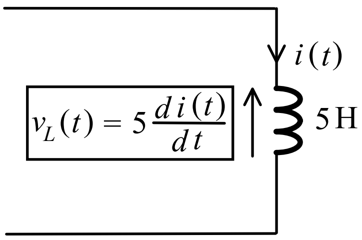
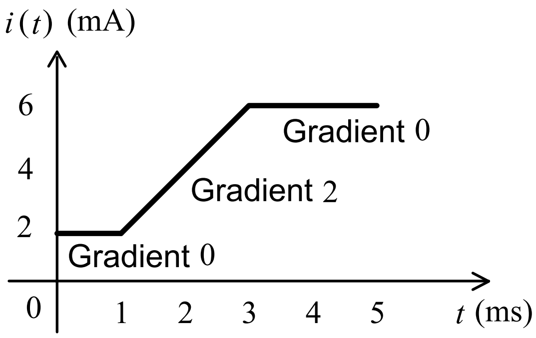
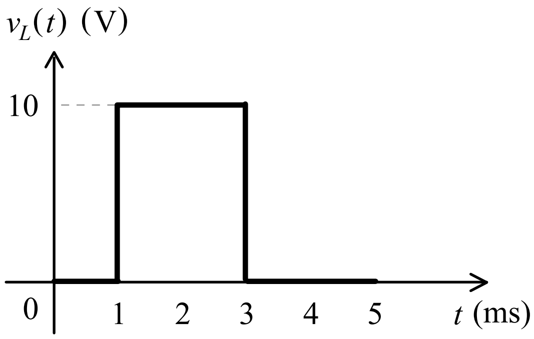
Solution to Tutorial Set 2

Q.1

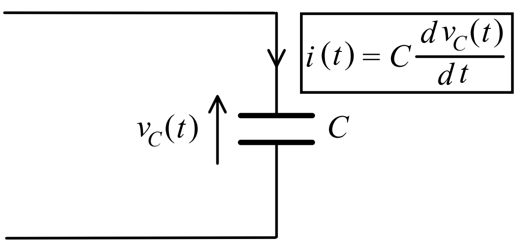
Voltages across inductor







Voltages across capacitor



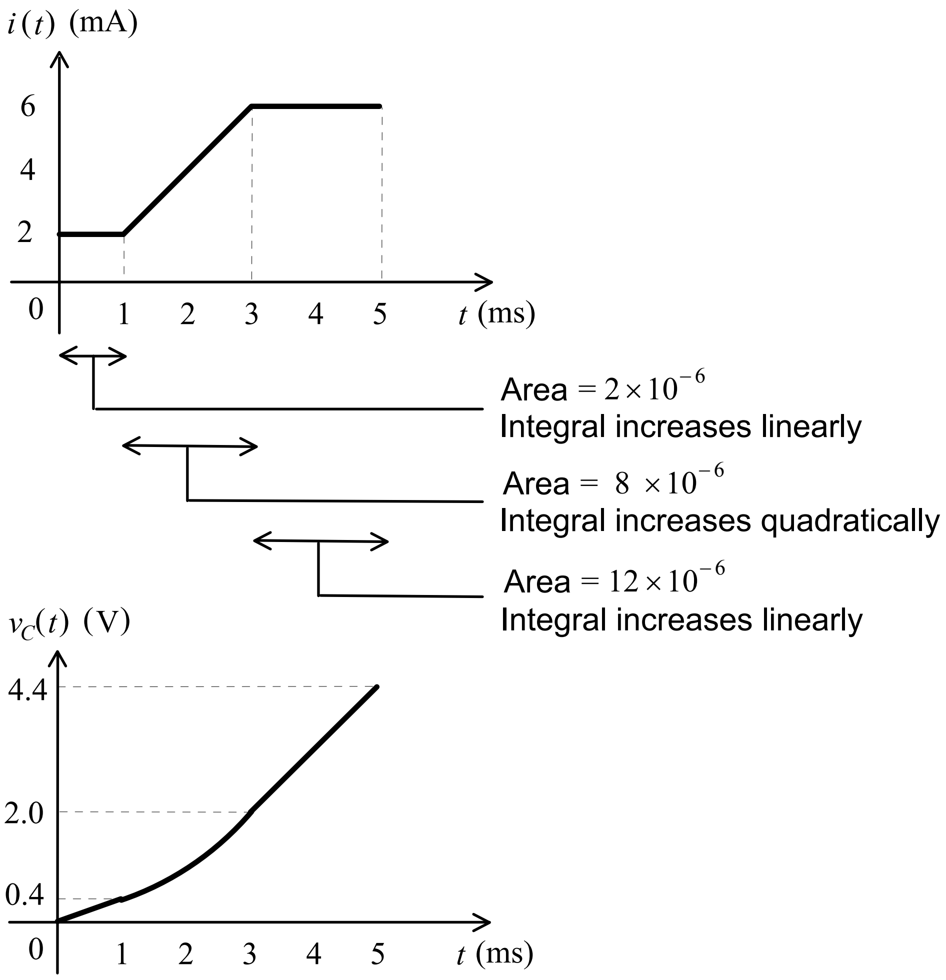
$$\int_0^t i(s)ds = \int_0^t C\frac{dv_C(s)}{ds}ds = \int_0^t C dv_C(s) = [Cv_C(s)]_0^t = C[v_C(t) - v_C(0)]$$

$$v_C(t) = v_C(0) + \frac{1}{C}\int_0^t i(s)\,ds$$

$$= [\text{initial voltage at } t=0] + \frac{1}{C}[\text{area under } i(t) \text{ from } 0 \text{ to } t]$$

For this problem:

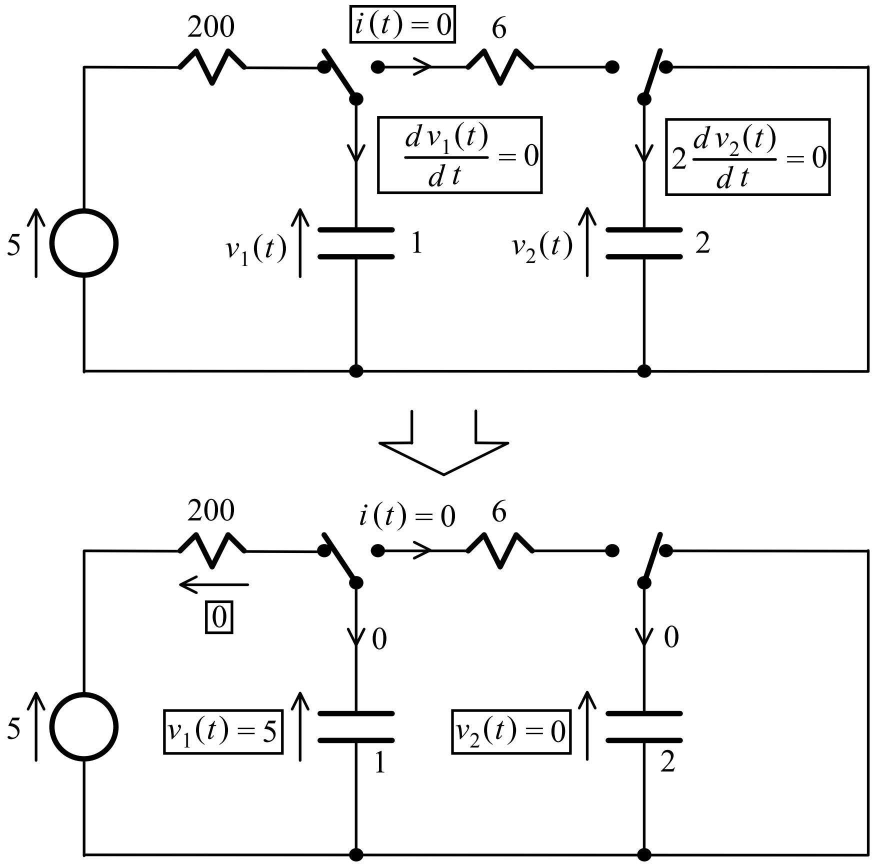
$$v_C(t) = \frac{1}{5\times 10^{-6}}[\text{area under } i(t) \text{ from } 0 \text{ to } t]$$



Q.2

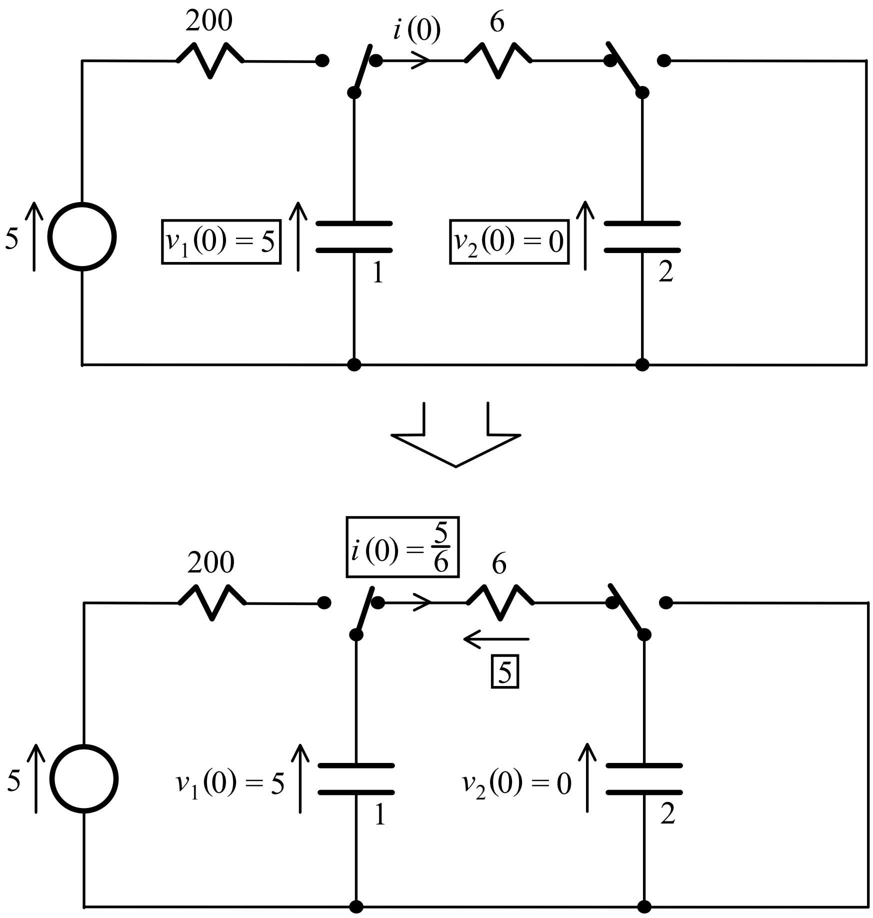
(a) Voltages and currents for $t < 0$

With the source being a dc one and taking the switches to be in the positions shown starting from $t = -\infty$, all the voltages and currents will have settled down to constant values for practically all $t < 0$:

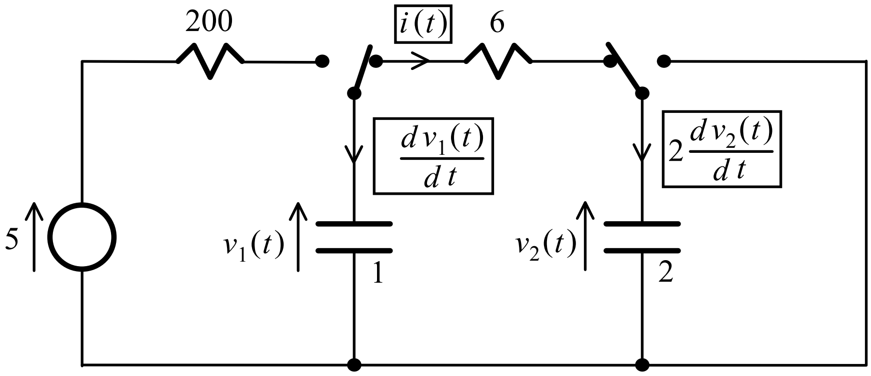


(b) Voltages and currents at $t = 0$ *(just after the switches are thrown)*

Since the voltages across capacitors must be continuous, $v_1(0)$ and $v_2(0)$ must have the same values before the switches are thrown:



(c) Voltages and current for $t \geq 0$



From KCL and KVL:

$$i(t) = 2\frac{dv_2(t)}{dt} = -\frac{dv_1(t)}{dt}$$

$$v_1(t) - 6i(t) - v_2(t) = 0$$

Eliminating $v_1(t)$ and $v_2(t)$:

$$\frac{dv_1(t)}{dt} - 6\frac{di(t)}{dt} - \frac{dv_2(t)}{dt} = \left(-1 - \frac{1}{2}\right)i(t) - 6\frac{di(t)}{dt} = 0 \;\Rightarrow\; \frac{di(t)}{dt} + \frac{i(t)}{4} = 0$$

Solving this homogeneous equation:

$$i(t) = ke^{-\frac{t}{4}},\ t \ge 0$$

Applying initial condition:

$$i(0) = k = \frac{5}{6} \ \Rightarrow\ i(t) = \frac{5}{6}e^{-\frac{t}{4}},\ t \ge 0$$

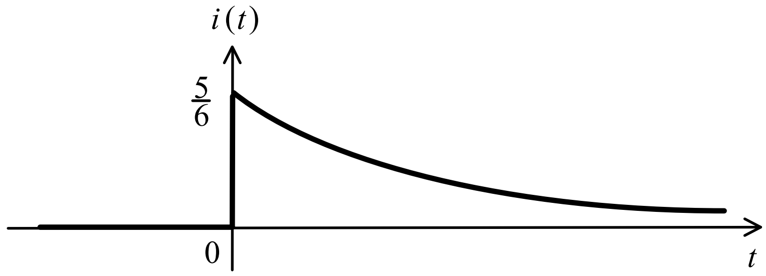
Solving for $v_1(t)$ and $v_2(t)$:

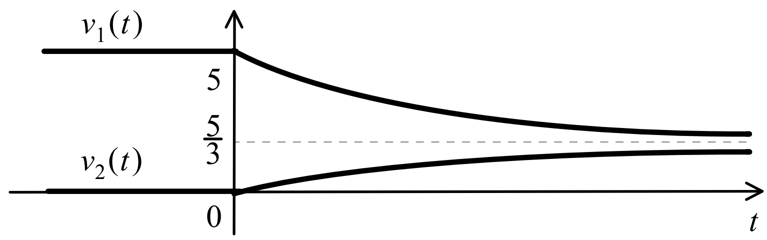
$$\frac{dv_1(t)}{dt} = -i(t) = -\frac{5}{6}e^{-\frac{t}{4}} \ \Rightarrow\ v_1(t) = k_1 + \frac{10}{3}e^{-\frac{t}{4}}$$

$$v_1(0) = k_1 + \frac{10}{3} = 5 \ \Rightarrow\ k_1 = \frac{5}{3} \ \Rightarrow\ v_1(t) = \frac{5}{3} + \frac{10}{3}e^{-\frac{t}{4}},\ t \ge 0$$

$$v_2(t) = v_1(t) - 6i(t) = \frac{5}{3} + \frac{10}{3}e^{-\frac{t}{4}} - 5e^{-\frac{t}{4}} = \frac{5}{3} - \frac{5}{3}e^{-\frac{t}{4}},\ t \ge 0$$

(d) Voltages and current waveforms

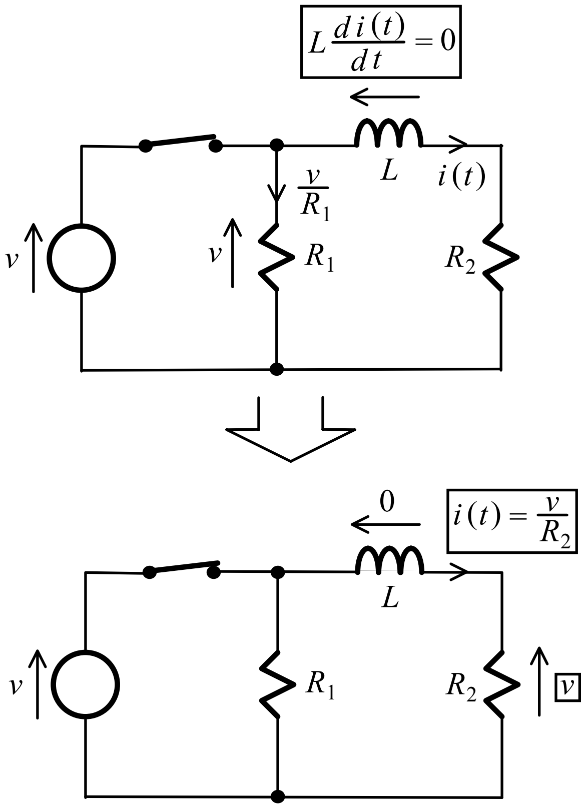




Q.3

(a) Voltages and current and energy stored for $t < 0$

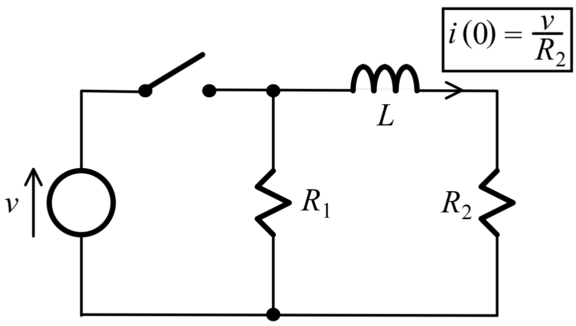
With the source being a dc one and taking the switches to be in the positions shown starting from $t = -\infty$, all the voltages and currents will have settled down to constant values for practically all $t < 0$:



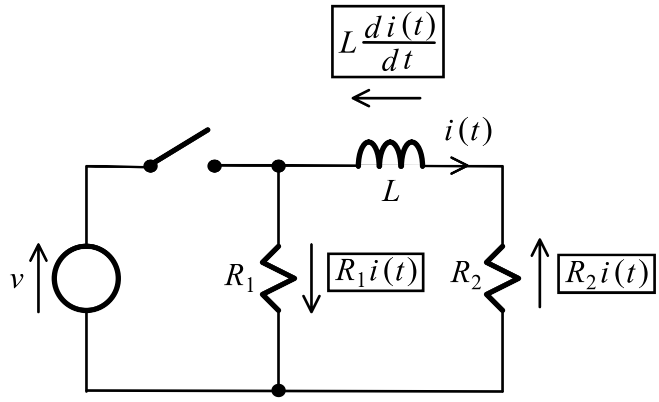
Energy stored in inductor $= \dfrac{Li^2(t)}{2} = \dfrac{Lv^2}{2R_2^{\,2}}$

(b) Current at $t = 0$ *(just after the switches is opened)*

Since the current in the inductor must be continuous, $i(0)$ must have the same value before the switch is opened:



(c) Current for $t \geq 0$



$$L\frac{di(t)}{dt} + R_1 i(t) + R_2 i(t) = 0 \;\Rightarrow\; \frac{di(t)}{dt} + \left(\frac{R_1 + R_2}{L}\right) i(t) = 0$$

$$i(t) = ke^{-\left(\frac{R_1+R_2}{L}\right)t}, \; t \geq 0$$

$$i(0) = k = \frac{v}{R_2} \;\Rightarrow\; i(t) = \frac{v}{R_2} e^{-\left(\frac{R_1+R_2}{L}\right)t}, \; t \geq 0$$

(d) Energy lost for $t \geq 0$

Instantaneous power lost in resistors$= \left(R_1 + R_2\right) i^2(t) = \frac{v^2\left(R_1 + R_2\right)}{R_2^{\,2}} e^{-2\left(\frac{R_1+R_2}{L}\right)t}, \; t \geq 0$

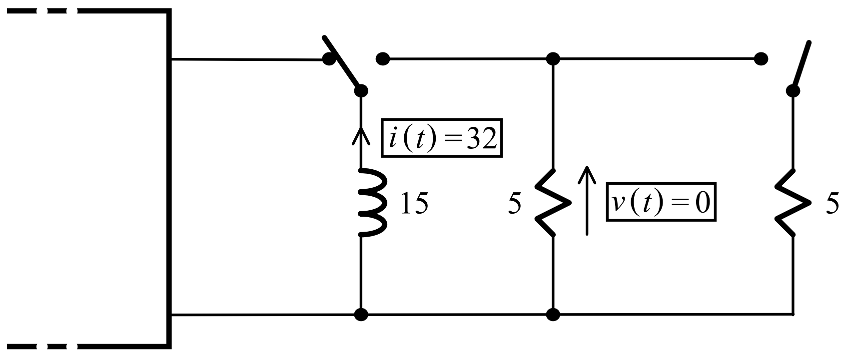
Total energy lost $= \int_0^\infty \frac{v^2\left(R_1 + R_2\right)}{R_2^{\,2}} e^{-2\left(\frac{R_1+R_2}{L}\right)t} dt = \frac{v^2\left(R_1 + R_2\right)}{R_2^{\,2}} \left[\frac{e^{-2\left(\frac{R_1+R_2}{L}\right)t}}{-2\left(\frac{R_1+R_2}{L}\right)} \right]_0^\infty = \frac{Lv^2}{2R_2^{\,2}}$

From the conservation of energy, this must also be equal to the decrease in energy stored by the inductor:

Decrease in energy stored by inductor $= \frac{Li^2(0)}{2} - \frac{Li^2(\infty)}{2} = \frac{Lv^2}{2R_2^{\,2}}$

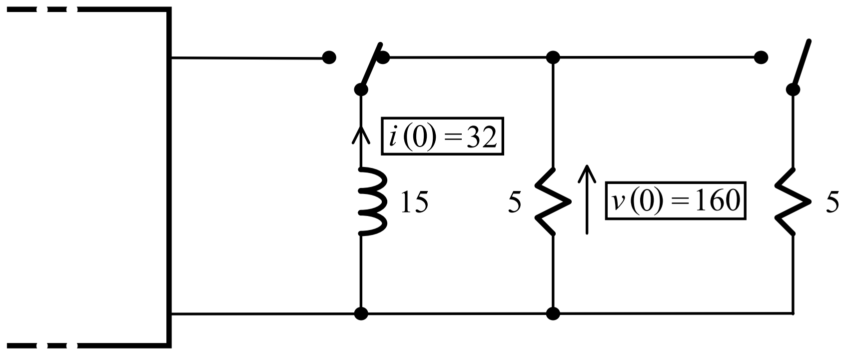
Q.4

Time $t < 0$

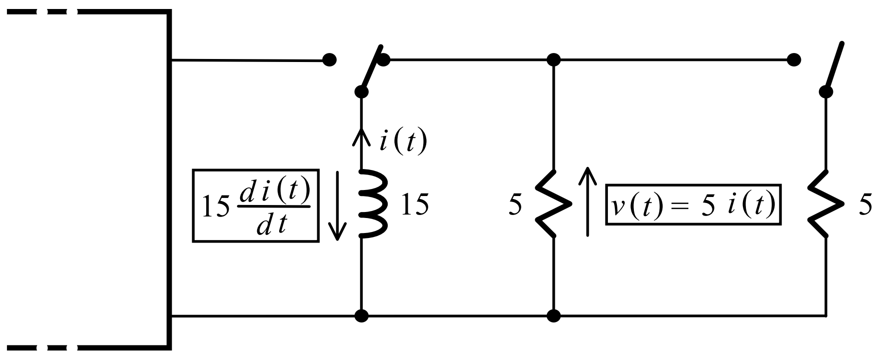


Time $t = 0$ *just after the first switch is activated*

Since the current in the inductor must be continuous, the inductor must be carrying the same 32A of current just after the first switch is activated:



Time $t \geq 0$ *and* $t < 3$ *after activating 1st switch but before closing 2nd switch*



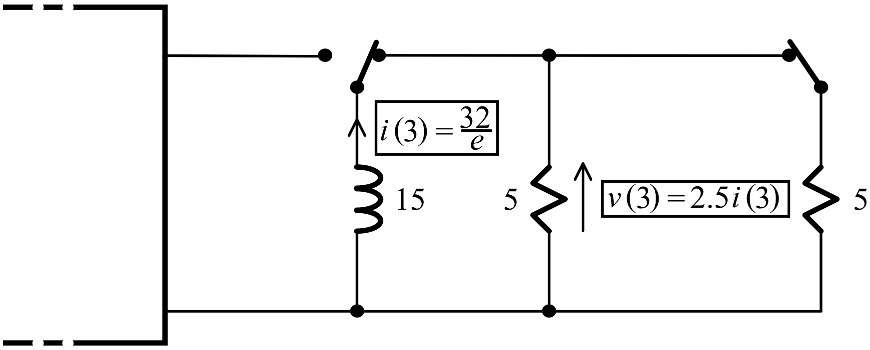
$$15\frac{di(t)}{dt} + 5i(t) = 0 \;\Rightarrow\; \frac{di(t)}{dt} + \frac{i(t)}{3} = 0 \;\Rightarrow\; i(t) = ke^{-\frac{t}{3}}, 0 \leq t < 3$$

$$i(0) = k = 32 \;\Rightarrow\; i(t) = 32e^{-\frac{t}{3}}, 0 \leq t < 3$$

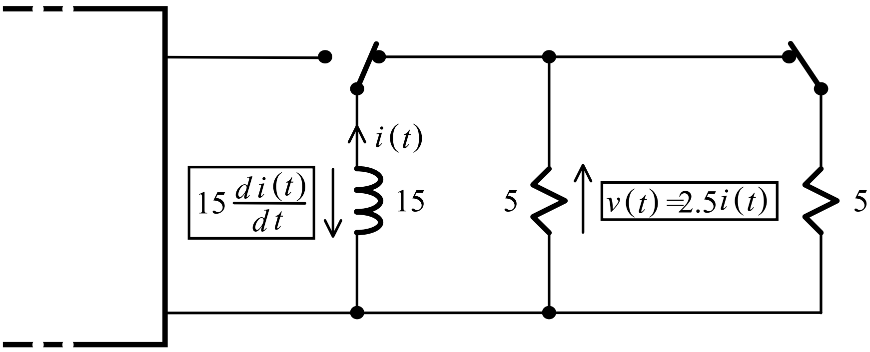
$$v(t) = 5i(t) = 160e^{-\frac{t}{3}}, 0 \leq t < 3$$

Time $t = 3$ just after closing second switch

Since the current in the inductor must be continuous, $i(t)$ must be the same as $32e^{-3/3} = 32e^{-1}\,\text{A}$ just before closing this switch:



Time $t \geq 3$ after closing second switch

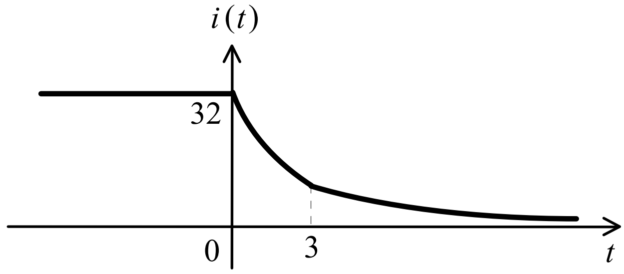


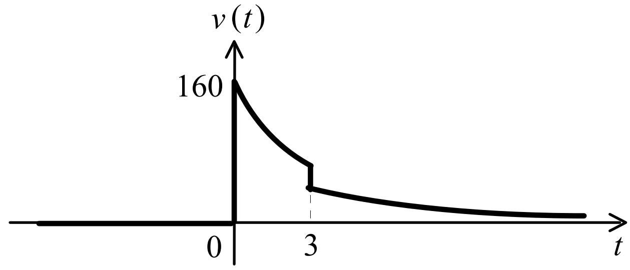
$$15\frac{di(t)}{dt} + 2.5i(t) = 0 \;\Rightarrow\; \frac{di(t)}{dt} + \frac{i(t)}{6} = 0 \;\Rightarrow\; i(t) = he^{-\frac{t}{6}},\, 3 \leq t$$

$$i(3) = he^{-\frac{3}{6}} = 32e^{-1} \;\Rightarrow\; h = 32e^{-\frac{1}{2}} \;\Rightarrow\; i(t) = 32e^{-1}e^{-\frac{(t-3)}{6}} \quad, 3 \leq t$$

$$v(t) = 2.5i(t) = 80e^{-1}e^{-\frac{(t-3)}{6}},\, 3 \leq t$$

Voltage and current waveforms


i(t)
32
0
3
t


v(t)
160
0
3
t