E-E 448 ANALOG FILTER DESIGN
One Hour Examination
Dr. R. A. Birgenheier
October 14, 1986

97/100

BENMEI CHEN

(20) 1. $|H(j\omega)|^2 = \dfrac{1}{1+\omega^6}$

Find the minimum phase function, H(s).

(60) 2. Realize the following DP functions by LC networks of the form indicated. If the function is not realizable, clearly state the reasons.

a) <u>Foster's 1st Form</u>

(i) $Z(s) = \dfrac{s^3+10s}{2s^2+4}$, (ii) $Y(s) = \dfrac{s^2+2s+1}{s^3+s}$

b) <u>Cauer's 1st Form</u>

(i) $Y(s) = \dfrac{s^3+s}{s^4+5s^2+6}$, (ii) $Z(s) = \dfrac{2s^2+16}{s^3+16s}$

c) <u>Cauer's 2nd Form</u>

(i) $Z(s) = \dfrac{s^3+2s}{s^4+2s^2+1}$, (ii) $Y(s) = \dfrac{s^2+1}{s^3+2s}$

(20) 3. Determine which of the following are realizable RC DP functions. If not, clearly state the reason.

a) $Z(s) = \dfrac{s^3+6s^2+8s}{s^2+4s+3}$

b) $Z(s) = \dfrac{s^2+4s+3}{s^2+6s+8}$ not

c) $Y(s) = \dfrac{s^2+2s+0.75}{s^2+3s+2}$

d) $Y(s) = \dfrac{s^3+5s^2+6s}{s^2+5s+4}$

1. (1) $C(s)/D(s) \triangleq |H(j\omega)|^2 \Big|_{\omega = \frac{s}{j}} = \frac{1}{1+\omega^6}\Big|_{\omega=\frac{s}{j}} = \frac{1}{1-s^6} = \frac{-1}{s^6-1}$

(2) $C(s) = -1 \quad \Rightarrow \quad A(s) = 1$

(3) $D(s) = s^6 - 1 = (s-1)(s^5+s^4+s^3+s^2+s+1) =$

$= (s-1)(s+1)(s-0.5-j\frac{\sqrt{3}}{2})(s-0.5+j\frac{\sqrt{3}}{2})(s+0.5-j\frac{\sqrt{3}}{2})(s+0.5+j\frac{\sqrt{3}}{2})$

$B(s) = (s+1)(s+0.5-j\frac{\sqrt{3}}{2})(s+0.5+j\frac{\sqrt{3}}{2})$

$= (s+1)(s^2+s+1) = (s^3+2s^2+2s+1)$

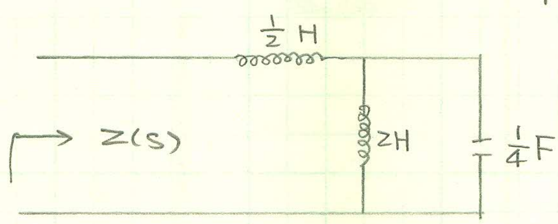
$\therefore \quad H(s) = 1 \Big/ (s^3+2s^2+2s+1)$

2. a) FOSTER'S 1st FORM

(i) $Z(s) = (s^3+10s)/(2s^2+4)$

$\frac{Z(s)}{s}\Big|_{s^2=p} = \frac{p+10}{2p+4} = \frac{\frac{1}{2}(2p+4)+8}{2p+4} = \frac{1}{2} + \frac{4}{p+2}$

$Z(s) = \frac{1}{2}s + \frac{4s}{s^2+2} = \frac{1}{2}s + \frac{1}{\frac{1}{4}s + \frac{1}{2s}}$



(ii) $Y(s) = (s^2+2s+1)/(s^3+s)$

$= (s+1)^2 / s(s^2+1)$

Y(s) is unrealizalible, Because Y(s) has two zeros at $s=-1$. (not on $j\omega$ axis also).

b) Cauer's 1st FORM

(i) $Y(s) = (s^3+s)/(s^4+5s^2+6) = s(s+j)(s-j)/(s+j\sqrt{2})(s-j\sqrt{2})(s+j\sqrt{3})(s-j\sqrt{3}$

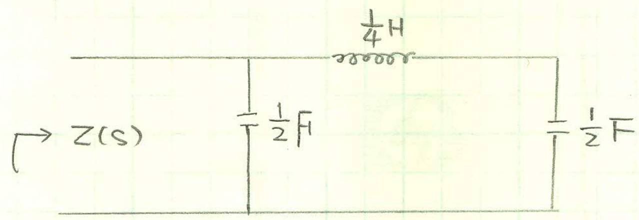
$Y(s)$ is unrealizable because the zeros and poles are

$j\sqrt{3}$ (pole), $j\sqrt{2}$) (pole), j (zero), 0 (zero), $-j$ (zero), ...

doesn't satisfy the properties.

(ii) $Z(s) = (2s^2+16)/(s^3+16s)$, $Y(s) = (s^3+16s)/(2s^2+16)$

$$2s^2+16 \,\big)\, s^3+16s \quad \rightarrow \tfrac{1}{2}s$$
$$s^3+8s$$
$$8s \,\big)\, 2s^2+16 \quad \rightarrow \tfrac{1}{4}s$$
$$2s^2$$
$$16 \,\big)\, 8s \quad \rightarrow \tfrac{1}{2}s$$
$$8s$$
$$0$$



c) Cauer's 2nd FORM

(i) $Z(s) = (s^3+2s)/(s^4+2s^2+1)$

$= s(s^2+2)/(s^2+1)(s^2+1)$

$Z(s)$ is unrealizable because $Z(s)$ has two poles

at $s = \pm j$.

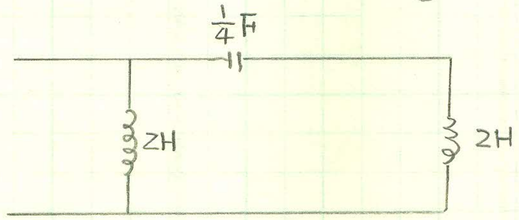
(ii) $Y(s) = (s^2+1)/(s^3+2s)$

$$2s + s^3 \,\overline{)\, 1 + s^2} \quad \frac{1}{2s} \;\cdots\cdots\to\; \frac{1}{2s}$$

$$\frac{1+\frac{1}{2}s^2}{\frac{1}{2}s} \;\overline{)\, 2 + s^2} \quad \frac{4}{s} \;\cdots\; \frac{1}{\frac{1}{4}s}$$

$$\frac{2}{s} \;\overline{)\, \frac{1}{2}} \quad \frac{1}{2s} \;\cdots\; \frac{1}{2s}$$

$$\frac{\frac{1}{2}}{0}$$



3. a) $Z(s) = (s^3+6s^2+8s)/(s^2+4s+3) \triangleq A(s)/B(s)$

$Z(s)$ is unrealizable because the degree of $A(s)=3$ > the degree of $B(s)=2$.

b) $Z(s) = (s^2+4s+3)/(s^2+6s+8)$

$= (s+1)(s+3)/[(s+2)(s+4)]$

$Z(s)$ is unrealizable because $2 > 1$ doesn't satisfy the properties of ZRC. which property? 3

c) $Y(s) = (s^2+2s+0.75)/(s^2+3s+2)$

$= (s+0.5)(s+1.5)/[(s+1)(s+2)]$

what about poles and zeros?

$Y(s) = (s^2+3s+2) - (s+1.25)/[s^2+3s+2)$

$= 1 - (s+1.25)/[(s+1)(s+2)]$

$= 1 + \frac{-0.25}{s+1} + \frac{-0.75}{s+2}$ $\quad (1>0,\ -0.25 \text{ and } -0.75 < 0)$

$Y(\sigma) = 1 + \frac{-0.25}{\sigma+1} + \frac{-0.75}{\sigma+2}$

$\frac{dY(\sigma)}{d\sigma} = \frac{0.25}{(\sigma+1)^2} + \frac{0.75}{(\sigma+2)^2} > 0$ (except at the poles)

So, $Y(s)$ is realizable.

3. d) $Y(s) = (s^3+5s^2+6s)/(s^2+5s+4)$

$= s(s^2+5s+6)/(s^2+5s+4)$

$= s(s+2)(s+3)/[(s+1)(s+4)]$

Y(S) is unrealizable because the poles and zeros of Y(S) are like:

-4 (pole), -3 (zero), -2 (zero), -1 (pole), 0 (zero)

doesn't satisfy the properties of YRC.

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EE 448 ANALOG FILTER DESIGN

Benmei Chen

1 HR. EXAMINATION
DR. R. A. BIRGENHEIER

25 pts each

One Sheet of Notes

(Clearly draw and label a circuit diagram for each problem.)

1. Realize Y(s) by (a) Foster 1st Form, (b) Foster 2nd Form.

$$Y(s) = \frac{s^2 + 4s + 3}{s + 2}$$

2. Realize H(s) by an R-C ladder network.

$$H(s) = \frac{ks}{s^2 + 7s + 10}$$

3. Realize H(s) with a loss-less 2-port network terminated in a 1 ohm load resistance

$$H(s) = \frac{ks^3}{(s + 1)^3}$$

4. Realize H(s) as a loss-less 2-port network terminated at both ends with $R_s = 1\Omega$, $R_L = 2\Omega$. (Carry at least 4 digits.)

$$H(s) = \frac{k}{s^2 + 5s + 4}$$

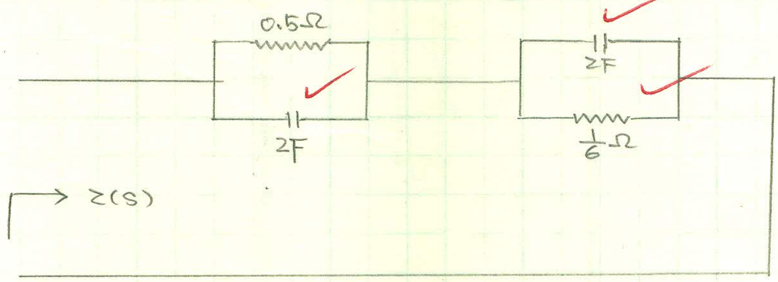
1. $Y(s) = \dfrac{s^2+4s+3}{s+2}$

(a) FOSTER'S 1st FORM.

$$Z(s) = \frac{s+2}{s^2+4s+3} = \frac{(s+2)}{(s+1)(s+3)}$$

$$= \frac{\frac{1}{2}}{s+1} + \frac{\frac{1}{2}}{s+3}$$

$$= \frac{1}{2s+2} + \frac{1}{2s+6}$$



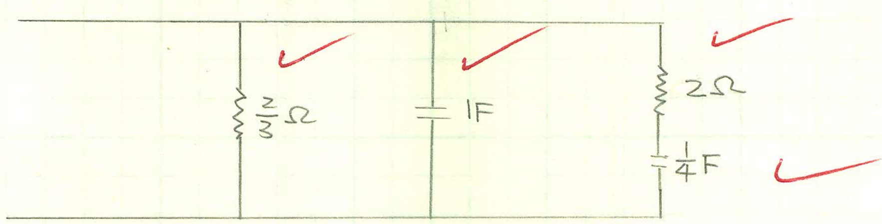
(b) FOSTER'S 2nd FORM:

$$Y(s) = \frac{s^2+4s+3}{s+2}$$

$$\frac{Y(s)}{s} = \frac{s^2+4s+3}{s(s+2)} = \frac{s^2+4s+3}{s^2+2s} = \frac{(s^2+2s)+(2s+3)}{s^2+2s}$$

$$= 1 + \frac{2s+3}{s^2+2s} = 1 + \frac{2s+3}{s(s+2)} = 1 + \frac{\frac{3}{2}}{s} + \frac{\frac{1}{2}}{s+2}$$

$$Y(s) = s + \frac{3}{2} + \frac{\frac{1}{2}s}{s+2} = s + \frac{3}{2} + \frac{1}{2 + 1/\frac{1}{4}s}$$



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2. $H(s) = ks/(s^2+7s+10)$

LET $Z_{11}(s) = \dfrac{(s+2)(s+5)}{(s+1)(s+4)}$, $Z_{12} = \dfrac{ks}{s^2+7s+10}$

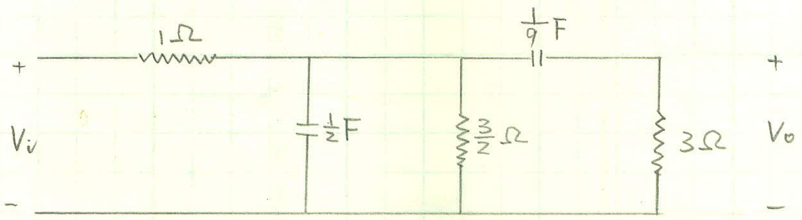
$$= \frac{s^2+7s+10}{s^2+5s+4}$$

$$\begin{array}{r} 1 \\ s^2+5s+4 \,\big)\, s^2+7s+10 \\ s^2+5s+4 \\ \hline 2s+6 \end{array} \qquad \begin{array}{r} \frac{1}{2}s \\ 2s+6 \,\big)\, s^2+5s+4 \\ s^2+3s \\ \hline 2s+4 \end{array}$$

$$Z_{11}(s) = 1 + \cfrac{1}{\frac{1}{2}s + \frac{2s+4}{2s+6}}$$

$$y'(s) = \frac{2s+4}{2s+6}$$

$$\begin{array}{r} \frac{2}{3} \\ 6+2s \,\big)\, 4+2s \\ 4+\frac{4}{3}s \\ \hline \frac{2}{3}s \end{array} \qquad \begin{array}{r} 9/s \\ \frac{2}{3}s \,\big)\, 6+2s \\ 6 \\ \hline 2 \end{array} \qquad \begin{array}{r} \frac{1}{3} \\ 2 \,\big)\, \frac{2}{3} \\ \frac{2}{3} \\ \hline 0 \end{array}$$



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3. $H(s) = ks^3/(s+1)^3 = ks^3/(s^3+3s^2+3s+1)$

$$= \frac{ks^3/(3s^2+1)}{1 + (s^3+3s)/(3s^2+1)}$$

LET $y_{22}(s) = \dfrac{s^3+3s}{3s^2+1}$ $\quad y_{12} = -\dfrac{ks^3}{3s^2+1}$

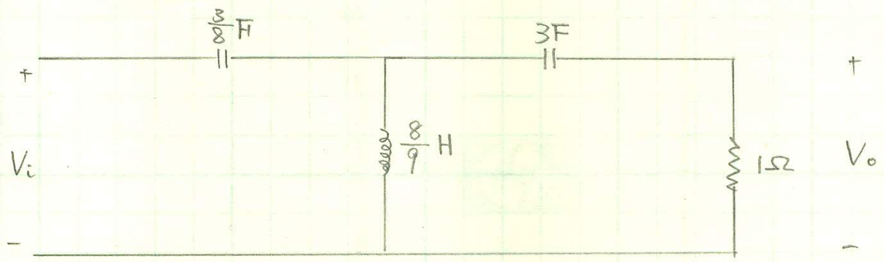
LET $Z(s) = \dfrac{1}{y_{22}(s)} = \dfrac{3s^2+1}{s^3+3s}$

$$3s + s^3 \overline{\big)\, 1 + 3s^2} \quad \frac{1}{3s}$$

$$\frac{1+\frac{1}{3}s^2}{\frac{8}{3}s} \quad \frac{8}{3}s \overline{\big)\, 3 + s^2} \quad \frac{9}{8s}$$

$$\frac{3}{s} \quad s \overline{\big)\, \frac{8}{3}} \quad \frac{8}{3s}$$

$$\frac{\frac{8}{3}}{0}$$



4. $H(s) = k / (s^2+5s+4)$, $R_S = 1\Omega$, $R_L = 2\Omega$

$$H(0) = \frac{R_L}{R_L+R_S} = \frac{2}{3} = \frac{k}{4} \quad , \quad k = \frac{8}{3}$$

(1) $$\rho(s)\rho(-s) = 1 - \frac{4}{R_L} H(s)H(-s)$$

$$= 1 - 2 \cdot \frac{k^2}{(s^2+5s+4)(s^2-5s+4)}$$

$$= \frac{s^4-17s^2+16-2\times\frac{64}{9}}{(s^2+5s+4)(s^2-5s+4)}$$

$$= \frac{s^4-17s^2+1.77778}{(s^2+5s+4)(s^2-5s+4)}$$

$$= \frac{(s+4.11033)(s+0.32439)(s-4.11033)(s-0.32439)}{(s^2+5s+4)(s^2-5s+4)}$$

$$= \frac{(s^2+4.43472s+1.33335)(s^2-4.43472s+1.33335)}{(s^2+5s+4)(s^2-5s+4)}$$

LET $$\rho(s) = \frac{s^2+4.43472s+1.33335}{s^2+5s+4}$$

(2) $$Z_{in1}(s) = \frac{1+\rho(s)}{1-\rho(s)} = \frac{2s^2+9.43472s+5.33335}{0.56528s+2.66665}$$

$$Z_{in2}(s) = \frac{1-\rho(s)}{1+\rho(s)} = \frac{0.56528s+2.66665}{2s^2+9.43472s+5.33335}$$

$Z_{in}(0) = R_L = 2$, $Z_{in1}(0) \doteq 2$, $Z_{in2}(0) = 0.5$

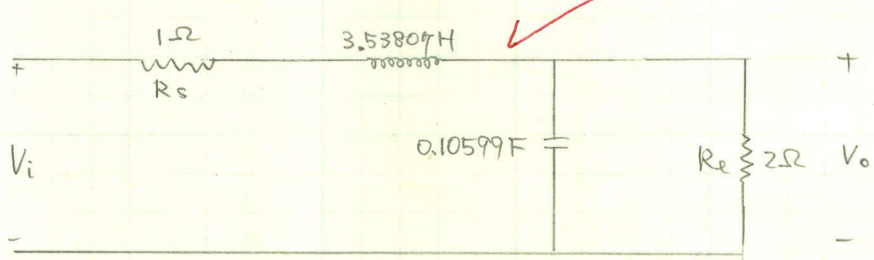
$\therefore Z_{in}(s) = Z_{in1}(s)$

$$= \frac{2s^2+9.43472s+5.33335}{0.56528s+2.66665}$$

$0.56528s+2.66665 \overline{)\,2s^2+9.43472s+5.33335}$ quotient $3.53807s$
$2s^2+9.43479s$
5.33335

$5.33335 \overline{)\,0.56528s+2.66665}$ quotient $0.10599s$
$0.56528s$
2.66665

$2.66665 \overline{)\,5.33335}$ quotient 2
5.33335
0



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E-E 448 Analog Filter Design
One Hour Examination
R. A. Birgenheier
December 11, 1986

100/100

Benmei Chen

One Sheet of Notes

Points

30 1. Determine the (a) order, (b) pole locations, (c) H(s) (in factored form) for a Butterworth filter that satisfies the following specifications:

i) R_s = 1 ohm, R_L = .5 ohms.

ii) Passband attenuation

<0.5 dB, $|\omega| < 0.8$ rps

iii) Stopband attenuation

>40 dB, $|\omega| > 3.0$ rps

30 2. Determine the (a) order, (b) pole locations, (c) H(s) (in factored form) for a Chebyshev filter that satisfies the following specifications:

i) R_s = 1 ohm, R_L = 1.5 ohms

ii) Passband attenuation

<0.5 dB, $|\omega| < 1.0$ rps

iii) Stopband attenuation

>40 dB, $|\omega| > 3.0$ rps

30 3. Realize a 2nd order Bessel low-pass filter for which $\tau(0) = 0.5$ seconds, R_s = 1 ohm and R_L = 2ohms. i.e., find (a) H(s), (b) $\rho(s)$, (c) $Z_{in}(s)$, (d) realization (clearly draw circuit diagram - use 5 significant figures).

10 4. Find the transfer function, H(s), for a transitional filter obtained from Butterworth and Bessel filters with the following pole locations:

Butterworth pole locations: $-.7071 \pm j.7071$

Bessel pole locations: $-1.5 \pm j0.866$

(be sure to normalize the Bessel poles so that they are on the unit circle).

EXAM #3 Analog and Digital Filter Benmei Chen

CHAPTER 8

BUTTERWORTH : $|H(j\omega)|^2 = B_n(\omega) = \dfrac{1}{1+\omega^{2n}}$

POLES = $S_k = -\sin\theta_k + j\cos\theta_k$; $\theta_k = \dfrac{2k-1}{2n}\pi$, $k = 1, 2, 3, \cdots, n$

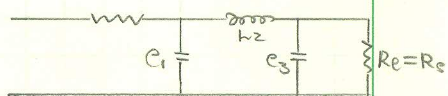
H(S) : $H(S) = \prod_{k=1}^{n/2} \dfrac{1}{S^2 + (2\sin\theta_k)S + 1}$ n is even

$H(S) = \left[\prod_{k=1}^{(n-1)/2} \dfrac{1}{S^2 + (2\sin\theta_k)S + 1}\right] \cdot \dfrac{1}{(S+1)}$

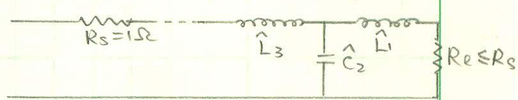
CIR. REALIZ. : $\lambda = \left(\dfrac{R_e - 1}{R_e + 1}\right)^{1/n}$ or $\lambda = \left(\dfrac{1 - R_e}{R_e + 1}\right)^{1/n}$

$\alpha_i = 2\sin\dfrac{\pi i}{2n}$, $\beta_i = 2\cos\dfrac{\pi i}{2n}$, $\tilde{C}_1 = \dfrac{\alpha_1}{R_e(1-\lambda)}$, $\hat{L}_1 = \dfrac{\alpha_1 R_e}{1-\lambda}$

$\tilde{C}_{2m-1}\tilde{L}_{2m} \{\hat{L}_{2m-1}\hat{C}_{2m}\} = \dfrac{\alpha_{4m-3}\,\alpha_{4m-1}}{1-\lambda\beta_{4m-2}+\lambda^2}$



$\tilde{C}_{2m+1}\tilde{L}_{2m} \{\hat{L}_{2m+1}\hat{C}_{2m}\} = \dfrac{\alpha_{4m-1}\,\alpha_{4m+1}}{1-\lambda\beta_{4m}+\lambda^2}$

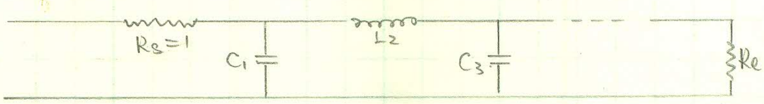


CHEBYSHEV : $T_n(\omega) = \begin{cases} \cos(n\cos^{-1}\omega) & |\omega| \leq 1 \\ \cosh(n\cosh^{-1}\omega) & |\omega| \geq 1 \end{cases}$

$|H(j\omega)|^2 = \dfrac{1}{1+\epsilon^2 T_n^2(\omega)}$, $\epsilon = \sqrt{10^{(A_{max}\,dB/10)} - 1}$

POLES : $S = -\sinh\left[\frac{1}{n}\sinh^{-1}\frac{1}{\epsilon}\right]\sin\dfrac{2k-1}{2n}\pi + \cosh\left[\frac{1}{n}\sinh^{-1}\frac{1}{\epsilon}\right]\cos\dfrac{2k-1}{2n}\pi$, $k = 1, 2, \cdots, n$

CIR : $a = \dfrac{4R_e}{(R_e+1)^2}$ WHEN n is odd $a = \dfrac{4R_e}{(R_e+1)^2}[1+\epsilon^2] \leq 1$ WHEN n IS EVEN



BESSEL : $H(S) = \dfrac{k}{B_n(\tau_0 S)}$ $\tau_0 = \tau(0)$, $\hat{B}_n(S) = (2n-1)\hat{B}_{n-1}(S) + S^2\hat{B}_{n-2}(S)$

$\hat{B}_1(S) = S+1$, $\hat{B}_2(S) = S^2+3S+3$, $\hat{B}_3(S) = S^3+6S^2+15S+15$, $\hat{B}_4(S) = S^4+10S^3+45S^2+105S+105$

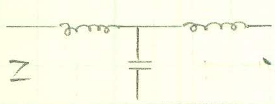
TRANSITIONAL : POLES = $\frac{1}{2}$(POLES OF BUTT. + POLES OF BESSEL)

DARLINGTON REALIZATION :

(1) $\rho(S)\rho(-S) = 1 - \dfrac{4}{R_e}H(S)H(-S)$

(2) $Z_{in}(S)$, $Z_{in}(0) = R_e$

(3) LONG DIVISION GET



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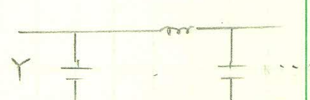

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1. $|H(j\omega)|^2 = \dfrac{1}{1+\omega^{2n}}$

$|H(j\omega)|_{dB} = -10 \log (1+\omega^{2n})$

ii) $10 \log (1+ 0.8^{2n}) < 0.5$

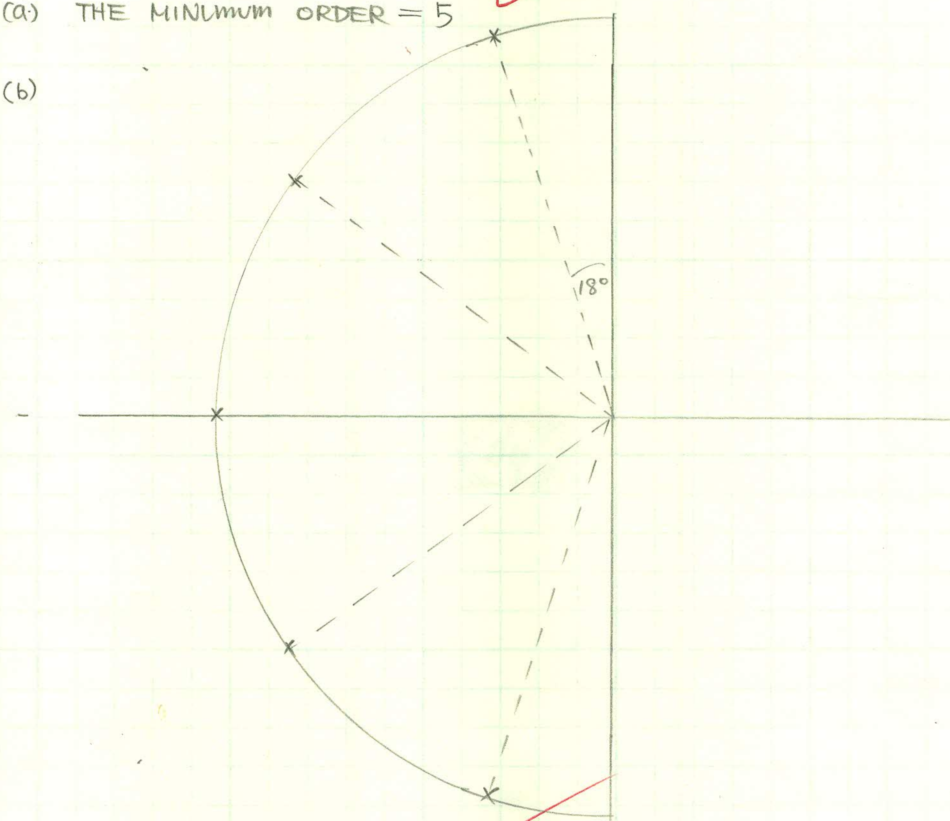
$0.8^{2n} < 0.1220 \quad \Rightarrow \quad n \geq 4.7135 \quad \therefore \; n \geq 5$

iii) $10 \log (1+ 3^{2n}) > 40$

$3^{2n} > 9999 \quad \Rightarrow \quad n \geq 4.1918 \quad \therefore \; n \geq 5$

(a) THE MINIMUM ORDER = 5

(b)



$S_{1,2} = -0.3090 \pm j0.9511$, $S_{3,4} = -0.8090 \pm j0.5878$, $S_5 = -1$

(c) $H(s) = \dfrac{1}{(S+1)(S^2+0.6180S+1)(S^2+1.6180S+1)}$

2. $T_n(\omega) = \begin{cases} \cos(n\cos^{-1}\omega) & |\omega| \le 1 \\ \cosh(n\cosh^{-1}\omega) & |\omega| \ge 1 \end{cases}$

$$|H(j\omega)|^2 = \frac{1}{1+\epsilon^2 T_n^2(\omega)}$$

$$|H(j\omega)|_{dB} = -10\log(1+\epsilon^2 T_n^2(\omega))$$

ii) Passband attenuation < 0.5 dB $|\omega| < 1.0$ rad/s

FOR CHEBYSHEV FILTER, THIS MEANS $A_{max} < 0.5$ dB

$$\epsilon \le \sqrt{10^{0.5/10} - 1} = 0.3493$$

iii) $10\log(1+0.3493^2 T_n^2(3.0)) > 40$

$$T_n(3.0) > 286.2632$$

$$\cosh(n\cosh^{-1}3.0) > 286.2632 \Rightarrow n > 3.6024 \Rightarrow n \ge 4$$

(a) MINIMUM ORDER = 4

(b) $S_k = -\sinh[\frac{1}{4}\sinh^{-1}\frac{1}{0.3493}]\sin\frac{2k-1}{2n}\pi + \cosh[\frac{1}{4}\sinh^{-1}\frac{1}{0.3493}]\cos\frac{2k-1}{2n}\pi$

$= -0.4582\sin\frac{2k-1}{8}\pi + 1.1000\cos\frac{2k-1}{8}\pi \qquad k = 1, 2, 3, 4$

$S_{1,4} = -0.1753 \pm j1.0163 \qquad S_{2,3} = -0.4233 \pm j0.4210$

(c)

$$H(s) = \frac{1}{(s+0.1753+j1.0163)(s+0.1753-j1.0163)(s+0.4233+j0.4210)(s+0.4233-j0.4210)}$$

$$= \frac{1}{(s^2+0.3506s+1.0636)(s^2+0.8466s+0.3564)}$$

3. FROM 'SHEET' : $\hat{B}_2(S) = S^2+3S+3$

(a) $$H(S) = \frac{k}{B_2(\tau(0)S)} = \frac{k}{(0.5S)^2+3(0.5S)+3} = \frac{k}{0.25S^2+1.5S+3} = \frac{k'}{S^2+6S+12}$$

$k=3$

(b) $$H(0) = \frac{R_\ell}{R_s+R_\ell} = \frac{2}{1+2} = \frac{k'}{12} \Rightarrow k'=8$$

$$\rho(S)\rho(-S) = 1 - \frac{4}{2}H(S)H(-S)$$
$$= 1 - \frac{128}{(S^2+6S+12)(S^2-6S+12)}$$
$$= \frac{S^4-12S^2+16}{(S^2+6S+12)(S^2-6S+12)}$$
$$= \frac{(S^2+4.47214S+4)(S^2-4.47214S+4)}{(S^2+6S+12)(S^2-6S+12)}$$

$$\therefore \rho(S) = \frac{S^2+4.47214S+4}{(S^2+6S+12)}$$

(c) $$Z_{in}(S) = \frac{1+\rho(S)}{1-\rho(S)} = \frac{2S^2+10.47214S+16}{1.52786S+8}$$

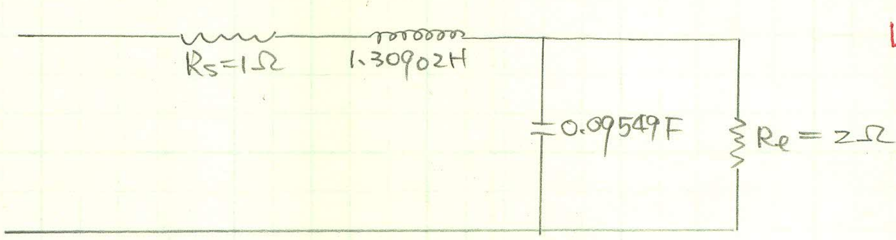
$Z_{in}(0) = 2 = R_\ell$

(d)

$$\begin{array}{r}
1.30902S \\
1.52786S+8 \;\big)\; 2S^2+10.47214S+16 \\
2S^2+10.47216S \\
\hline
16
\end{array}$$

$$\begin{array}{r}
0.09549S \\
16 \;\big)\; 1.52786S+8 \\
1.52786S \\
\hline
8
\end{array}$$

$$\begin{array}{r}
2 \\
8 \;\big)\; 16 \\
16 \\
\hline
0
\end{array}$$



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4.

X POLES OF TRANSITIONAL FILTER

$$S_{1,2} = -\frac{0.7071+1.5}{2} \pm j\,\frac{0.7071+0.866}{2}$$

$$= -1.10355 \pm j\,0.78655$$

$$H(S) = \frac{k}{(S+1.10355+j0.78655)(S+1.10355-j0.78655)}$$

$$= \frac{k}{S^2 + 2.20710S + 1.83648}$$

2 . THE NORMALIZE BESSEL POLES:

$$= -0.866 \pm 0.5$$

POLES OF TRAN. FILTER:

$$S_{1,2} = -\frac{0.866+0.7071}{2} \pm j\,\frac{0.7071+0.5}{2}$$

$$= -0.78657 \pm j0.60355$$

SO,

$$H(S) = \frac{k}{(S+0.78657-j0.60355)(S+0.78657+j0.60355)}$$

$$= \frac{0.98176}{S^2+1.57314+0.98176}$$

E-E 448 PASSIVE FILTER DESIGN
Final Examination
Dr. R. A. Birgenheier
December 18, 1986

172/200

Bennei Chen

OPEN NOTES

50 points each

1. Consider a LPF to meet the following specifications:
   - (i) Maximally-flat passband response.
   - (ii) 3dB cut-off frequency: f_c = 15 KHz
   - (iii) Stopband Attenuation: >20 dB for f>37.5KHz
   - (iv) $R_s = R_L$ = 1 K ohm

   Determine (a) minimum order n; (b) H(s); (c) realization (start with a normalized LP prototype).

2. Consider a HPF to meet the following specifications:
   - (i) Maximum passband ripple: 0.25 dB for f>120 KHz
   - (ii) Minimum stopband attenuation: >50 dB for f<30 KHz
   - (iii) $R_s = R_L$ = 300 ohms

   CHB

   Determine (a) minimum order n; (b) H(s); (c) realization (start with a normalized LPF prototype).

3. Consider a Butterworth BPF to meeth the following specifications:
   - (i) 3 dB bandwidth: 1 MHz
   - (ii) Center frequency: 10 MHz
   - (iii) Stopband attenuation at least 20 dB |f| <5MHz and |f| >15MHz
   - (iv) $R_s = R_L$ = 600 ohms.

   Determine (a) Q_0 of filter; (b) minimum order n; (c) H(s); (d) realization (start with a normalized LPF prototype).

4. Consider a Bessel LPF to meet the following specifications:

(i) $\tau(0) = 10^{-6}$ seconds

(ii) $\tau(\omega)$ has less than 2% error for $\omega < 2.5$ Mrps

(iii) $R_s = R_L = 1$K ohm

Determine (a) minimum order n; (b) H(s); (c) realization (use filter structure and element values given below).

1. First, consider the condition (ii)

$$f_c = 15\ KHz \quad , \quad \omega_c = 2\pi f_c = 30\pi\ K\ rad/s .$$

Now, consider condition (iii), THIS means

Stopband Attenuation : $> 20dB$ for $\omega > 2.5$ rad/s

WE CAN USE A NORMALIZED LP BUTTERWORTH FILTER (FOR condition (i))

$$|H(j\omega)|^2 = \frac{1}{1+\omega^{2n}}$$

$$|H(j\omega)|_{dB} = -10 \log (1+\omega^{2n})$$

$$\therefore \quad 10 \log (1 + 2.5^{2n}) > 20 \quad \Rightarrow \quad n > 2.50746$$

SO, (a) minimum order $n = 3$

(b) TRANSF. FUN.OF THE NORMALIZED FILTER : $\theta_1 = \frac{\pi}{6}$

$$H_N(S) = \frac{1}{(S+1)(S^2+(2\sin\theta_1)S+1)} = \frac{1}{(S+1)(S^2+S+1)} = \frac{1}{S^3+2S^2+2S+1}$$

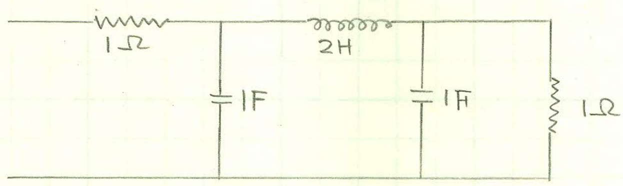
$$SO, \ H(S) = \frac{1}{(\frac{S}{\omega_c})^3 + 2(\frac{S}{\omega_c})^2 + 2(\frac{S}{\omega_c}) + 1}$$

$$= \frac{\omega_c^3}{S^3 + 2\omega_c S^2 + 2\omega_c^2 S + \omega_c^3}$$

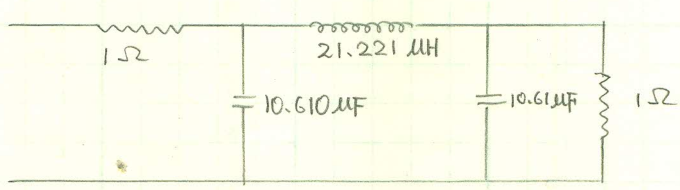
$$= \frac{(30\times\pi\times10^3)^3}{S^3 + 2\times30\times\pi\times10^3 S^2 + 2\times(30\pi\times10^3)^2 S + (30\times\pi\times10^3)^3}$$

$$= \frac{8.37169\times10^{14}}{S^3 + 188495.5592 S^2 + 1.77653\times10^{10} S + 8.37169\times10^{14}}$$

(c) THE LP REALIZ. CIRCUIT :



NOW SCALING TO THE 3dB AT $\omega_c = 30\pi\ K\ rad/s$



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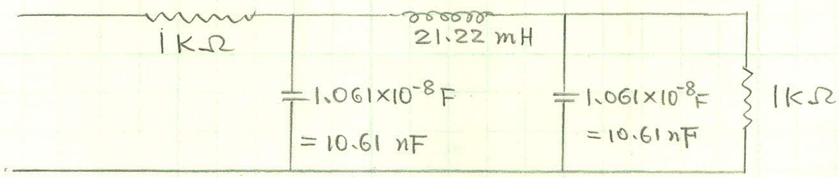
AND THEN, IMPEDANCE SCALING BY 1000



THIS IS THE REALIZATION WE NEED.

2. FIRST, LET US BUILD A HPF TO MEET THE FOLLOWING CONDITIONS:

(i)' Maximum passband ripple = 0.25 dB FOR $\omega = 1$ rad/s

(ii)' Minimum Stopband atten. = > 50 dB FOR $\omega < 0.25$ rad/s

USING FREQ. SCALING $K_f = 120 \times 2 \times \pi \times 10^3 = 240\pi \times 10^3$, THE CONDITIONS

(i)' and (ii)' WILL BE THE SAME AS CON. (i), (ii)

THEN, WE CONSIDER CHEBYSHEV LPF WITH

(i)'' Maximum passband ripple = 0.25 dB FOR $\omega = 1$ rad/s

(ii)'' minimum Stopband atten = > 50 dB FOR $\omega > 4$ rad/s

$$\epsilon = \sqrt{10^{0.025} - 1} = 0.2434$$

$$|H(j\omega)|_{dB} = -10 \log (1 + \epsilon^2 T_n^2(\omega))$$

$$10 \log (1 + \epsilon^2 T_n^2(4)) > 50 \quad \Rightarrow \quad T_n(4) > 1299.0923$$

$$T_n(4) = \cosh(n \cosh^{-1} 4) > 1299.0923 \quad \Rightarrow \quad n > 3.8104$$

SO, (a) minimum order = 4

POLES OF NOR. CHEB. LPF = $= -0.5553 \sin \frac{2k-1}{8}\pi + j1.1439 \cos \frac{2k-1}{8}\pi$

$$S_{1,4} = -0.2125 \pm j1.0568, \quad S_{2,3} = -0.5130 \pm j0.4378$$

$$H_N(s) = \frac{1}{(s^2 + 0.4250s + 1.1620)(s^2 + 1.0260s + 0.4548)}$$

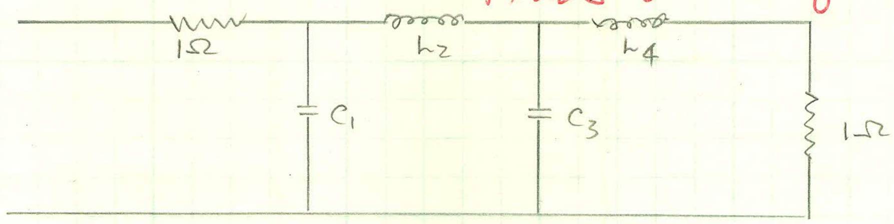
$$H_{HP,N}(s) = H_N\left(\frac{1}{s}\right) = \frac{s^4}{(1.1620s^2 + 0.4250s + 1)(0.4548s^2 + 1.0260s + 1)}$$

$$\underline{H(s) = H_{HP,\text{SCALED}}} = H_{HP,N}\left(\frac{s}{\omega_f}\right) = \frac{(s/240\pi \times 10^3)^4}{(1.1620/(240\pi \times 10^3)^2 s^2 + 0.4250/240\pi \times 10^3 \, s + 1)}$$

$$\times \frac{1}{(0.4548/(240\pi \times 10^3)^2 s^2 + 1.0260/240\pi \times 10^3 \, s + 1)}$$

(C) NORMALIZED CHEB. LPF CIR.:

need 5th order. Find values of normalized LPF



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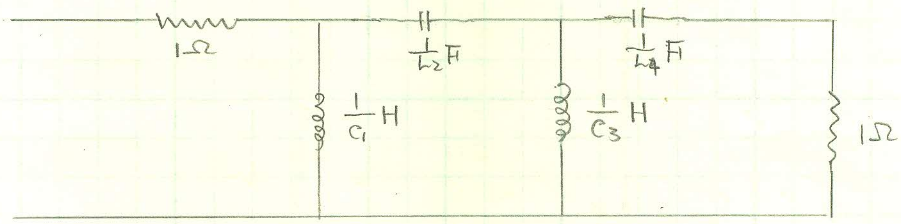
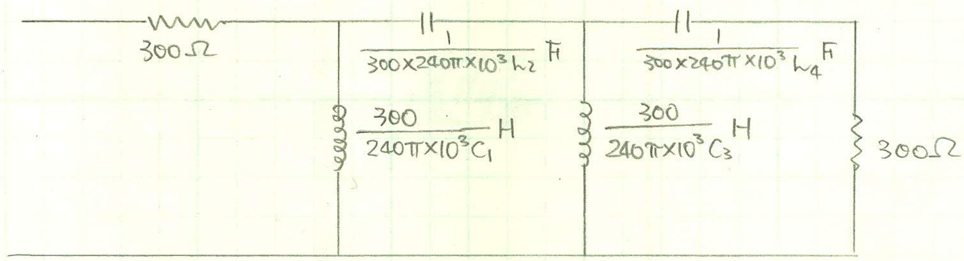
THEN HP NOR. CHEB: CIR.:



SALING TO $k_f = 240\pi \times 10^3$, $A = 300$



3. $B = 1\,\text{MHz} \longrightarrow 2\pi\,M\ \text{rad/s} = 2\pi \times 10^6\ \text{rad/s}$

$\omega_0 = 10\,\text{MHz} \longrightarrow \omega_0 = 2\pi \times 10 \times 10^6 = 2\pi \times 10^7\ \text{rad/s}$

(a) $Q_0 = \dfrac{\omega_0}{B} = \dfrac{2\pi \times 10^7}{2\pi \times 10^6} = 10$

(b) FROM LPF TO BPF, THE FREQ. $S \longmapsto \dfrac{S^2+\omega_0^2}{BS}$.

SO, WE HAVE

$$\omega_{LP} = \frac{\omega_{BP}^2 - \omega_0^2}{B\,\omega_{BP}}$$

$f_{1BP} = 5\text{MHz}$, $\omega_{1BP} = 2\pi \times 5 \times 10^6$ rad/s $\Rightarrow \omega_{LP1} = -15$ rad/s

$f_{1BP} = 15\text{MHz}$, $\omega_{2BP} = 2\pi \times 15 \times 10^6$ rad/s $\Rightarrow \omega_{LP2} = 8.3333$ rad/s

Condition (iii) is same as

Stopband attnu. > 20dB WHEN $w > 8.3333$ rad/s FOR LP

$10 \log (1 + (8.3333)^{2n}) > 20$ $n > 1.0836$

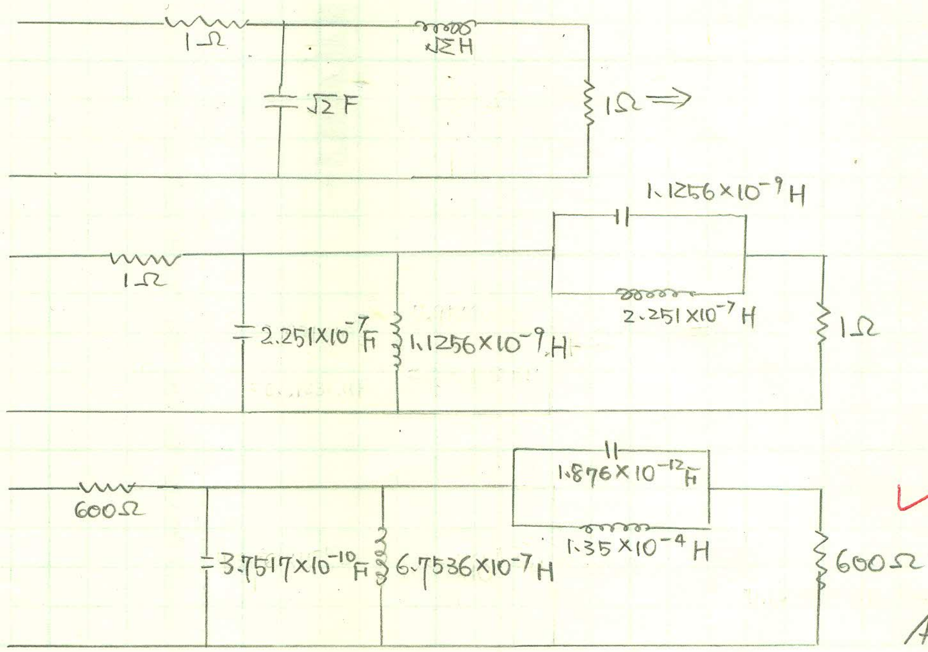
minimum order = 2

(c) NORMAL. BUTT. LP $H(S) = \dfrac{1}{S^2 + \sqrt{2}S + 1}$

$$BP = H(S) = \frac{1}{\left(\frac{S^2+2\pi\times10^7}{2\pi\times10^6 S}\right)^2 + \sqrt{2}\left(\frac{S^2+2\pi\times10^7}{2\pi\times10^6 S}\right)S + 1}$$

−5

(d) NORM. BUTT. LP : CIR. REAL. AS BELOW



Answer!!

4. condition (ii) is same as

$\tau(\omega)$ has less than 2% error for $\omega < 2.5 \times 10^6 \times \tau(0) = 2.5$ rps

LOOKING AT THE FIG 8-21, WE FIND THAT

(a) minimum order = 5

(b) $\hat{B}_3(s) = s^3 + 6s^2 + 15s + 15$ $\qquad \hat{B}_4(s) = s^4 + 10s^3 + 45s^2 + 105s + 105$

$$\hat{B}_5(s) = 9\hat{B}_4(s) + s^2\hat{B}_3(s)$$

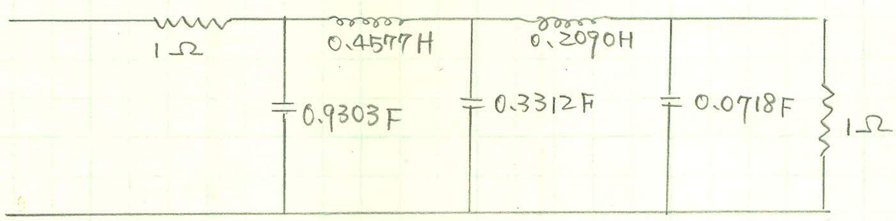
$$= 9(s^4 + 10s^3 + 45s^2 + 105s + 105) + s^2(s^3 + 6s^2 + 15s + 15)$$

$$= s^5 + 15s^4 + 105s^3 + 420s^2 + 945s + 945$$

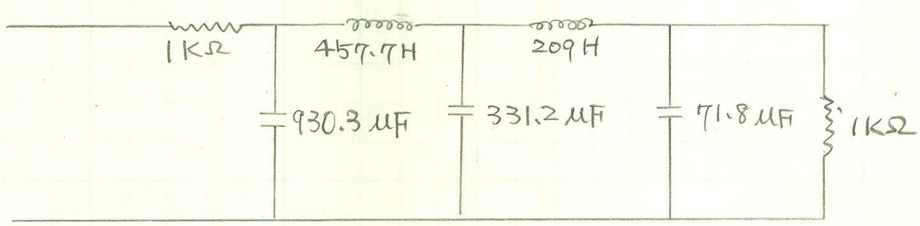
$$H(s) = \frac{k}{\hat{B}_5(10^{-6}s)}$$

$$= \frac{k}{(10^{-6}s)^5 + 15\times(10^{-6}s)^4 + 105\cdot(10^{-6}s)^3 + 420(10^{-6}s)^2 + 945(10^{-6}s) + 945}$$

(c) NORMAL. CIR.:



CIR. WITH $R_s = R_L = 1K\Omega$



freq. scale

—8

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