19/20

Benmei Chen

E-E 423 COMMUNICATION SYSTEMS
September 12, 1986
15 Minute Quiz
Dr. R. A. Birgenheier

Complete the Table

	f(t)	F(s)
1.	$e^{-at}u(t)$	$1/(a+j\omega)$
2.	$e^{-a\|t\|}$	$\frac{2a}{a^2 + \omega^2}$
3.	sgn(t)	$2/(j\omega)$
4.	$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
5.	1	$2\pi\delta(\omega)$
6.	$\cos\omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
7.	rec(t/T)	$T\,Sa(\omega T/2)$
8.	$\frac{W}{2\pi} Sa(Wt/2)$	rect(ω/W)
9.	Λ(t/T)	$T[Sa(\omega T/2)]^2$
10.	$\delta_T(t)$	$\omega_0\,\delta_{\omega_0}(\omega)$ $\omega_0 = \frac{2\pi}{T}$ X

(continue over)

−1

If the Fourier transform of f(t) is $F(\omega) = \dfrac{j\omega}{1 + j\omega}$

Determine the Fourier transform of

1. f(2t)

$$\mathcal{F}\{f(2t)\} = \frac{1}{2} F(\omega/2) = \frac{1}{2} \cdot \frac{j\omega/2}{1+j\omega/2} = \frac{j\omega}{2(2+j\omega)}$$

2. f(t-2)

$$\mathcal{F}\{f(t-2)\} = e^{-j\omega \cdot 2} F(\omega) = e^{-j2\omega} \cdot \frac{j\omega}{1+j\omega}$$

3. $e^{j\pi t} f(t-1)$

$$F_1(\omega) = \mathcal{F}\{f(t-1)\} = e^{-j\omega} \cdot \frac{j\omega}{1+j\omega}$$

$$\mathcal{F}\{e^{j\pi t} f(t-1)\} = F_1(\omega - \pi) = e^{-j(\omega-\pi)} \cdot \frac{j(\omega-\pi)}{1+j(\omega-\pi)}$$

4. $f(t)\cos(10\pi t)$

$$\mathcal{F}\{f(t)\cos(10\pi t)\} = \frac{1}{2} F(\omega - 10\pi) + \frac{1}{2} F(\omega + 10\pi)$$

$$= \frac{1}{2} \cdot \frac{j(\omega-10\pi)}{1+j(\omega-10\pi)} + \frac{1}{2} \cdot \frac{j(\omega+10\pi)}{1+j(\omega+10\pi)}$$

5. $\dfrac{d^2 f(t)}{dt^2}$ $\xrightarrow{\mathcal{F}}$ $(j\omega)^2 \cdot F(\omega) = (j\omega)^2 \cdot \dfrac{j\omega}{1+j\omega} = -\dfrac{j\omega^3}{1+j\omega}$

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BENMEI CHEN

E E 423 COMMUNICATION SYSTEMS

1 Hr. Exam.
October 5, 1986
Dr. R.A. Birgenheier

Closed Book - One Sheet of notes

20 points each

1. Clearly and concisely answer the following questions.

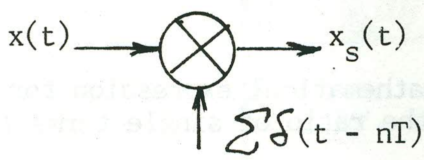
(a) What are the requirements for distortionless transmission?

(b) For a practical low-pass filter of 3-rd order or greater, approximately what is the relationship between the 10% to 90% rise-time and the -3 dB bandwidth of the filter?

(c) What are the consequences of observing a function through a finite rectangular window when computing a Fourier transform?

(d) Why are tapered window functions used? What is the penalty for using a tapered window?

2. Suppose we wish to sample a band-limited signal, $x(t)$, for which $|X(\omega)| = 0$ for $|\omega| > \omega_M$



(a) What is the minimum sampling frequency that can be used if we wish to reconstruct $x(t)$ from $x_s(t)$ without distortion?

(b) How do we reconstruct $x(t)$ from $x_s(t)$?

3. Consider a filter for which the unit-step response is

$$g(t) = [1 - e^{-10t}]u(t)$$

Determine (a) the unit-impulse response function, $h(t)$, (b) the transfer function, $H(s)$, (c) the frequency response function, $H(j\omega)$, and (d) the -3 dB bandwidth.

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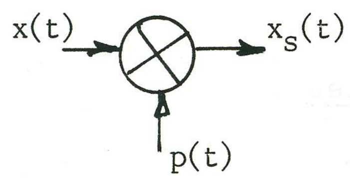
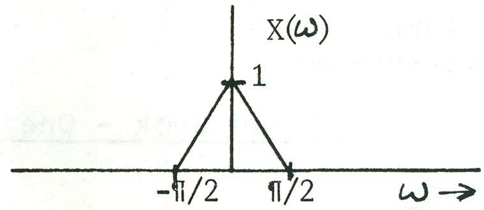
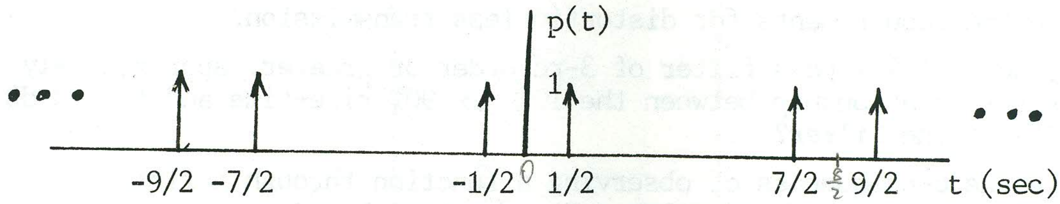
4.







(a) Determine the Fourier transform, $X_s(\omega)$, of the sampled signal. Sketch $|X_s(\omega)|$ for $|\omega| < 2\pi$.

(b) Can the original spectrum, $X(\omega)$, be recovered from $X_s(\omega)$? Why?

5. $f[k] = 1$, $k = 0,1,2,\ldots,7$
$= 0$, $k = 8,9,10,\ldots,15$

For $N = 16$ and $T = 1$ sec. obtain a mathematical expression for the magnitude of the DFT, $|F_D[n]|$. (Simplify to the ratio of single terms !)

1. (a) $H(\omega) = K e^{-j\omega t_0}$

(b) $t_r \doteq 1/(2B)$

(c) DFT. EASY APPLICATION IN COMPUTER. sidelobes, limited res. -5

(d) WE USE the TAPERED WINDOW FUNCTIONS IN ORDER TO REDUCE THE SIDELOBES. Penalty? poorer res. width. mainlobes. -2

2. $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) = \frac{1}{T} \sum_{n=-\infty}^{\infty} e^{jn\omega_0 t}$, $\omega_0 = \frac{2\pi}{T}$

$x_s(t) = x(t)\,p(t) = \frac{1}{T} \sum_{n=-\infty}^{\infty} x(t) e^{jn\omega_0 t}$

$X_s(\omega) = \frac{1}{T} \sum_{n=-\infty}^{\infty} X(\omega - n\omega_0)$

(a) $T_{max} = \frac{1}{2B}$

$\omega_{s,min} = \frac{2\pi}{T_{max}} = 4\pi B = 2\cdot(2\pi B) = 2\omega_M$

(b) WE CAN use THE FOLLOWING SYSTEM TO RECOVER $x(t)$ from $x_s(t)$

$$x_s(t) \longrightarrow \boxed{\begin{matrix}\text{LPF}\\ W=\omega_M\end{matrix}} \longrightarrow x(t)$$

3. $g(t) = [1 - e^{-10t}]\,u(t)$

(a) $h(t) = \frac{dg(t)}{dt} = \frac{d}{dt}[(1 - e^{-10t})\,u(t)]$

$= (10e^{-10t})\,u(t) + (1 - e^{-10t})\,\delta(t)$

$= 10e^{-10t}\,u(t)$

(b) $H(s) = \mathcal{L}\{h(t)\} = \mathcal{L}\{10e^{-10t}u(t)\} = 10/(s+10)$

(c) $H(j\omega) = \mathcal{F}\{h(t)\} = 10/(10+j\omega)$

(d) $|H(j\omega)| = 10/\sqrt{100+\omega^2}$ $\qquad |H(0)|\ dB = 0\ dB$

$-3\ dB = 20\log_{10}|H(j\omega)| \Rightarrow |H(j\omega)| = 0.707$

$\therefore \omega = 9.976$ rps

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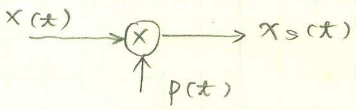
4. 

$$p(t) = \sum_{n=-\infty}^{\infty} P_n e^{jn\omega_0 t}$$

$$\omega_0 = \frac{2\pi}{T} \quad , \quad T = 4 \text{ sec}$$

$$\therefore \omega_0 = \frac{2\pi}{4} = \frac{\pi}{2} \text{ rps}$$

$$P_n = \frac{1}{4}\int_0^4 [\delta(t-\tfrac{1}{2}) + \delta(t-\tfrac{7}{2})] e^{-jn\omega_0 t} dt$$

$$= \frac{1}{4}\left[\int_0^4 \delta(t-\tfrac{1}{2}) e^{-jn\omega_0 t} dt + \int_0^4 \delta(t-\tfrac{7}{2}) e^{-jn\omega_0 t} dt \right]$$

$$= \frac{1}{4}\left[e^{-j\frac{n\omega_0}{2}} + e^{-j\frac{7n\omega_0}{2}} \right]$$

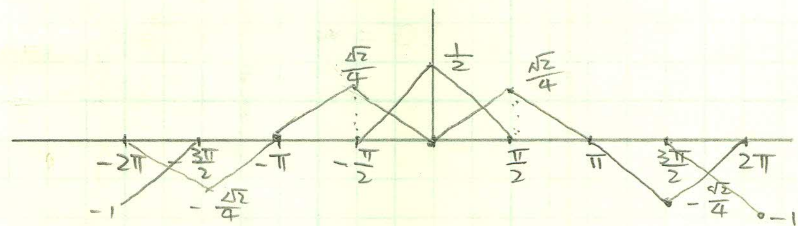
$$= \frac{1}{4}\left[e^{-j\frac{n}{2}\cdot\frac{\pi}{2}} + e^{-j\frac{7n}{2}\cdot\frac{\pi}{2}} \right]$$

$$= \frac{1}{4}\left[\cos(\tfrac{n\pi}{4}) - j\sin(\tfrac{n\pi}{4}) + \cos(\tfrac{7n\pi}{4}) - j\sin\tfrac{7n\pi}{4} \right]$$

$$= \frac{1}{4}\left[\cos\frac{n\pi}{4} - j\sin\frac{n\pi}{4} + \cos\frac{n\pi}{4} + j\sin\frac{n\pi}{4} \right]$$

$$= \frac{1}{2}\cos\frac{n\pi}{4}$$

(a)



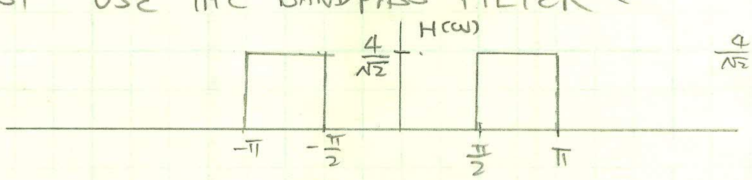
$$** : \left[x_s(t) = x(t)\cdot p(t) = x(t) \sum_{n=-\infty}^{\infty} P_n e^{jn\omega_0 t} \right.$$

$$X_s(\omega) = \frac{1}{2}\sum_{n=-\infty}^{\infty} \cos\frac{n\pi}{4}\, X(\omega - n\omega_0)$$

$$= \frac{1}{2}\sum_{n=-\infty}^{\infty} \cos\frac{n\pi}{4} \cdot X(\omega - \frac{n\pi}{2})$$

(b) WE CAN RECOVER $X(\omega)$ FROM $X_s(\omega)$,

FIRST USE THE BANDPASS FILTER :



$$\frac{4}{\sqrt{2}} = 2\sqrt{2}$$

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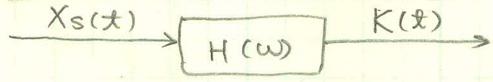
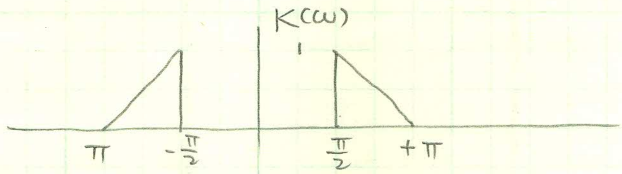
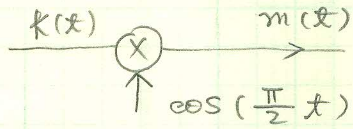
4 (b) CONT.

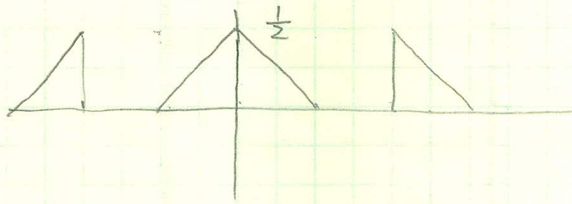




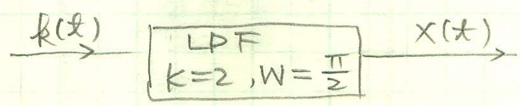


$$M(\omega) = \mathcal{F}\{k(t)\cos(\tfrac{\pi}{2}t)\}$$

$$= \tfrac{1}{2}K(\omega+\tfrac{\pi}{2}) + \tfrac{1}{2}K(\omega-\tfrac{\pi}{2})$$



USING LPF



5.

$$F_D[n] = \sum_{k=0}^{N-1} f[k]\, e^{-j\frac{2\pi}{N}\cdot n\cdot k}$$

$$= \sum_{k=0}^{15} f[k]\, e^{-j\frac{2\pi}{N} n\cdot k}$$

$$= \sum_{k=0}^{7} e^{-j\frac{2\pi}{16}\cdot n k}$$

$$= \sum_{k=0}^{7} e^{-j\frac{\pi}{8} n\cdot k}$$

$$= \begin{cases} 8, & \text{WHEN } n=0 \\ \dfrac{1-e^{-j\frac{\pi}{8}\cdot n\cdot 8}}{1-e^{-j\frac{\pi}{8}\cdot n}} = \dfrac{1-e^{-jn\pi}}{1-e^{-j\frac{\pi}{8}\cdot n}} = \dfrac{1-\cos n\pi}{1-e^{-j\frac{\pi}{8}n}} \end{cases}$$

$$= \begin{cases} 8 ; & n=0 \\ 2 ; & n=2m+1 \\ 0 ; & n=2m \end{cases} \qquad m=0, \pm 1, \ldots$$

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Benmei Chen

E-E 423 COMMUNICATION SYSTEMS
One Hour Examination
R. A. Birgenheier
November 12, 1986

One Sheet of Notes

25 pts each

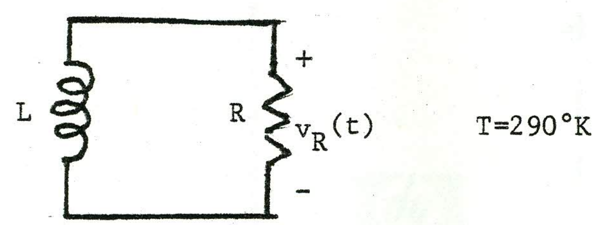
1. A periodic signal has Fourier series

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{4}{(2n-1)\pi} \exp\left[-j\frac{(2n-1)}{4}\pi t\right]$$

a) Determine the power spectral density, $S_f(\omega)$.

b) Determine the autocorrelation function, $R_f(\tau)$.

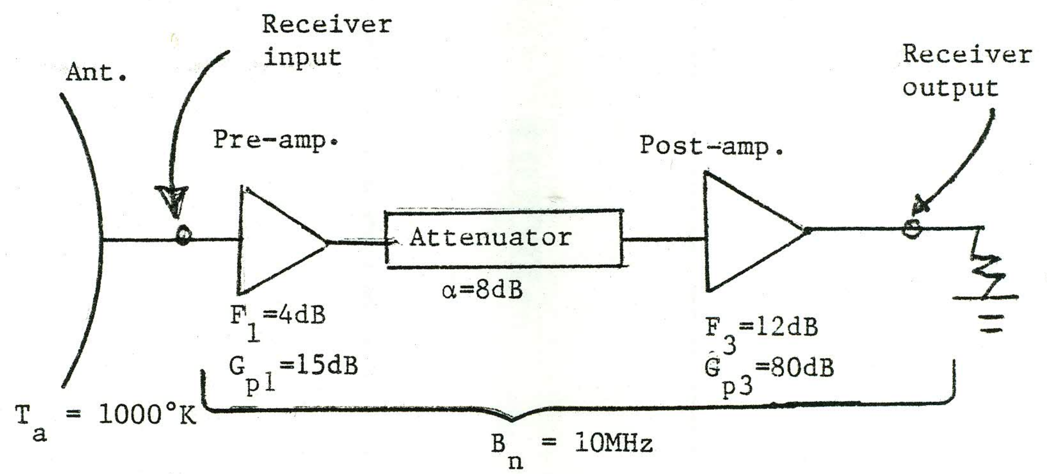
2.



a) Determine an expression for the power spectral density of $v_R(t)$.

b) For $R = 1K\Omega$ and $L = 1mH$, find the RMS value of $v_R(t)$.

3.



Determine the signal level necessary at the receiver input for an SNR of 40 dB at the receiver output.

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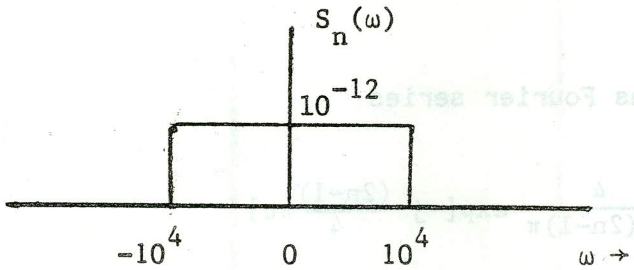
4. $y(t) = s(t) + n(t)$

where $s(t) = 2 \cos 10^3 t$ is the signal

and $n(t)$ is band limited white noise with power spectral density shown.



a) If $s(t)$ and $n(t)$ are uncorrelated, determine the autocorrelation function of $y(t)$.

b) Determine the signal-to-noise power ratio (express in dB).

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① ENERGY SPECTRAL DENSITY: $E(\omega)=|F(\omega)|^2$, $E_v=\frac{1}{2\pi R}\int_{-\infty}^{\infty}E(\omega)d\omega$ joules

Parseval's theorem: $\int_{-\infty}^{\infty}|f(t)|^2dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}|F(\omega)|^2d\omega$. $P=\frac{1}{T}\int_{-T/2}^{T/2}|f(t)|^2dt=\sum_{n=-\infty}^{\infty}|F_n|^2$

② POWER SPECTRAL DENSITY:

$P\triangleq\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}|f(t)|^2dt\triangleq\frac{1}{2\pi}\int_{-\infty}^{\infty}S_f(\omega)d\omega$, $S_f(\omega)=\lim_{T\to\infty}|F_T(\omega)|^2/T$ ✓, $F_T(\omega)=\mathcal{F}\{f(t)\cdot rect(t/T)\}$

IF $f(t)=\sum_{n=-\infty}^{\infty}F_ne^{jn\omega_0t}$, $\omega_0=2\pi/T$ & $F_n=\frac{1}{T}\int_{-T/2}^{T/2}f(t)e^{-jn\omega_0t}dt$; THEN $S_f(\omega)=2\pi\sum_{n=-\infty}^{\infty}|F_n|^2\delta(\omega-n\omega_0)$

THE POWER DENSITY OF OUTPUT SIGNAL: $\overline{g^2(t)}\triangleq P_{out}=\frac{1}{2\pi}\int_{-\infty}^{\infty}S_f(\omega)\cdot|H(\omega)|^2d\omega$; $S_g(\omega)=S_f(\omega)\cdot|H(\omega)|^2$

③ TIME-AVERAGED NOISE REPRE. SIGNAL-TO-NOISE RATIO IN dB: $[S/N]_{dB}=10\cdot\log_{10}[\overline{s^2(t)}/\overline{n^2(t)}]$

mean value, $\overline{n(t)}=\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}n(t)dt$, mean-square value, $\overline{n^2(t)}=\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}|n(t)|^2dt$

④ CORRELATION FUNCTIONS:

Autocorrelation function OF $f(t)$: $R_f(\tau)=\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}f^*(t)f(t+\tau)dt$

$S_f(\omega)=\mathcal{F}\{R_f(\tau)\}$. Crosscorrelation function OF $f(t)$ and $g(t)$: $R_{fg}(\tau)=\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}f^*(t)g(t+\tau)$

PROPERTIES: ① $R_f(-\tau)=R_f^*(\tau)$, ② $|R_f(\tau)|\leq R_f(0)$, ③ $R_f(\tau+T)=R_f(\tau)$, IF $f(t+T)=f(t)$

④ $R_f(0)=\overline{f^2(t)}$ // $S_f(\omega)=2\pi\sum_{n=-\infty}^{\infty}R_n\delta(\omega-n\omega_0)$, $R_n=\frac{1}{T}\mathcal{F}\{R_{fT}(\tau)\}|_{\omega=n\omega_0}$

⑤ CORRELATION FUNCTIONS FOR FINITE-ENERGY SIGNAL

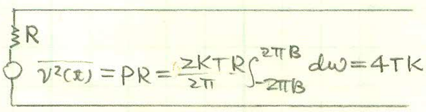
$r_f(\tau)=\int_{-\infty}^{\infty}f^*(t)\cdot f(t+\tau)dt$; $r_{fg}(\tau)=\int_{-\infty}^{\infty}f^*(t)g(t+\tau)dt$; $\mathcal{F}\{r_f(\tau)\}=|F(\omega)|^2$

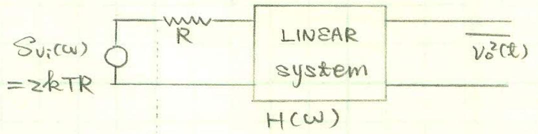
⑥ BAND-LIMITED WHITE NOISE: $S_n(\omega)=\eta/2$ W/Hz (measured over + freq.) $P_n=\eta B$ (Bandwidth)

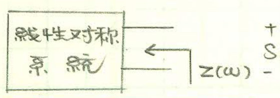
THE OUTPUT POWER: $\overline{n_o^2(t)}=\eta/2\pi\cdot\int_0^{\infty}|H(\omega)|^2d\omega$ W

⑦ THERMAL NOISE: $S_n(\omega)=h|\omega|/[\pi[\exp(h|\omega|/2\pi kT)-1]]$; $S_n(\omega)\cong 2kT$ W/Hz for $|\omega|\ll 2\pi kT/h$

$k=1.38\times10^{-23}$ joule/K, $h=6.625\times10^{-34}$ joule-sec







$S_v(\omega)=2kTR(\omega)$, $R(\omega)=Re\{Z(\omega)\}$

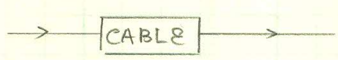
⑧ Equivalent Noise Bandwidth $B_N=\frac{1}{2\pi}[\int_0^{\infty}|H(\omega)|^2d\omega]/|H(\omega_0)|^2$, $\omega_0=0$ FOR LOW-PASS.

⑨ Available Power and Noise Temperature. $P_a=kTB$

⑩ Noise Figure: $F\triangleq(S/N)_i/(S/N)_o=(1+T_e/T_o)$, $T_e=(F-1)T_o$, $T_o=290°K$

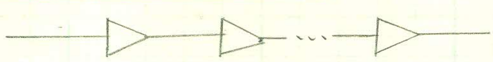
A resistive attenuator: $F=\alpha=\frac{1}{G_p}$.

Example:



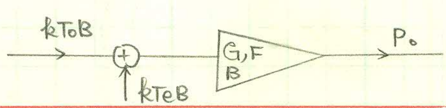
LOSS 6dB. $G_p=-6dB$ $\alpha=F=\frac{1}{G_p}=6dB$

A multistage amplifier:



$F=F_1+[(F_2-1)/G_1]+[(F_3-1)/G_1G_2]+\cdots$

A typical amplifier:



1. $$f(t) = \sum_{n=-\infty}^{\infty} \frac{4}{(2n-1)\pi} e^{-j\frac{2n-1}{4}\pi t}$$

$$= \sum_{n=-\infty}^{\infty} \frac{4}{(2n-1)\pi} e^{-j(2n-1)\cdot\frac{\pi}{4}t}$$

$$F_k = \begin{cases} \dfrac{4}{(2n-1)\pi} & k = 2n-1 \\ 0 & k \neq 2n-1 \end{cases} \qquad \omega_0 = \frac{\pi}{4}$$

① $$S_f(\omega) = 2\pi \sum_{k=-\infty}^{\infty} |F_k|^2 \, \delta(\omega - k\omega_0)$$

$$= 2\pi \sum_{n=-\infty}^{\infty} \frac{16}{(2n-1)^2\pi^2} \delta\left(\omega - (2n-1)\cdot\frac{\pi}{4}\right)$$ ✓

② $$R_f(\tau) = \mathcal{F}^{-1}\{S_f(\omega)\}$$

$$= \mathcal{F}^{-1}\left\{2\pi \sum_{n=-\infty}^{\infty} \frac{16}{(2n-1)^2\pi^2} \delta\left(\omega - (2n-1)\cdot\frac{\pi}{4}\right)\right\}$$

$$= 2\pi \sum_{n=-\infty}^{\infty} \frac{16}{(2n-1)^2\pi^2} \left[\mathcal{F}^{-1}\left\{\delta\left(\omega - (2n-1)\cdot\frac{\pi}{4}\right)\right\}\right.$$

$$= 2\pi \sum_{n=-\infty}^{\infty} \frac{16}{(2n-1)^2\pi^2} \cdot \frac{1}{2\pi} e^{j\frac{\pi}{4}(2n-1)\tau}$$

$$= \sum_{n=-\infty}^{\infty} \frac{16}{(2n-1)^2\pi^2} e^{j\frac{\pi}{4}(2n-1)\tau}$$

notes: $$R_f(\tau) = \sum_{n=-\infty}^{\infty} \frac{16}{(2n-1)^2\pi^2} e^{j\frac{\pi}{4}(2n-1)\tau}$$ ✓

the same as $$R_f(\tau) = \sum_{n=-\infty}^{\infty} \frac{16}{(2n-1)^2\pi^2} e^{-j\frac{\pi}{4}(2n-1)\tau}$$

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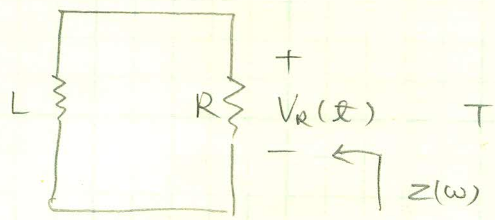

$T = 290°K$

$$Z(\omega) = \frac{R \cdot jL\omega}{R + jL\omega}$$

$$= \frac{R \cdot jL\omega\,(R - jL\omega)}{R^2 + L^2\omega^2} = \frac{RL^2\omega^2 + jR^2L\omega}{R^2 + L^2\omega^2}$$

$$R_e(\omega) = Re\{Z(\omega)\} = \frac{RL^2\omega^2}{R^2 + L^2\omega^2}$$

ⓐ $$S_{V_R}(\omega) = 2kTR(\omega) = 2kT \cdot \frac{RL^2\omega^2}{R^2 + L^2\omega^2}$$ ✓

ⓑ $R = 1000\Omega$. $L = 0.001$ mH.

$$S_{V_R}(\omega) = 2 \times 1.38 \times 10^{-23} \times 290° \cdot \frac{1000 \times 10^{-6}\,\omega^2}{10^6 + 10^{-6}\omega^2}$$

$$= 8.004 \times 10^{-24}\,\frac{\omega^2}{10^6 + 10^{-6}\omega^2}$$

$$\overline{v_R^2(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{V_R}(\omega)\,d\omega$$

$$= \frac{1}{\pi} \times 8.004 \times 10^{-24}\int_0^{\infty} \frac{\omega^2}{10^6 + 10^6\omega^2}\,d\omega = \infty$$ ✓

$$\sqrt{\overline{v_R^2(t)}} = \infty$$

something wrong here, unbelievable.

72

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③

$F_1 = 4\,dB = 2.512 \qquad F_2 = \alpha = 8\,dB = 6.31\ , \quad F_3 = 12\,dB = 15.849$

$G_1 = 15\,dB = 31.623 \qquad G_2 = \frac{1}{\alpha} = 0.158\ , \qquad G_3 = 80\,dB = 10^8$

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2}$$

$$= 2.512 + \frac{6.31 - 1}{31.623} + \frac{15.849 - 1}{31.623 \times 0.158} = 5.652$$

$$F = \frac{(S/N)_i}{(S/N)_o} \qquad (S/N)_o = 40\,dB = 10^4$$

$$(S/N_o)_i = F \cdot (S/N)_o = 5.652 \times 10^4 = 56518.35 = 47.522\ dB$$

−10

signal level?

④ ⓐ $R_y(\tau) = R_s(\tau) + R_n(\tau)$

2

(

)?

−10

ⓑ $s(t) = 2\cos 10^3 t.$

$S_m = 2\ V. \qquad S_{RMS} = \frac{2}{\sqrt{2}} = \sqrt{2}$

$\therefore\ \overline{S^2(t)} = (S_{RMS})^2 = 2$

$$\overline{n^2(t)} = \frac{1}{2\pi}\int_{-10^4}^{10^4} S_n(\omega)\, d\omega$$

$$= \frac{1}{2\pi} \times 10^{-12} \times 2 \times 10^4 = 3.183 \times 10^{-9}$$

$$SNR = \overline{S^2(t)} / \overline{n^2(t)} = 2 / 3.183 \times 10^{-9} = 87.982\ dB.$$

2

73

$\frac{80}{100}$ 85

E-E 423 COMMUNICATION SYSTEMS
One Hour Examination
R. A. Birgenheier
November 20, 1986

$\frac{80+80}{2} = 80$ Benmei Chen

One Sheet of Notes

25 pts each

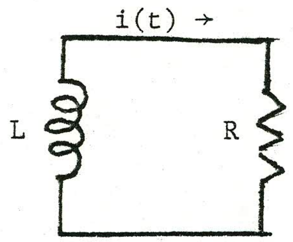
1. A periodic signal has Fourier series

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{4}{(2n-1)\pi} \exp\left[-j\frac{(2n-1)}{4}\pi t\right]$$

a) Determine the power spectral density, $S_f(\omega)$.

b) Determine the autocorrelation function, $R_f(\tau)$.

2.

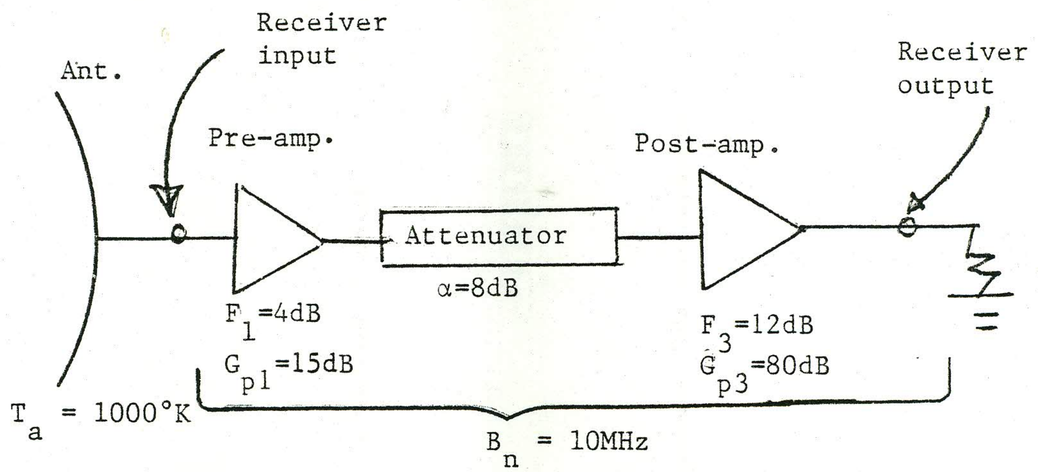


T=290°K

a) Determine an expression for the power spectral density of i(t).

b) For R = 1KΩ and L = 1mH, find the RMS value of i(t)

3.



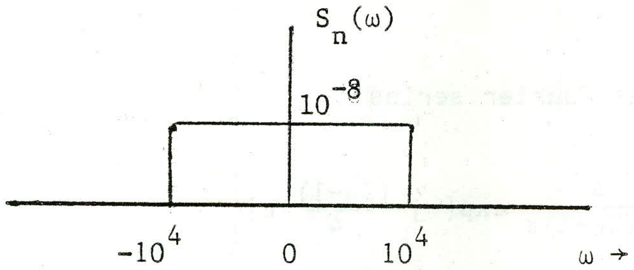
Determine the signal level necessary at the receiver input for an SNR of 40 dB at the receiver output.

66

4. $y(t) = s(t) + n(t)$

where $s(t) = 2 \cos 10^3 t$ is the signal

and $n(t)$ is band limited white noise with power spectral density shown.



a) If $s(t)$ and $n(t)$ are uncorrelated, determine the autocorrelation function of $y(t)$.

b) Determine the signal-to-noise power ratio (express in dB).

1. $f(t)=\sum_{n=-\infty}^{\infty}\frac{4}{(2n-1)\pi}e^{-j\frac{(2n-1)}{4}\pi t}=\sum_{n=-\infty}^{\infty}\frac{4}{(2n-1)\pi}e^{j\cdot[-(2n-1)]\cdot\frac{\pi}{4}t}$

$$F_{-k}=\begin{cases}\frac{4}{(2n-1)\pi} & k=(2n-1)\\ 0 & k=2n\end{cases}\quad , n \text{ is a integel from } -\infty, \text{to } \infty$$

$$f(t)=\sum_{n=-\infty}^{\infty}F_k e^{jk\frac{\pi}{4}t}\quad , \quad \omega_0=\frac{\pi}{4}$$

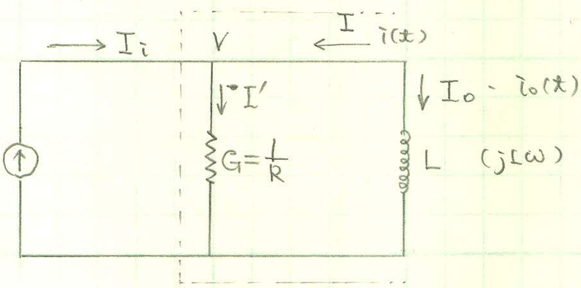
(a) $S_f(\omega)=2\pi\sum_{k=-\infty}^{\infty}|F_k|^2\delta(\omega-k\omega_0)$

$$=2\pi\sum_{n=-\infty}^{\infty}\frac{16}{(2n-1)^2\pi^2}\delta(\omega-(2n-1)\cdot\frac{\pi}{4})=2\pi\sum_{n=-\infty}^{\infty}\frac{16}{(2n-1)^2\pi^2}\delta(\omega+(2n-1)\cdot\frac{\pi}{4})$$

(b) $R_f(\tau)=\mathcal{F}^{-1}\{S_f(\omega)\}=\sum_{n=-\infty}^{\infty}\frac{16}{(2n-1)^2\pi^2}e^{j\frac{2n-1}{4}\pi\tau}=\sum_{n=-\infty}^{\infty}\frac{16}{(2n-1)^2\pi^2}e^{-j\cdot\frac{2n-1}{4}\pi\tau}$

2.

$S_{I_i}(\omega)$
$=2kTG$



IN ORDER TO FIND OUT THE $H(\omega)=\frac{I_0}{I_i}$, LET $I_0=1A$

$$V=jL\omega\quad , \quad I'=V\cdot G=jLG\omega\quad , \quad I_i=I_0+I'=1+jLG\omega$$

$$H(\omega)=\frac{I_0}{I_i}=\frac{1}{1+jLG\omega}\quad , \quad i(t)=-i_0(t)$$

$$H'(\omega)=\frac{I}{I_i}=-\frac{1}{1+jLG\omega}$$

(a) $S_i(\omega)=S_{I_i}(\omega)\cdot|H'(\omega)|^2=2kTG\cdot\frac{1}{1+L^2G^2\omega^2}=\frac{2kTR}{R^2+L^2\omega^2}$

(b) $\overline{i^2(t)}=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{2kTG}{1+L^2G^2\omega^2}d\omega=\frac{kTG}{L\pi G}\int_{-\infty}^{\infty}\frac{d(LG\omega)}{1+(LG\omega)^2}$

$$=\frac{kT}{L}\quad , \quad T=290\,^\circ K \qquad L=1mH$$

$$\sqrt{\overline{i^2(t)}}=\sqrt{kT/L}=\sqrt{1.38\times10^{-23}\times290/0.001}\doteq 2\times10^{-9}\text{ A rms.}$$

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3. $F_1 = 4dB = 2.5119$ $\quad F_2 = \alpha = 8dB = 6.3096$ $\quad F_3 = 12dB = 15.8489$

$G_{p1} = 15dB = 31.6228$ $\quad G_{p2} = \frac{1}{\alpha} = -8dB = 0.1585$ $\quad G_{p3} = 80dB.$

$$F = F_1 + \frac{F_2 - 1}{G_{p1}} + \frac{F_3 - 1}{G_{p1}G_{p2}} = 2.5119 + \frac{6.3096-1}{31.6228} + \frac{15.8489-1}{31.6228 \times 0.1585} = 5.6423 \approx 7.51dB$$

$F \triangleq (SNR)_i / (SNR)_o$ WHERE $N_i = kT_oB_n$, $T_o = 290°K$, $B_n = 10MHz$

$(SNR)_i = F \cdot (SNR)_o = F_{dB} + (SNR)_{o\,dB} = 7.51dB + 40dB = 47.51dB = 56423$

$S_i = N_i \cdot 56423 = kT_oB_N \times 56423 = 1.38 \times 10^{-23} \times 290 \times 10 \times 10^6 \times 56423 = 2.258 \times 10^{-9}$

$= -86.46$ dB.

misuse of noise figure

−10

4. (a) $R_y(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} y^*(t)\, y(t+\tau)\, dt$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} [s(t)+n(t)]^* \cdot [s(t+\tau)+n(t+\tau)]\, dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} [s^*(t)s(t+\tau) + s^*(t)n(t+\tau) + n^*(t)s(t+\tau) + n^*(t)n(t+\tau)]\, dt$$

$$= R_s(\tau) + R_{sn}(\tau) + R_{ns}(\tau) + R_n(\tau)$$

−10

$= R_s(\tau) + R_n(\tau)$ (Because $s(t)$ and $n(t)$ are uncorrelated)

(b) $s(t) = 2\cos 10^3 t$ $\quad \omega = 10^3$ $\quad T = \frac{2\pi}{\omega} = \frac{2\pi}{10^3}$

$$\overline{s^2(t)} = \frac{1}{T}\int_0^T [2\cos 10^3 t]^2\, dt$$

$$= \frac{10^3}{2\pi} \cdot \int_0^{\frac{2\pi}{10^3}} 4\cos^2 10^3 t\, dt = \frac{10^3}{2\pi}\int_0^{\frac{2\pi}{10^3}} 2(1+\cos(2\times 10^3 t))\, dt$$

$$= 2 \cdot \frac{10^3}{2\pi}\left[t + \sin(2\times10^3 t)\cdot\frac{1}{2\times10^3}\right]\Big|_0^{\frac{2\pi}{10^3}} = 2 \quad V^2$$

$$\overline{n^2(t)} = \frac{1}{2\pi}\int_{-10^4}^{10^4} 10^{-8} \cdot d\omega = \frac{1}{2\pi} \times 10^{-8} \times 2\times10^4 = 3.1831\times10^{-5} \quad V^2$$

$$SNR = \overline{s^2(t)}/\overline{n^2(t)} = 2/3.1831\times10^{-5} = 62831.8531 = 47.9818 \text{ dB}.$$

E-E 423 Communication Systems
One hour Examination
Dr. R.A. Birgenheier
December 12, 19''6

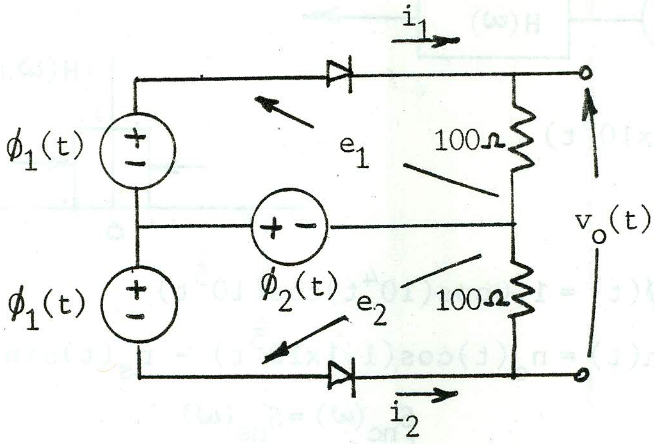
92/100

Benmei Chen

One Sheet of Notes

25 points each

1.



$\phi_1(t) = 0.1\cos(100t)$ volts

$\phi_2(t) = 1.0\cos(1000t)$ volts

$i_1(t) = 5e_1^2$ mA

$i_2(t) = 5e_2^2$ mA

Determine the output voltage, $v_o(t)$

2. The output voltage of a modulator is given as

5-71°

$$\phi(t) = 2.0\cos(\omega_c + \omega_m)t + 0.2\sin(\omega_c + \omega_m)t$$
$$+ .02\cos(\omega_c - \omega_m)t + .16\sin(\omega_c - \omega_m)t$$

Determine the ratio of the magnitude of the lower sideband to the magnitude of the upper sideband. (Express in dB to 4 significant figures)

3. The following AM voltage is developed across a 73 ohm resistor:

$$\phi(t) = [100 + 90\cos(3\text{x}10^4t)]\cos(10^7t) \quad \text{volts}$$

Determine (a) modulation index, (b) total average power, (c) percentage of power in sidebands, (d) peak value of envelope.

126

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42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE
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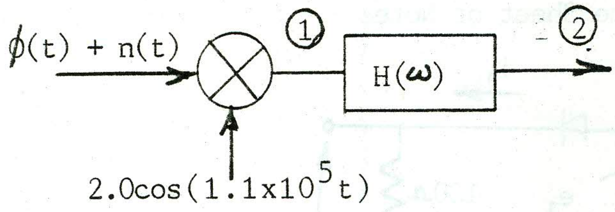
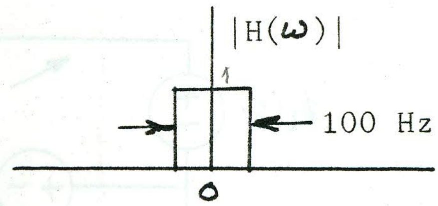
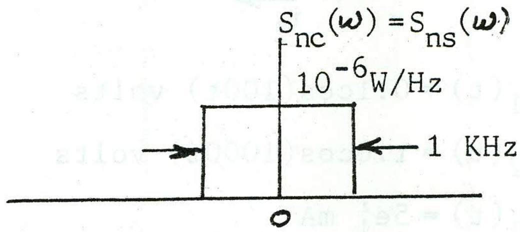
4.





$$\phi(t) = 1.0\cos(10^4 t)\cos(10^5 t)$$

$$n(t) = n_c(t)\cos(1.1\text{x}10^5 t) - n_s(t)\sin(1.1\text{x}10^5 t)$$



(a) Determine time domain expressions for the signal at points 1 and 2 (input and output of the LPF)

(b) Determine a time domain expression for the noise at point 1 .

(c) Determine the SNR at point 2 .

Notes For Exam.3 -- Communication

1. AM - DSB-LC: $P_t=\frac{1}{2}A^2+\frac{1}{2}\overline{f(t)*f(t)} = P_c + P_s$

$$\mu=m*m/(2+m*m)=P_s/P_t$$

2. SSB: Hilbert Transform:

$$F(w)=\begin{cases} -jF(w) & w>0 \\ jF(w) & w<0 \end{cases} =-jF(w)\ \mathrm{sgn}(w)$$

If $f(t)=\cos w_m t$, then $\hat{f}(t)=\sin w_m t$

$$\Phi_{SSB\mp} = f(t)\cos w_c t \pm \hat{f}(t)\sin w_c t$$

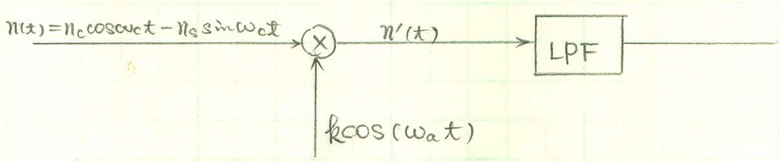
3. $n(t)= n_c(t)\cos w_c t-n_s(t)\sin w_c t$

4. Noise: DsB-SC $(S_o/N_o)=2(S_i/N_i)$

SSB-SC $(S_o/N_o)=(S_i/N_i)$

DSB-LC: Synhronous and Envelope Detector

$$(S_o/N_o)=\frac{2\,\overline{f^2(t)}}{A^2+\overline{f^2(t)}}(S_i/N_i)$$

5. 

$$S_{n'(t)}(\omega)=[S_n(\omega-\omega_a)+S_n(\omega+\omega_a)] \qquad k=2$$

$$=\frac{1}{4}[S_n(\omega-\omega_a)+S_n(\omega+\omega_a)] \qquad k=1$$

$$=\frac{k^2}{4}[S_n(\omega-\omega_a)+S_n(\omega+\omega_a)]$$

Because $n'(t)=n(t)\cdot k\cos\omega_a t$ $\quad N'_T(\omega)=\frac{k}{2}[N_T(\omega+\omega_a)+N_T(\omega-\omega_a)]$

$$|N'_T(\omega)|^2=\frac{k^2}{4}[\,|N_T(\omega+\omega_a)|^2+|N_T(\omega-\omega_a)|^2+\cancelto{0}{2|N_T(\omega+\omega_a)|\,|N_T(\omega-\omega_a)|}\,]$$

$$=\frac{k^2}{4}[\,|N_T(\omega+\omega_a)|^2+|N_T(\omega-\omega_a|\,].$$

1. $e_1 = \phi_1(t) + \phi_2(t) = 0.1\cos(100t) + 1.0\cos(1000t)$

$e_2 = \phi_2(t) - \phi_1(t) = 1.0\cos(1000t) - 0.1\cos(100t)$

$i_1(t) = 5e_1^2 = 5(0.1\cos(100t) + 1.0\cos(1000t))^2$

$= 5[0.01\cos^2(100t) + 0.2\cos(100t)\cos(1000t) + \cos^2(1000t)]$

$i_2(t) = 5e_2^2 = 5[-0.1\cos(100t) + 1.0\cos(1000t)]^2$

$= 5[0.01\cos^2(100t) - 0.2\cos(100t)\cos(1000t) + \cos^2(1000t)]$

$i_1(t) - i_2(t) = 5\times 0.4\cos(100t)\cos(1000t)$ mA

$V_o(t) = i_1(t)\times 100 - i_2(t)\times 100 = 100\times(i_1(t) - i_2(t))$

$= 100\times 0.4\times 5\cos(100t)\cos(1000t)$

$= 200\cos(100t)\cos(1000t)$ mV ✓

2. $\phi(t) = 2.0\cos(\omega_c+\omega_m)t + 0.2\sin(\omega_c+\omega_m)t$

$+ 0.02\cos(\omega_c-\omega_m)t + 0.16\sin(\omega_c-\omega_m)t$

$= 2.0100[0.9950\cos(\omega_c+\omega_m)t + 0.0995\sin(\omega_c+\omega_m)t]$

$+ 0.1612[0.1240\cos(\omega_c-\omega_m)t + 0.9923\cos(\omega_c-\omega_m)t]$

$= 2.0100\cos((\omega_c+\omega_m)t - 5.71^\circ) + 0.1612\cos((\omega_c-\omega_m)t - 82.88^\circ)$

$\frac{\text{Magnitude of the lower sideband}}{\text{magnitude of the upper sideband}} = \frac{0.1612}{2.0100} = 0.0802$

$= -21.91$ dB

-5

128

3.

$$\phi(t) = [100 + 90\cos(3\times10^4 t)]\cos(10^7 t)$$

$$= 100\cos(10^7 t) + 0.9\times100\cos(3\times10^4 t)\cos(10^7 t)$$

(a) Modulation index $= 0.9 = 90\%$

(b) $P_t = (\frac{1}{2}A^2 + \frac{1}{2}\cdot\frac{1}{2}(mA)^2)/R_L$

$$= [\tfrac{1}{2}\times100^2 + \tfrac{1}{4}\times90^2]/73$$

$$= 96.2329 \text{ W}$$

(c) $\mu = m^2/(2+m^2) = 0.9^2/(2+0.9^2) = 0.2883\times100\% = 28.83\%$

(d) peak value of envelope $= 100+90 = 190$ V

4. (a) $\phi(t) = 1.0\cos(10^4 t)\cos(10^5 t)$

$$\phi_{①}(t) = \underline{1.0\cos(10^4 t)\cos(10^5 t)\times 2.0\cos(1.1\times10^5 t)} \quad ①$$

$$= 1.0\cos(10^4 t)[\cos(2.1\times10^5 t) + \cos(10^4 t)]$$

$$= 1.0\cos(10^4 t)\cos(2.1\times10^5 t) + 1.0\cos^2(10^4 t)$$

$$= 0.5\cos(2.2\times10^5 t) + 0.5\cos(2.0\times10^5 t) + 0.5 + 0.5\cos(2\times10^4 t)$$

$$\phi_{②}(t) = \phi_{①}(t)|_{LP}$$

$$= 0.5$$ (Because 2×10^4 rad/s $= 3.18$ KHz > 100 Hz)

(b) $n_{①}(t) = n(t)\times 2.0\cos(1.1\times10^5 t)$

$$= [n_c(t)\cos(1.1\times10^5 t) - n_s(t)\sin(1.1\times10^5 t)]\times 2.0\cos(1.1\times10^5 t)$$

$$= n_c(t)\cdot 2\cos^2(1.1\times10^5 t) - n_s(t)\cdot 2\sin(1.1\times10^5 t)\cos(1.1\times10^5 t)$$

$$= n_c(t)(1 + \cos(2.2\times10^5 t)) - n_s(t)\sin(2.2\times10^5 t)$$

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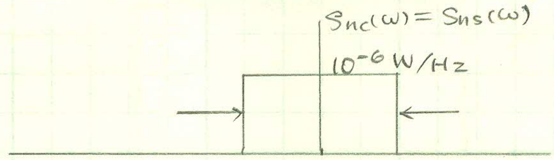
(c)



$$S_n(\omega) = \frac{1}{2} S_{nc}(\omega - 1.1\times10^5) + \frac{1}{2} S_{nc}(\omega + 1.1\times10^5)$$

According to the notes:

$$S_{n\phi(t)}(\omega) = S_n(\omega - 1.1\times10^5) + S_n(\omega + 1.1\times10^5)$$

$$S_{n\phi(t)}(\omega)\Big|_{LP} = S_{nc}(\omega) = S_{n\otimes(t)}(\omega)$$

SO, $N_o = 100 \times 10^{-6} = 10^{-4}$ W

$S_o = 0.5$ W $\quad \frac{1}{4}$ $\qquad -3$

$SNR_{\otimes} = 0.5 / 10^{-4} = 5000 = 37$ dB

130

187/200

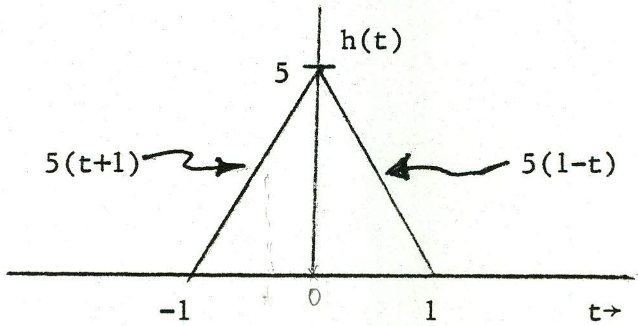
Benmei Chen

E-E 423 COMMUNICATION SYSTEMS
Final Examination
Dr. R. A. Birgenheier
December 17, 1986

Closed Book - Sheet of Notes

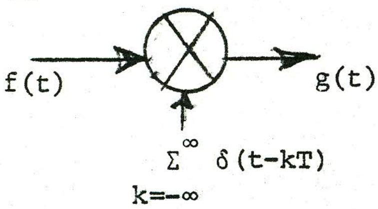
Points

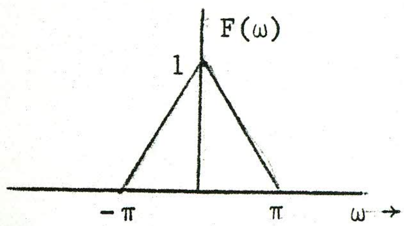
30 1. Consider a filter with unit impulse response function given as



Determine (a) unit step response, (b) 10% to 90% rise-time.

30 2.

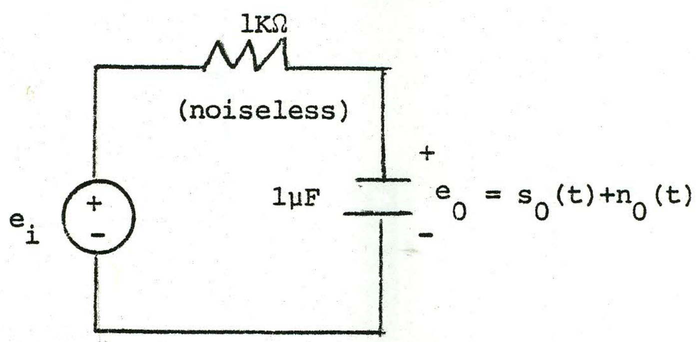




a) For $T = \frac{2}{3}$ sec., determine and sketch the Fourier transform, $G(\omega)$.

b) What is the maximum value of T to avoid aliasing?

30 3.



$e_i = s_i(t) + n_i(t)$

$s_i(t)$ is the input signal voltage given as

$s_i(t) = 2\cos(1000t)$ volts

and $n_i(t)$ is the input noise with two-sided power spectral density

$S_{ni}(\omega) = 10^{-6}$ W/Hz

Determine (a) $\overline{s_0^2(t)}$, (b) $\overline{n_0^2(t)}$, (c) SNR_0

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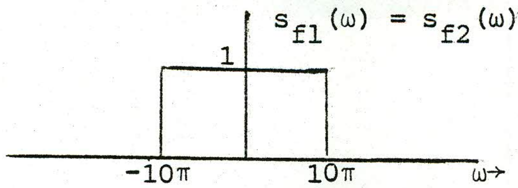
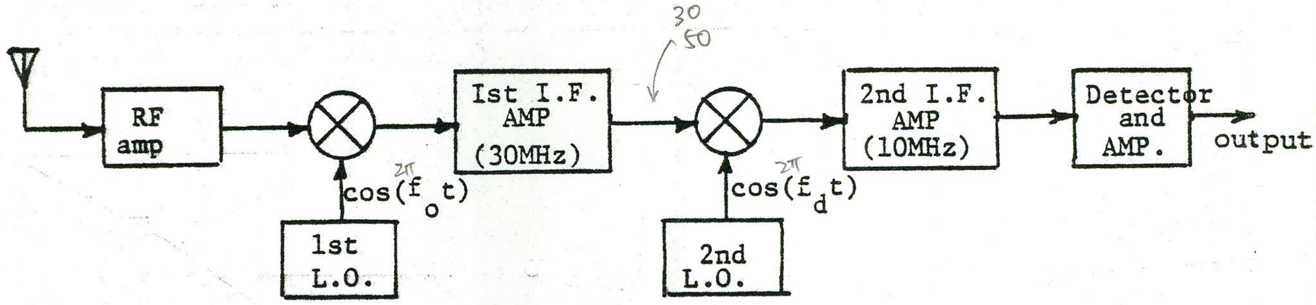
30 4. $g(t) = f_1(t)\cos(100\pi t) + f_2(t)\sin(100\pi t)$



Determine and sketch (a) autocorrelation function, $R_g(\tau)$, (b) Power spectral density, $s_g(\omega)$.

30 5.

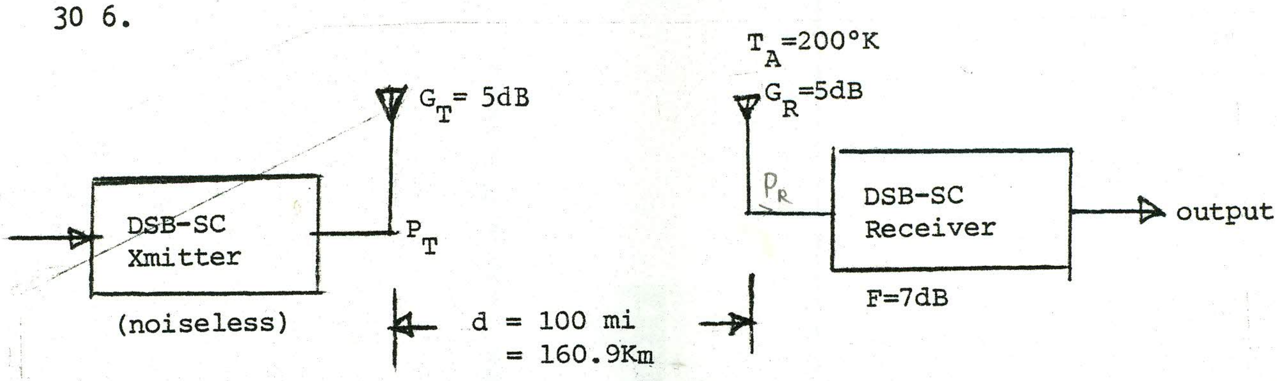


The first local oscillator of the dual-conversion receiver is adjusted to receive a signal at 148MHz.

a) Give two possible values for f_0 and f_d.

b) Select the lower value of f_0 and the higher value of f_d determined in part (a). Determine all possible image frequencies. (Do not assume ideal RF and IF filters; i.e., assume the images can pass through the amplifiers.)

30 6.



Carrier frequency: 2.0 GHz

Input (baseband) signal bandwidth: 5KHz

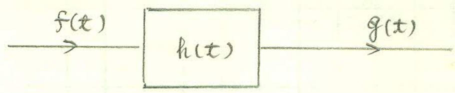
a) Determine the equivalent noise level at the receiver input (B_n = 10KHz).

b) Determine the path loss, α.

c) Determine the power transmitted, P_T, to obtain an output SNR of 40dB.

20 7. **MERRY CHRISTMAS!!!!**

1. FILTER:



$g(t) = f(t) \circledast h(t) = \int_{-\infty}^{\infty} f(\tau) \times h(t-\tau)\, d\tau$

FOR TRANSVERSAL FILTER: $g(t) = \int_0^t f(t-\tau)\, h(\tau)\, d\tau$; $G(\omega) = F(\omega) \cdot H(\omega)$

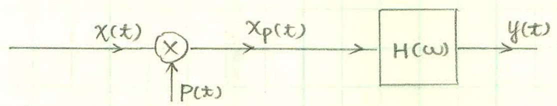
FOR DISTORTIONLESS FILTER: $g(t) = K f(t - t_0)$; $H(\omega) = K e^{-j\omega t_0}$

SPECIAL CASE: $g(t) = h(t)$ WHEN $f(t) = \delta(t)$; $g(t) = g(t)$ WHEN $f(t) = u(t)$

$h(t) = \dfrac{dg(t)}{dt}$; $\delta(t - t_0) = \dfrac{d}{dt} u(t - t_0)$; $f(t)\,\delta(t - t_0) = f(t_0)\,\delta(t - t_0)$

ORDER > 2 LPF : rise-time $t_r \approx 1/2B$; $\angle H(\omega) = -\tan^{-1} \dfrac{X(\omega)}{R(\omega)}$; $\tau_g = -\dfrac{d}{d\omega} \angle H(\omega)$

2. SAMPLING:

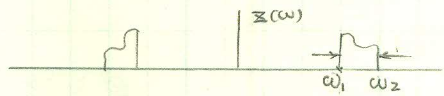


THEOREM: *A real-valued band-limited signal having no spectral components above a freq. of B Hz is determined uniquely by its values at uniform intervals spaced no greater than 1/2B sec. apart.* $T \leq \frac{1}{2B}$, $W = 2\pi B$

$p(t) = \sum_{n=-\infty}^{\infty} P_n e^{jn\omega_0 t}$; $P_n = \frac{1}{T} \int_{-T/2}^{T/2} p(t) e^{-jn\omega_0 t} dt$, $\omega_0 = \frac{2\pi}{T}$

FOR $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$ $P_n = \frac{1}{T}$, $p(t) = \sum_{n=-\infty}^{\infty} \frac{1}{T} \cdot e^{jn\omega_0 t}$.

MIN. SAMPLING RATE FOR BANDPASS:



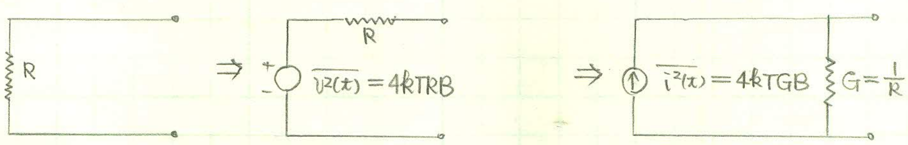
$\omega_{0,min} = \dfrac{2\omega_2}{\left[\frac{\omega_1}{\omega_2 - \omega_1}\right] + 1}$

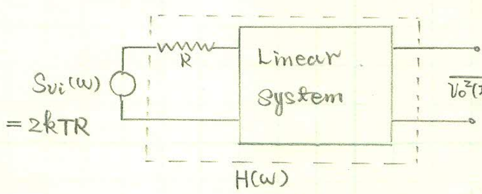
3. DFT : $F[n] = \sum_{k=0}^{N-1} f[k]\, e^{-j\frac{2\pi}{N} \cdot n \cdot k}$ $n = 0, 1, \ldots, N-1$

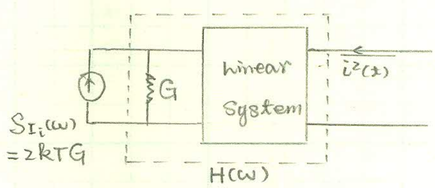
IDFT : $f[k] = \frac{1}{N} \sum_{n=0}^{N-1} F_D[n]\, e^{j\frac{2\pi}{N} \cdot n \cdot k}$ $k = 0, 1, \ldots, N-1$

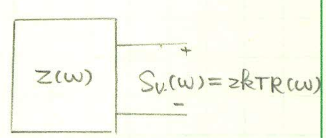
DFT = $N \times N = N^2$ complex multiplications ; FFT = $\frac{N}{2} \log_2 N$ complex multiplications.

4. Signal and noise through a linear system









$B_N = \frac{1}{2\pi} \left[\int_0^{\infty} |H(\omega)|^2 d\omega \right] / |H(\omega_0)|^2$, $B_N \approx B_{-3dB}$. $k = 1.38 \times 10^{-23}$

5. Autocorrelation functions and power spectral density.

$S_f(\omega) = \lim_{T \to \infty} \dfrac{|F_T(\omega)|^2}{T}$; IF $f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$, THEN $S_f(\omega) = 2\pi \sum_{n=-\infty}^{\infty} |F_n|^2 \delta(n - n\omega_0)$

$R_f(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} f^*(t) f(t+\tau) dt$, $S_f(\omega) = \mathcal{F}\{R_f(\tau)\}$ 133

$S_g(\omega) = S_f(\omega) |H(\omega)|^2$; $E_g(\omega) = E_f(\omega) \cdot |H(\omega)|$; $E_f(\omega) = |F(\omega)|^2$

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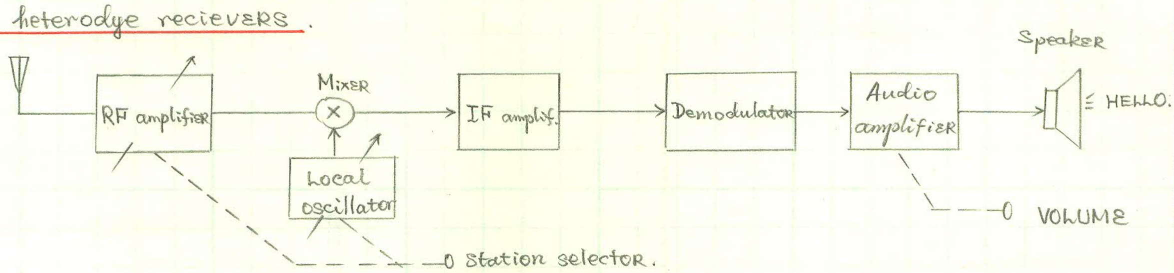
PROPERTIES OF $R_f(\tau)$: ① $R_f(-\tau) = R_f^*(\tau)$; ② $|R_f(\tau)| \le R_f(0)$; ③ $R_f(\tau+T) = R_f(\tau)$, IF $f(t+T) = f(t)$

④ $R_f(0) = \overline{f^2(t)}$

6. Super heterodye recievers.



If the intermediate freq. (IF) is lower than the received carrier freq. but above the final output signal freq., it is called a superheterodye receiver.

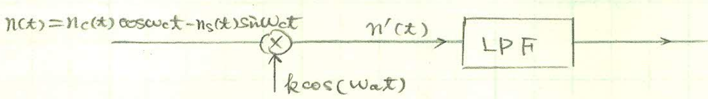
7. Modulation :

DSB − SC : $\phi(t) = f(t)\cos\omega_c t$

DSB − LC (AM): $\phi(t) = A\cos\omega_c t + f(t)\cos\omega_c t$; $\overline{\phi^2_{AM}(t)} = \frac{1}{2}A^2 + \frac{1}{2}\overline{f^2(t)} = P_c + P_s$

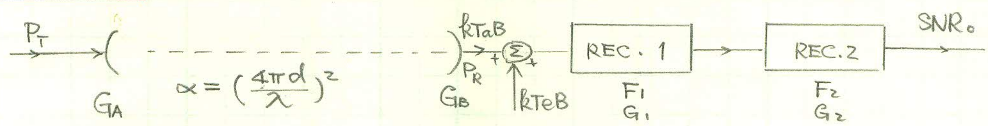
FOR $f(t) = \cos\omega_m t$, $\mu = m^2/(2+m^2) = P_s/P_T$

SSB − SC : FOR SPECIAL CASE , $f(t) = \cos\omega_m t$; $\Phi_{SSB\mp}(t) = \cos\omega_m t\cos\omega_c t \pm \sin\omega_m t\sin\omega_c t$



$$S_{n'(t)} = [S_n(\omega-\omega_a) + S_n(\omega+\omega_a)]$$

8. Link Calculation :



$T_e = (F-1)T_0$; $F = F_1 + \frac{F_2-1}{G_1} + \frac{F_3-1}{G_1G_2} + \cdots$; $\lambda = \frac{c}{f} = \frac{3\times10^8}{f}$ m/s

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THESE AREN'T ON FINAL, BUT THEY MAY BE USEFUL SOMEDAY.

DSB−SC : ADVAN. : THE TRANSMISSION EFFCIENCY IS 100% OR SLIGHTLY LESS 100% IF A PILOT CARRIER IS TRANSMITTED.

DISADV : IN ORDER TO AVOID THE FREQ. ERRORS AND PHASE ERRORS, WE NEED A SYNCHRONOUS DETECTOR TO RECOVER THE ORIGINAL SIGNAL. IT BECOMES A MOJOR PROBLEM WHEN THE RECEIVER IS REMOTELY LOCATED.

DSB−LC : ADVAN. : EASY TO BE DETECTED BY USING A CHEAP ENVELOPE DETECTOR.

DISADV. : THE TRANSMISSION EFFCIENCY IS AT BEST 33%

SSB−SC : ADVAN. : ONLY NEEDS UPPER-SIDE BAND OR LOWER-SIDE BAND OF SIGNAL TO BE TRAN.

DISADV. : THE GENERATION OF SSB SIGNAL IS DIFFICULT.

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1. (a) $g(t) = u(t) \otimes h(t) = \int_{-\infty}^{\infty} u(\tau) h(t-\tau)\, d\tau$

$= \int_{0}^{\infty} h(t-\tau)\, d\tau$

$= \int_{-\infty}^{t} h(\tau)\, d\tau$

$= \int_{-1}^{t} h(\tau)\, d\tau$ WHEN $t \geq -1$; $g(t) = 0$ WHEN $t < -1$

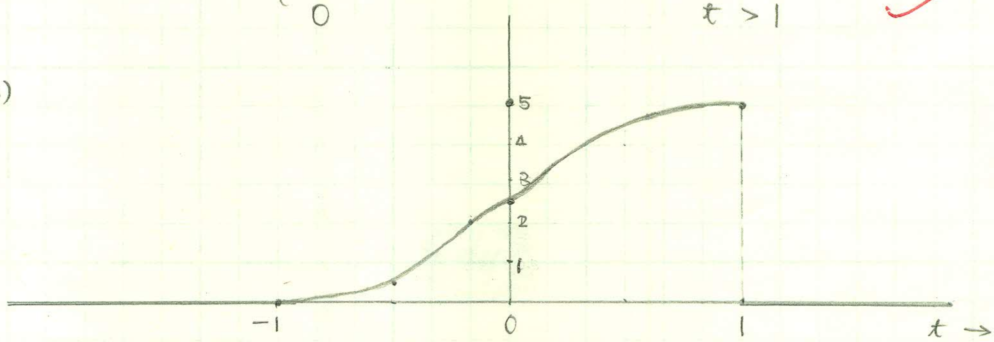
$\therefore\ g(t) = \int_{-1}^{t} 5(\tau+1)\, d\tau = \frac{5}{2}\tau^2 + 5\tau \Big|_{-1}^{t} = \frac{5}{2}t^2 + 5t + \frac{5}{2}$ WHEN $-1 \leq t < 0$

$g(t) = \int_{-1}^{0} 5(\tau+1)\, d\tau + \int_{0}^{1} 5(1-\tau)\, d\tau = \frac{5}{2} + 5t - \frac{5}{2}t^2$ WHEN $0 \leq t \leq 1$

SO,

$$g(t) = \begin{cases} 0 & \text{WHEN } t < -1 \\ \frac{5}{2}t^2 + 5t + \frac{5}{2} & -1 \leq t < 0 \\ \frac{5}{2} + 5t - \frac{5}{2}t^2 & 0 \leq t \leq 1 \\ 0 & t > 1 \end{cases}$$

(b)



$g(t) < g(1) = 5$

$\frac{5}{2}t^2 + 5t + \frac{5}{2} = 5 \times 10\% \Rightarrow 2.5t^2 + 5t + 2 = 0 \Rightarrow$

$\Rightarrow t_1 = -0.55279$ (sec.?)

$-\frac{5}{2}t^2 + 5t + \frac{5}{2} = 5 \times 90\% = -2.5t^2 + 5t - 2 = 0$

$\Rightarrow t_2 = 0.55279$ (sec.?)

SO, $t_{r\,(10\%-90\%)} = t_2 - t_1 = 0.55279 - (-0.55279) = 1.10557$ (sec?)

-3

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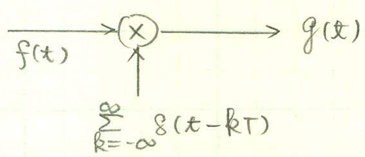
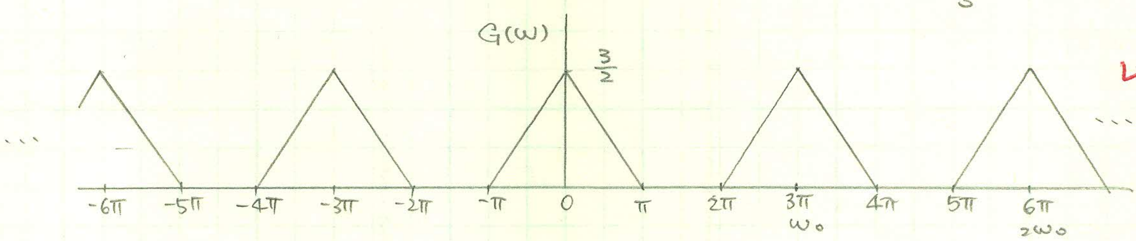
2.



a) $\sum_{k=-\infty}^{\infty} \delta(t-kT) = \frac{1}{T}\sum_{n=-\infty}^{\infty} e^{jn\omega_0 t}$

$g(t) = f(t)\cdot \frac{1}{T}\sum_{n=-\infty}^{\infty} e^{jn\omega_0 t} = \frac{1}{T}\sum_{n=-\infty}^{\infty} f(t)e^{jn\omega_0 t}$

$G(\omega) = \frac{1}{T}\sum_{n=-\infty}^{\infty} F(\omega - n\omega_0)$, WHERE $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{\frac{2}{3}} = 3\pi$, $\frac{1}{T} = \frac{3}{2}$

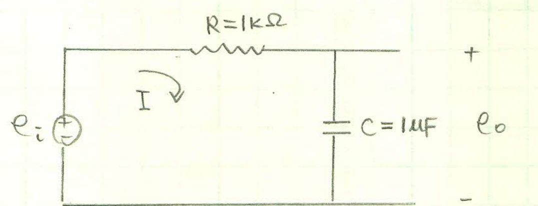


b) $\omega_0 \geq 2W_{F(\omega)} = 2\pi$

$\omega_{0,min} = 2\pi = \frac{2\pi}{T_{max}}$

SO, $T_{max} = 2\pi/2\pi = 1$ sec.

3.



$I = \frac{e_i}{R - j\frac{1}{\omega C}}$

$e_o = I\cdot(-j\frac{1}{\omega C}) = \frac{e_i}{R-j\frac{1}{\omega C}}\cdot(-j\frac{1}{\omega C})$

$= e_i \cdot \frac{-j}{RC\omega - j}$

$= e_i \cdot \frac{1}{1 + jRC\omega}$

$H(\omega) = \frac{e_o}{e_i} = \frac{1}{1+jRC\omega}$

$|H(\omega)|^2 = \frac{1}{1+R^2C^2\omega^2} = \frac{1}{1+(1000\times10^{-6})^2\omega^2}$

a) $S_i(t) = 2\cos(1000t) = e^{j1000t} + e^{-j1000t}$

$S_{ei}(\omega) = 2\pi[\delta(\omega-1000) + \delta(\omega+1000)]$

$S_o(\omega) = S_{ei}(\omega)\cdot|H(\omega)|^2 = 2\pi\cdot[\delta(\omega-1000)+\delta(\omega+1000)]\cdot\frac{1}{1+R^2C^2\omega^2}$

$\overline{S_o^2(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_o(\omega)\,d\omega = \frac{1}{1+(1000RC)^2} + \frac{1}{1+(-1000RC)^2}$

$= \frac{2}{1+(1000\times1000\times10^{-6})^2} = 1$ V^2 OR W

3. CONT. b) $S_{ni}(\omega) = 10^{-6}$ W/Hz

$$S_{no}(\omega) = S_{ni}(\omega)\cdot|H(\omega)|^2 = \frac{10^{-6}}{1+R^2C^2\omega^2}$$

$$\overline{n_o^2(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{10^{-6}}{1+R^2C^2\omega^2} = \frac{10^{-6}}{\pi\cdot RC}\int_0^{\infty}\frac{1}{1+(RC\omega)^2}\,d(RC\omega)$$

$$= \frac{10^{-6}}{\pi\cdot RC}\cdot\arctan(RC\omega)\Big|_0^{\infty} = \frac{10^{-6}}{\pi\cdot RC}\cdot\frac{\pi}{2} = \frac{10^{-6}}{2\times1000\times10^{-6}} = 0.0005\ V^2 \text{ or } W$$

c) $SNR_o = 1/0.0005 = 2000 = 33.0103\,dB$

4. $g(t) = f_1(t)\cos(100\pi t) + f_2(t)\sin(100\pi t)$

According the defination

a) $$R_g(\tau) = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} g^*(t)\,g(t+\tau)\,dt$$

$$g(t)g(t+\tau) = [f_1(t)\cos(100\pi t) + f_2(t)\sin(100\pi t)][f_1(t+\tau)\cos(100\pi t+100\pi\tau) + f_2(t+\tau)\sin(100\pi t+100\pi\tau)]$$

$$= f_1(t)f_1(t+\tau)\cdot\cos(100\pi t)\cos(100\pi t+100\pi\tau)$$
$$+ f_1(t)f_2(t+\tau)\cos(100\pi t)\sin(100\pi t+100\pi\tau) \quad (\to 0)$$
$$+ f_2(t)f_2(t+\tau)\cdot\sin(100\pi t)\sin(100\pi t+100\pi\tau)$$
$$+ f_2(t)f_1(t+\tau)\sin(100\pi t)\cos(100\pi t+100\pi\tau) \quad (\to 0)$$

$$= f_1(t)f_1(t+\tau)\cdot\tfrac{1}{2}[\cos(200\pi t+100\pi t) + \cos(100\pi\tau)]$$
$$+ f_1(t)\cdot f_2(t+\tau)\cdot\tfrac{1}{2}[\sin(200\pi t+100\pi\tau) + \sin(100\pi\tau)]$$
$$+ f_2(t)f_2(t+\tau)\cdot\tfrac{1}{2}[\cos(100\pi\tau) - \cos(200\pi t+100\pi\tau)]$$
$$+ f_2(t)\cdot f_1(t+\tau)\cdot\tfrac{1}{2}[\sin(200\pi t+100\pi\tau) - \sin(100\pi\tau)]$$

SO, $R_g(\tau) = R_{f_1}(\tau)\cdot\frac{1}{2}\cos(100\pi\tau) + R_{f_2}(\tau)\cdot\frac{1}{2}\cos(100\pi\tau)$

FROM THE FIG. ON TEST SHEET

$$R_{f_1}(\tau) = R_{f_2}(\tau) = \mathcal{F}^{-1}\{S_{f_1}(\omega)\} = \frac{10\pi}{\pi}Sa(10\pi\tau) = 10\,Sa(10\pi\tau)$$

$$\therefore R_g(\tau) = \tfrac{1}{2}\cos(100\pi\tau)\cdot 2\times10\,Sa(10\pi\tau) = 10\cos(100\pi\tau)\,Sa(10\pi\tau)$$

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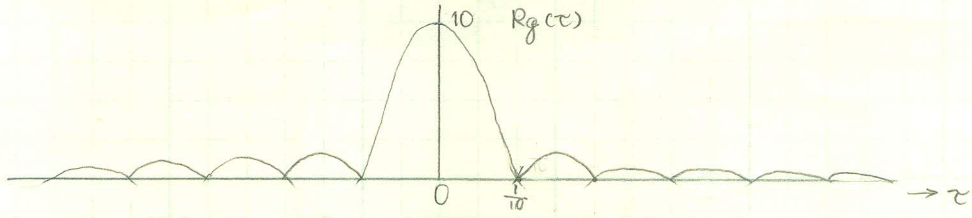
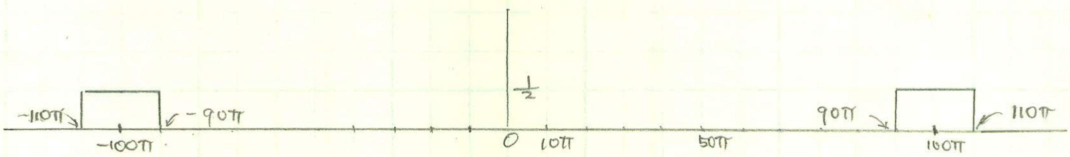


b) $S_g(\omega) = \mathfrak{F}\{R_f(\tau)\} = \mathfrak{F}\{10\,Sa(10\pi\tau)\cos(100\pi\tau)\}$

$$\mathfrak{F}\{Sa(10\pi\tau)\} = \frac{\pi}{10\pi}\,rect[\omega/20\pi] = \frac{1}{10}\,rect[\omega/20\pi]$$

$$\therefore\ S_g(\omega) = \frac{1}{2}\left[rect\left[\tfrac{1}{20\pi}(\omega-100\pi)\right] + rect\left[\tfrac{1}{20\pi}(\omega+100\pi)\right]\right]$$



5. a) $f_0 = 148\,MHz - 30\,MHz = 118\,MHz$ or 178MHz

$f_d = 30\,MHz + 10\,MHz = 40\,MHz$ or 20MHz

−5

b) $f_{IM1} = 118 - 30 = 88\,MHz$

$f_{IM2} = 118 + 50 = 168\,MHz$

$f_{IM2} = 118 - 50 = 68\,MHz$

6. a) $kT_aB_n = 1.38\times10^{-23}\times200\times10^4 = 2.76\times10^{-17}\ W$

$kT_eB_n = k(F-1)T_0B_n = 1.38\times10^{-23}\times(5.0119-1)\times290\times10^4 = 1.6056\times10^{-16}\ W$

$N_i = 2.76\times10^{-17} + 1.6056\times10^{-16} = 1.8816\times10^{-16}\ W = -157.255\ dBW$

b) $\alpha = \left(\frac{4\pi d}{\lambda}\right)^2, \quad \lambda = \frac{c}{f_c}$

$$\alpha = \left(\frac{4\pi d\cdot f_c}{c}\right)^2 = \left(\frac{4\times3.14159\times160.9\times10^3\times2.0\times10^9}{3\times10^8}\right)^2$$

$$= 1.817\times10^{14} = 142.593\ dB$$

c) $P_R = (SNR)_{dB} + N_{i\,dB} = 40 + (-157.255) = -117.255\ dBW$

37

−5

$P_{R\,dB} = P_{T\,dB} + G_{T\,dB} - \alpha_{dB} + G_{R\,dB}$

$\therefore\ P_{T\,dB} = P_{R\,dB} - G_{T\,dB} + \alpha_{dB} - G_{R\,dB} = -117.255 - 5 + 142.593 - 5 = 15.338\ dB$

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