EE 425GU/451WSU Digital Communication System
One hour 20 minute examination
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March 2, 1988

135/150

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One Sheet of Notes

(25 points each)

1. Consider a source with five symbols x_1, x_2, x_3, x_4, x_5 with probabilities $P(x_1) = .1$, $P(x_2) = .15$, $P(x_3) = .20$, $P(x_4) = .25$, $P(x_5) = .3$. Find:

   a) Entropy of the source

   b) a Huffman code

   c) efficiency of the Huffman code.

2. A channel has transition matrix

$$\begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}$$

The input probabilities are

$P_1 = 0.55$, $P_2 = 0.45$

Find: a) the mutual information

b) the probability of error.

3. The Shannon-Hartley law may be written as

$$\frac{C}{W} = \log_2 \left[1 + \frac{E_b}{N_0} \left(\frac{R}{W}\right)\right]$$

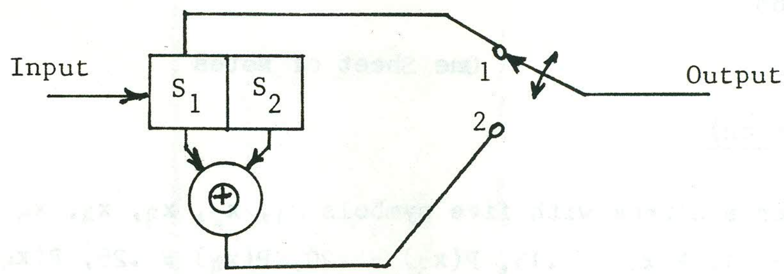
Write a short paragraph describing the significance of this law.

4. The parity check matrix for a (7,4) code is given as

$$H = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Find the (a) corresponding generator matrix, (b) codeword for information word 1 0 1 1, (c) Is 1 1 1 0 0 0 1 a codeword? Why? (d) If the word in (c) is received, what is the most likely transmitted codeword?

5. A convolutional encoder is shown. $k = 1$, $n = 2$, $L = 2$



Find (a) state diagram, (b) trellis diagram (use a solid line to denote an information bit "0" and a dashed line to denote an information bit "1"; indicate transmitted sequence as we move from one state to the next). (c) d_{free} and number of correctable errors, t. (d) transmitted sequence for information sequence 1 0 1 1 and initial state "0".

6. A certain bandpass filter has transfer function

$$H(f) = \frac{1}{1+j\frac{(f - 10^5)}{1,000}} + \frac{1}{1+j\frac{(f + 10^5}{1,000}}$$

The input to the filter is

$$x(t) = u(t)\sin(2\pi \times 10^5 t)$$

determine and sketch the filter ouput, y(t).

TABLE 2-4. Fourier Transform Theorems

Name of Theorem	Signal	Transform
1. Superposition (a_1 and a_2 arbitrary constants)	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(f) + a_2X_2(f)$
2. Time delay	$x(t - t_0)$	$X(f)e^{-j2\pi f t_0}$
3a. Scale change	$x(at)$	$\|a\|^{-1}X\left(\frac{f}{a}\right)$
3b. Time reversal[a]	$x(-t)$	$X(-f) = X^*(f)$
4. Duality	$X(t)$	$x(-f)$
5a. Frequency translation	$x(t)e^{j\omega_0 t}$	$X(f - f_0)$
5b. Modulation	$x(t)\cos\omega_0 t$	$\frac{1}{2}X(f - f_0) + \frac{1}{2}X(f + f_0)$
6. Differentiation	$\frac{d^n x(t)}{dt^n}$	$(j2\pi f)^n X(f)$
7. Integration	$\int_{-\infty}^{t} x(t')\,dt'$	$(j2\pi f)^{-1}X(f) + \frac{1}{2}X(0)\delta(f)$
8. Convolution	$\int_{-\infty}^{\infty} x_1(t - t')x_2(t')\,dt' = \int_{-\infty}^{\infty} x_1(t')x_2(t - t')\,dt'$	$X_1(f)X_2(f)$
9. Multiplication	$x_1(t)x_2(t)$	$\int_{-\infty}^{\infty} X_1(f - f')X_2(f')\,df' = \int_{-\infty}^{\infty} X_1(f')X_2(f - f')\,df$

[a] $\omega_0 = 2\pi f_0$; $x(t)$ is assumed to be real in 3b.

TABLE 2-5. Fourier Transform Pairs

Pair Number	$x(t)$	$X(f)$	Comments on Derivation
1.	$\Pi\left(\frac{t}{\tau}\right)$	$\tau\ \mathrm{sinc}\,(\tau f)$	Direct evaluation
2.	$2W\ \mathrm{sinc}(2Wt)$	$\Pi\left(\frac{f}{2W}\right)$	Duality with pair 1
3.	$\Lambda\left(\frac{t}{\tau}\right)$	$\tau\ \mathrm{sinc}^2\,(\tau f)$	Convolution with pair 1
4.	$\exp\,(-\alpha t)u(t),\ \alpha > 0$	$\dfrac{1}{\alpha + j2\pi f}$	Direct evaluation
5.	$t\exp\,(-\alpha t)u(t),\ \ \alpha > 0$	$\dfrac{1}{(\alpha + j2\pi f)^2}$	Differentiation of pair 4 with respect to α
6.	$\exp\,(-\alpha\lvert t\rvert),\ \ \alpha > 0$	$\dfrac{2\alpha}{\alpha^2 + (2\pi f)^2}$	Direct evaluation
7.	$\delta(t)$	1	Sifting property of $\delta(t)$
8.	1	$\delta(f)$	Duality with pair 7
9.	$\delta(t - t_0)$	$\exp\,(-j2\pi f t_0)$	Shift and pair 7
10.	$\exp\,(j2\pi f_0 t)$	$\delta(f - f_0)$	Duality with pair 9
11.	$\cos 2\pi f_0 t$	$\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$	Exponential representation of cos and sin and pair 10
12.	$\sin 2\pi f_0 t$	$\dfrac{1}{2j}\,\delta(f - f_0) - \dfrac{1}{2j}\,\delta(f + f_0)$	
13.	$u(t)$	$(j2\pi f)^{-1} + \frac{1}{2}\delta(f)$	Integration and pair 7
14.	$\mathrm{sgn}\,(t)$	$(j\pi f)^{-1}$	Pair 8 and pair 13 with superposition
15.	$\dfrac{1}{\pi t}$	$-j\ \mathrm{sgn}\,(f)$	Duality with pair 14
16.	$\hat{x}(t) = \dfrac{1}{\pi}\displaystyle\int_{-\infty}^{\infty} \dfrac{x(\lambda)}{t - \lambda}\,d\lambda$	$-\,j\ \mathrm{sgn}\,(f)X(f)$	Convolution and pair 15
17.	$\displaystyle\sum_{m=-\infty}^{\infty} \delta(t - mT_s)$	$f_s \displaystyle\sum_{m=-\infty}^{\infty} \delta(f - mf_s),\ \ f_s = T_s^{-1}$	Example 2-2

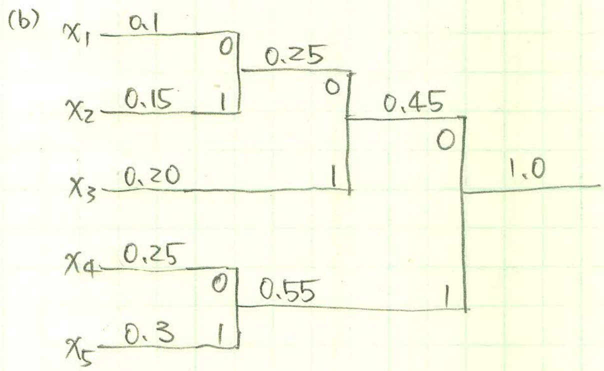
PROBLEM 1.

(a) $H(X) = -P(X_1)\log_2 P(X_1) - P(X_2)\log_2 P(X_2) - P(X_3)\log_2 P(X_3) - P(X_4)\log_2 P(X_4) - P(X_5)\log_2 P(X_5)$

$= -0.1\log_2 0.1 - 0.15\log_2 0.15 - 0.20\log_2 0.20 - 0.25\log_2 0.25 - 0.3\log_2 0.3$

$= 2.22821$ bits/symbol

(b)



Symbol	code
x_1	000
x_2	001
x_3	01
x_4	10
x_5	11

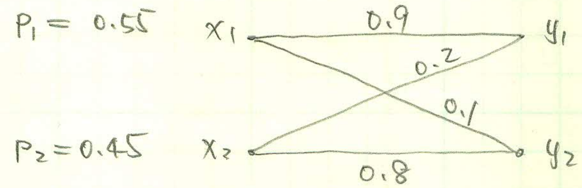
(c) $L = 3\times 0.1 + 3\times 0.15 + 2\times(0.2 + 0.25 + 0.3)$

$= 2.25$

$\text{Effic.} = \dfrac{H(X)}{L} = 99.03\%$

PROBLEM 2.

$$P(\underset{\sim}{y}|\underset{\sim}{x}) = \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix} = \begin{bmatrix} P(y_1|x_1) & P(y_2|x_1) \\ P(y_1|x_2) & P(y_2|x_2) \end{bmatrix}$$



(a) $[P(y_1),\ P(y_2)] = [0.55 \quad 0.45] \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}$

$$= [0.585 \quad 0.415]$$

$$H(\underline{Y}) = -0.585 \log_2 0.585 - 0.415 \log_2 0.415$$

$$= 0.97905$$

$$-H(\underline{Y}|\underline{X}) = \sum_{i=1}^{2} \sum_{j=1}^{2} P(x_i) P(y_j|x_i) \log_2 P(y_j|x_i)$$

$$= P_1 \left(0.9 \times \log_2 0.9 + 0.1 \times \log_2 0.1\right)$$

$$+ P_2 \left(0.2 \log_2 0.2 + 0.8 \log_2 0.8\right)$$

$$= 0.55 \times (-0.469) + 0.45\,(-0.72193)$$

$$= -0.58282$$

$I(\underline{X};\underline{Y}) = H(\underline{Y}) - H(\underline{Y}|\underline{X}) = 0.39623$ bits / symbol.

(b) $P_E = 0.1 \times 0.55 + 0.2 \times 0.45 = 0.145$

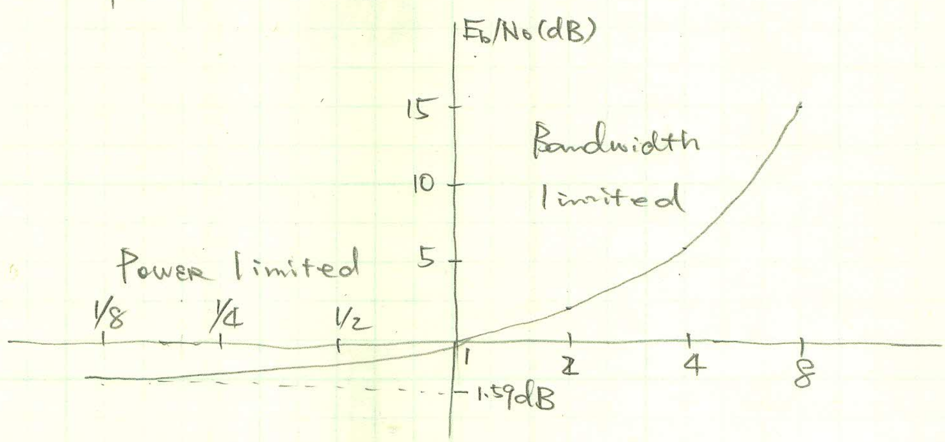
PROBLEM 3.

Although Shannon-Hartley law does not provide us how to find a error-free code. It does prove theorically that for a particular channel, we can find a suitable code for a soure to be transmitted by the probability of error as small as we can. Also, Shannon-Hartley law states that no-source can be transmitted without error when the signal-noise-ratio is below -1.59 dB.

If we let C=R at the equation given, we can rewrite it as

$$\frac{E_b}{N_o} = \frac{\left[2^{R/W}-1\right]}{R/W}$$

And we can plot this equation as below



From the plot, we can see that the region above the curve is error-free-possible, under the curve, it is impossible to do so. By the way, we can devide the error-free-possible region by Bandwidth limited and power-limited.

What about trade-off of $\frac{E_b}{N_o}$ vs. $\frac{R}{W}$? -10

PROBLEM 4.

$$H = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

(a)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

(b)

$$\underset{\sim}{T} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

(c)

$$\underset{\sim}{S} = H \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \neq \underset{\sim}{0}$$

So, 1110001 is not a codeword.

(d) from (c), Compared $\underset{\sim}{S}$ to H, we know

$$\underset{\sim}{e} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \underset{\sim}{T} = \underset{\sim}{R} \oplus \underset{\sim}{e} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

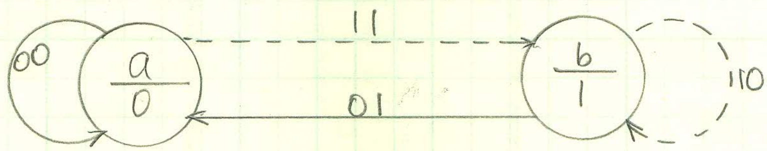
PROBLEM 5.

S_1	S_2	Output 1	Output 2
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

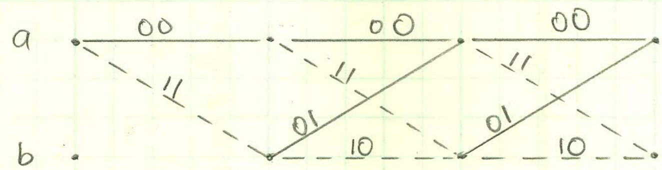
Define

states	S_1
a	0
b	1

(a)



(b)



(c) $d_{free} = W(1101) = 3$

$t \le \frac{d_{free} - 1}{2} = 1$

(d)

INFORMATION SEQ.	TRANSMITTED SEQ.
1011	11011110

PROBLEM 6.

$$H_e(f) = \frac{1}{1 + j\frac{f}{1000}} \qquad f_0 = 10^5, \Rightarrow \omega_0 = 2\pi \times 10^5$$

$$x(t) = u(t)\sin(2\pi\times 10^5 t) \qquad \text{Let } \omega_0 = 2\pi \times 10^5$$

$$= u(t)\sin(\omega_0 t)$$

$$= \mathrm{Re}\{-j u(t) e^{j\omega_0 t}\}$$

$$\therefore \tilde{x}(t) = -j u(t)$$

$$\tilde{X}(f) = -j\left[\frac{1}{j2\pi f} + \frac{1}{2}\delta(f)\right]$$

$$= \frac{-1}{2\pi f} - \frac{1}{2} j\delta(f)$$

$$\tilde{Y}(f) = H_e(f)\tilde{X}(f) = \frac{-1}{1 + j\frac{f}{1000}}\left[\frac{1}{2\pi f} + \frac{1}{2} j\delta(f)\right]$$

$$= -\frac{1}{(2\pi f)(1 + j\frac{f}{1000})} - \frac{1}{1 + j\frac{f}{1000}} \cdot \frac{1}{2} j\delta(f)$$

$$= \frac{j/2\pi\times 1000}{1 + j\frac{f}{1000}} - \frac{1}{2\pi f} - \frac{1}{2} j\delta(f)$$

$$= j\frac{1}{2000\pi + j2\pi f} - \frac{1}{2\pi f} - \frac{1}{2} j\delta(f)$$

$$\therefore \tilde{y}(t) = j e^{-2000\pi t} u(t) - j u(t)$$

$$= j[e^{-2000\pi t} u(t) - u(t)]$$

$$y(t) = \mathrm{Re}\{\tilde{y}(t) e^{j\omega_0 t}\}.$$

$$= \mathrm{Re}\{ j[e^{-2000\pi t} u(t) - u(t)][\cos(2\pi\times 10^5 t) + j\sin(2\pi\times 10^5 t)]$$

$$= [1 - e^{-2000\pi t}]\sin(2\pi\times 10^5 t)\, u(t)$$

sketch? −5

E-E 425GU/451WSU Digital Communication Systems
1 hour 20 minute Examination
April 25, 1988
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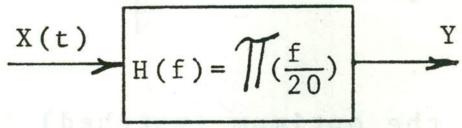
Ben Chen

1. (30pts)



X(t) is a Gaussian random process with two-sided power spectral density

$$S_x(f) = \tfrac{1}{2}\ \text{W/Hz}$$

Find (a) mean of Y(t), (b) autocorrelation function of Y(t), (c) the joint pdf of Y(t) at time instants t_1 and $t_1 + 0.05$ sec.

2. (35pts)

$$x_c(t) = z(t)\cos(2\pi f_o t)$$

with

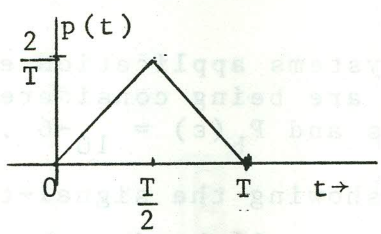
$$z(t) = \sum_{k=-\infty}^{\infty} A_k p(t-kT-\Delta)$$

A_k can take on 4-levels 0, A, 2A and 3A with equal probability.

$$E\{A_k A_{k+m}\} = E\{A_k^2\},\ m=0$$
$$= 0\ ,\ m \neq 0$$

Δ is a random variable uniformly distributed between 0 and T.

p(t) is the triangular pulse shown.



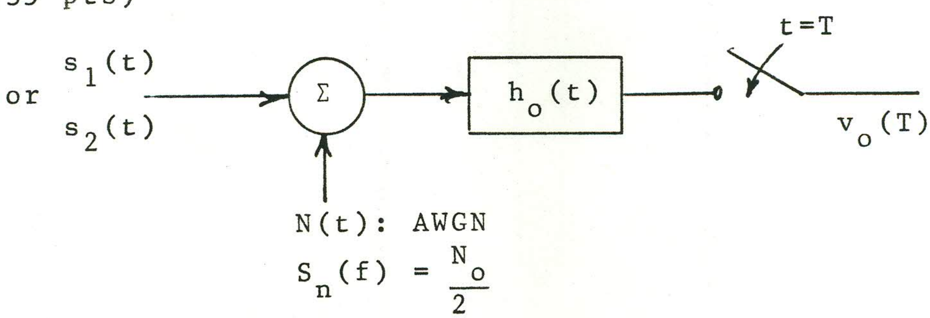
$\frac{T}{2} \cdot \frac{m}{f_0}$

Determine and sketch the power spectral density of (a) z(t), (b) $x_c(t)$.

$(0.1 X_1,\ 0.1 X_2)\begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$

$0.1 X_1^2 + 0.1 X_2^2$

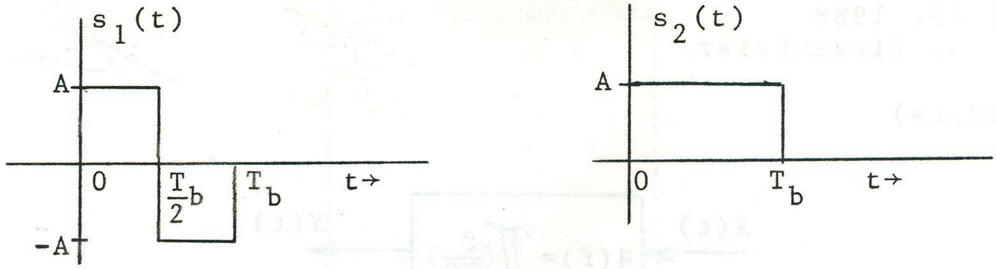
3. (35 pts)



$\sin\left(\frac{T}{2} \cdot \frac{m}{T}\pi\right)$

$\left(\frac{m}{2}\right)$

3.(cont.)



Find (a) and sketch the optimum (matched) filter impulse response, $h_o(t)$, for detection of $s_1(t)$ and $s_2(t)$.

(b) the correlation coefficient for $s_1(t)$ and $s_2(t)$.

(c) the probability of detection error (in terms of A, T_b, and N_o).

(d) the probability of bit error for $\frac{E_b}{N_o} = 10$ dB.

4. (30 pts)

Coherent FSK signals

$$\left.\begin{aligned} s_1(t) &= A_c \cos(2\pi f_o t) \\ s_2(t) &= A_c \cos[2\pi(f_o+\Delta f)t] \end{aligned}\right\} \quad 0 \le t \le T_b$$

are present with additive white Gaussian noise with two-sided power spectral density $S_N(f)=N_o/2$.

(a) For a matched filter receiver, derive an expression for the probability of bit error in terms of A_c, T_b, N_o and Δf. (25pts)

(b) For what value of Δf is $P_b(\varepsilon)$ minimum? (5pts)

5. (20 pts)

For a particular systems application either BPSK or OQPSK modulation schemes are being considered. For either modulation scheme R_b = 16 Kbps and $P_b(\varepsilon) = 10^{-6}$.

Construct a table showing the signal-to-noise ratios $(\frac{E_b}{N_o})$ in dB and the null-to-null bandwidths of the two modulation schemes.

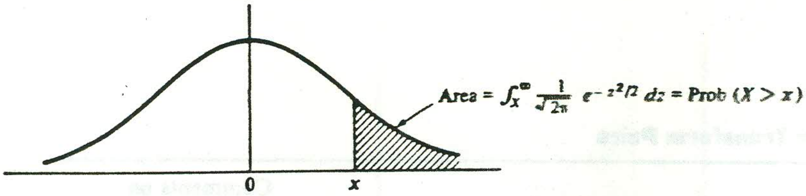
TABLE 2-5. Fourier Transform Pairs

Pair Number	$x(t)$	$X(f)$	Comments on Derivation
1.	$\Pi\left(\frac{t}{\tau}\right)$	$\tau \operatorname{sinc}(\tau f)$	Direct evaluation
2.	$2W \operatorname{sinc}(2Wt)$	$\Pi\left(\frac{f}{2W}\right)$	Duality with pair 1
3.	$\Lambda\left(\frac{t}{\tau}\right)$	$\tau \operatorname{sinc}^2(\tau f)$	Convolution with pair 1
4.	$\exp(-\alpha t)u(t),\ \alpha > 0$	$\frac{1}{\alpha + j2\pi f}$	Direct evaluation
5.	$t \exp(-\alpha t)u(t),\quad \alpha > 0$	$\frac{1}{(\alpha + j2\pi f)^2}$	Differentiation of pair 4 with respect to α
6.	$\exp(-\alpha\|t\|),\quad \alpha > 0$	$\frac{2\alpha}{\alpha^2 + (2\pi f)^2}$	Direct evaluation
7.	$\delta(t)$	1	Sifting property of $\delta(t)$
8.	1	$\delta(f)$	Duality with pair 7
9.	$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$	Shift and pair 7
10.	$\exp(j2\pi f_0 t)$	$\delta(f - f_0)$	Duality with pair 9
11.	$\cos 2\pi f_0 t$	$\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$	Exponential representation of cos and sin and pair 10
12.	$\sin 2\pi f_0 t$	$\frac{1}{2j}\delta(f - f_0) - \frac{1}{2j}\delta(f + f_0)$	
13.	$u(t)$	$(j2\pi f)^{-1} + \frac{1}{2}\delta(f)$	Integration and pair 7
14.	$\operatorname{sgn}(t)$	$(j\pi f)^{-1}$	Pair 8 and pair 13 with superposition
15.	$\frac{1}{\pi t}$	$-j \operatorname{sgn}(f)$	Duality with pair 14
16.	$\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\lambda)}{t - \lambda}\, d\lambda$	$-j \operatorname{sgn}(f) X(f)$	Convolution and pair 15
17.	$\sum_{m=-\infty}^{\infty} \delta(t - mT_s)$	$f_s \sum_{m=-\infty}^{\infty} \delta(f - mf_s),\ f_s = T_s^{-1}$	Example 2-2

$$Q(x) = \operatorname{Erfc}(x) \approx \left[\frac{1}{(1 - a)x + a\sqrt{x^2 + b}}\right] \frac{1}{\sqrt{2\pi}} e^{-x^2/2},$$

where $a = 1/\pi$ and $b = 2\pi$.

Table G.1 Values of Erfc (x) vs. x.†

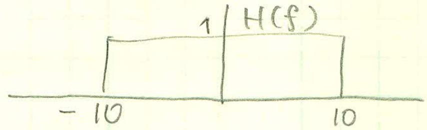


	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.00990	.00964	.00939	.00914	.00889	.00866	.00842
2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139

Table G.2 Values of Erfc (x) for large x.

x	10 log x	Erfc (x)	x	10 log x	Erfc (x)	x	10 log x	Erfc (x)
3.00	4.77	1.35E-03	4.00	6.02	3.17E-05	5.00	6.99	2.87E-07
3.05	4.84	1.14E-03	4.05	6.07	2.56E-05	5.05	7.03	2.21E-07
3.10	4.91	9.68E-04	4.10	6.13	2.07E-05	5.10	7.08	1.70E-07
3.15	4.98	8.16E-04	4.15	6.18	1.66E-05	5.15	7.12	1.30E-07
3.20	5.05	6.87E-04	4.20	6.23	1.33E-05	5.20	7.16	9.96E-08
3.25	5.12	5.77E-04	4.25	6.28	1.07E-05	5.25	7.20	7.61E-08
3.30	5.19	4.83E-04	4.30	6.33	8.54E-06	5.30	7.24	5.79E-08
3.35	5.25	4.04E-04	4.35	6.38	6.81E-06	5.35	7.28	4.40E-08
3.40	5.31	3.37E-04	4.40	6.43	5.41E-06	5.40	7.32	3.33E-08
3.45	5.38	2.80E-04	4.45	6.48	4.29E-06	5.45	7.36	2.52E-08
3.50	5.44	2.33E-04	4.50	6.53	3.40E-06	5.50	7.40	1.90E-08
3.55	5.50	1.93E-04	4.55	6.58	2.68E-06	5.55	7.44	1.43E-08
3.60	5.56	1.59E-04	4.60	6.63	2.11E-06	5.60	7.48	1.07E-08
3.65	5.62	1.31E-04	4.65	6.67	1.66E-06	5.65	7.52	8.03E-09
3.70	5.68	1.08E-04	4.70	6.72	1.30E-06	5.70	7.56	6.00E-09
3.75	5.74	8.84E-05	4.75	6.77	1.02E-06	5.75	7.60	4.47E-09
3.80	5.80	7.23E-05	4.80	6.81	7.93E-07	5.80	7.63	3.32E-09
3.85	5.85	5.91E-05	4.85	6.86	6.17E-07	5.85	7.67	2.46E-09
3.90	5.91	4.81E-05	4.90	6.90	4.79E-07	5.90	7.71	1.82E-09
3.95	5.97	3.91E-05	4.95	6.95	3.71E-07	5.95	7.75	1.34E-09

1.



$$S_Y(f) = S_X(f)\cdot|H(f)|^2 = \frac{1}{2}\cdot\Pi^2\left(\frac{f}{20}\right) = \frac{1}{2}\Pi\left(\frac{f}{20}\right)$$

$$R_Y(\tau) = \mathcal{F}^{-1}\{S_Y(f)\} = \frac{1}{2}\times 20\cdot \mathrm{sinc}(20\tau)$$

$$= 10\ \mathrm{sinc}(20\tau)$$

(a) $m_Y(t) = \lim_{\tau\to\infty} 10\ \mathrm{sinc}(20\tau) = 0$ ✓

(b) $R_Y(\tau) = 10\ \mathrm{sinc}(20\tau)$ ✓

(c) $$C_{X_1X_2} = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}$$

$$C(\tau) = R(\tau) = 10\ \mathrm{sinc}(20\tau)$$

$$C_{11} = C_{22} = C(0) = R(0) = 10$$

$$C_{12} = C_{12} = R(0.05) = 0$$

$$C_{X_1X_2} = \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix} \qquad C_{X_1X_2}^{-1} = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix}$$

$$|C_{X_1X_2}| = 100$$

pdf: $$f_{X_1X_2}(x_1, t_1;\ x_2, t_1+0.05) = \frac{1}{2\pi\times 10}\, e^{-\frac{1}{2}(x_1, x_2)\begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}}$$

Y_1 Y_2

$$= \frac{1}{20\pi}\, e^{-\frac{1}{20}(x_1^2 + x_2^2)}$$

y_1 y_2

2. $p(t) = \frac{2}{T} \cdot \Lambda\left(\frac{t - \frac{T}{2}}{T/2}\right)$

$$P(f) = e^{-j2\pi f \cdot \frac{T}{2}} \cdot \frac{2}{T} \cdot \frac{T}{2}\, \mathrm{sinc}^2\left(\frac{T}{2} \cdot f\right)$$

$$= e^{-j\pi f T}\, \mathrm{sinc}^2\left(\frac{T}{2} f\right)$$

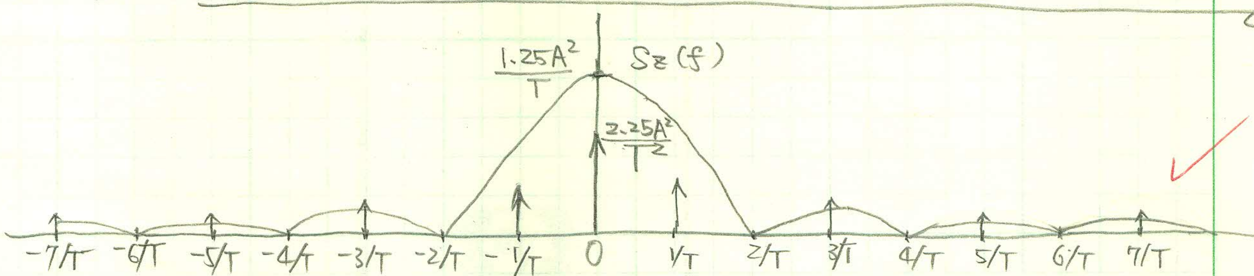
$$|P(f)|^2 = \mathrm{sinc}^4\left(\frac{T}{2} f\right)$$

$$a = E\{A_k\} = 0.25 \times 0 + 0.25A + 0.5A + 0.75A = 1.5A$$

$$\sigma_A^2 = 0.25\left[(0 - 1.5A)^2 + (A - 1.5A)^2 + (2A - 1.5A)^2 + (3A - 1.5A)^2\right]$$

$$= 1.25A^2$$

(a) $$S_Z(f) = \underline{\frac{1.25A^2}{T} \cdot \mathrm{sinc}^4\left(\frac{T}{2} f\right) + \left(\frac{1.5A}{T}\right)^2 \cdot \sum_{m=-\infty}^{\infty} \left|\mathrm{sinc}^4\left(\frac{m}{2}\right)\right|^2 \delta\left(f - \frac{m}{T}\right)}$$

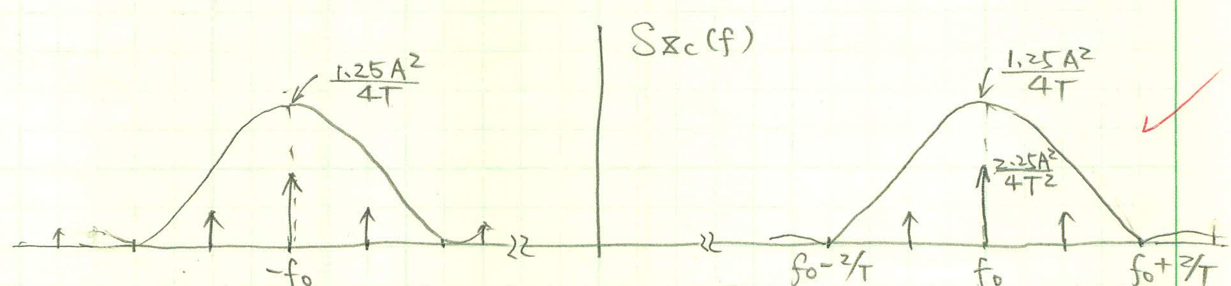


(b) $$S_{X_c}(f) = \frac{1}{4} S_Z(f - f_0) + \frac{1}{4} S_Z(f + f_0)$$

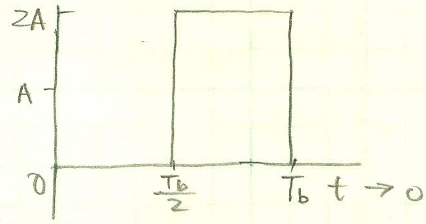
$$= \frac{1.25A^2}{4T} \mathrm{sinc}^4\left[\frac{T}{2}(f - f_0)\right] + \frac{1.25A^2}{4T} \mathrm{sinc}^4\left[\frac{T}{2}(f + f_0)\right]$$

$$+ \frac{2.25A^2}{4T^2} \cdot \sum_{m=-\infty}^{\infty} \left|\mathrm{sinc}^4\left[\frac{T}{2}\left(\frac{m}{T} - f_0\right)\right]\right|^2 \delta\left(f - f_0 - \frac{m}{T}\right)$$

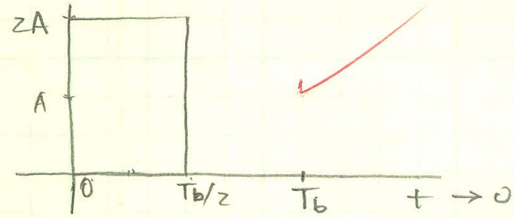
$$+ \frac{2.25A^2}{4T^2} \cdot \sum_{m=-\infty}^{\infty} \left|\mathrm{sinc}^4\left[\frac{T}{2}\left(\frac{m}{T} + f_0\right)\right]\right|^2 \delta\left(f + f_0 - \frac{m}{T}\right)$$



3. $g(t) = s_2(t) - s_1(t)$



the optimum filter impulse $h_o(t) = g(T_b - t)$



(a) $$h_o(t) = \Pi\left(\frac{t - \frac{T_b}{4}}{T_b/2}\right) = \Pi\left(\frac{4t - T_b}{2T_b}\right)$$

(b) $$\int_0^{T_b} s_1(t)s_2(t)\,dt = \int_0^{T_b/2} A^2\,dt - \int_{T_b/2}^{T_b} A^2\,dt = 0.$$

$$\rho_{12} = 0$$

(c) $$E_1 = \int_0^{T_b} s_1^2(t)\,dt = \int_0^{T_b} A^2\,dt = A^2 T_b = E_b$$

$$E_2 = A^2 T_b = E_b$$

$$P_\varepsilon = Q\left(\sqrt{\frac{2A^2T_b}{2N_0}}\right) = Q\left(\sqrt{\frac{A^2T_b}{N_0}}\right)$$

(d) $$P_\varepsilon = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$\frac{E_b}{N_0} = 10 \text{ dB}$$

$$P_\varepsilon = 0.000781376$$

4.

$$E_1 = \int_0^{T_b} s_1^2(t) \cos^2(2\pi f_0 t)\, dt$$

$$= A_c^2 \cdot \int_0^{T_b} \frac{1}{2}\, dt + A_c^2 \cdot \int_0^{T_b} \frac{1}{2} \cos(4\pi f_0 t)\, dt$$

$$= \frac{A_c^2}{2} T_b$$

$$E_2 = \frac{A_c^2}{2} T_b$$

$$\int_0^{T_b} s_1(t)\, s_2(t)\, dt = \int_0^{T_b} A_c^2 \cos(2\pi f_0 t) \cos(2\pi f_0 t + 2\pi \Delta f t)\, dt$$

$$= \int_0^{T_b} A_c^2 \cdot \frac{1}{2} \left[\cos(2\pi \Delta f t) + \cos(4\pi f_0 t + 2\pi \Delta f t) \right] dt$$

$$= \frac{A_c^2}{2} \cdot \frac{1}{2\pi \Delta f} \sin(2\pi \Delta f t) \Big|_0^{T_b} + 0.$$

$$= \frac{A_c^2}{4\pi \Delta f} \sin(2\pi \Delta f T_b) = \frac{A_c^2 T_b}{2} \operatorname{sinc}(2\Delta f T_b)$$

(a)

$$P_\varepsilon = Q\left(\sqrt{\frac{A_c^2 T_b - A_c^2 T_b \operatorname{sinc}(2\Delta f T_b)}{2N_0}} \right)$$

$$= Q\left(\sqrt{\frac{A_c^2 T_b}{2N_0} \left[1 - \operatorname{sinc}(2\Delta f T_b) \right]} \right)$$

(b) $2\Delta f\, T_b = 1.43$

$$\Delta f = \frac{0.715}{T_b}$$

5. $R_b = 16$ Kbps $P_b(\varepsilon) = 10^{-6}$

	BPSK	OQPSK
E_b/N_o	10.53 dB	10.53 dB
Bandwidth null-to-null	32 Kbps	16 Kbps