ \end{cases}$$

$\therefore\ \Phi_H(\omega) = 2\Phi_p(\omega) + k\cdot180^\circ$, $k=0,1,-1$ ✓

SORRY FOR LATE-HAND IN THIS HOMEWORK, BECAUSE I THOGHT IT WAS DUE THURSDAY.

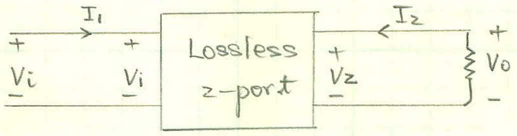
OK ✓

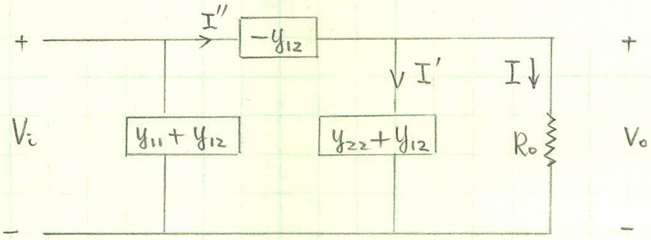
EE 448 H.W. #10 | Analog and digital Filter | Benmei Chen

Derive THE TRASFER FUNCTION:

20/20

1





SEE THE EQUIVALENT CIRCUIT ABOVE.

LET $I = 1$, $V_o = R_o$

$$I' = (y_{22}+y_{12})\cdot R_o$$

$$I'' = (y_{22}+y_{12})R_o + 1$$

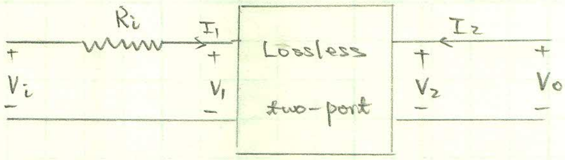
$$V_i = I''/(-y_{12}) + V_o$$

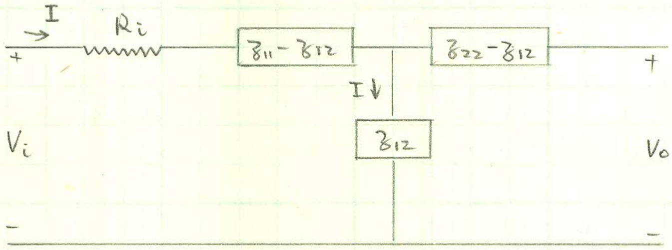
$$= \frac{(y_{22}+y_{12})\cdot R_o + 1}{-y_{12}} + R_o$$

$$= \frac{y_{22}R_o + 1}{-y_{12}}$$

$$H(S) = \frac{V_o}{V_i} = \frac{-y_{12}R_o}{y_{22}R_o+1} = \frac{-y_{12}}{\frac{1}{R_o} + y_{22}}$$

2.





LET $I = 1A$; $V_o = z_{12}$, $V_i = R_i + z_{11}$

$$H(S) = \frac{V_o}{V_i} = \frac{z_{12}}{R_o + z_{11}}$$

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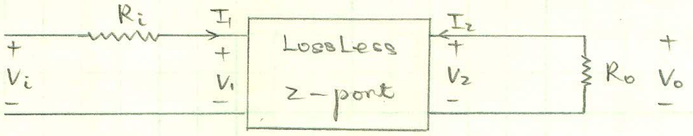
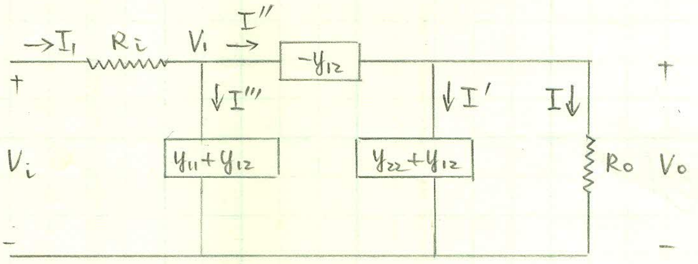
3.





LET $I = 1A$, $V_o = R_o$

$$I' = (y_{22}+y_{12})R_o$$

$$I'' = (y_{22}+y_{12})R_o + 1$$

$$V_1 = I''/(-y_{12}) + V_o$$

$$= (y_{22}R_o+1)/(-y_{12})$$

$$I''' = V_1(y_{11}+y_{12})$$

$$= (y_{11}+y_{12})(y_{22}R_o+1)/(-y_{12})$$

$$I_1 = I''+I''' = (y_{22}+y_{12})R_o + 1 + (y_{11}+y_{12})(y_{22}R_o+1)/(-y_{12})$$

$$V_i = I_1R_i + V_1 = (y_{22}+y_{12})R_iR_o + R_i + (y_{11}+y_{12})(y_{22}R_o+1)R_i/(-y_{12})$$

$$= V_1 + (-y_{12}y_{22}R_iR_o - y_{12}^2R_iR_o - y_{12}R_i + y_{11}y_{22}R_oR_i + y_{12}y_{22}R_oR_i + y_{11}R_i + y_{12}R_i)/(-y_{12})$$

$$H(s) = \frac{-y_{12}}{-y_{12}V_1/R_o + (-y_{12}y_{22}R_i - y_{12}^2\cdot R_i - y_{12}R_i/R_o + y_{11}y_{22}R_i + y_{12}y_{22}R_i + y_{11}R_i/R_o + y_{12}R_i/R_o)}$$

$$\doteq \frac{-y_{12}}{(y_{22}+1/R_o) - y_{12}R_i(y_{22}+1/R_o) + y_{11}R_i(y_{22}+1/R_o) + y_{12}R_i(y_{22}+1/R_o) - y_{12}^2\cdot R_i}$$

$$= \frac{-y_{12}}{(\frac{1}{R_o}+y_{22})\cdot[\,y_{11}R_i - y_{12}^2\cdot R_i/(\frac{1}{R_o}+y_{22}) + 1\,]}$$

$$= \frac{-y_{12}}{(\frac{1}{R_o}+y_{22})} \cdot \left[\frac{1}{R_i[y_{11} - y_{12}^2/(\frac{1}{R_o}+y_{22})] + 1}\right]$$

✓

$\frac{60}{60}$

Analog and Digital Filter (EE448)

Homework # 10

Nov. 9, 1986

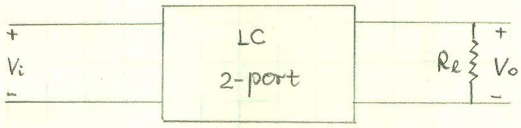
Benmei Chen

Chapter 7: Problem 7.12 (e), 7.13 (c)

7.14 (d) 7.16 (d)

7.17 (d)

7.12 Realize each of the following transfer functions by a lossless 2-port terminated at only one end by a load resistor, as shown in Fig below.



(e) $$H(S) = \frac{kS}{S^3+6S^2+15S+15}$$

$$= \frac{kS/(6S^2+15)}{1+\frac{S^3+15S}{6S^2+15}}$$

$$y_{22}(S) = \frac{S^3+15S}{6S^2+15} \quad , \quad y_{12} = -\frac{kS}{6S^2+15}$$

$$y_{22}(S) = \frac{S^3+15S}{6S^2+15} = \frac{1}{1/S + Z_R(S)}$$

$$Z_R(S) \triangleq \frac{1}{y_{22}(S)} - \frac{1}{S} = \frac{6S^2+15}{S^3+15S} - \frac{1}{S} = \frac{5S}{S^2+15}$$

$$Y_R(S) \triangleq \frac{1}{Z_R(S)} = \frac{S^2+15}{5S}$$

$\frac{1}{5}S$

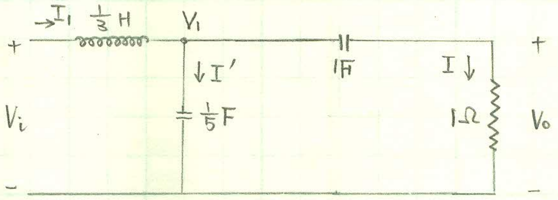
$5S \,\big)\, S^2+15$

S^2

$15 \,\big)\, 5S$ $\quad \frac{1}{3}S$

$5S$

0



CHECK: LET $I = 1\ A$, $V_o = 1A \times 1\Omega = 1\ V$

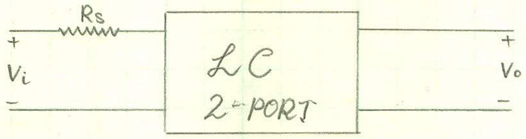
$$V_1 = \frac{1}{S} + 1 \quad , \quad I' = V_1 \cdot \frac{1}{5}S = \frac{1}{5} + \frac{1}{5}S$$

$$I_1 = I + I' = 1 + \frac{1}{5} + \frac{1}{5}S = \frac{6}{5} + \frac{1}{5}S$$

$$V_i = I_1 \cdot \frac{1}{3}S + V_1 = \frac{2}{5}S + \frac{1}{15}S^2 + \frac{1}{S} + 1 = \frac{6S^2+S^3+15+15S}{15S}$$

$$H(S) = \frac{V_o}{V_i} = \frac{15S}{S^3+6S^2+15S+15}$$ CHECKED!

7-13. Realize each of the trasfer functions in Problem 9-12 by a lossless 2-port terminated at only one end by a source resistance, as shown in Fig. below.

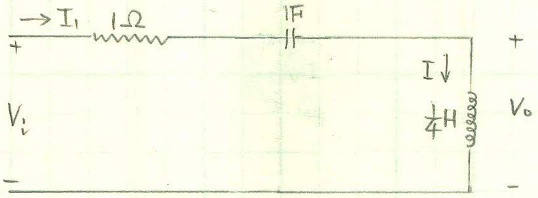


(c) $H(S) = kS^2/(S^2+4S+4)$

$$= \frac{kS^2/(4S)}{1+\dfrac{S^2+4}{4S}}$$

$$Z_{11}(S) \triangleq \frac{S^2+4}{4S} \qquad Z_{12}(S) = \frac{kS^2}{4S} = \frac{k}{4}S$$

$$\begin{array}{r|l} & \frac{1}{S} \\ 4S & 4+S^2 \\ & 4 \\ \hline & S \;\big)\; 4 \quad \frac{4}{S} \\ & \quad\;\; 4 \\ \hline & \quad\;\; 0 \end{array}$$



✓

CHECK: LET $I = 1\,A$, $V_o = \frac{1}{4}S$

$$V_i = 1 + \frac{1}{S} + \frac{1}{4}S$$

$$H(S) = \frac{S/4}{(4S+4+S^2)/4S}$$

$$= \frac{S^2}{S^2+4S+4}$$ CHECKED!

✓

7.14 (d) Realize each of the following transfer functions by a lossless 2-port terminated at both ends with $R_s = R_e = 1\Omega$, as shown in Fig. below.

$$H(S) = kS / (S^2 + \sqrt{2} S + 1)$$

$$\rho(s)\cdot\rho(-s) = 1 - \frac{4}{R_e} H(S)H(-S)$$

$$= 1 + \frac{4k^2 S^2}{(S^2+\sqrt{2}S+1)(S^2-\sqrt{2}S+1)}$$

$$= \frac{S^4+4k^2S^2+1}{(S^2+\sqrt{2}S+1)(S^2-\sqrt{2}S+1)}$$

LET $(s^2+as+b)(s^2-as+b) = s^4+4k^2s^2+1$, $a>0$, $b>0$

$$\therefore \; s^4+(2b-a^2)s^2+b^2 = s^4+4k^2s^2+1$$

$$b=1 , \quad 2-a^2=4k^2 , \quad a=\sqrt{2-4k^2}$$

$$\rho(s) = \frac{S^2+\sqrt{2-4k^2}+1}{S^2+\sqrt{2}S+1}$$

$$Z_{in1}(s) = \frac{1+\rho(s)}{1-\rho(s)} = \frac{S^2+\sqrt{2}S+1+S^2+\sqrt{2-4k^2}+1}{S^2+\sqrt{2}S+1-S^2-\sqrt{2-4k^2}-1}$$

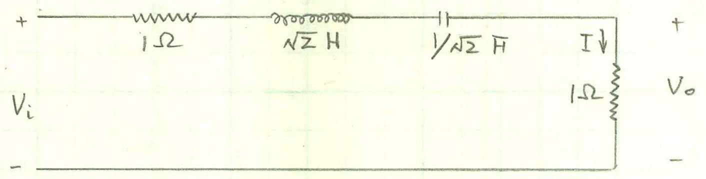
$$= \frac{2S^2+(\sqrt{2}+\sqrt{2-4k^2})S+2}{(\sqrt{2}-\sqrt{2-4k^2})S}$$

$$= \underbrace{\frac{2}{\sqrt{2}-\sqrt{2-4k^2}}}_{\text{series L}} S + \underbrace{\frac{2}{\sqrt{2}-\sqrt{2-4k^2}}}_{\text{SERIES C}}\cdot\frac{1}{S} + \underbrace{\frac{\sqrt{2}+\sqrt{2-4k^2}}{\sqrt{2}-\sqrt{2-4k^2}}}_{R_e}$$

$$R_e = 1 = \frac{\sqrt{2}+\sqrt{2-4k^2}}{\sqrt{2}-\sqrt{2-4k^2}}$$

$$\sqrt{2-4k^2} = 0 , \quad k=0.707=\frac{\sqrt{2}}{2}$$

$$C = 1/\sqrt{2}\ \text{F} , \quad L = \sqrt{2}\ \text{H}$$



CHECK: $I = 1$ A , $V_o = 1$ V

$$V_i = 1+\sqrt{2}S+\sqrt{2}/s+1 = (\sqrt{2}s^2+2s+\sqrt{2})/s$$

$\therefore H(S) = \frac{1}{\sqrt{2}} s / (S^2+\sqrt{2}S+\sqrt{2})$ CHECKED!

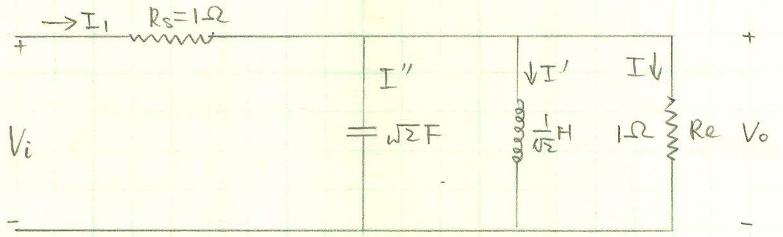
7.14 (d) (CONT.)

$$Z_{in2}(S) = (\sqrt{2} - \sqrt{2-4k^2})S / (2S^2 + (\sqrt{2}+\sqrt{2-4k^2})S + 2)$$

$$Y_{in2}(S) = (2S^2 + (\sqrt{2}+\sqrt{2-4k^2})S + 2) / (\sqrt{2}-\sqrt{2-4k^2})S$$

$$= \underbrace{\frac{2}{\sqrt{2}-\sqrt{2-4k^2}}}_{\text{SHUNT } C} S + \underbrace{\frac{2}{\sqrt{2}-\sqrt{2-4k^2}}}_{\text{SHUNT } L} \cdot \frac{1}{S} + \underbrace{\frac{\sqrt{2}+\sqrt{2-4k^2}}{\sqrt{2}-\sqrt{2-4k^2}}}_{\frac{1}{R_\ell} = 1}$$

$$\therefore C = \sqrt{2}\ F \qquad L = \frac{1}{\sqrt{2}}\ H$$



CHECK: LET $I = 1A$., $V_o = 1V$, $I' = \sqrt{2}/s$, $I'' = \sqrt{2}S$

$I_1 = 1 + \sqrt{2}/s + \sqrt{2}S$, $V_i = 1 + \sqrt{2}/s + \sqrt{2}S + 1 = 2 + \sqrt{2}/s + \sqrt{2}S$

$H(S) = \frac{1}{\sqrt{2}} S / (S^2 + \sqrt{2}S + 1)$ CHECKED! ✓

7.16 (d) *Realize each of the following transfer functions with a lossless 2-port terminated at both ends by $R_s = 1$ and $R_\ell = 2\Omega$*

$$H(S) = \frac{kS^3}{(S+1)^3} = \frac{kS^3}{S^3 + 3S^2 + 3S + 1}$$

$$\rho(S) \cdot \rho(-S) = 1 - \frac{4}{2} \cdot H(S) \cdot H(-S)$$

$$= 1 - 2 \cdot \frac{k^2 S^6}{(S^3+3S^2+3S+1)(S^3-3S^2+3S-1)}$$

$$= \frac{(1-2k^2)S^6 - 3S^4 + 3S^2 - 1}{(S^3+3S^2+3S+1)(S^3-3S^2+3S-1)}$$

FROM $H(\infty) = \frac{R_\ell}{R_s + R_\ell} = \frac{2}{3} = k$, WE HAVE

$$\rho(S) \cdot \rho(-S) = \frac{\frac{1}{9}S^6 - 3S^4 + 3S^2 - 1}{(S^3+3S^2+3S+1)(S^3-3S^2+3S-1)}$$

$$= \frac{1}{3} \cdot \frac{S^3 + 6.58S^2 + 8.15S + 3}{S^3 + 3S^2 + 3S + 1} \cdot \frac{1}{3} \cdot \frac{-S^3 + 6.58S^2 - 8.15S + 3}{-S^3 + 3S^2 - 3S + 1}$$

$$\rho(S) = \frac{S^3 + 6.58S^2 + 8.15S + 3}{3S^3 + 9S^2 + 9S + 3}$$

7.16 (d) (CONT.)

$$Z_{in1} = \frac{1-\rho(s)}{1+\rho(s)}$$

$$= \frac{3s^3+9s^2+9s+3-s^3-6.58s^2-8.15s-3}{3s^3+9s^2+9s+3+s^3+6.58s^2+8.15s+3}$$

$$= \frac{2s^3+2.42s^2+0.85s}{4s^3+15.58s^2+17.15s+6}$$

$$Z_{in2}(s) = \frac{4s^3+15.58s^2+17.15s+6}{2s^3+2.42s^2+0.85s}$$

HENCE, $Z_{in}(\infty) = R_l = 2$, $Z_{in1}(\infty) = \frac{1}{2}$ and $Z_{in2}(\infty) = 2$

SO, WE choose $Z_{in}(s) = Z_{in2}(s)$

$$0.85s+2.42s^2+2s^3 \overline{\big) 6+17.15s+15.58s^2+4s^3} \quad 7.09/s$$

$$\underline{6+17.15s+14.18s^2}$$

$$1.4\ s+4s^2 \overline{\big) 0.85+2.42s+2s^2} \quad 0.61/s$$

$$\underline{0.85+2.42s}$$

$$2s \overline{\big) 1.4+4s} \quad 0.7/s$$

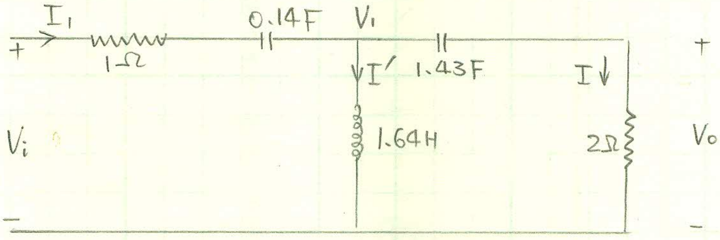
$$\underline{1.4}$$

$$4 \overline{\big) 2} \quad \frac{1}{2}$$

$$\underline{2}$$

$$0$$

$$Z_{in}(s) = \frac{7.09}{s} + \cfrac{1}{\cfrac{0.61}{s} + \cfrac{1}{\cfrac{0.7}{s} + \cfrac{1}{\cfrac{1}{2}}}}$$



CHECK: LET $I = 1A$, $V_o = 2V$,

$$V_1 = 2 + \frac{0.7}{s}, \quad I' = 1.22/s + 0.43/s^2$$

$$I' = 1 + 1.22/s + 0.43/s^2$$

$$V_i = (1+\frac{7.09}{s})(1+1.22/s+0.43/s^2) + 2 + 0.7/s$$

$$= 1 + 1.22/s + 0.43/s^2 + 7.09/s + 8.65/s^2 + 3.05/s^3 + 2 + 0.7/s$$

$$= 3 + 9.01/s + 9.08/s^2 + 3.05/s^3$$

$$\doteq 3(s^3+3.00s^2+3.03s+1.02)/s^3$$

$$H(s) = \tfrac{2}{3}s^3/(s^3+3.00s^2+3.03s+1.02)$$ close enough

CHECKED!

Benmei Chen

7.17. (d) $H(s) = k/(s^3+4s^2+6s+3)$, $R_s = 1\Omega$, $R_e = 0.25$

$$\rho(s)\rho(-s) = 1 - \frac{4}{R_e} H(s)H(-s) = 1 - 16\, H(s)H(-s)$$

$$= 1 + 16 \frac{k^2}{(s^3+4s^2+6s+3)(s^3-4s^2+6s-3)}$$

$$= \frac{s^6 - 4s^4 + 12s^2 - 9 + 16k^2}{(s^3+4s^2+6s+3)(s^3-4s^2+6s-3)}$$

$$H(0) = \frac{R_e}{R_e+R_s} = \frac{1}{5} = \frac{k}{3} \quad , \quad k = \frac{3}{5}$$

$$\rho(s)\rho(-s) = \frac{s^6 - 4s^4 + 12s^2 - 3.24}{(s^3+4s^2+6s+3)(s^3-4s^2+6s-3)}$$

$$= \frac{s^3 + 3.755s^2 + 5.052s + 1.8}{s^3 + 4s^2 + 6s + 3} \cdot \frac{-s^3 + 3.755s^2 - 5.052s + 1.8}{-s^3 + 4s^2 - 6s + 3}$$

$$\rho(s) = \frac{s^3 + 3.755s^2 + 5.052s + 1.8}{s^3 + 4s^2 + 6s + 3}$$

$$Z_{in1}(s) = \frac{1 - \rho(s)}{1 + \rho(s)} = \frac{0.245s^2 + 0.948s + 1.2}{2s^3 + 7.755s^2 + 11.052s + 4.8}$$

$$Z_{in2}(s) = \frac{1 + \rho(s)}{1 - \rho(s)} = \frac{2s^3 + 7.755s^2 + 11.052s + 4.8}{0.245s^2 + 0.948s + 1.2}$$

HENCE , $Z_{in}(0) = 0.25$, $Z_{in1}(0) = 0.25$ and $Z_{in2}(0) = 4$

SO, WE CHOOSE $Z_{in}(s) = Z_{in1}(s)$

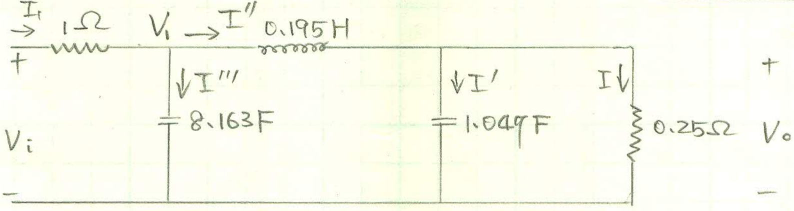
$$Y_{in1}(s) = \frac{2s^3 + 7.755s^2 + 11.052s + 4.8}{0.245s^2 + 0.948s + 1.2}$$

$$\begin{array}{r|l} & 8.163s \\ \hline 0.245s^2+0.948s+1.2 & 2s^3 + 7.755s^2 + 11.052s + 4.8 \\ & 2s^3 + 7.74s^2 + 9.796s \\ \hline & 1.256s + 4.8 \end{array}$$

$$\begin{array}{r|l} & 0.195s \\ \hline & 0.245s^2 + 0.948s + 1.2 \\ & 0.245s^2 + 0.940s \\ \hline & 1.2 \end{array}$$

$$\begin{array}{r|l} & 1.047s \\ \hline 1.2 & 1.256s + 4.8 \\ & 1.256s \\ \hline & 4.8 \end{array}$$

$$\begin{array}{r|l} & 0.25 \\ \hline 4.8 & 1.2 \\ & 1.2 \\ \hline & 0 \end{array}$$



CHECK : $I_1 = 1A$, $V_o = 0.25\,V$, $I' = 0.262s$

$I'' = 1 + 0.262s$, $V_1 = 0.195s + 0.051s^2 + 0.25$

$I''' = 0.417s^3 + 1.592s^2 + 2.041s$, $I_1 = 0.417s^3 + 1.592s^2 + 2.303s + 1$

$V_i = 0.417s^3 + 1.592s^2 + 2.303s + 1 + 0.195s + 0.051s^2 + 0.25$
$= 0.417s^3 + 1.643s^2 + 2.498s + 1.25 = 0.417(s^3 + 3.94s^2 + 5.99s + 2.998)$

$H(s) = 0.6/(s^3 + 3.94s^2 + 5.99s + 2.998)$ close enough

Checked!

20/20

Analog and Digital Filter EE448

Homework #11

Benmei Chen

Nov.20,1986

8-2. (a) Find the transfer function of the n-th-order normalized low-pass Butterworth filter, where n=4, 5.

(b) Sketch the pole-zero patterns of the nth-order normalized low-pass Butterworth Filter, where n=4, 5

SOLUTION: (a) n=4 even. $\theta_K = \frac{2K-1}{2n}\pi = \frac{2K-1}{8}\pi$, k=1, 2, 3, 4,

$\theta_1 = \frac{\pi}{8}$, $\theta_2 = \frac{3\pi}{8}$, $\theta_3 = \frac{5\pi}{8}$, $\theta_4 = \frac{7\pi}{8}$

$$H(S) = \frac{1}{S^2+(2\sin\theta_1)S+1} \cdot \frac{1}{S^2+(2\sin\theta_2)S+1}$$

$$= \frac{1}{S^2+0.7654S+1} \cdot \frac{1}{S^2+1.8478S+1}$$

$$= \frac{1}{S^4+2.6132S^3+3.4142S^2+2.6132S+1}$$

n=5 odd, $\theta_K = \frac{2k-1}{10}\pi$, k=1, 2, 3, 4, 5

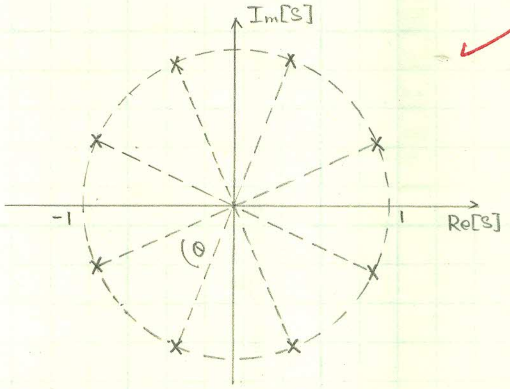
$\theta_1 = \frac{\pi}{10}$, $\theta_2 = \frac{3\pi}{10}$, $\theta_3 = \frac{5\pi}{10}$, $\theta_4 = \frac{7\pi}{10}$, $\theta_5 = \frac{9\pi}{10}$

$$H(S) = \frac{1}{S+1} \cdot \frac{1}{S^2+0.6180S+1} \cdot \frac{1}{S^2+1.6180S+1}$$

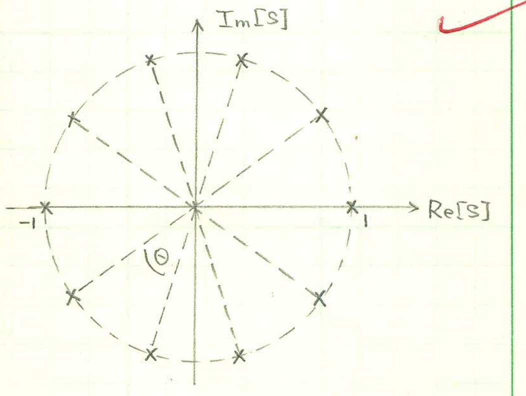
$$= \frac{1}{S+1} \cdot \frac{1}{S^4+2.2360S^3+3S^2+2.2360S+1}$$

$$= \frac{1}{S^5+3.2360S^4+5.2360S^3+5.2360S^2+3.2360S+1}$$

(b)



n=4, θ = 45°



n=5, θ = 36°

8-3. (a) Find the minimum order of Butterworth filter to meet the follwing specifications:

(i) The 3dB cutoff frequency is at 1 rad/sec.

(ii) The pass-band requirment is $|H(j\omega)| \geq 0.99$ for $0 \leq \omega \leq 0.25$ rad/sec.

(iii) The stopband requirment is $|H(j\omega)| \leq 0.001$ for $\omega \geq 2$ rad/s.

(b) Find the transfer function of the desired filter.

(c) Realized the filter in (b) with $R_s = 1\Omega$ and $R_\ell = 1\Omega$.

(d) Realized the filter in (b) with $R_s = 1\Omega$ and $R_\ell = 3\Omega$

(e) Realized the filter in (b) with $R_s = 1\Omega$ and $R_\ell = 0.5\Omega$

SOLUTION: (a) $|H(j\omega)|^2 = \dfrac{1}{1+\omega^{2n}}$

(i) $|H(j0)|^2 = 1 = 0\text{dB}$. $|H(j\cdot 1)|^2 = 0.5$, $|H(j\cdot 1)| = 0.707 = -3\text{dB}$.

(ii) $|H(j\omega)| \geq 0.99$, $|H(j\omega)|^2 \geq 0.9801$

$\dfrac{1}{1+\omega^{2n}} \geq 0.9801$, $1+\omega^{2n} \leq 1.0203$, $\omega^{2n} \leq 0.0203$

$\omega^{2n} \leq 0.25^{2n} \leq 0.0203$, $n \geq 2$

(iii) $|H(j\omega)| \leq 0.001$, $|H(j\omega)|^2 \leq 0.001^2 = 0.000001$

$1+\omega^{2n} \geq 10^6$, $\omega^{2n} \geq 2^{2n} \geq 10^6 - 1$, $n \geq 10$

SO, 10th-order of Butterworth filter needed to meet the conditions.

(b) $n = 10$ even, $\theta_K = \frac{2k-1}{20}\pi$, $k = 1, 2, \cdots, 10$

$\theta_1 = \frac{\pi}{20}$, $\theta_2 = \frac{3\pi}{20}$, $\theta_3 = \frac{5\pi}{20}$, $\theta_4 = \frac{7\pi}{20}$, $\theta_5 = \frac{9\pi}{20}$, $\cdots$, $\theta_{10} = \frac{19\pi}{20}$

$$H(S) = \frac{1}{S^2+0.3129S+1}\cdot\frac{1}{S^2+0.9080S+1}\cdot\frac{1}{S^2+1.4142S+1}\cdot\frac{1}{S^2+1.7820S+1}\cdot\frac{1}{S^2+1.9754S+1}$$

$$= \frac{1}{S^4+1.2209S^3+2.2841S^2+1.2209S+1}\cdot\frac{1}{S^4+3.1962S^3+4.5201S^2+3.1962S+1}\cdot\frac{1}{S^2+1.9754S+1}$$

$$= \frac{1}{S^2+1.9754S+1}\cdot\frac{1}{S^8+4.4171S^7+10.7064S^6+17.2361S^5+18.1288S^4+17.2361S^3+10.7064S^2+4.4171S+1}$$

$$= \frac{1}{S^{10}+6.3925S^9+20.4319S^8+42.8026S^7+62.8834S^6+70.2838S^5+62.8834S^4+42.8026S^3+20.4319S^2+6.3925S+1}$$

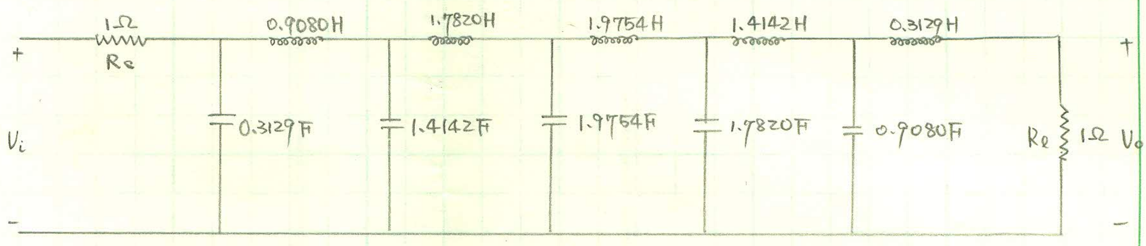
8.3 CONT. (c) $n=10$, FROM the equation 8-46 in the text book

$$C_m = 2\sin\left[\frac{(2m-1)}{20}\pi\right] \qquad m = 1, 3, 5, 7, 9$$

$C_1 = 0.3129$ F, $C_3 = 1.4142$F, $C_5 = 1.9754$F, $C_7 = 1.7820$F, $C_9 = 0.9080$F

$$L_m = 2\sin[(2m-1)\pi/20] \qquad m = 2, 4, 6, 8, 10$$

$L_2 = 0.9080$ H, $L_4 = 1.7820$H, $L_6 = 1.9754$ H, $L_8 = 1.4142$H, $L_{10} = 0.3129$H



(d) $R_s = 1\Omega$ and $R_e = 3\Omega$

$$\lambda = \left(\frac{R_e - 1}{R_e + 1}\right)^{1/n} = 0.9330$$

$\alpha_1 = 0.3129$, $\alpha_2 = 0.6180$, $\alpha_3 = 0.9080$, $\alpha_4 = 1.1756$, $\alpha_5 = 1.4142 \cdots$

$\beta_1 = 1.9754$, $\beta_2 = 1.9021$, $\beta_3 = 1.7820$, $\beta_4 = 1.6180$, $\beta_5 = 1.4142 \cdots$

$$\tilde{C}_1 = \frac{\alpha_1}{3(1-\lambda)} = \frac{0.3129}{3 \times 0.067} = 1.5567 \text{ F}$$

$$\tilde{L}_2 = \frac{1}{\tilde{C}_1} \cdot \frac{\alpha_1 \cdot \alpha_3}{1 - \lambda\beta_2 + \lambda^2} = 1.9045 \text{ H}$$

$$\tilde{C}_3 = \frac{1}{\tilde{L}_2} \cdot \frac{\alpha_3 \cdot \alpha_5}{1 - \lambda\beta_4 + \lambda^2} = 1.8682 \text{ F}$$

$$\tilde{L}_4 = \frac{1}{\tilde{C}_3} \cdot \frac{\alpha_5 \cdot \alpha_7}{1 - \lambda\beta_6 + \lambda^2} = 1.7435 \text{ H}$$

$$\tilde{C}_5 = \frac{1}{\tilde{L}_4} \cdot \frac{\alpha_7 \cdot \alpha_9}{1 - \lambda\beta_8 + \lambda^2} = 1.5604 \text{ F}$$

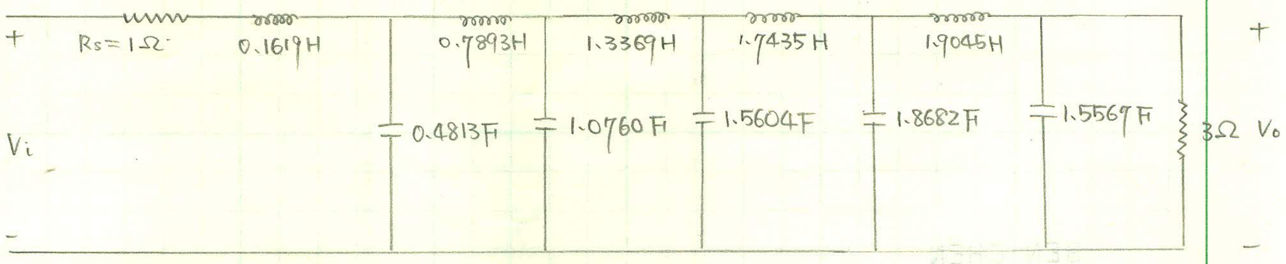
$$\tilde{L}_6 = \frac{1}{\tilde{C}_5} \cdot \frac{\alpha_9 \cdot \alpha_{11}}{1 - \lambda\beta_{10} + \lambda^2} = 1.3369 \text{ H}$$

$$\tilde{C}_7 = \frac{1}{\tilde{L}_6} \cdot \frac{\alpha_{11} \cdot \alpha_{13}}{1 - \lambda\beta_{12} + \lambda^2} = 1.0760 \text{ F}$$

$$\tilde{L}_8 = \frac{1}{\tilde{C}_7} \cdot \frac{\alpha_{13} \cdot \alpha_{15}}{1 - \lambda\beta_{14} + \lambda^2} = 0.7893 \text{ H}$$

$$\tilde{C}_9 = \frac{1}{\tilde{L}_8} \cdot \frac{\alpha_{15} \cdot \alpha_{17}}{1 - \lambda\beta_{16} + \lambda^2} = 0.4813 \text{ F}$$

$$\tilde{L}_{10} = \frac{1}{C_9} \cdot \frac{\alpha_{17} \cdot \alpha_{19}}{1 - \lambda\beta_{18} + \lambda^2} = 0.1619 \text{ H}$$



(e) $R_s = 1\Omega$ and $R_e = 0.5\Omega$

$$\lambda = \left(\frac{1-0.5}{1+0.5}\right)^{1/10} = 0.8960$$

$$\hat{L}_1 = \frac{\alpha_1 R_e}{1-\lambda} = 1.5036 \text{ H}$$

$$\hat{C}_2 = \frac{1}{\hat{L}_1}\cdot\frac{\alpha_1\cdot\alpha_3}{1-\lambda\beta_2+\lambda^2} = 1.9177 \text{ F}$$

$$\hat{L}_3 = \frac{1}{\hat{C}_2}\cdot\frac{\alpha_3\cdot\alpha_5}{1-\lambda\beta_4+\lambda^2} = 1.8966 \text{ H}$$

$$\hat{C}_4 = \frac{1}{\hat{L}_3}\cdot\frac{\alpha_5\cdot\alpha_7}{1-\lambda\beta_6+\lambda^2} = 1.7729 \text{ F}$$

$$\hat{L}_5 = \frac{1}{\hat{C}_4}\cdot\frac{\alpha_7\cdot\alpha_9}{1-\lambda\beta_8+\lambda^2} = 1.5896 \text{ H}$$

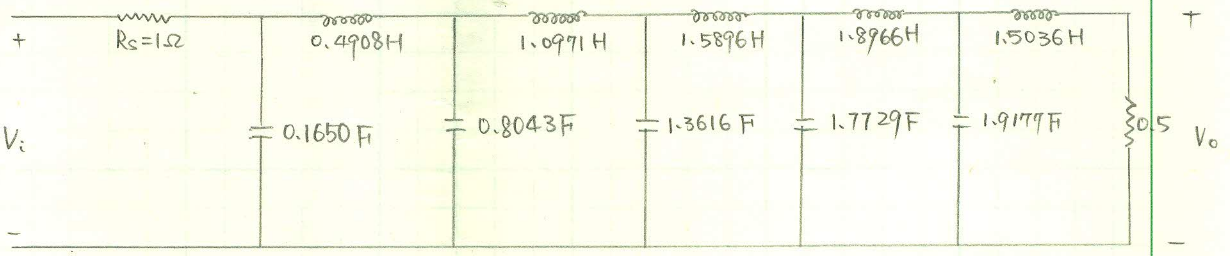
$$\hat{C}_6 = \frac{1}{\hat{L}_5}\cdot\frac{\alpha_9\cdot\alpha_{11}}{1-\lambda\beta_{10}+\lambda^2} = 1.3616 \text{ F}$$

$$\hat{L}_7 = \frac{1}{\hat{C}_6}\cdot\frac{\alpha_{11}\cdot\alpha_{13}}{1-\lambda\beta_{12}+\lambda^2} = 1.0971 \text{ H}$$

$$\hat{C}_8 = \frac{1}{\hat{L}_7}\cdot\frac{\alpha_{13}\cdot\alpha_{15}}{1-\lambda\beta_{14}+\lambda^2} = 0.8043 \text{ F}$$

$$\hat{L}_9 = \frac{1}{\hat{C}_8}\cdot\frac{\alpha_{15}\cdot\alpha_{17}}{1-\lambda\beta_{16}+\lambda^2} = 0.4908 \text{ H}$$

$$\hat{C}_{10} = \frac{1}{\hat{L}_9}\cdot\frac{\alpha_{17}\cdot\alpha_{19}}{1-\lambda\beta_{18}+\lambda^2} = 0.1650 \text{ F}$$



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EE 448 FILTER DESIGN

HOMEWORK #"FORGOT"

Butterworth and Chebyshev Lowpass

Dec. 3 , 1986 Wendesday

Benmei Chen

Benmei Chen.

E-E 448 FILTER DESIGN
R. A. Birgenheier
November 20, 1986

FIRST THUR. NEXT WEEK

Homework

Design a normalized Butterworth LPF of minimum order to meet the following specs.

Filter	Passband Attenuation	Stopband Attenuation	Output Load
A	<.5dB for ω<.8 rps	>40dB for ω>2 rps	.25Ω
B	<.8dB for ω<.85 rps	>30dB for ω>1.6 rps	0.5Ω
C	<.1dB for ω<.75 rps	>20dB for ω>1.25 rps	0.5Ω
D	<.4dB for ω<.85 rps	>30dB for ω>1.4 rps	2Ω
E	<.1dB for ω<.75 rps	>30dB for ω>1.5 rps	2Ω

a) Determine and plot pole locations.

b) Determine H(s)

c) Obtain a Realization (schematic)

d) Determine the frequency response from a SPICE run for (i) 8 significant figures and (ii) 3 significant figures for component values.

(A)	(B)	(C)	(D)	(E)
G.A.	R.H.	F.H.	K.P.	H.R.
L.A.	A.H.	T.M.	T.R.	T.W.
C.C.	B.K.	S.N.	M.R.	D.Y.
B.C.	B.M.	M.P.		

SOLUTION: A Normalized Butterworth : $|H(j\omega)|^2 = \frac{1}{1+\omega^{2n}}$

$$|H(j\omega)|_{dB} = 10 \log \frac{1}{1+\omega^{2n}} = -10 \log (1+\omega^{2n})$$

FOR FILTER A : $-10 \log \frac{1}{1+(0.8)^{2n}} < 0.5 \Rightarrow n \geqslant 5$

$-10 \log \frac{1}{1+ 2^{2n}} > 40 \Rightarrow n \geqslant 7$

SO, FILTER 'A' IS A 7th-ORDER.

FOR FILTER B: $10 \log [1+(0.85)^{2n}] < 0.8 \Rightarrow n \geqslant 5$

$10 \log [1+(1.6)^{2n}] > 30 \Rightarrow n \geqslant 8$

SO, FILTER 'B' IS A 8th-ORDER.

FOR FILTER C: $10 \log [1+(0.75)^{2n}] < 0.1 \Rightarrow n \geqslant 7$

$10 \log [1+(1.25)^{2n}] > 20 \Rightarrow n \geqslant 11$

SO, FILTER 'C' IS A 11-th-ORDER

FOR FILTER D : $10 \log (1+(0.85)^{2n}) < 0.4 \Rightarrow n \geqslant 8$

$10 \log (1+(1.4)^{2n}) > 30 \Rightarrow n \geqslant 11$

SO, FILTER 'D' IS A 11th-ORDER TOO.

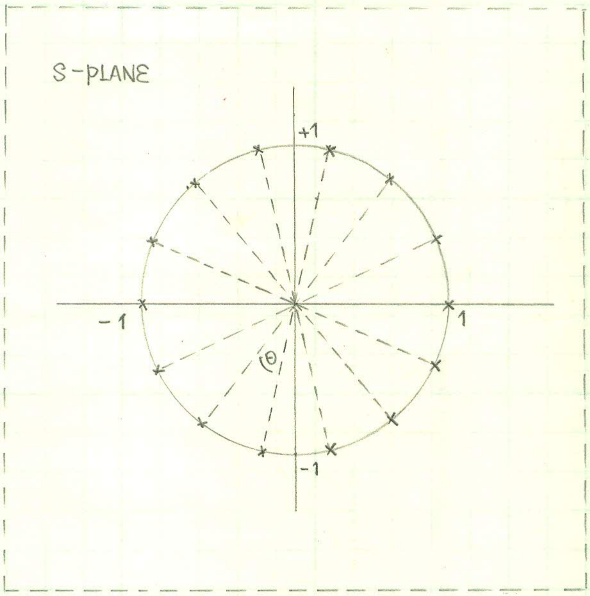
FOR FILTER E : $10 \log [1+(0.75)^{2n}] < 0.1 \Rightarrow n \geqslant 7$

$10 \log [1+ 1.5^{2n}] > 30 \Rightarrow n \geqslant 9$

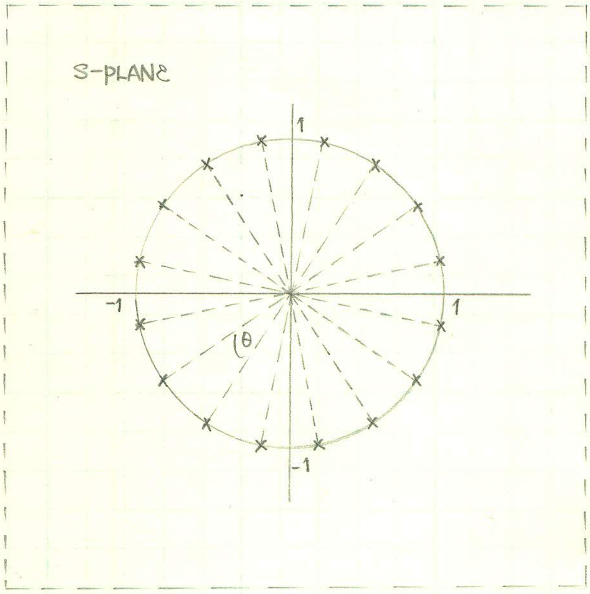
SO, FILTER 'E' IS A 9th-ORDER.

FILTER	A	B	C	D	E
ORDER	7	8	11	11	9

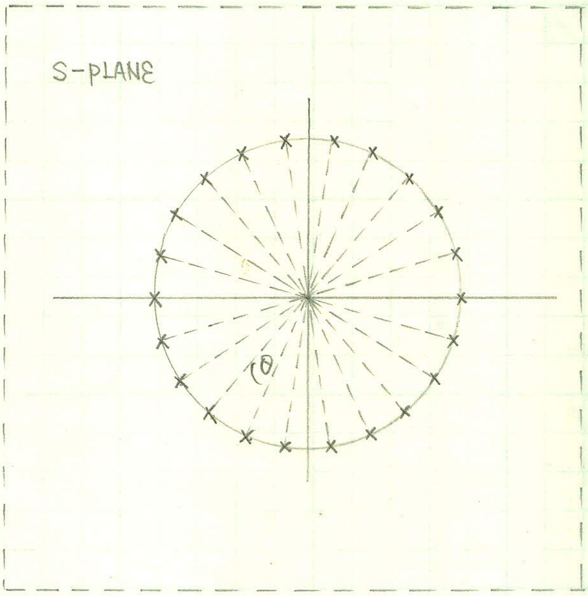
a) Determine and plot pole locations.



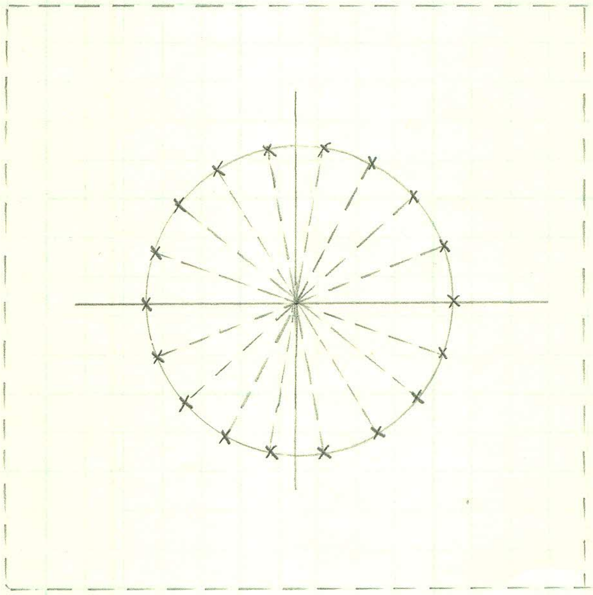
FILTER A , $n=7$, $\theta=25.7°$



FILTER B , $n=8$, $\theta = 22.5°$



FILTER C AND D , $n=11$, $\theta=16.36°$



FILTER E , $n=9$, $\theta=20°$

(b) Determine H(S) : (WE CAN FIND H(S) IN TABLE A8-1 ON PAGE 301 WITH $n \leq 9$)

FILTER A :

$$H(S) = \frac{1}{S^7 + 4.49396S^6 + 10.09783S^5 + 14.59179S^4 + 14.59179S^3 + 10.09783S^2 + 4.49396S + 1}$$

$$= \frac{1}{(S^2+0.44504S+1)(S^2+1.24698S+1)(S^2+1.80194S+1)(S+1)}$$

FILTER B :

$$H(S) = \frac{1}{S^8 + 5.1258S^7 + 13.1371S^6 + 21.8462S^5 + 25.6884S^4 + 21.8462S^3 + 13.1371S^2 + 5.1258S + 1}$$

$$= \frac{1}{(S^2+0.3902S+1)(S^2+1.1111S+1)(S^2+1.6629S+1)(S^2+1.9616S+1)}$$

FILTER C AND D :

$$H(S) = \frac{1}{(S+1)(S^2+0.2846S+1)(S^2+0.8308S+1)(S^2+1.3097S+1)(S^2+1.6825S+1)(S^2+1.9190S+1)}$$

FILTER E :

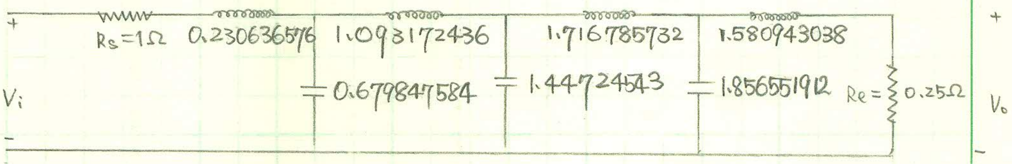
$$H(S) = \frac{1}{(S^2+0.3473S+1)(S^2+S+1)(S^2+1.5321S+1)(S^2+1.8794S+1)(S+1)}$$

(c) Obtain a Realization (schematic) :

FILTER A:

$R_e = 0.25\Omega$

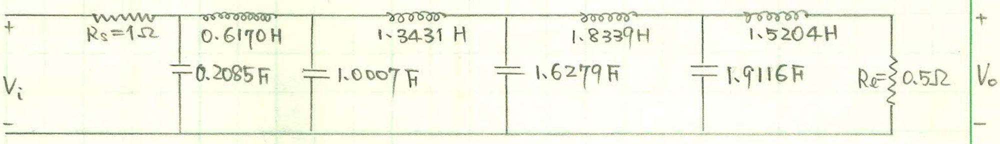
$n = 7$



FILTER B:

$R_e = 0.5\Omega$

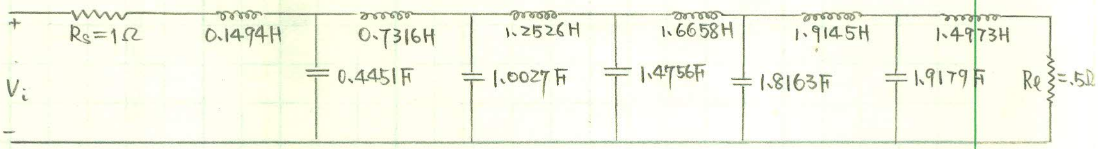
$n = 8$



FILTER C:

$R_e = 0.5\Omega$

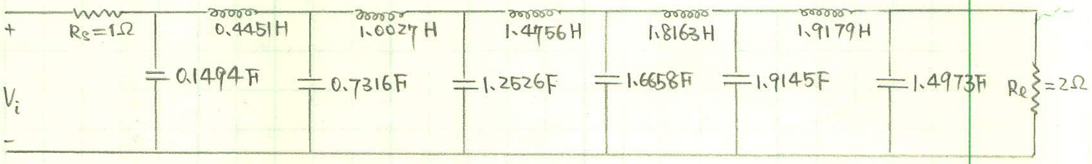
$n = 11$



FILTER D:

$R_e = 2\Omega$

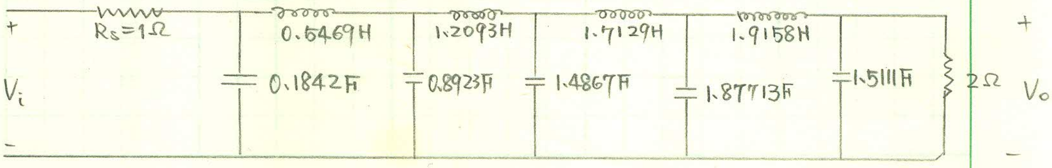
$n = 11$



FILTER E:

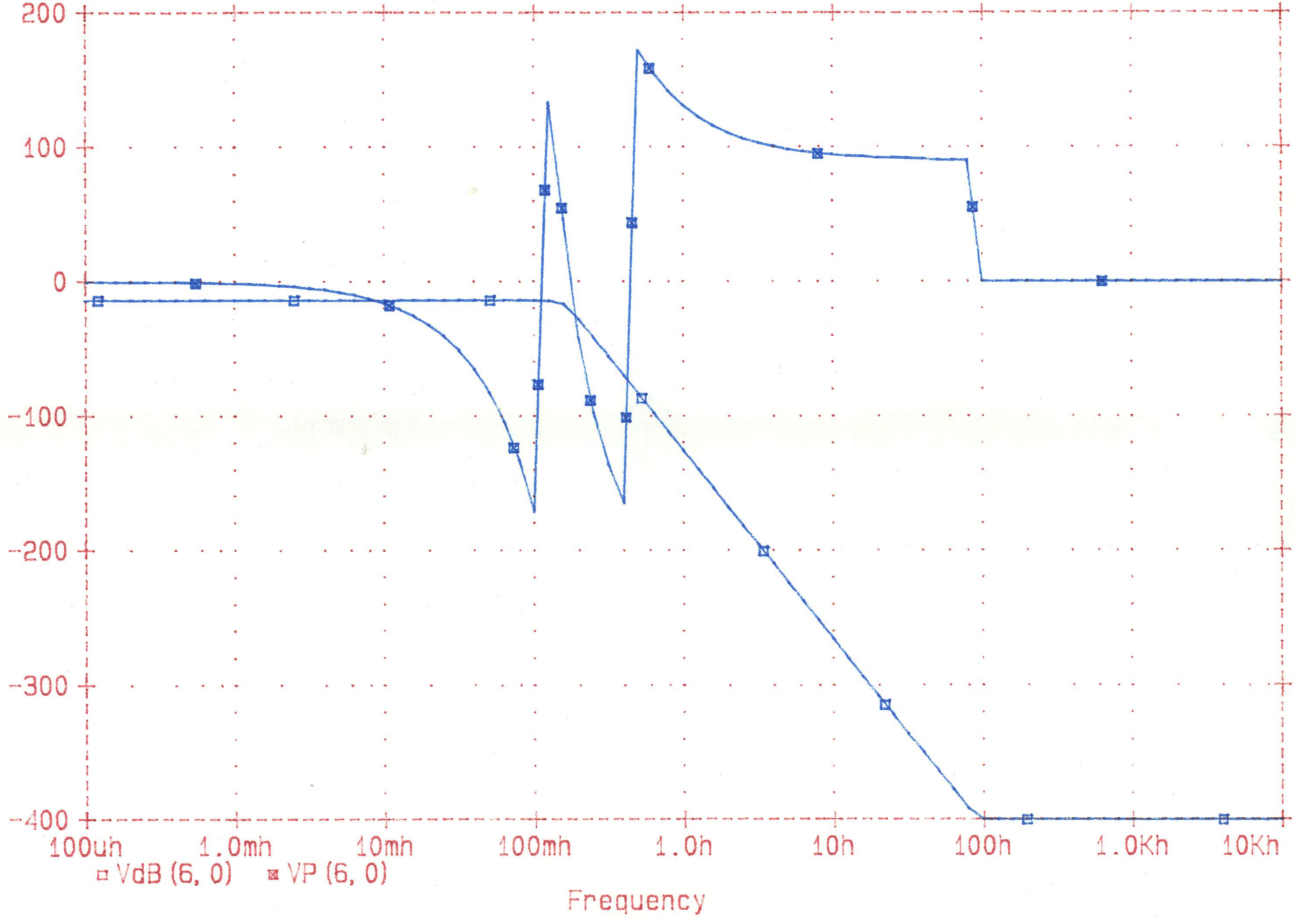
$R_e = 2\Omega$

$n = 9$



BUTTERWORTH LOWPASS FILTER -- 7TH-ORDER

Date/Time run: 12/ 3/86 22:00:29 Temperature: 27.0



E-E 448 PASSIVE AND ACTIVE FILTER DESIGN
Homework
November 25, 1986

Design a normalized L-P chebyshev filter that satisfies the following specifications:

Filter	Max. Passband Ripple	Stopband Attenuation	Load Resistance
A	.15 dB	>40 dB for $\omega>1.5$ rps	(a) .5Ω (b) 1.5Ω
B	.25 dB	>50 dB for $\omega>1.8$ rps	(a) .75Ω (b) 1.2Ω
C	.4 dB	>80 dB for $\omega>2.0$ rps	(a) .25Ω (b) 1.25Ω
D	1.2 dB	>80 dB for $\omega>1.8$ rps	(a) .75Ω (b) 2.0Ω
E	0.8 dB	>40 dB for $\omega>1.3$ rps	(a) .5Ω (b) 1.2Ω

a) Determine and plot pole locations.

b) Determine the transfer function (you may leave it in factored form).

c) Use SPICE to determine the frequency response, and thereby verify the specifications.

A	B	C	D	E
G.A.	R.H.	F.H.	K.P.	T.W.
L.A.	A.H.	T.M.	M.R.	D.Y.
C.C.	B.K.	S.J.	H.R.	M.P.
B.C.	B.M.	T.R.		

SOLUTION: A normalized L-P Chebyshev Filter:

$$|H(j\omega)|^2 = \frac{1}{1+\epsilon^2 T_n^2(\omega)}$$

WHERE $T_n(\omega) = \begin{cases} \cos(n\cos^{-1}\omega) & |\omega| \leq 1 \\ \cosh(n\cosh^{-1}\omega) & |\omega| \geq 1 \end{cases}$

$$|H(j\omega)|_{dB} = -10\log\left(1+\epsilon^2 T_n^2(\omega)\right)$$

FOR FILTER A:

$\epsilon = \sqrt{10^{0.015}-1} = 0.1875$

$10\log\left(1+0.1875^2 T_n^2(1.5)\right) > 40 \Rightarrow T_n(1.5) > 533.3067 \Rightarrow n \geq 8$

FOR FILTER B:

$\epsilon = \sqrt{10^{0.025}-1} = 0.2434$

$10\log\left[1+0.2434^2 T_n^2(1.8)\right] > 50 \Rightarrow T_n(1.8) > 1299.2037 \Rightarrow n \geq 7$

FOR FILTER C:

$\epsilon = \sqrt{10^{0.04}-1} = 0.3106$

$10\log\left[1+0.3106^2 T_n^2(2.0)\right] > 80 \Rightarrow T_n(2) > 31645.5695 \Rightarrow n \geq 9$

FOR FILTER D:

$\epsilon = \sqrt{10^{0.12}-1} = 0.5641$

$10\log\left[1+0.3183\, T_n^2(1.8)\right] > 80 \Rightarrow T_n(1.8) > 17724.8137 \Rightarrow n \geq 9$

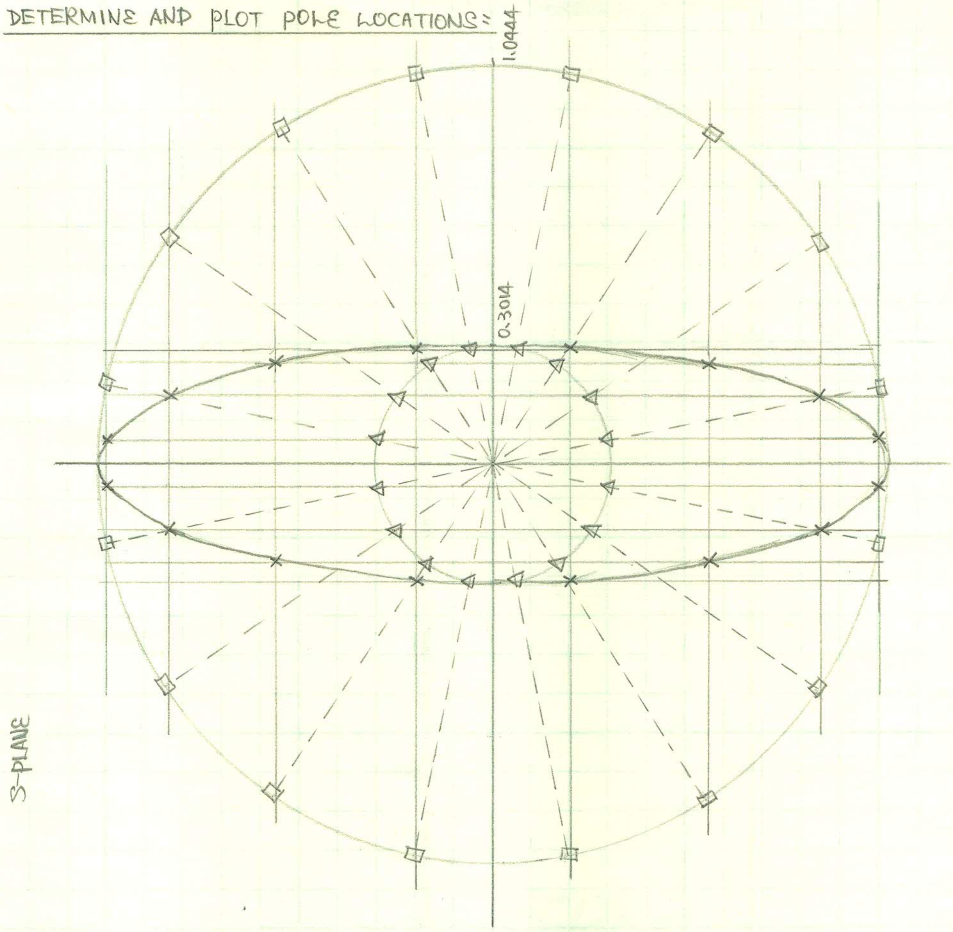
FOR FILTER E:

$\epsilon = \sqrt{10^{0.08}-1} = 0.4497$

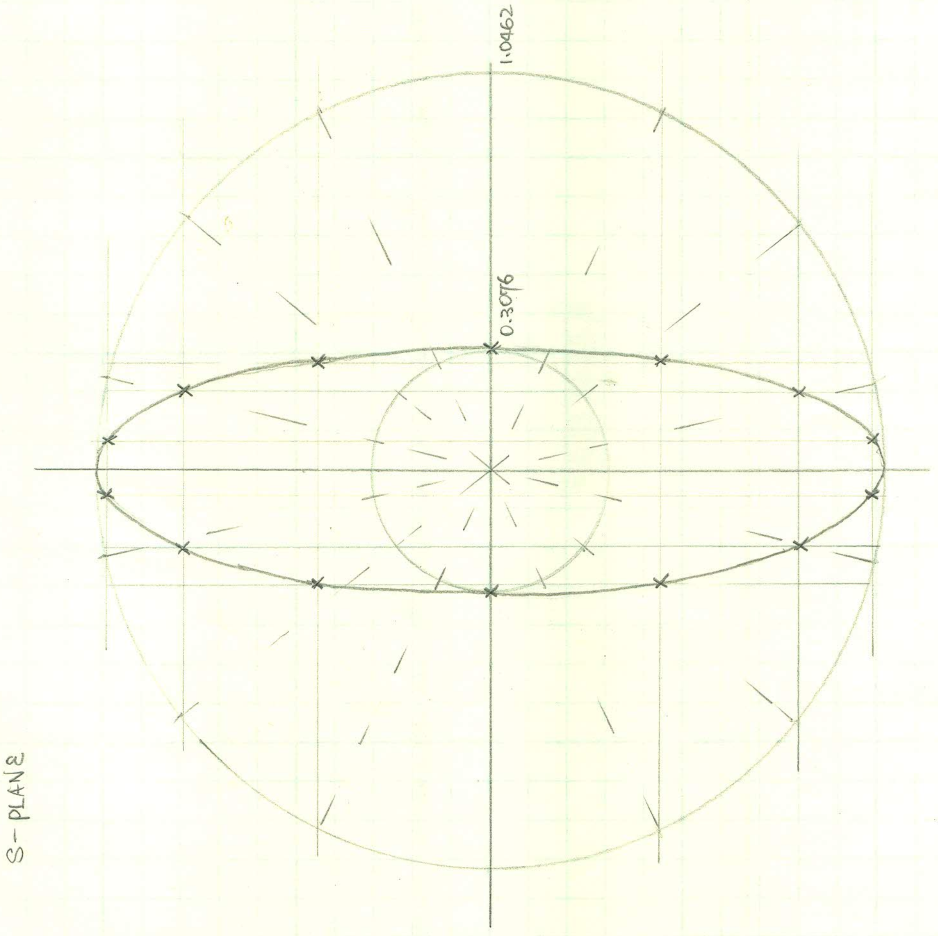
$10\log\left[1+0.2023\, T_n^2(1.3)\right] > 40 \Rightarrow T_n(1.3) > 222.3209 \Rightarrow n \geq 9$

FILTER	A	B	C	D	E
ORDER	8	7	9	9	9
ϵ	0.1875	0.2434	0.3106	0.5641	0.4497

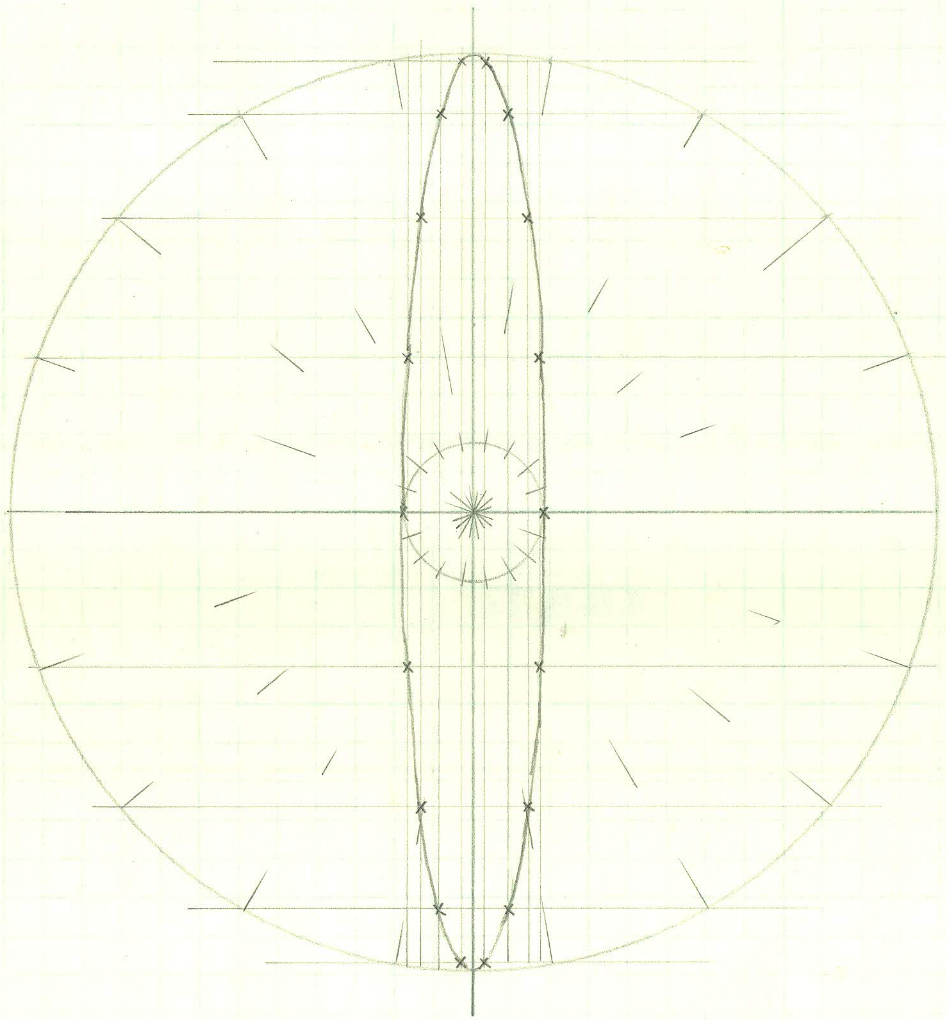
(a) DETERMINE AND PLOT POLE LOCATIONS:



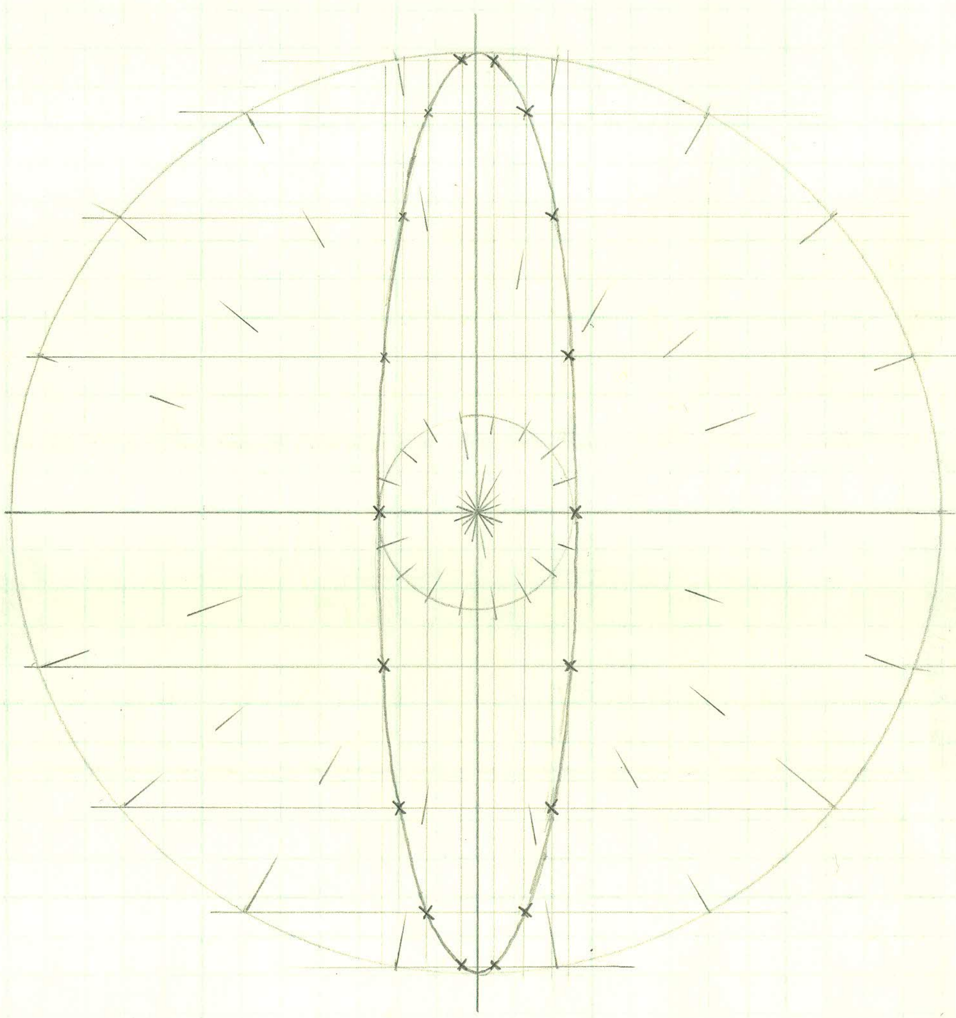
FILTER A , n=8 , θ=22.5° , a=0.3014 b=1.0444



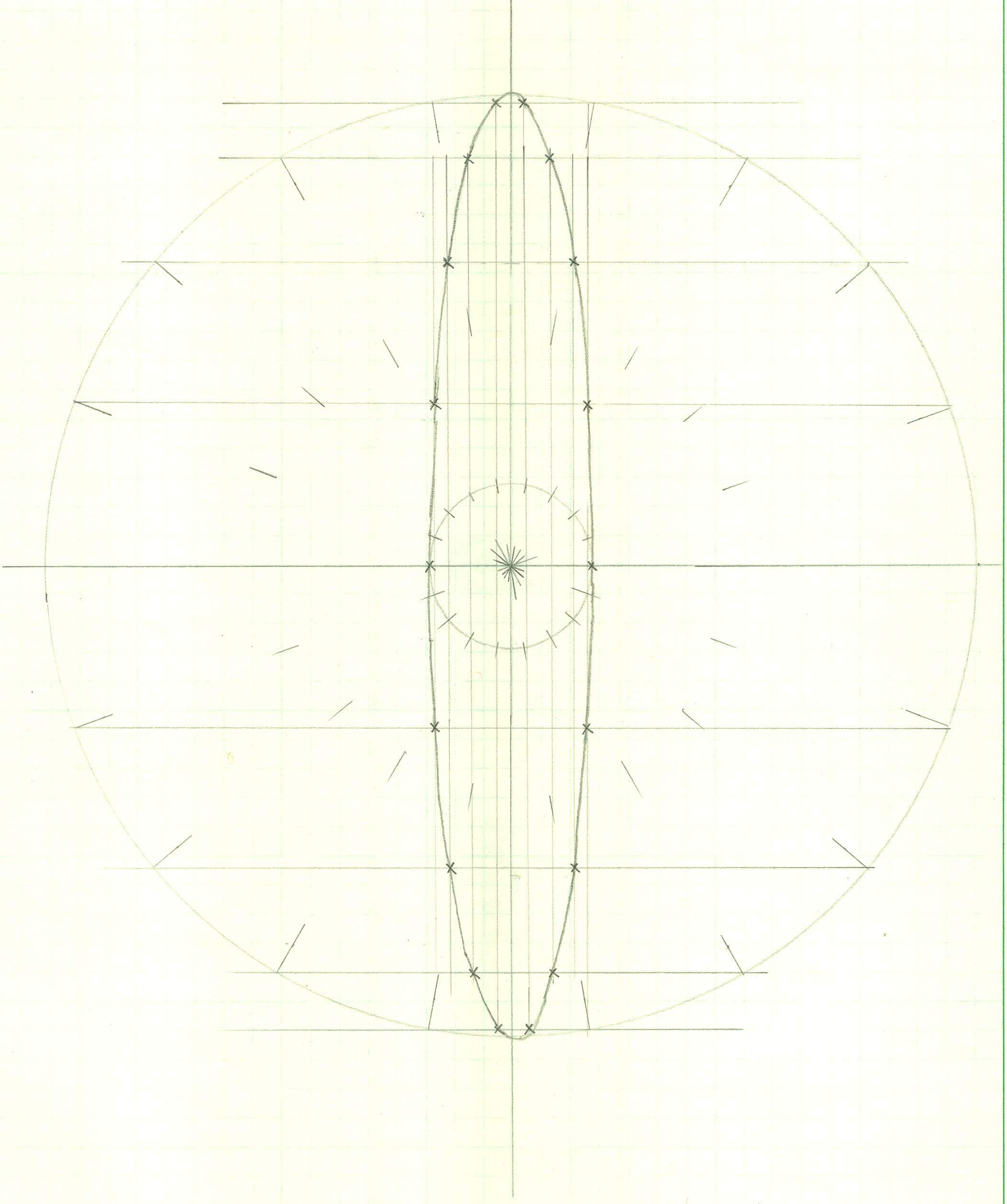
FILTER B n=7 . θ=25.71° , a=0.3076 , b=1.0462



FILTER D : $n=9$, $\theta=20°$ $a=0.1491$, $b=1.0111$



FILTER C : $n=9$, $\theta=20°$, $a=0.2111$, $b=b=1.0220$



FILTER E: $n=9$, $\theta=20°$, $a=0.1719$, $b=1.0307$

(b)

FILTER A: $\epsilon = 0.1875$, $n = 8$

$$\sigma_K = -\sinh\left[\frac{1}{n}\sinh^{-1}\frac{1}{\varepsilon}\right]\sin\frac{2k-1}{2n}\pi = -0.3014 \cdot \sin[(2k-1)\times 11.25°]$$

$$\omega_K = \cosh\left[\frac{1}{n}\sinh^{-1}\frac{1}{\varepsilon}\right]\cos\frac{2k-1}{2n}\pi = 1.0444 \cdot \cos[(2k-1)\times 11.25°]$$

k	1	2	3	4	5	6	7	8
σ_K	-0.0588	-0.1674	-0.2506	-0.2956	-0.2956	-0.2506	-0.1674	-0.0588
ω_k	1.0243	0.8684	0.5802	0.2038	-0.2038	-0.5802	-0.8684	-1.0243

$$H(S) = \frac{1}{(S+0.0588-j1.0243)(S+0.0588+j1.0243)(S+0.1674-j0.8684)(S+0.1674+j0.8684)}$$

$$\cdot \frac{1}{(S+0.2506-j0.5802)(S+0.2506+j0.5802)(S+0.2956-j0.2038)(S+0.2956-j0.2038)}$$

$$= \frac{1}{(S^2+0.1176S+1.0526)(S^2+0.3348S+0.7821)(S^2+0.5012S+0.3994)(S^2+0.5912S+0.1289)}$$

FILTER B: $\epsilon = 0.2434$, $n = 7$

$$\sigma_K = -\sinh\left[\frac{1}{n}\sinh^{-1}\frac{1}{\varepsilon}\right]\sin\frac{2k-1}{2n}\pi = -0.3076 \cdot \sin[(2k-1)\times 12.85°]$$

$$\omega_K = \cosh\left[\frac{1}{n}\sinh^{-1}\frac{1}{\varepsilon}\right]\cos\frac{2k-1}{2n}\pi = 1.0462 \cdot \cos[(2k-1)\times 12.85°]$$

k	1	2	3	4	5	6	7
σ_K	-0.0684	-0.1917	-0.2771	-0.3076	-0.2771	-0.1917	-0.0684
ω_K	1.0200	0.8182	0.4545	0	-0.4545	-0.8182	-1.0200

$$H(S) = \frac{1}{(S+0.0684-j1.0200)(S+0.0684+j1.0200)(S+0.1917-j0.8182)}$$

$$\cdot \frac{1}{(S+0.1917+j0.8182)(S+0.3076)(S+0.2771-j0.4545)(S+0.2771+j0.4545)}$$

$$= \frac{1}{(S^2+0.1368S+1.0451)(S^2+0.3834S+0.7062)(S^2+0.5542S+0.2834)(S+0.3076)}$$

FILTER C: $\epsilon = 0.3106$, $n = 9$

$$\sigma_K = -\sinh\left[\frac{1}{n}\sinh^{-1}\frac{1}{\varepsilon}\right]\sin\frac{2k-1}{2n}\pi = -0.2111\sin[(2k-1)\times 10°]$$

$$\omega_K = \cosh\left[\frac{1}{n}\sinh^{-1}\frac{1}{\varepsilon}\right]\cos\frac{2k-1}{2n}\pi = 1.0220\cos[(2k-1)\times 10°]$$

k	1	2	3	4	5	6	7	8	9
σ_K	-0.0367	-0.1056	-0.1617	-0.1984	-0.2111	-0.1984	-0.1617	-0.1056	-0.0367
ω_K	1.0065	0.8851	0.6569	0.3495	0	-0.3495	-0.6569	-0.8851	-1.0065

$$H(S) = \frac{1}{(S+0.0367-j1.0065)(S+0.0367+j1.0065)(S+0.1056-j0.8851)(S+0.1056+j0.8851)(S+0.2111)}$$

$$\frac{1}{(S+0.1617-j0.6569)(S+0.1617+j0.6569)(S+0.1984-j0.3495)(S+0.1984+j0.3495)}$$

$$= \frac{1}{(S^2+0.0734S+1.0144)(S^2+0.2112S+0.7946)(S^2+0.3234S+04577)(S^2+0.3968S+0.1615)(S+0.2111)}$$

FILTER D: $\epsilon = 0.5641$, $n = 9$

$$\sigma_K = -0.1491\sin[(2k-1)\times 10°]$$

$$\omega_K = 1.0111\cos[(2k-1)\times 10°]$$

k	1	2	3	4	5	6	7	8	9
σ_K	-0.0259	-0.0746	-0.1142	-0.1401	-0.1491	-0.1401	-0.1142	-0.0746	-0.0259
ω_K	0.9957	0.8756	0.6499	0.3458	0	-0.3458	-0.6499	-0.8756	-0.9957

$$H(S) = \frac{1}{(S+0.0259-j0.9957)(S+0.0259+j0.9957)(S+0.0746-j0.8756)(S+0.0746+j0.8756)(S+0.1491)}$$

$$\times\frac{1}{(S+0.1142-j0.6499)(S+0.1142+j0.6499)(S+0.1401-j0.3458)(S+0.1401+j0.3458)}$$

$$= \frac{1}{(S^2+0.0518S+0.9921)(S^2+0.1492S+0.7722)(S^2+0.2284S+0.4354)(S^2+0.2802S+0.1392)(S+0.1491)}$$

FILTER E : $\epsilon = 0.4497$, $n=9$

$$\sigma_K = -0.1719 \sin[(2k-1)\times 10^\circ]$$

$$\omega_K = 1.0307 \cos[(2k-1)\times 10^\circ]$$

K	1	2	3	4	5	6	7	8	9
	−0.0299	−0.0860	−0.1317	−0.1615	−0.1719	−0.1615	−0.1317	−0.0860	−0.0299
	1.0150	0.8926	0.6625	0.3525	0	−0.3525	−0.6625	−0.8926	−1.0150

$$H(S) = \frac{1}{(S+0.0299-j1.0150)(S+0.0299+j1.0150)(S+0.0860-j0.8926)(S+0.0860+j0.8926)(S+0.1719)}$$

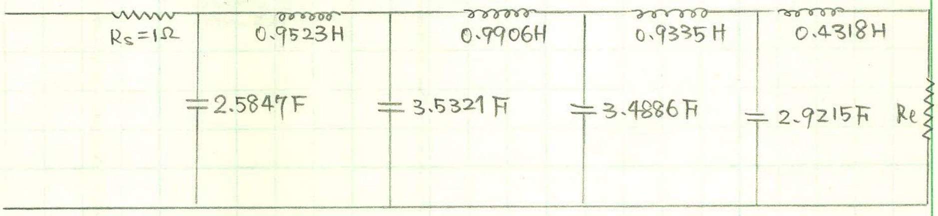
$$\times \frac{1}{(S+0.1317-j0.6625)(S+0.1317+j0.6625)(S+0.1615-j0.3525)(S+0.1615+j0.3525)}$$

$$= \frac{1}{(S^2+0.0598S+1.0311)(S^2+0.1720S+0.8041)(S^2+0.2634S+0.4563)(S^2+0.3230S+0.1503)(S+0.1719)}$$

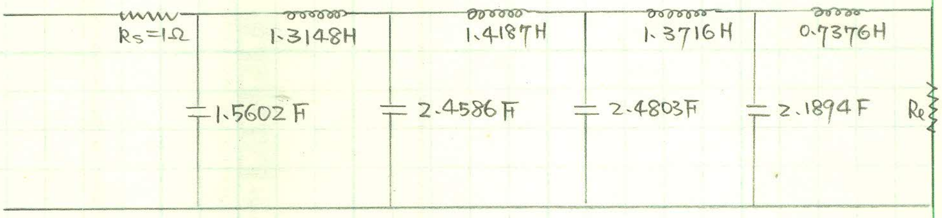
** <u>Circuit Realization</u> =

FILTER A :

$R_l = 0.5\Omega$ (a)

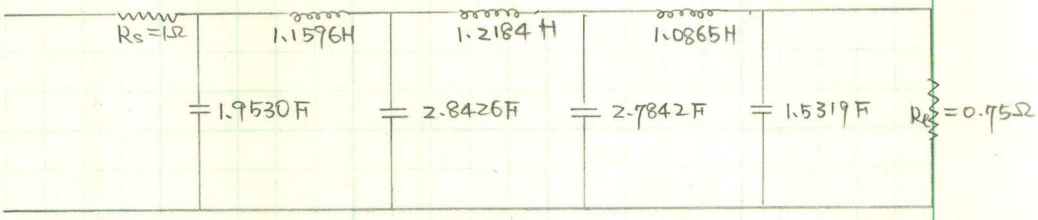


$R_l = 1.5\Omega$ (b)



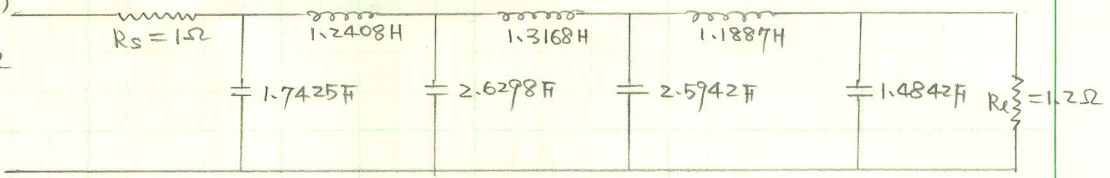
FILTER B : (a)

$R_l = 0.75\Omega$



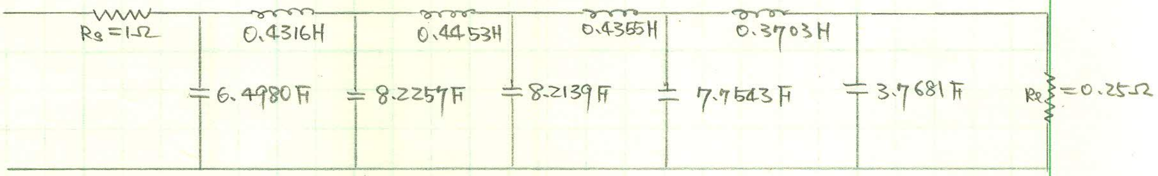
FILTER B: (b)

$R_\ell = 1.2\Omega$

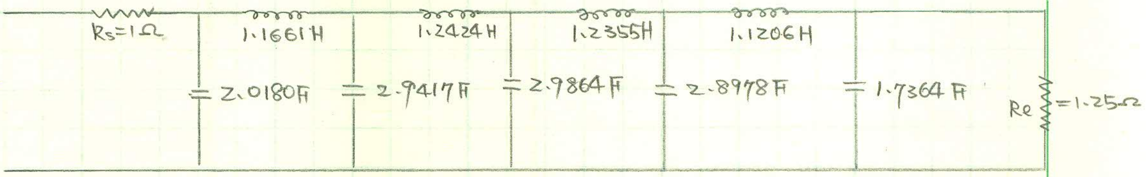


FILTER C:

(a) $R_\ell = 0.25\Omega$

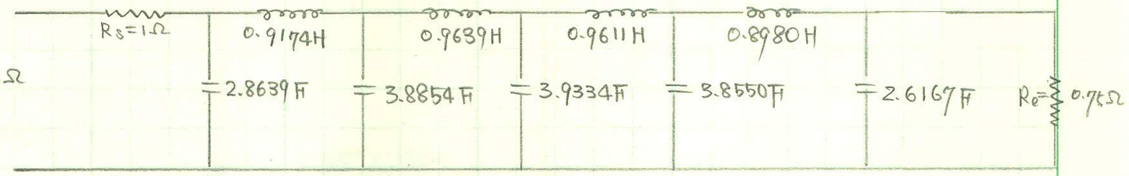


(b) $R_\ell = 1.25\Omega$

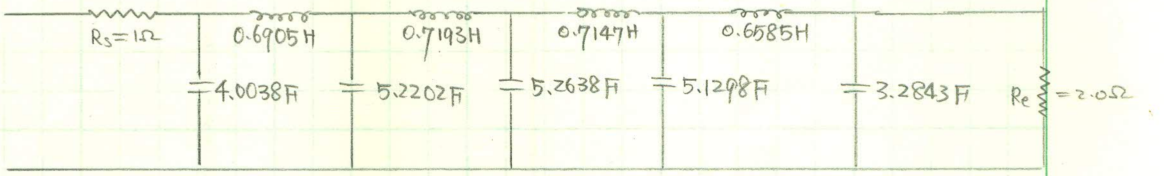


FILTER D:

(a) $R_\ell = 0.75\Omega$

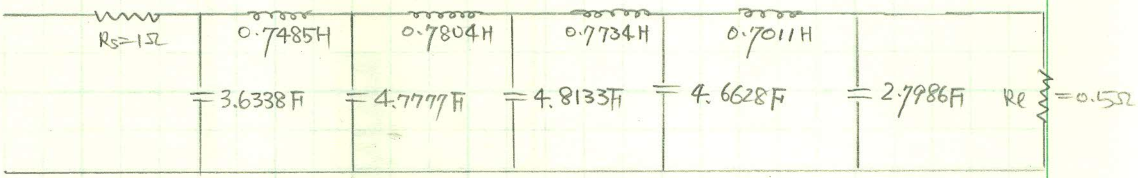


(b) $R_\ell = 2.0\Omega$

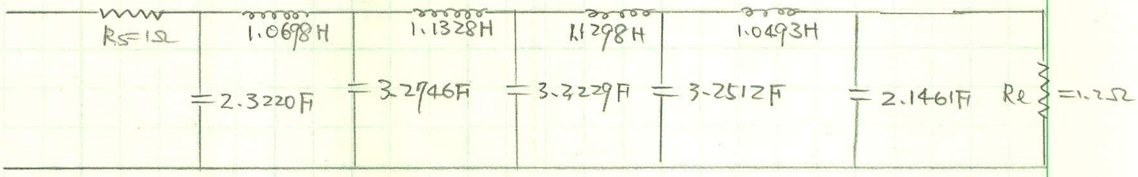


FILTER E:

(a) $R_\ell = 0.5\Omega$

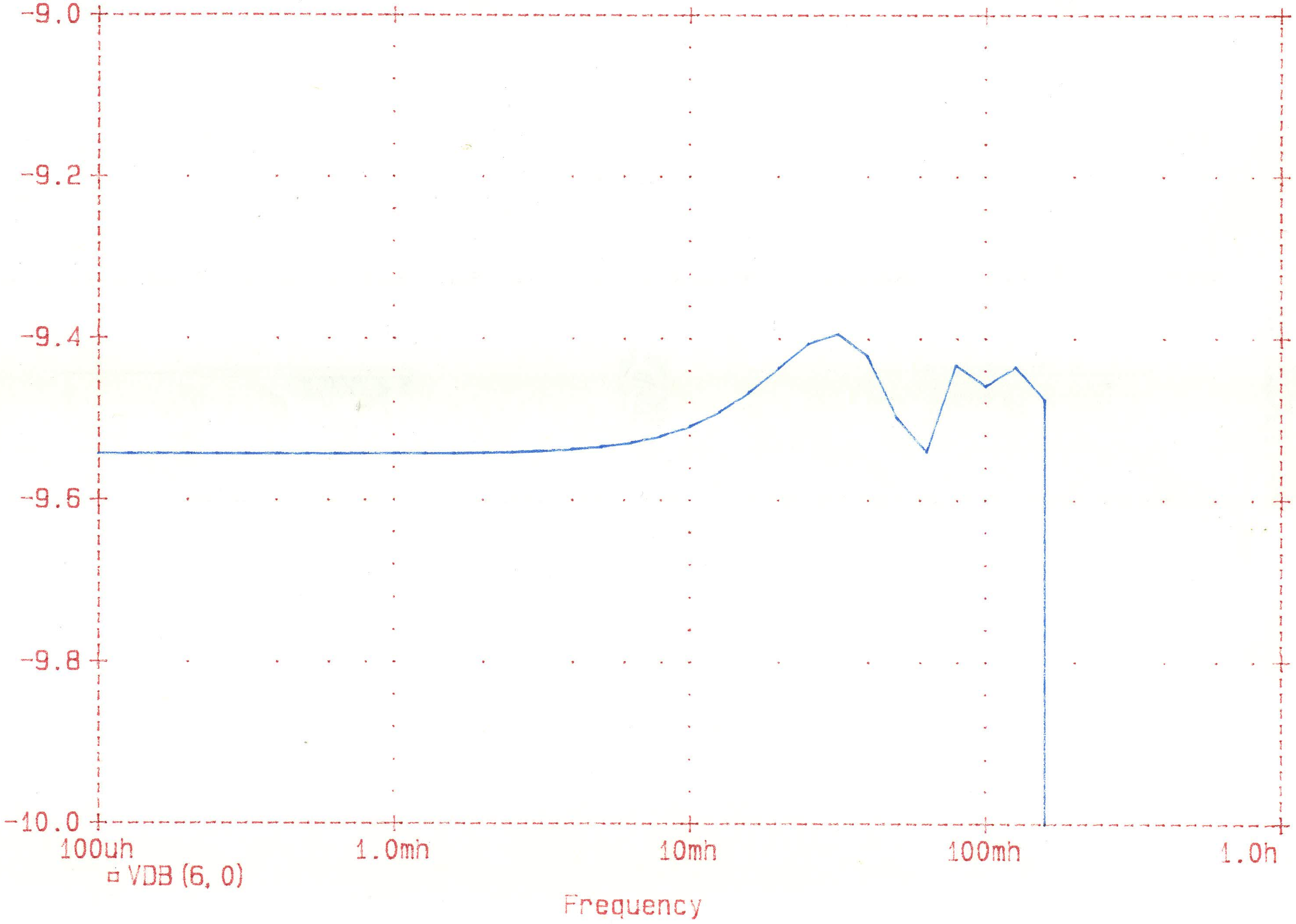


(b) $R_\ell = 1.2\Omega$



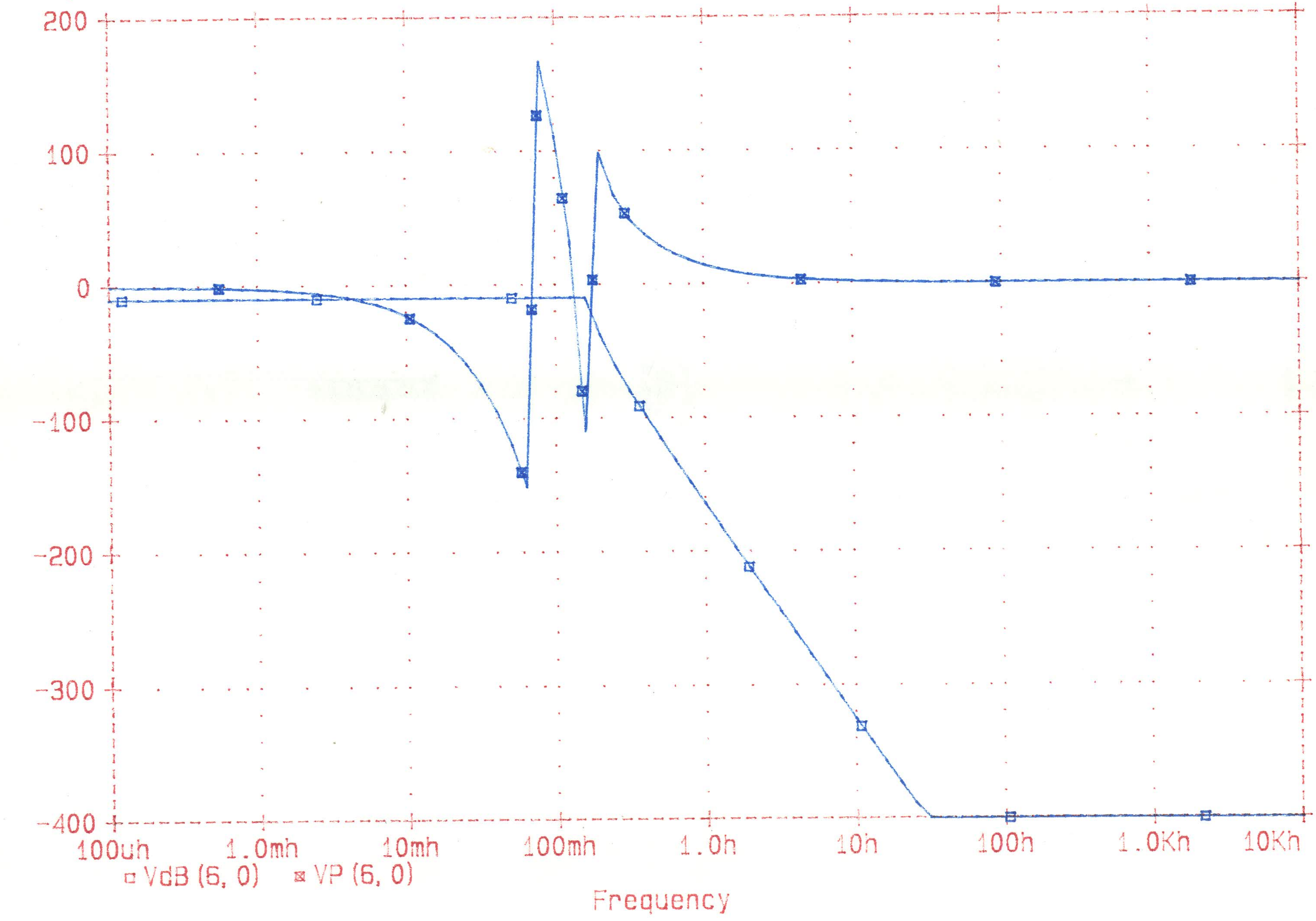
CHEBYSHEV 8TH-ORDER LOWPASS FOLTER $R_0 = 0.5$

Date/Time run: 1/ 1/80 0: 32: 22 Temperature: 27.0



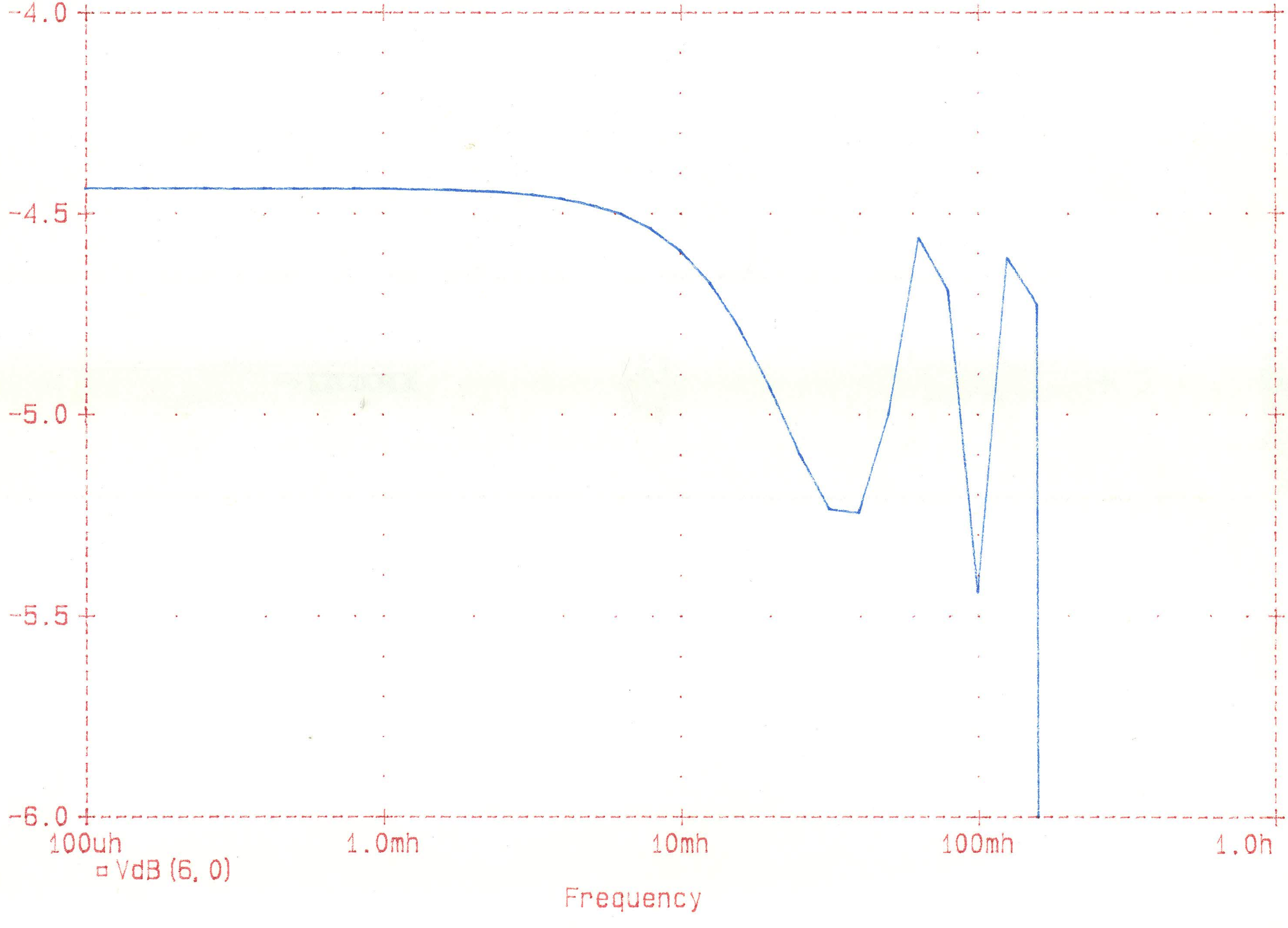
CHEBYSHEV 8TH-ORDER LOWPASS FOLTER $R_o = 0.5$

Date/Time run: 1/ 1/80 0: 32: 22 Temperature: 27.0



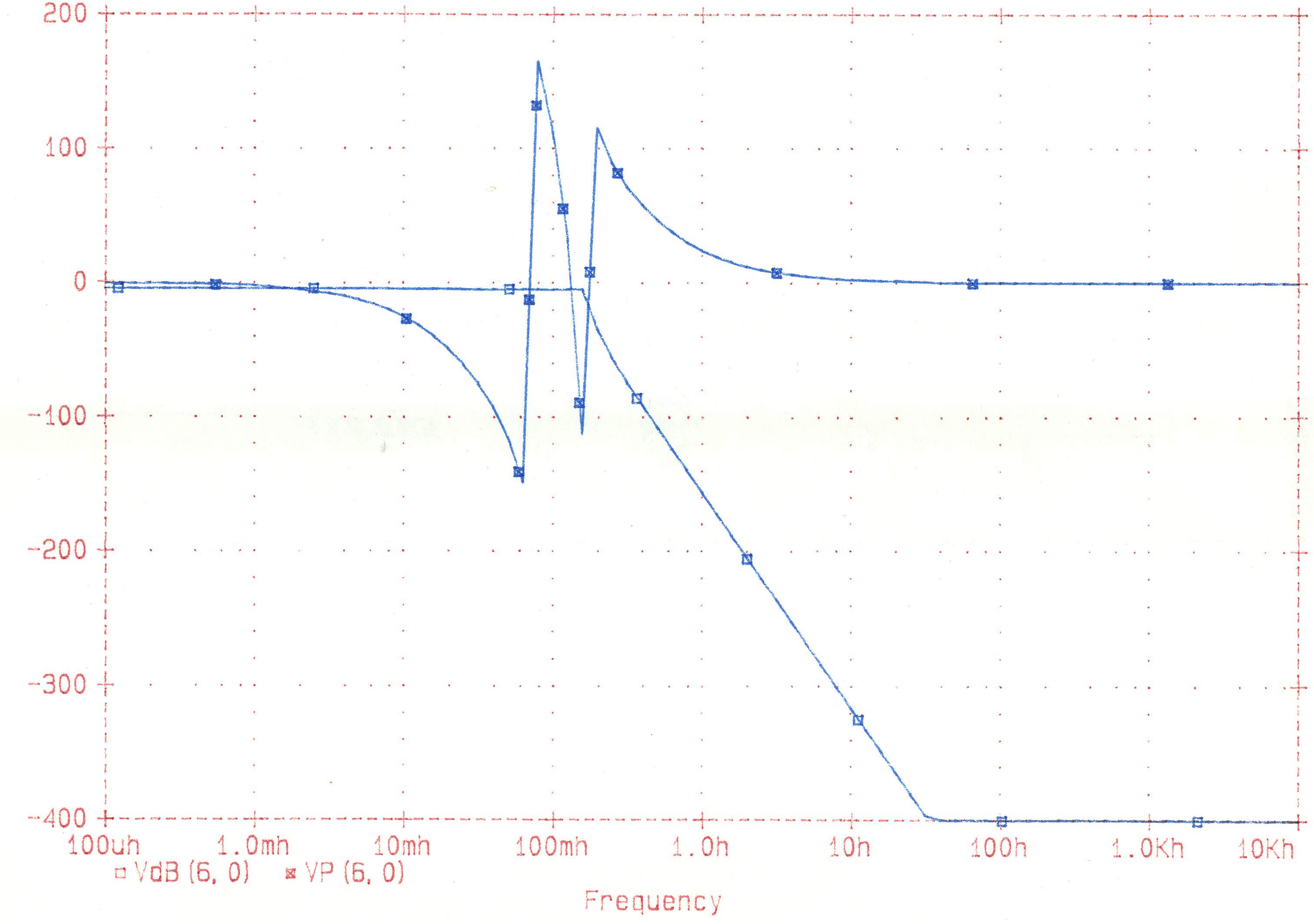
CHEBYSHEV LOWPASS FILTER -- 8TH-ORDER   Ro=1.5

Date/Time run: 12/ 3/86 22:13:36   Temperature: 27.0



CHEBYSHEV LOWPASS FILTER -- 8TH-ORDER Ro=1.5

Date/Time run: 12/ 3/86 22:13:36 Temperature: 27.0



$\frac{100}{105}$

E-E 448 Passive and Active Filter Design
Homework

BENMEI CHEN

Compare the magnitude and phase responses of 4-th order Bessel, Butterworth and transitional filters. For each of the three filters, determine the following:

a) H(s), b) pole locations, c) realization (schematic), and d) from SPICE

$|H(j\omega)|^2$ (dB) and $\phi(\omega)$ (degrees) for $0\leq\tau_0\omega<10$ rad.

Make a statement about the relative merits of the three filters.

The group delays (for Bessel filters) and the load resistances are given below:

Filter	Group Delay, $\tau(0)$ (sec.)	Load Resistance (ohms)
A	2	.5
B	2	1.5
C	2.5	2.0
D	2.5	.75
E	3	.80
F	3	2.0

A	B	C	D	E	F
G.A.	B.C.	F.H.	T.M.	K.P.	H.R.
L.A.	R.H.	B.K.	S.N.	T.R.	T.W.
C.C.	A.H.	B.M.	M.P.	M.R.	D.Y.

Compare the magnitude and phase responeses of 4-th order Bessel, Butterworth and transitional filters. For each of these three filters, determine the

a) H(S)

BESSEL : B : $\tau(0) = 2$

$$\hat{B}_4(S) = (2\times4-1)\hat{B}_3(S) + S^2\hat{B}_2(S) = 7\hat{B}_3(S) + S^2 \cdot \hat{B}_2(S)$$

$$= 7(S^3+6S^2+15S+15) + S^2(S^2+3S+3)$$

$$= 7S^3 + 42S^2 + 105S + 105 + S^4 + 3S^3 + 3S^2$$

$$= S^4 + 10S^3 + 45S^2 + 105S + 105$$

$$H(S) = \frac{k}{\hat{B}_4(2S)}$$

$$k = \hat{B}_4(0) = 105$$

$$\hat{B}_4(2S) = 16S^4 + 80S^3 + 180S^2 + 210S + 105$$

$$H(S) = \frac{105}{16S^4 + 80S^3 + 180S^2 + 210S + 105}$$

Butterworth : $H(S) = \dfrac{1}{S^4 + 2.61312593S^3 + 3.41421356S^2 + 2.61312593S + 1}$

TRANSITIONAL : POLES OF BESSEL FILTER : $S_{1,2} = -1.448105302 \pm j0.433617062$

$S_{3,4} = -1.051894699 \pm 1.328709021$

POLES OF Butterwoth FILTER : $S_{1,2} = -0.38268343 \pm j0.92387953$

$S_{3,4} = -0.92387953 \pm j0.38268343$

POLES FOR TRAN. FILTER

$$S_{1,2} = \frac{-1.051894699 - 0.38268343}{2} \pm j \cdot \frac{1.328709021 + 0.92387953}{2}$$

$$= -.717289065 \pm j1.126294276$$

$$S_{3,4} = \frac{-1.448105302 - 0.92387953}{2} \pm j \cdot \frac{0.433617062 + 0.38268343}{2}$$

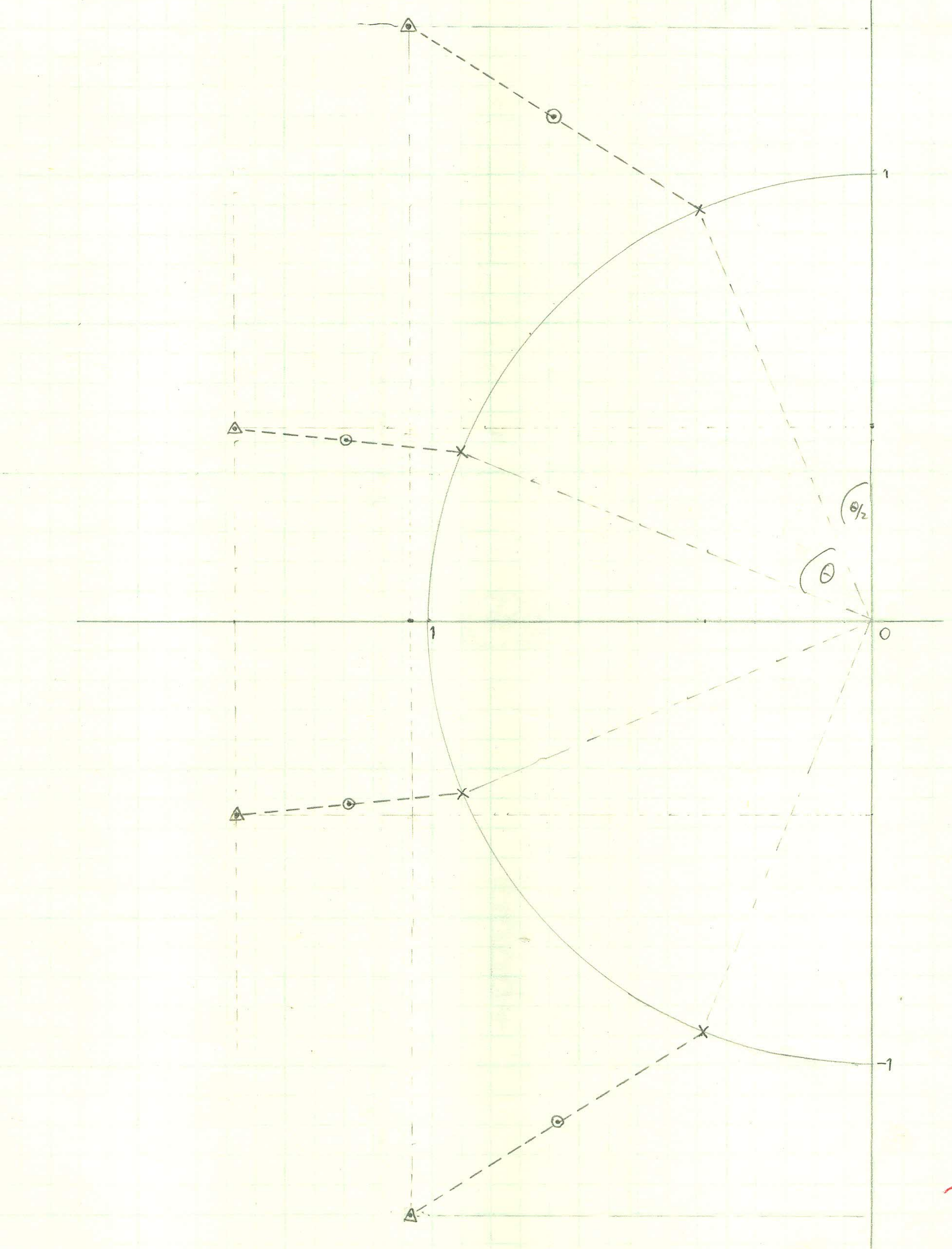
$$= -1.185992416 \pm j\,0.408150246$$

$$H(S) = \frac{k}{(S^2 + 1.434578130S + 1.783042399) \times (S^2 + 2.371984832S + 1.573164634)}$$

$$= \frac{2.805019243}{S^4 + 3.806562962S^3 + 6.759004598S^2 + 6.486177104S + 2.805019243}$$

b) POLE LOCATIONS :

S-PLANE :



X -- POLES OF BUTTERWORTH FILTER

△ POLES OF BESSEL FILTER

⊙ POLES OF TRANSITIONAL FIL.

↑ normalize

-5

C) REALIZATION (SCHEMATIC):

BESSEL FILTER: $H(S) = \dfrac{k}{16S^4+80S^3+180S^2+210S+105}$

$$H(0) = \frac{k}{105} = \frac{1.5}{1+1.5} \Rightarrow k = 63$$

(1) $$\rho(S)\rho(-S) = 1 - \frac{4}{1.5} \cdot \frac{3969}{(16S^4+80S^3+180S^2+210S+105)(16S^4-80S^3+180S^2-210S+105)}$$

$$= \frac{256S^4 - 640S^6 + 2160S^4 - 6300S^2 + 441}{(16S^4+80S^3+180S^2+210S+105)(16S^4-80S^3+180S^2-210S+105)}$$

$$= \frac{16(S^4+4.29924S^3+7.99175S^2+6.75185S+1.31251) \times 16\,(S^4-4.29924S^3+\cdots)}{(16S^4+80S^3+180S^2+210S+105)(16S^4-80S^3+180S^2-210S+105)}$$

$$\rho(S) = \frac{S^4+4.29924S^3+7.99175S^2+6.75185S+1.31251}{S^4+5.S^3+11.25S^2+13.125S+6.5625}$$

$$Z_{in}(S) = \frac{1+\rho(S)}{1-\rho(S)} = \frac{2S^4+9.29924S^3+19.24175S^2+20.00185S+7.87501}{0.70076S^3+3.25825S^2+6.37315S+5.24999}$$

$$Z_{in}(0) = 1.5 = R_\ell$$

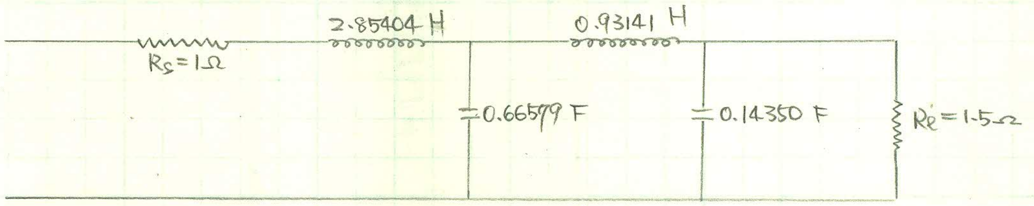
$$\begin{array}{r|l} & 2.85404S \\ 0.70076S^3+3.25825S^2+6.37315S+5.24999 & 2S^4+9.29924S^3+19.24175S^2+20.00185S+7.87501 \\ & 2S^4+9.29919S^3+18.18923S^2+14.98368S \\ \hline & 1.05252S^2+5.01817S+7.87501 \end{array}$$

$$\begin{array}{r|l} & 0.66579S \\ 1.05252S^2+5.01817S+7.87501 & 0.70076S^3+3.25825S^2+6.37315S+5.24999 \\ & 0.70076S^3+3.34105S^2+5.24312S \\ \hline \end{array}$$

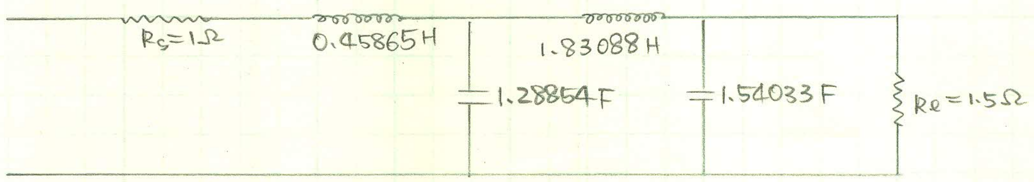
$$\begin{array}{r|l} & 0.93141S \\ 1.13003S+5.24999 & 1.05252S^2+5.01817S+7.87501 \\ & 1.05252S^2+4.88990S \\ \hline & 7.87501 \end{array}$$

$$\begin{array}{r|l} & 0.14350S \\ 7.87501 & 1.13003S+5.24999 \\ & 1.13003S \\ \hline & 5.24999 \end{array}$$

$$\begin{array}{r|l} & 1.5 \\ 5.24999 & 7.87501 \\ & 7.87501 \\ \hline & 0 \end{array}$$



BUTTERWORTH FILTER:



TRANSITIONAL FILTER:

$$H(S) = \frac{k}{S^4 + 3.80563S^3 + 6.75900S^2 + 6.48618S + 2.80502}$$

$$H(0) = \frac{k}{2.80502} = \frac{1.5}{1+1.5} \quad \Rightarrow \quad k = 1.68301$$

$$\rho(S)\rho(-S) = 1 - \frac{4}{1.5} \cdot \frac{2.83253}{(S^4+3.80563S^3+6.75900S^2+6.48618S+2.80502)(S^4-3.80563S^3+6.759S^2-6.48618S+2.80502)}$$

$$= \frac{S^8 - 0.96482S^6 + 1.92734S^4 - 4.15227S^2 + 0.31473}{(S^4+3.80563S^3+6.75900S^2+6.48618S+2.80502)(S^4-3.80563S^3+6.759S^2-6.48618S+2.80502)}$$

$$= \frac{(S^4+3.14080S^3+4.44991S^2+3.02414S+0.56103)(S^4-3.14080S^3+4.44991S^2-\cdots)}{(S^4+3.80563S^3+6.75900S^2+6.48618S+2.80502)(S^4-3.80563S^3+6.75900S^2-\cdots)}$$

$$\rho(S) = \frac{S^4+3.14080S^3+4.44991S^2+3.02414S+0.56103}{S^4+3.80563S^3+6.75900SS^2+6.48618S+2.80502}$$

$$Z_{in}(S) = \frac{1+\rho(S)}{1-\rho(S)} = \frac{2S^4+6.94643S^3+11.20891S^2+9.51032S+3.36605}{0.66483S^3+2.30909S^2+3.46204S+2.24399}$$

$$Z_{in}(0) = 1.50003 \approx 1.5\,\Omega = R_\ell$$

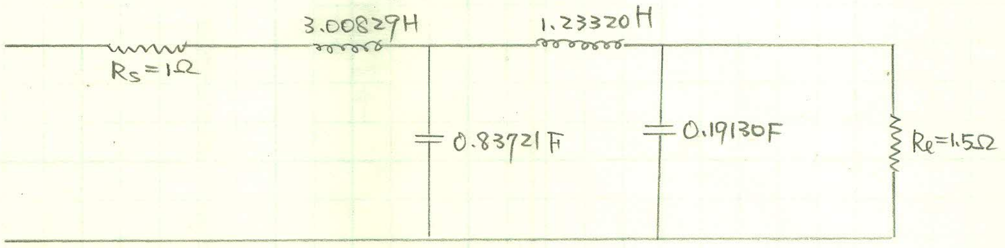
$$\begin{array}{r|l}
 & 3.00829S \\
0.66483S^3+2.30909S^2+3.46204S+2.24399 & 2S^4+6.94643S^3+11.20891S^2+9.51032S+3.36605 \\
 & 2S^4+6.94641S^3+10.41481S^2+6.75057S \\
\hline
 & 0.79410S^2+2.75975S+3.36605
\end{array}$$

$$\begin{array}{r|l}
 & 0.83721S \\
0.79410S^2+2.75975S+3.36605 & 0.66483S^3+2.30909S^2+3.46204S+2.24399 \\
 & 0.66483S^3+2.31050S^2+2.81811S \\
\hline
 & 0.64393S+2.24399
\end{array}$$

$$\begin{array}{r|l}
 & 1.23320S \\
0.64393S+2.24399 & 0.79410S^2+2.75975S+3.36605 \\
 & 0.79410S^2+2.76730S \\
\hline
 & 3.36605
\end{array}$$

$$\begin{array}{r|l}
 & 0.19130S \\
3.36605 & 0.64393S+2.24399 \\
 & 0.64393S \\
\hline
 & 2.24399
\end{array}$$

$$\begin{array}{r|l}
 & 1.50003 \\
2.24399 & 3.36605 \\
 & 3.36605 \\
\hline
 & 0
\end{array}$$



d) FROM THE GRAPHACIS WE GOT, IT IS EASY TO SEE THAT:

THE FREQUENCY RESPONE OF BESSEL FILTER DECREASE MORE SLOWLY THAN THE OTHER TWO FILTERS. AT $f = 1$ Hz

BESSEL = $|H(\omega)|_{dB} \doteq -53$ dB

TRANSITIONAL = $|H(\omega)|_{dB} \doteq -59$ dB

BUTTERWORTH = $|H(\omega)|_{dB} \doteq -64$ dB

BUT, IN PHASE RESPONE; BESSEL FILTER HAS A MOST-WIDE-LINEAR-PHASE-BAND.

LINEAR PHASE BAND:

BESSEL = $|f| < 0.225$ Hz

TRANSITIONAL = $|f| < 0.200$ Hz

BUTTERWORTH = $|f| < 0.179$ Hz

SO, IN MAGNITUDE CASE, BUTTERWORTH FILTER IS THE BEST.

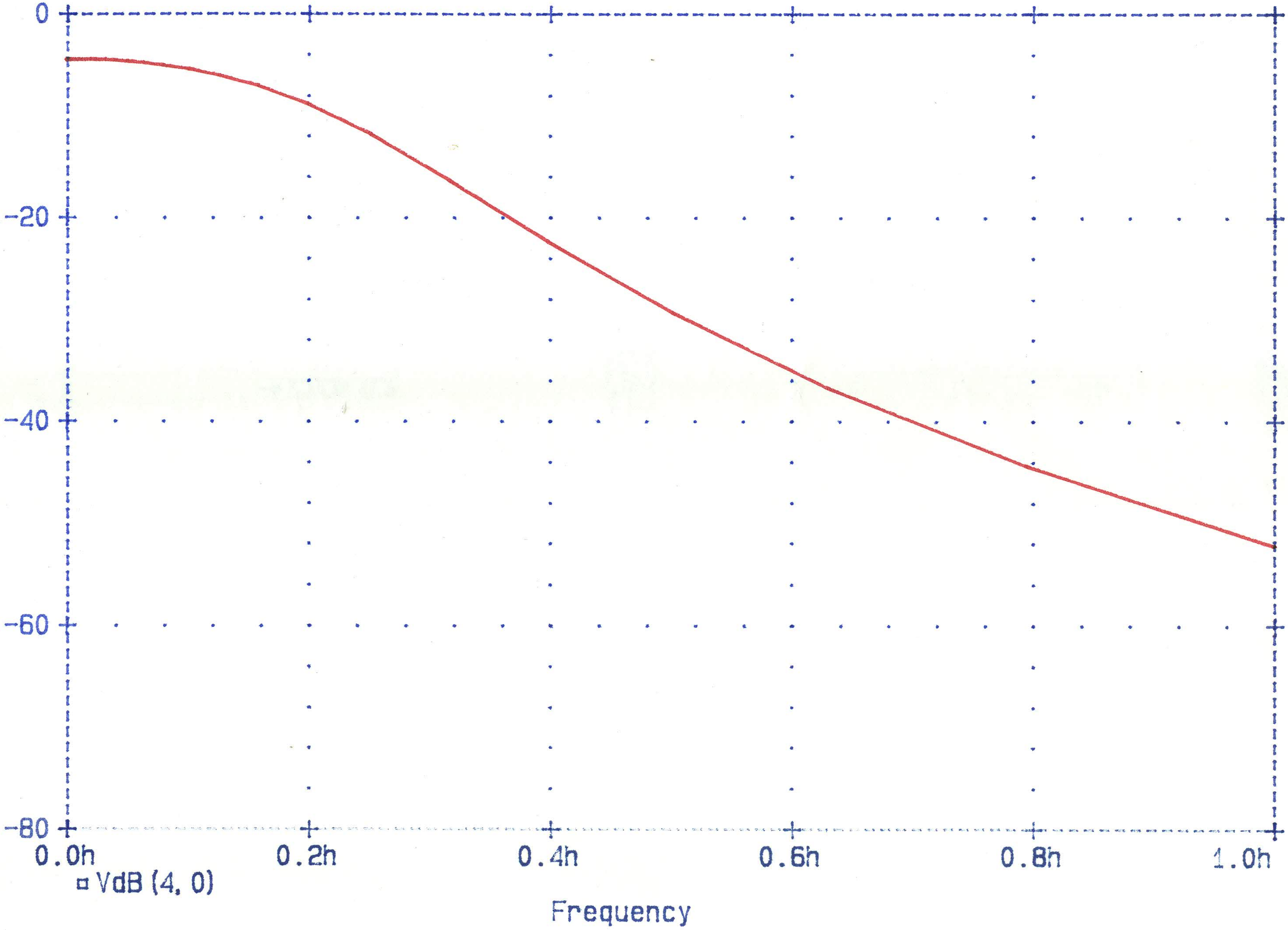
BESSEL FILTER IS THE WORST.

TRANSITIONAL FILTER IS BETWEEW THOSE TWO.

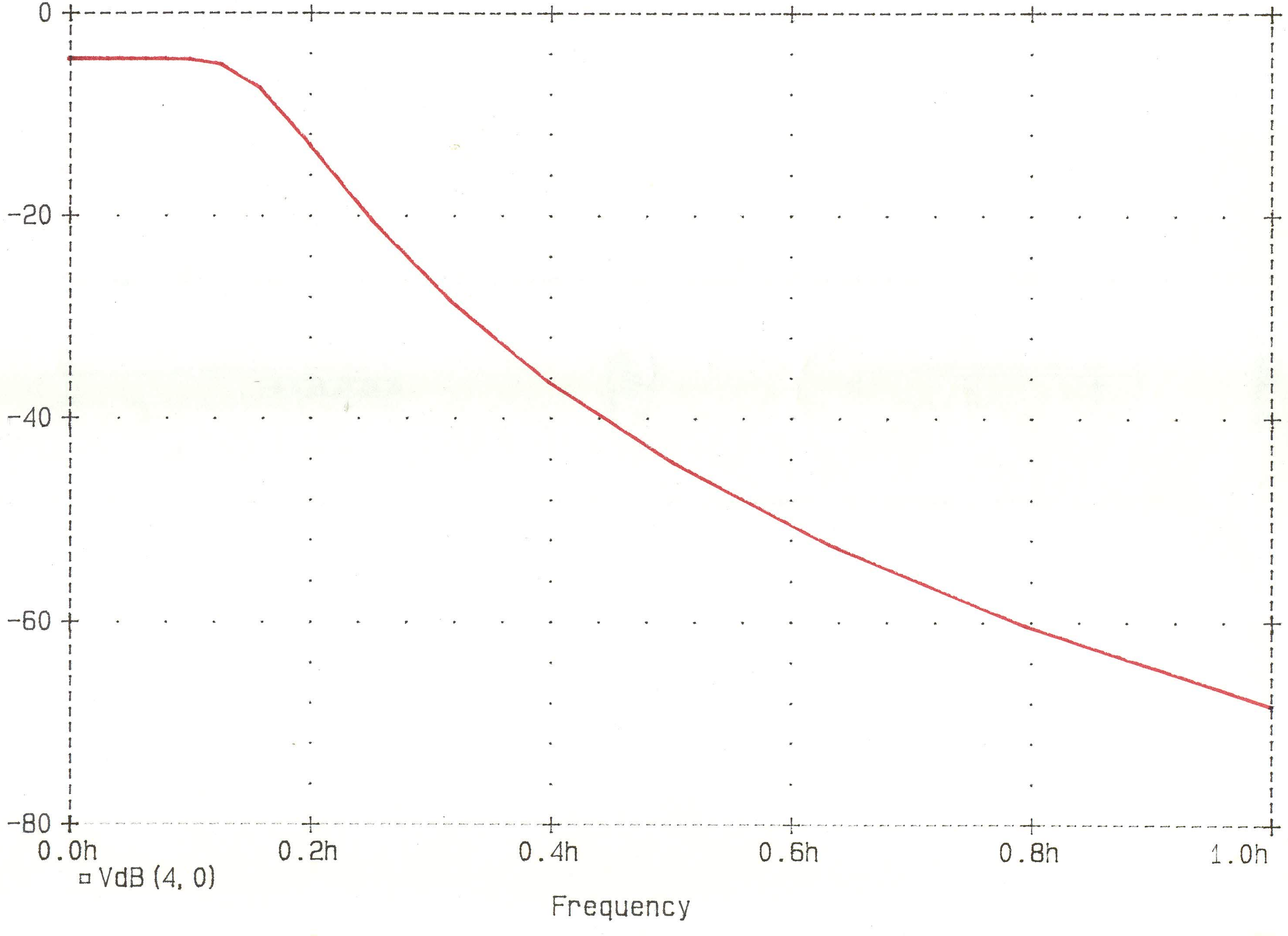
IN PHASE CASE, THEY ARE opposite.

BESSEL 4th-ORDER LOWPASS FILTER WITH Ro=1.5ohm

Date/Time run: 12/ 8/86 22: 07: 08 Temperature: 27.0

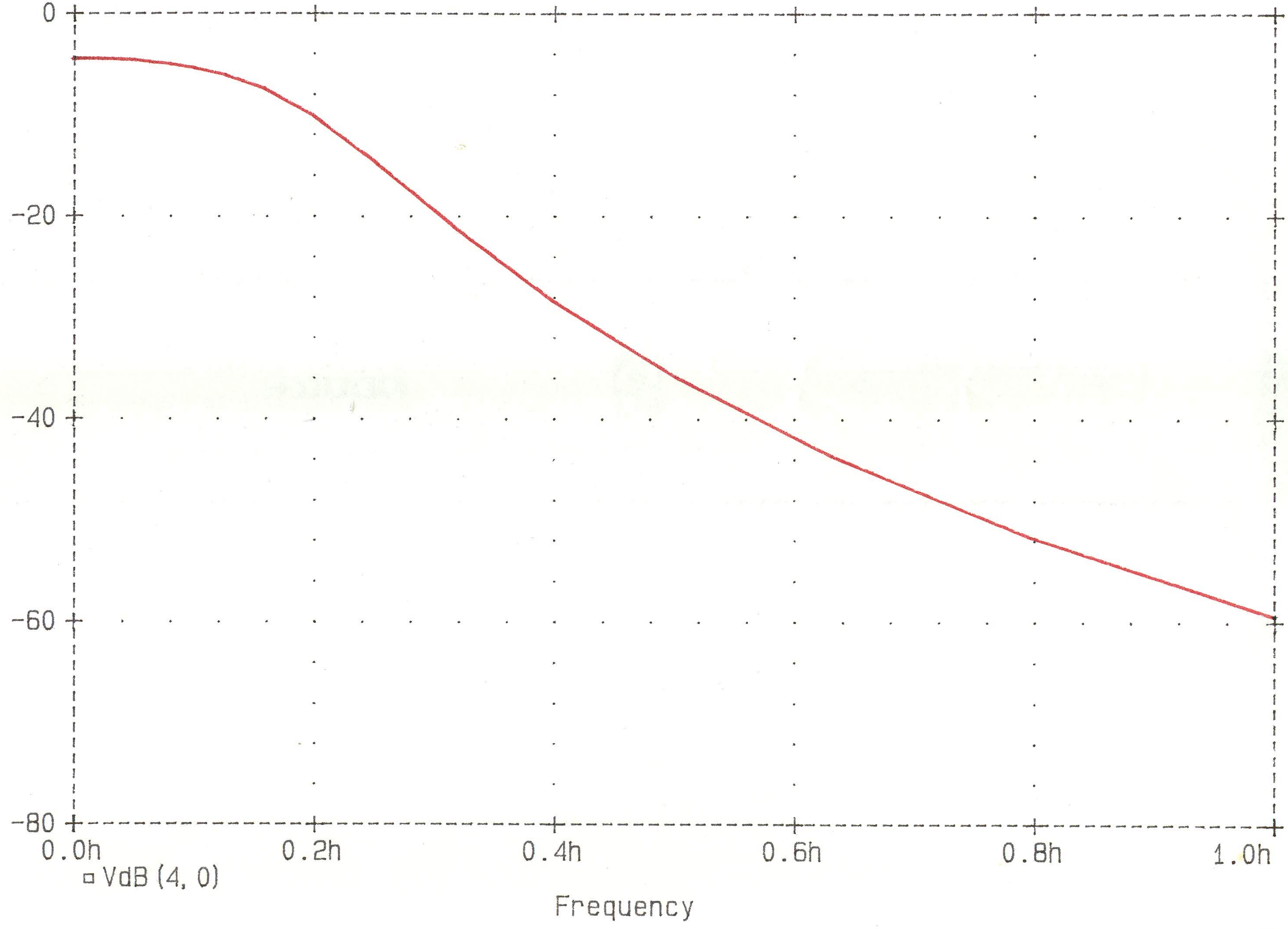


BUTTERWORTH 4th-ORDER LOWPASS FILTER WITH Ro=1.5ohm
Date/Time run: 12/ 8/86 21: 42: 54 Temperature: 27.0



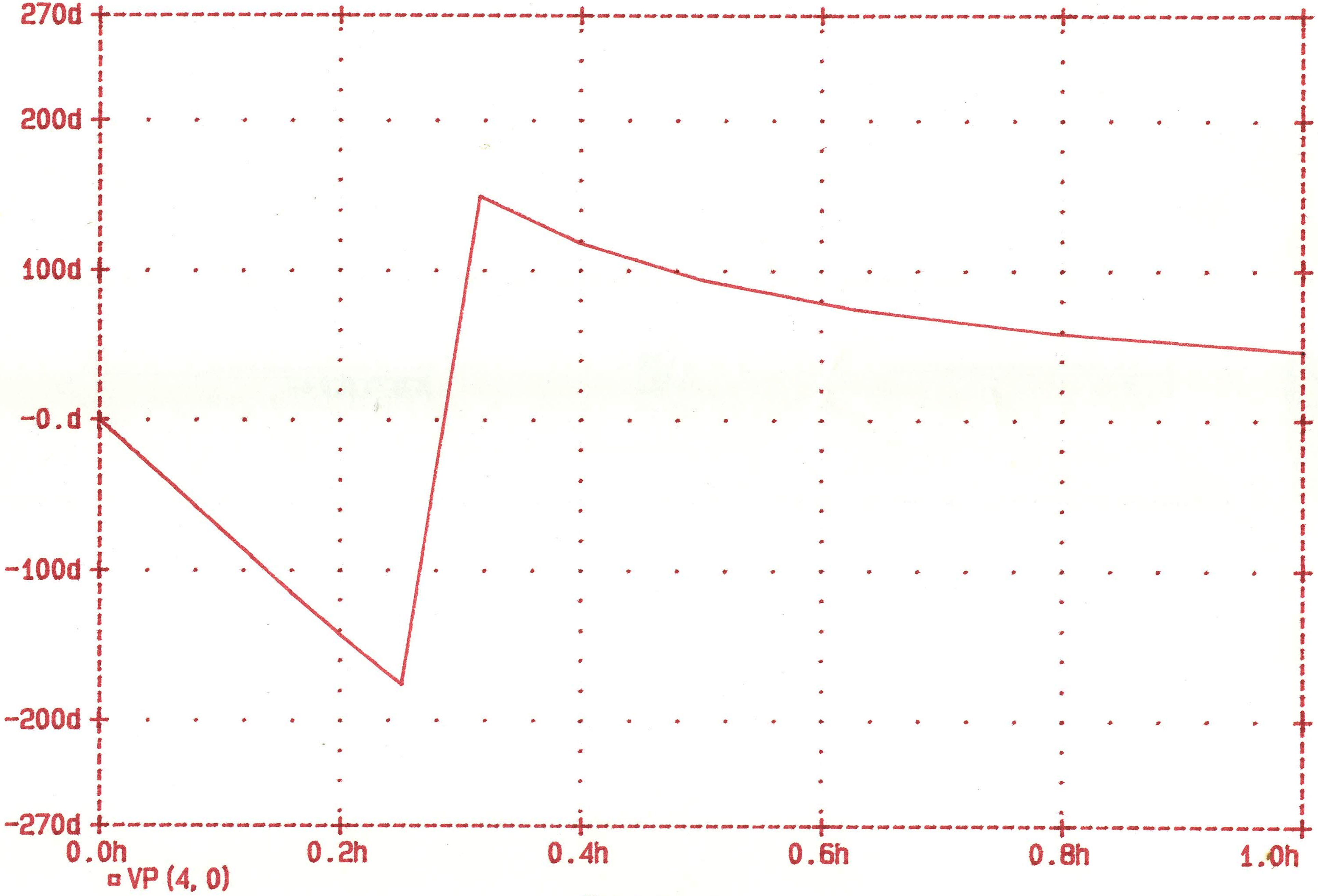
TRANSITIONAL 4th-ORDER LOWPASS FILTER WITH Ro=1.5ohm

Date/Time run: 12/ 8/86 22: 26: 32 Temperature: 27.0



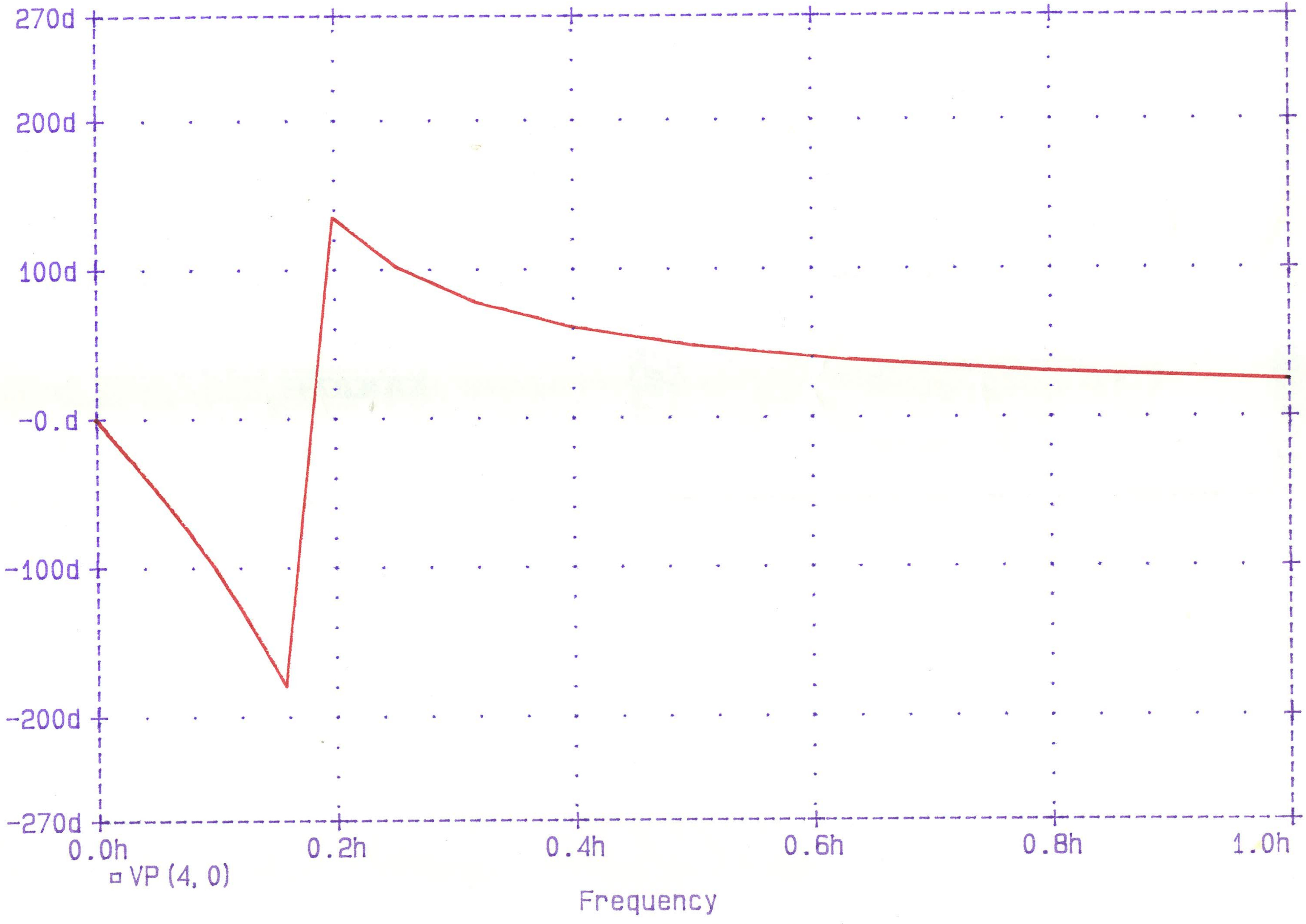
BESSEL 4th-ORDER LOWPASS FILTER WITH Ro=1.5ohm

Date/Time run: 12/ 8/86 22: 07: 08 Temperature: 27.0



BUTTERWORTH 4th-ORDER LOWPASS FILTER WITH Ro=1.5ohm

Date/Time run: 12/ 8/86 21: 42: 54 Temperature: 27.0



TRANSITIONAL 4th-ORDER LOWPASS FILTER WITH Ro=1.5ohm

Date/Time run: 12/ 8/86 22: 26: 32 Temperature: 27.0