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E-E 431 Digital Control Systems Test 1

OPEN BOOK 1.5 Hours

Please put your name on the BACK of the last page only.

1. Given:

$$F(z) = \frac{z^2-z+1}{(z-1)(z-0.5)}$$

Find: f_k using the inversion integral.

SOLUTION: For the causal system. R.O.C of $F(z)$ is $|z|>1$

Using the inversion integral

$$f_k = \frac{1}{2\pi j}\oint_c F(z)\cdot z^{k-1}\,dz \qquad , \quad c \text{ is IN R.O.C.}$$

$$= \frac{1}{2\pi j}\oint_c \frac{z^2-z+1}{(z-1)(z-0.5)}\cdot z^{k-1}\,dz$$

$$= \text{Res}\left(\frac{z^2-z+1}{(z-1)(z-0.5)}\cdot z^{k-1}, 1\right) + \text{Res}\left(\frac{z^2-z+1}{(z-1)(z-0.5)} z^{k-1}, 0.5\right)$$

$$\text{Res}\left(\frac{z^2-z+1}{(z-1)(z-.5)}\cdot z^{k-1}, 1\right) = \lim_{z\to 1} (z-1)\cdot\frac{z^2-z+1}{(z-1)(z-0.5)}\cdot z^{k-1}$$

$$= 2$$

$$\text{Res}\left(\frac{z^2-z+1}{(z-1)(z-.5)} z^{k-1}, 0.5\right) = \lim_{z\to 0.5} (z-0.5)\cdot\frac{z^2-z+1}{(z-1)(z-.5)}\cdot z^{k-1}$$

$$= -1.5\cdot 0.5^{k-1}$$

$$\therefore\quad f_k = 2 - 1.5\times 0.5^{k-1}$$

$$= 2 - \frac{3}{2^k}$$

$f_0 = 1$. because $z^{0-1} = \frac{1}{z}$ is a pole.

2. Determine F(z) for the sequence

$f_0 = 0$
$f_1 = 1$
$f_k = 2, \; k > 1.$

Write F(z) in simplest closed form.

SOLUTION:

$$F(z) = \sum_{k=-\infty}^{\infty} f_k \cdot z^{-k}$$

$$= \sum_{k=0}^{\infty} f_k \cdot z^{-k} \qquad f_k = 0 \qquad k < 0$$

$$= 0 + 1 \cdot z^{-1} + 2\,(\cdot z^{-2} + z^{-3} + \cdots + z^{-k} + \cdots)$$

$$= z^{-1} + 2 \cdot z^{-2} \cdot (1 + z^{-1} + z^{-2} + \cdots + z^{-k+2} + \cdots)$$

$$= z^{-1} + 2 \cdot z^{-2} \cdot \frac{1}{1 - z^{-1}} \qquad |z| > 1$$

$$= \frac{1}{z} + 2 \cdot \frac{1}{z^2 - z}$$

$$= \frac{z - 1 + 2}{z(z-1)}$$

$$= \frac{z+1}{z(z-1)} \checkmark \; , \quad |z| > 1$$

SOLUTION 2:

$$f(k) = 1(k-1) + 1(k-2)$$

$$F(z) = z^{-1} \cdot \frac{z}{z-1} + z^{-2} \cdot \frac{z}{z-1}$$

$$= \frac{z+1}{z(z-1)}$$

3. Find the transfer fucntion, $H(z)$, for a causal system whose difference equation is

$$y_k - y_{k-1} - 2y_{k-2} = 6u_k - 2u_{k-1},$$

where u_k is the input
y_k is the output.

Show the poles and zeros on the z-plane.
Is the system stable.

SOLUTION : $y_k - y_{k-1} - 2y_{k-2} = 6u_k - 2u_{k-1}$

take z-transform, we have

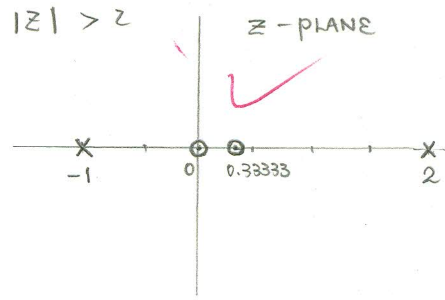
$$Y(z) - z^{-1}Y(z) - 2\cdot z^{-2}Y(z) = 6U(z) - 2\cdot z^{-1}U(z)$$

$$Y(z)\left(1 - z^{-1} - 2\cdot z^{-2}\right) = U(z)\left(6 - 2\cdot z^{-1}\right)$$

$$H(z) = \frac{Y(z)}{U(z)} = \frac{6 - 2\cdot z^{-1}}{1 - z^{-1} - 2\cdot z^{-2}}$$

$$= \frac{6z^2 - 2z}{z^2 - z - 2}$$

$$= \frac{6z(z - 0.33333)}{(z+1)(z-2)}, \quad |z| > 2$$



THE SYSTEM IS NOT STABLE BECAUSE H(z) HAS A POLE AT $z=2$ WHICH IS OUTSIDE UNIT CIRCLE. ALSO, IF WE TAKE INVERSE TRANSFORM OF H(z),

$$H(z) = \frac{6z^2 - 2z}{z^2 - z - 2} = \frac{6(z^2 - z - 2) + 4z + 12}{z^2 - z - 2}$$

$$= 6 + 4\cdot\frac{z+3}{(z+1)(z-2)} = 6 + \tfrac{4}{3}\left[\frac{-2}{z+1} + \frac{5}{z-2}\right]$$

$$= 6 - \frac{8}{3}\cdot z^{-1}\cdot\frac{1}{1-(-z^{-1})} + \frac{20}{3}\cdot z^{-1}\cdot\frac{1}{1-\frac{2}{z}}$$

$$= 6 - \frac{8}{3}z^{-1}\cdot\sum_{k=0}^{\infty}(-1)^k\cdot z^{-k} + \frac{20}{3}z^{-1}\sum_{k=0}^{\infty}2^k\cdot z^{-k}$$

$$h_k = -\frac{8}{3}(-1)^{k-1} + \frac{20}{3}2^{k-1} \qquad \therefore \sum_{k=0}^{\infty}|h_k| \longrightarrow +\infty$$

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E-E 431 Digital Control Systems   Test 1

4. A digital low pass filter is to be made with a cutoff frequency of 2 rad/sec. The continouous time system has a transfer function

$$H(s) = \frac{1}{(s+2)^2 + 4}.$$

a) Find the corresponding digital filter using Tustin's rule corrected for 2 radians/sec.

b) Find the corresponding digital filter using hold equivalence.

a)
$$H(z) = H(s)\Big|_{s = \frac{2}{\tan T}\cdot\frac{z-1}{z+1}}$$

$$= \frac{1}{\left(\frac{2}{\tan T}\cdot\frac{z-1}{z+1} + 2\right)^2 + 4}$$

$$= \frac{(z+1)^2}{\left[\frac{2}{\tan T}\cdot(z-1) + 2(z+1)\right]^2 + 4(z+1)^2}$$

$$= \frac{(z+1)^2}{4(1/\tan^2 T + 1/\tan T + 2)\,z^2 + 8(1/\tan^2 T - 2)\,z + 4(1/\tan^2 T + 1/\tan T + 2)}$$

b)
$$H(z) = \frac{z}{z-1}\,\mathcal{Z}\left[\frac{H(s)}{s}\right]$$

$$= \frac{z}{z-1}\,\mathcal{Z}\left[\frac{2^2+2^2}{8s\,((s+2)^2+2^2)}\right]$$

$$= \frac{z}{z-1}\cdot\frac{z\,(Az+B)}{8(z-1)(z^2 - 2e^{-2T}(\cos 2T)\,z + e^{-4T})}$$

$$= \frac{z^2\,(Az+B)}{8(z-1)^2(z^2 - 2e^{-2T}\cos 2T\cdot z + e^{-4T})}$$

WHERE $A = 1 - e^{-2T}\cos 2T - e^{-2T}\sin 2T$

$B = e^{-4T} + e^{-2T}\sin 2T - e^{-2T}\cos 2T$

T is THE SAMPLED SPACE.

E-E 431 Digital Control Systems   Test 1

5. Sketch the frequency response (gain and phase) of the digital filter

$$H(z) = \frac{2(z+1)}{z(z-0.5)}.$$

SOLUTION :
$$H(e^{j\omega T}) = \frac{2(e^{j\omega T}+1)}{e^{j\omega T}(e^{j\omega T}-0.5)}$$

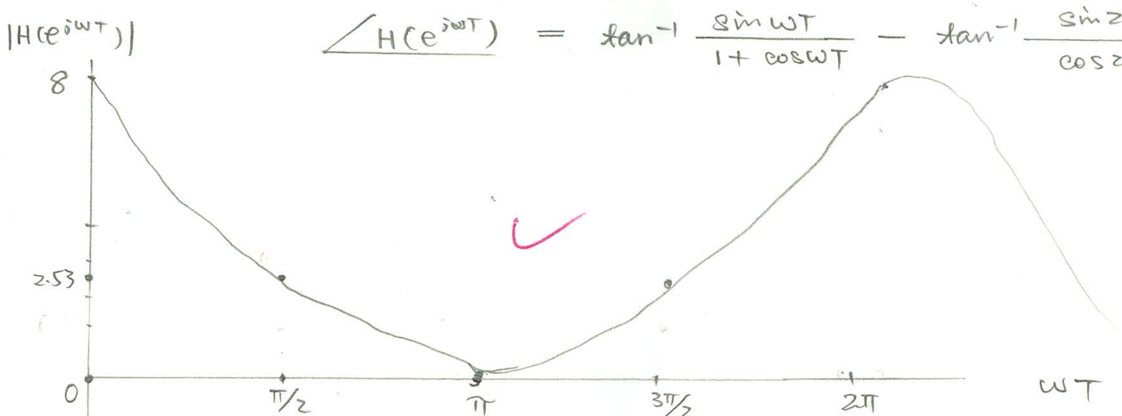
$$= \frac{2(\cos\omega T + 1 + j\sin\omega T)}{\cos 2\omega T + j\sin 2\omega T - 0.5\cos\omega T - j0.5\sin\omega T}$$

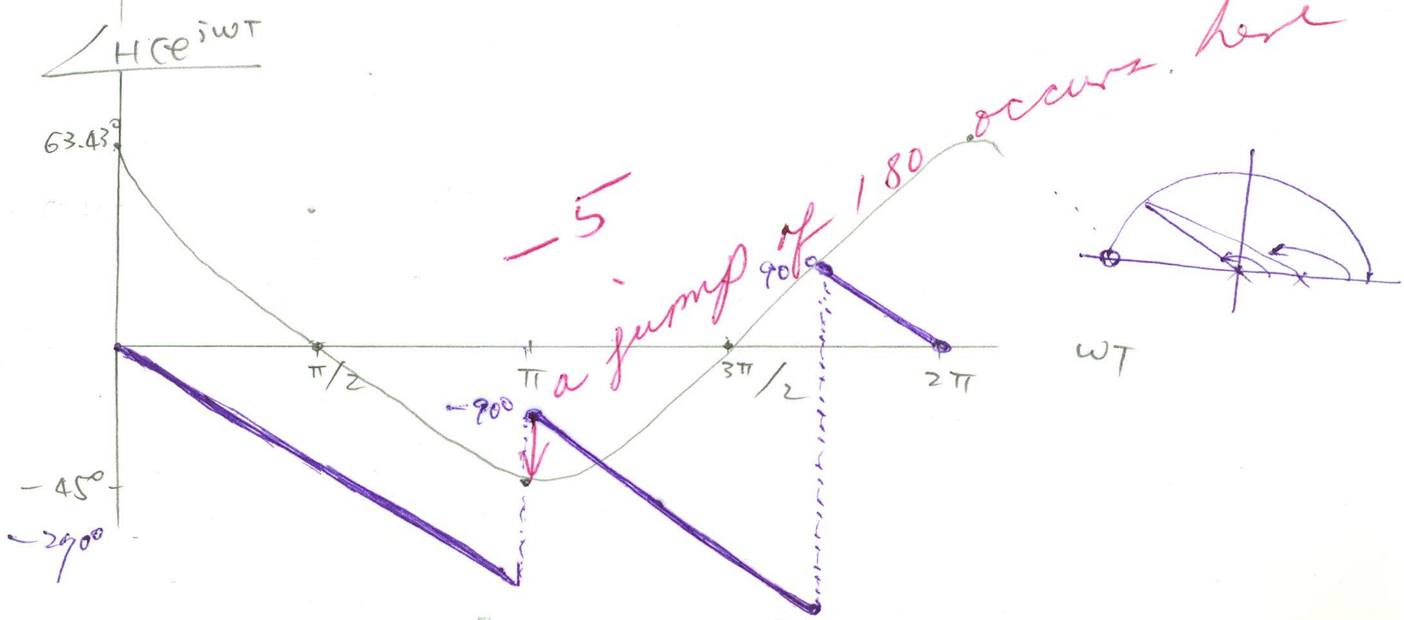
$$= \frac{2(1+\cos\omega T) + j2\sin\omega T}{(\cos 2\omega T - 0.5\cos\omega T) + j(\sin 2\omega T - 0.5\sin\omega T)}$$

$$|H(e^{j\omega T})|^2 = \frac{4(1+2\cos\omega T + \cos^2\omega T + \sin^2\omega T)}{(\cos\omega T - 0.5)^2 + \sin^2\omega T}$$

$$= \frac{8(1+\cos\omega T)}{1.25 - \cos\omega T}$$

$$\angle H(e^{j\omega T}) = \tan^{-1}\frac{\sin\omega T}{1+\cos\omega T} - \tan^{-1}\frac{\sin 2\omega T - 0.5\sin\omega T}{\cos 2\omega T - 0.5\cos\omega T}$$





-5

a jump of 180 occurs here

6. Given the difference equation $y_k = 0.5y_{k-1} + u_k$, and $u_k = 0.2^k$, find y_k using z transforms.

$$y_k = 0.5\, y_{k-1} + u_k$$

$$\therefore\ Y(z) = 0.5 \cdot z^{-1}\, Y(z) + U(z)$$

$$Y(z)\,(1 - 0.5 z^{-1}) = U(z)$$

$$Y(z) = \frac{1}{1-0.5z^{-1}} \cdot U(z) \quad \text{---- (1)}$$

$$u_k = 0.2^k \qquad k > 0$$

$$U(z) = \sum_{k=0}^{\infty} 0.2^k \cdot z^{-k}$$

$$= \sum_{k=0}^{\infty} (5z)^{-k} = \frac{1}{1-(5z)^{-1}} = \frac{1}{1-0.2z^{-1}}$$

$$\therefore\ Y(z) = \frac{1}{1-0.5z^{-1}} \cdot \frac{1}{1-0.2z^{-1}}$$ ✓

$$= \frac{z^2}{(z-0.5)(z-0.2)} \qquad |z| > 0.5$$

$$= \frac{z^2}{z^2 - 0.7z + 0.1}$$ expand in z^{-1}

$$= 1 + \frac{0.7z - 0.1}{(z-0.5)(z-0.2)}$$

$$= 1 + \frac{0.83333}{z-0.5} - \frac{0.13333}{z-0.2}$$

$$= 1 + 0.83333\, z^{-1} \cdot \frac{1}{1-0.5z^{-1}} - 0.13333 \cdot z^{-1} \cdot \frac{1}{1-0.2z^{-1}}$$

$$= 1 + 0.83333\, z^{-1}\,(1 + 0.5 \cdot z^{-1} + \cdots + 0.5^k \cdot z^{-k} + \cdots$$

$$- 0.13333\, z^{-1}\,(1 + 0.2z^{-1} + \cdots + 0.2^k \cdot z^{-k} + \cdots)$$

$$\therefore\ f_0 = 1$$

$$f_k = 0.83333 \times 0.5^{k-1} - 0.13333 \times 0.2^{k-1} \qquad k \geq 1$$

$$y_k = \frac{5}{3}\, 0.5^k - \frac{2}{3} \times 0.2^k$$

E-E 441 Digital Control Systems Test 2

OPEN BOOK 1¼ Hours

Please put your name on the BACK of the last page only.

1.(25pts)

Given:

$$G(s) = \frac{e^{-0.2s}}{s+1}$$

and sampling period, T = 0.5 sec.

Find: G(z)

$$G(z) = \mathcal{Z}[G(s)] = \mathcal{Z}\left[\frac{e^{-0.2s}}{s+1}\right]$$

$$= \mathcal{Z}\left[\frac{e^{-1\times 0.5s} \cdot e^{0.6\times 0.5\, s}}{s+1}\right]$$

$$= \frac{1}{z}\,\mathcal{Z}\left[\frac{e^{0.6Ts}}{s+1}\right]$$

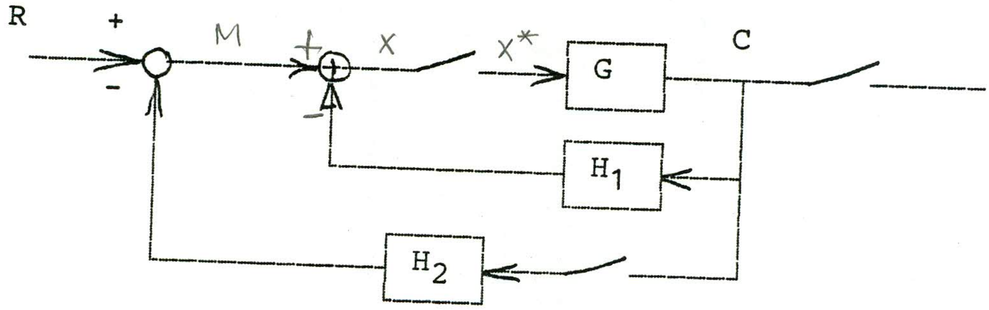
$$= \frac{1}{z}\,\mathcal{Z}\left[e^{-(t+0.6T)}\, 1(t+mT)\right]$$

$$= \frac{1}{z} \cdot \frac{e^{-0.6T} \cdot z}{z - e^{-T}}$$

$$= \frac{e^{-0.6\times 0.5}}{z - e^{-0.5}} = \frac{0.7408}{z - 0.6065}$$

E-E 441 Digital Control Systems Test 2

2.(25pts) Derive the input-output relationship for the given block diagram. Is a transfer function C^*/R^* possible?



$C = (M - H_1 C)^* \cdot G$ .... (1)

$M = R - C^* H_2$ .... (2)

FROM EQ (1),

$C^* = M^* G^* - (H_1 C)^*$

FROM EQ (2)

$M^* = R^* - C^* H_2^*$ −3

$\therefore C^* = (R^* - C^* H_2^*) G^* - (H_1 C)^*$

C^*/R^* IMPOSSIBLE

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② $C = X^* G$

$C^* = X^* G^*$

$X = M - CH_1$

② $X^* = M^* - (CH_1)^*$

$X^* = M^* - (X^* GH_1)^*$

③ $\therefore X^* = \dfrac{M^*}{1 + (GH_1)^*}$

$C = \dfrac{M^* \cdot G}{1 + (GH_1)^*}$

④ $C^* = \dfrac{E_1^* G^*}{1 + (GH_1)^*}$

OUTER LOOP

$M = R - C^* H_2$

④ $M^* = R^* - C^* H_2^*$

$C^* = (R^* - C^* H_2) G^* / 1 + (GH_1)^*$

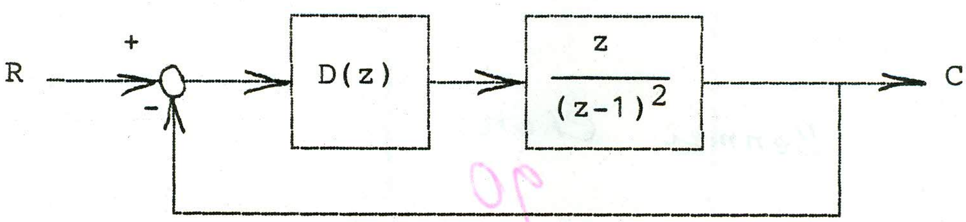
E-E 441 Digital Control Systems   Test 2

3.(50pts) For the system shown find a controller, D(z), so that the closed loop system has a 2% settling time of 4.0 sec and a peak overshoot of 5%. Plot the step response of the system. The sampling period of the system is 0.2 sec. After designing the controller determine the system type and the percent steady state error for an acceleration (parabolic) input, r(t).

$k_a = \dfrac{0.053(1-0.95)}{1-.625}$

$= .0071.$

$e_{ss} = \dfrac{1}{0.0071} \approx 141$



90

$s = -1 \pm j1$

$$\zeta \doteq (0.6)\left(1 - \frac{\%\text{overshoot}}{100}\right) = (0.6)\cdot\left(1 - \frac{5}{100}\right) = 0.57$$

$$\zeta\omega_n \doteq 4.6/t_s = 4.6/4.0 = 1.15 \quad , \quad \omega_n \doteq 2.0175$$

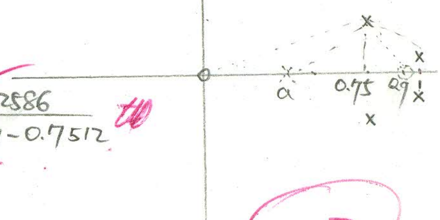
$$s = -\zeta\omega_n \pm j\,\omega_n\sqrt{1-\zeta^2} = -1.15 \pm j\,1.6577$$

$$z = e^{TS} = e^{0.2(-1.15 \pm j1.6577)} = 0.7945\,(\cos 0.3315 \pm j\sin 0.3315)$$

$$= 0.7512 \pm 0.2586$$

BETTER CHOICE $z_o = 0.95$ $z_p = .625$ $K = .053$

Let us choose $D(z) = \dfrac{K(z-0.90)}{z-a}$



$$\therefore -\tan^{-1}\frac{0.2586}{0.7512} + \tan^{-1}\frac{0.2586}{0.75-a} - 180° + \tan^{-1}\frac{0.2586}{0.9-0.7512}$$

$$+ 360° - 2\times\tan^{-1}\frac{0.2586}{1-0.7512} = 180°$$

$$\therefore \tan^{-1}\frac{0.2586}{0.7512-a} = 0.8923 \quad \Rightarrow \quad a = 0.5427$$

-2

TOO HIGH

$K = 3.1776$

$$\therefore C(z) = \frac{3.1776\cdot\dfrac{z-0.9}{z-0.5427}\cdot\dfrac{z}{(z-1)^2}}{1 + 3.1776\cdot\dfrac{z-0.9}{z-0.5427}\cdot\dfrac{z}{(z-1)^2}} \cdot \frac{z}{z-1}$$

$$= \frac{3.1776\,z^3 - 2.8598\,z^2}{(z^3 + 0.6349z^2 - 0.7744z - 0.54217)(z-1)}$$

SS ERROR -5

DIFFERENCE EQ a better way to do this

95

Please sign the following and turn in this page with your results. **This page must be signed to receive a grade for this test.**

I have not given nor received aid from others on this design.

Benmei Chen

Signed

EE 441 Design Problem Spring 1987

Design an estimator and state variable feedback for an inverted pendulum system as depicted in Figure 1. The pendulum dynamics is described by the equation:

$$\ddot{\Theta}(t) - \Theta(t) = f(t)$$

where: $\Theta(t)$ is the angle (radians) of the inverted pendulum and is the output,
$f(t)$ is the force (newtons) on the cart.

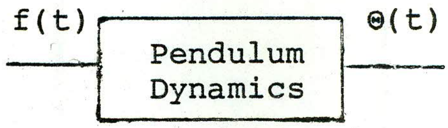


Figure 1 The Inverted Pendulum

The purpose of the controller is to drive $\Theta(t)$ to zero. You may assume that $\Theta(t)$ is available as a voltage and the $f(t)$ is an electrical signal supplied to drive the cart.

The following are the specifications and constraints on the design:

1. The system is to have a settling time of 1.0 ± 0.1 seconds,
2. The system is to have a peak overshoot of less than 5%.
3. The observer poles are to be at least 5 times faster than the closed loop poles of the system.
4. The sampling time is 0.2 seconds.

Include the following in your results:

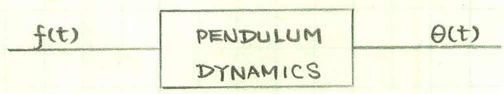
1. All the work used to completed the design,
2. The feedback matrix, K,
3. The observer matrix, L,
4. A graph showing the time response of $\Theta(t)$ with an initial angle error, $\Theta(0) = 0.1$ radians,
5. A signal flow graph which shows the feedback and estimator.

Your result will be judged on:

1. The accuracy with which the final design meets the specifications,
2. Clarity of presentation of the design. Explain clearly what you are doing and why you are doing it,
3. Neatness of presentation. Present the results on neatly written 8½ X 11 paper using graph paper where appropriate. Signal flow graphs should be neatly drawn using a straight edge,
4. Conciseness of presentation. Present your results in a brief and easily read report.

DESIGN PROBLEM FOR EE 441

GIVEN: CONSIDER THE INVERTED PENDULUM DYNAMICS SYSTEM BELOW



$$\ddot{\theta}(t) - \theta(t) = f(t) \qquad \text{------} \qquad (1)$$

WHERE $\theta(t)$ IS THE ANGLE (RADIANS) OF THE INVERTED PENDULUM AND IS THE OUTPUT.

$f(t)$ IS THE FORCE (NEWTONS) ON THE CART.

THE PURPOSE OF THE CONTROLLER IS TO DRIVE $\theta(t)$ TO ZERO.

SPECIFICATIONS AND CONSTRAINTS:

1. THE SYSTEM IS TO HAVE A SETTLING TIME OF 1.0 ± 0.1 SECONDS.
2. THE SYSTEM IS TO HAVE A PEAK OVERSHOOT OF LESS THAN 5%
3. THE OBSERVER POLES ARE TO BE AT LEAST 5 TIMES FASTER THAN THE CLOSED LOOP POLES OF THE SYSTEM.
4. THE SAMPLING TIME IS 0.2 SECONDS.

SOLUTION TO THE PROBLEM:

(1) From the specifications and the results in text and notes, we have

$$\xi \omega_n \doteq 4.6/t_s = 4.6/1.0 = 4.6$$

$$\text{PER CENT OVERSHOOT} = 100 e^{-\xi\pi/\sqrt{1-\xi^2}} \leq 5$$

$$\xi \geq 0.6901 , \qquad \text{LET } \xi = 0.7$$

So, the desired poles for continous system are given as

$$s = -\xi\omega_n \pm j\omega_n \cdot \sqrt{1-\xi^2}$$

$$= -4.6 \pm j \cdot 4.6929$$

The desired poles for discrete system,

$$z = e^{TS} = e^{0.2(-4.6 \pm j4.6929)}$$

$$= 0.3985(0.5909 \pm j0.8067)$$

$$= 0.2355 \pm j0.3215$$

$$a_c(z) = (z - 0.2355 + j0.3215)(z - 0.2355 - j0.3215)$$

OK

$$= z^2 - 0.4710z + 0.1588 \qquad \text{------} \qquad (2)$$

(2) Recall the equation (1) on page one and take the Laplace transform to both sides of the equation, we have

$$S^2\theta(S) - \theta(S) = F(S)$$

Then the transfer function for the continous system,

$$\frac{\theta(S)}{F(S)} = \frac{1}{S^2 - 1}$$

From this, we can write the control canonic form for the system

$$\dot{X} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} X + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot f(t)$$

$$\theta = \begin{pmatrix} 1 & 0 \end{pmatrix} X$$

$$\Phi(t) = \mathcal{L}^{-1}[SI - F]^{-1}$$

$$= \mathcal{L}^{-1}\left[\begin{pmatrix} S & 0 \\ 0 & S \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right]^{-1}$$

$$= \mathcal{L}^{-1}\begin{bmatrix} \frac{S}{(S+1)(S-1)} & \frac{1}{(S+1)(S-1)} \\ \frac{1}{(S+1)(S-1)} & \frac{S}{(S+1)(S-1)} \end{bmatrix} = 0.5\begin{bmatrix} e^{t}+e^{-t} & e^{t}-e^{-t} \\ e^{t}-e^{-t} & e^{t}+e^{-t} \end{bmatrix}$$

$$\Phi = \Phi(t)\Big|_{t=T=0.2SEC.}$$

$$= \begin{bmatrix} 1.0201 & 0.2013 \\ 0.2013 & 1.0201 \end{bmatrix}$$

$$\Gamma = \int_0^T \Phi(t)\, G\, dt = \int_0^{0.2} 0.5 \begin{pmatrix} e^t+e^{-t} & e^t-e^{-t} \\ e^t-e^{-t} & e^t+e^{-t} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} dt$$

$$= \begin{pmatrix} 0.0201 \\ 0.2013 \end{pmatrix}$$

The state varible form for the discrete system

$$\mathbf{X}_{n+1} = \begin{pmatrix} 1.0201 & 0.2013 \\ 0.2013 & 1.0201 \end{pmatrix} \mathbf{X}_n + \begin{pmatrix} 0.0201 \\ 0.2013 \end{pmatrix} \cdot f_n$$

$$\theta_n = (1 \quad 0)\, \mathbf{X}_n$$

$\frac{\Theta}{F} = ?$ → uncompensated system ↓ K_1' K_2'

From this system equations and the desired poles given by Eq. (2)

$$\det\left[z\mathbf{I} - (\Phi - \Gamma\mathbf{K})\right] = z^2 - 0.4710z + 0.1588 \;!$$

$$\det \begin{bmatrix} z - 1.0201 + 0.0201 \cdot K_1 & -0.2013 + 0.0201 \cdot K_2 \\ -0.2013 + 0.2013 \cdot K_1 & z - 1.0201 + 0.2013 K_2 \end{bmatrix} = z^2 - 0.4710z + 0.1588$$

$$z^2 - (2.0402 - 0.0201 \cdot K_1 - 0.2013 \cdot K_2) + (1.0000 - 0.2013 \cdot K_2 + 0.0201 \cdot K_1)$$

$$= z^2 - 0.4710z + 0.1588$$

$$\begin{cases} 2.0402 - 0.0201 \cdot K_1 - 0.2013 \cdot K_2 = 0.4710 \\ 1.0000 - 0.2013 \cdot K_2 + 0.0201 \cdot K_1 = 0.1588 \end{cases}$$

$\underline{\mathbf{K} = [18.1095 \quad 5.9871]}$ THE FEEDBACK MATRIX.

(18.8 5.51)

(3) From the specifications and constraints, we have to let the observer poles be at least 5 times faster than the closed loop poles. The closed loop poles have been given on page one as

$$s = -4.6 \pm j4.6929$$

So, the observer poles are at $z = e^{T(-4.6 \times 5)} = 0.01005$

$$\det\left[z\mathbf{I} - \Phi + \mathbf{L}\mathbf{H}\right] = (z - 0.01005)^2$$

$$\det\left[\begin{pmatrix} z & 0 \\ 0 & z \end{pmatrix} - \begin{pmatrix} 1.0201 & 0.2013 \\ 0.2013 & 1.0201 \end{pmatrix} + \begin{pmatrix} \ell_1 \\ \ell_2 \end{pmatrix}\begin{pmatrix} 1 & 0 \end{pmatrix}\right] = z^2 - 0.0201z + 0.00010$$

$$\det\begin{bmatrix} z - 1.0201 + \ell_1 & -0.2013 \\ -0.2013 + \ell_2 & z - 1.0201 \end{bmatrix} = z^2 - 0.02010z + 0.00010$$

$$z^2 + (-2.0402 + \ell_1)z + 1.0201\cdot(1.0201 - \ell_1) - 0.2013\,(0.2013 - \ell_2)$$

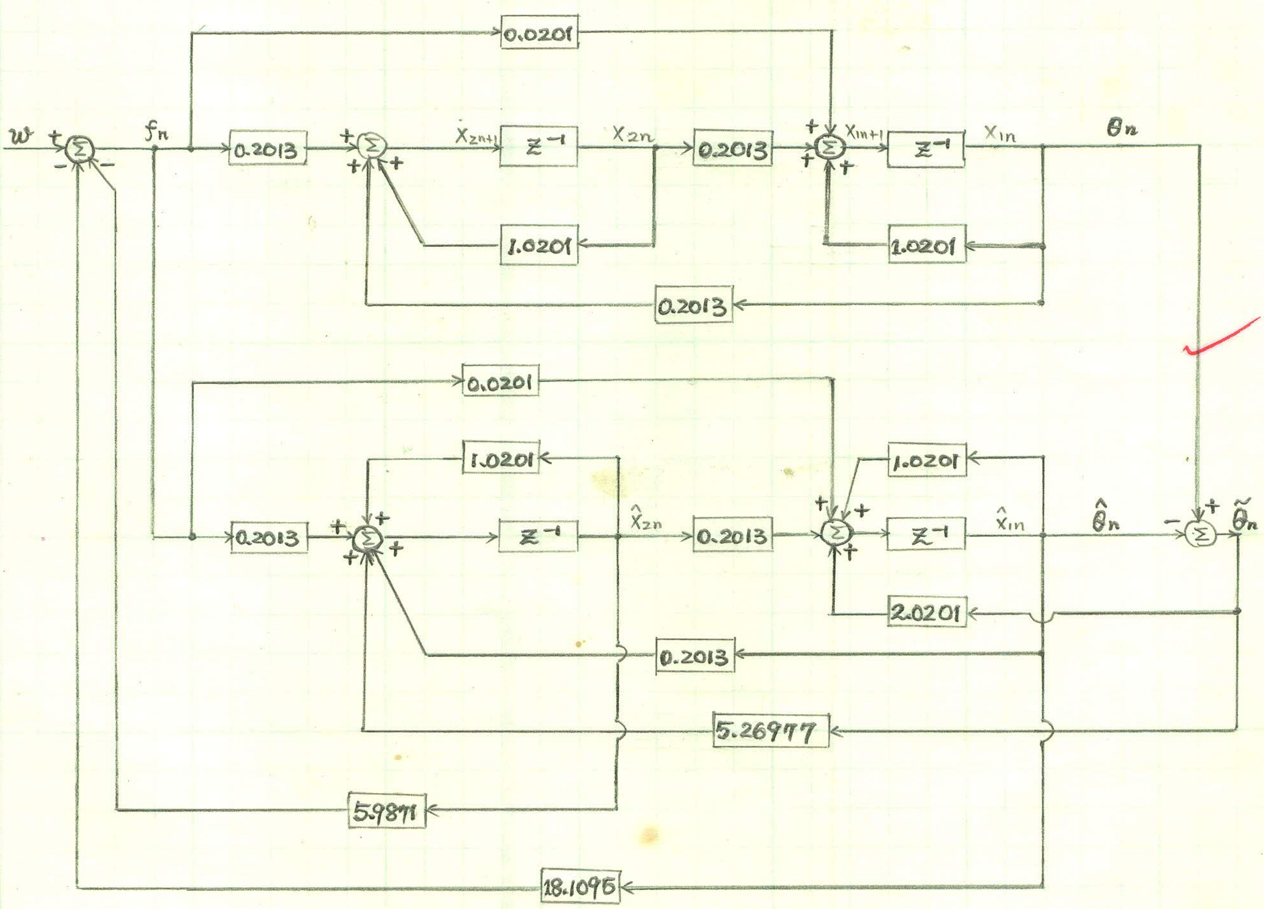
$$= z^2 - 0.02010z + 0.00010$$

$$\begin{cases} \ell_1 - 2.0402 = -0.02010 \\ 1 - 1.0201\cdot\ell_1 + 0.2013\cdot\ell_2 = 0.00010 \end{cases}$$

$$L = \begin{bmatrix} 2.02010 \\ 5.26977 \end{bmatrix}$$ close THE OBSERVER MATRIX.

(4) THE TIME RESPONSE OF $\theta(t)$ IS TO BE DRAWN ON NEXT PAGE.

(5) THE SIGNAL FLOW GRAPH:



(4) From the equations (6.63) & (6.64) on page 155 in text, we have

$$\hat{\mathbf{x}}(K+1) = (\Phi - \Gamma K - LH)\,\hat{\mathbf{x}}(K) + L\,\theta(K)$$

$$= \left[\begin{pmatrix} 1.0201 & 0.2013 \\ 0.2013 & 1.0201 \end{pmatrix} - \begin{pmatrix} 0.0201 \\ 0.2013 \end{pmatrix}(18.1095 \quad 5.9871) - \begin{pmatrix} 2.02010 \\ 5.26977 \end{pmatrix}(1 \quad 0)\right]\hat{\mathbf{x}}(K)$$

$$+ \begin{pmatrix} 2.02010 \\ 5.26977 \end{pmatrix}\theta(K)$$

$$= \begin{bmatrix} -1.36400 & 0.08096 \\ -8.71391 & -0.18510 \end{bmatrix}\cdot\hat{\mathbf{x}}(K) + \begin{bmatrix} 2.02010 \\ 5.26977 \end{bmatrix}\cdot\theta(K) \quad \cdots\cdots \ (a)$$

$$u(K) = -K\hat{\mathbf{x}}(K) = (-18.1095 \quad -5.9871)\cdot\hat{\mathbf{x}}(K)$$

$$\mathbf{x}(K+1) = \Phi\cdot\mathbf{x}(K) + \Gamma\,u(K)$$

$$= \begin{bmatrix} 1.0201 & 0.2013 \\ 0.2013 & 1.0201 \end{bmatrix}\cdot\mathbf{x}(K) + \begin{pmatrix} 0.0201 \\ 0.2013 \end{pmatrix}(-18.1095 \quad -5.9871)\cdot\hat{\mathbf{x}}(K)$$

$$= \begin{bmatrix} 1.0201 & 0.2013 \\ 0.2013 & 1.0201 \end{bmatrix}\cdot\mathbf{x}(K) + \begin{bmatrix} -0.3640 & -0.1203 \\ -3.6454 & -1.2052 \end{bmatrix}\cdot\hat{\mathbf{x}}(K) \quad \cdots\cdots \ (b)$$

$$\theta(K) = H\,\mathbf{x}(K) = (1 \quad 0)\cdot\mathbf{x}(K) \quad \cdots\cdots \ (c)$$

From the equations (a), (b) and (c), as well as the initial angle error, $\theta(0) = 0.1$ radians, we can compute the time response for $\theta(t)$.

(The following data are obtained from a BASIC program in VAX)

-5 starts at .1