Solutions to EE2008 Homework Assignment 1

Q.1 In the circuit of Fig. 1, the switch has been in the position shown for a long time and is thrown to the other position for time $t \geq 0$. Determine the values of the voltages and currents in the inductor and capacitor just before and just after the switch has been moved to the final position.

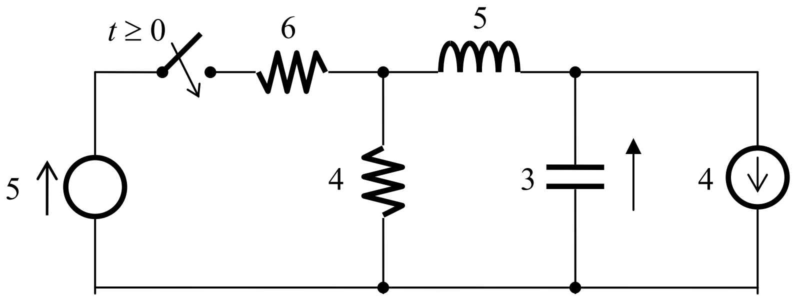


Fig. 1

Solution: Taking all the voltages and currents to be constants for $t < 0$, and measuring currents/voltages clockwise/anticlockwise for *L* and *C*:

$$i_C(t) = 3\frac{dv_C(t)}{dt} = 0$$

$$v_L(t) = 5\frac{di_L(t)}{dt} = 0$$

Using KCL:

$$i_L(t) = i_C(t) + 4 = 4$$

Using KVL:

$$v_C(t) + v_L(t) + 4i_L(t) = 0 \;\Rightarrow\; v_C(t) = -16$$

Applying continuity for $i_L(t)$ and $v_C(t)$ after switching and using KCL:

$$i_L(0) = 4 \;\Rightarrow i_C(0) = 0$$

$$i_L(0) + \frac{v_L(0) + v_C(0)}{4} + \frac{v_L(0) + v_C(0) - 5}{6} = 0$$

$$4 + \frac{5}{12}v_L(0) - \frac{15}{2} = 0 \;\Rightarrow\; v_L(0) = \frac{42}{5}$$

Q.2 Assuming the capacitor to be initially uncharged, derive the differential equation from which the inductor current can be found in the circuit of Fig. 2. For what range of values of C will the circuit be underdamped? If C is such that the circuit is critically damped, will a change in the source voltage make it underdamped?

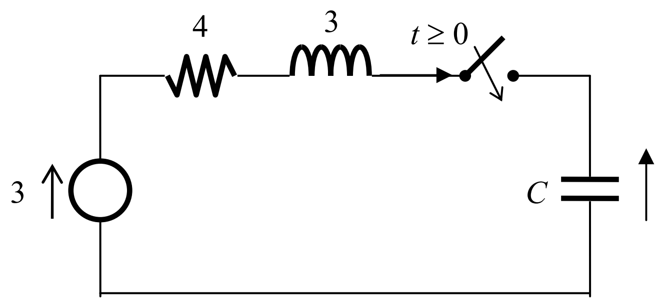


Fig. 2

Solution: After the switch is closed:

$$i(t) = i_L(t) = i_C(t)$$

$$i_C(t) = C\frac{dv_C(t)}{dt}$$

$$v_L(t) = 3\frac{di_L(t)}{dt}$$

Applying KVL:

$$v_L(t) + v_C(t) + 4i(t) = 3 \;\Rightarrow\; 3\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt + 4i(t) = 3$$

$$\frac{d^2 i(t)}{dt^2} + \frac{4}{3}\frac{di(t)}{dt} + \frac{i(t)}{3C} = 0$$

The characteristic equation for the transient response is:

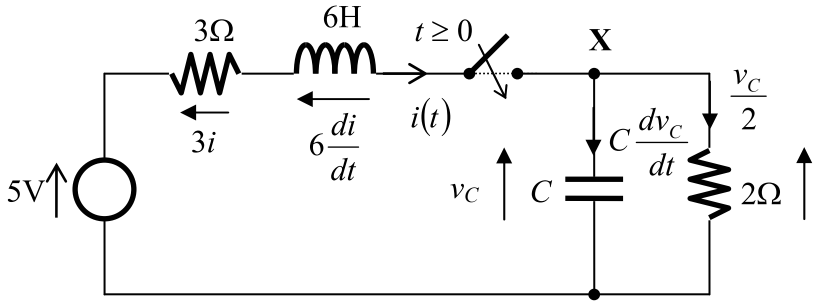
$$z^2 + \frac{4}{3}z + \frac{1}{3C} = 0$$

For under damped behavior, the roots must be complex:

$$\left(\frac{4}{3}\right)^2 < 4\left(\frac{1}{3C}\right) \;\Rightarrow\; C < \frac{3}{4}$$

A change in the source voltage has no effect on whether the system is under damped, critically damped or over damped.

Q.3 Assuming the capacitor to be initially uncharged, derive the differential equation from which the inductor current can be found in the circuit of Fig. 3. Determine the values of C for the circuit to be critically damped.



Solution: Applying KVL to the left loop of the above circuit (when the switch is closed), we obtain

$$3i + 6\frac{di}{dt} + v_C = 5 \quad \Rightarrow \quad v_C = 5 - 3i - 6\frac{di}{dt}$$

Then, applying KCL to node X, we have

$$i = C\frac{dv_C}{dt} + \frac{v_C}{2} = C\frac{d}{dt}\left(5 - 3i - 6\frac{di}{dt}\right) + \frac{1}{2}\left(5 - 3i - 6\frac{di}{dt}\right)$$

$$\Rightarrow \quad 6C\frac{d^2 i(t)}{dt^2} + 3(C+1)\frac{di(t)}{dt} + \frac{5}{2}i(t) = \frac{5}{2}$$

The corresponding characteristic polynomial is

$$6C z^2 + 3(C+1)z + \frac{5}{2} = 0$$

For critically damped response, the above polynomial should have two repeated roots, which imply

$$b^2 - 4ac = 9(C+1)^2 - 60C = 9C^2 - 42C + 9 = 0 \quad \Rightarrow \quad \begin{cases} C_1 = 4.4415 \\ C_2 = 0.2251 \end{cases}$$

Q.4. In the circuit of Fig. 4, the switches have been in the positions shown for a long time and are thrown to the other position for $t \ge 0$.

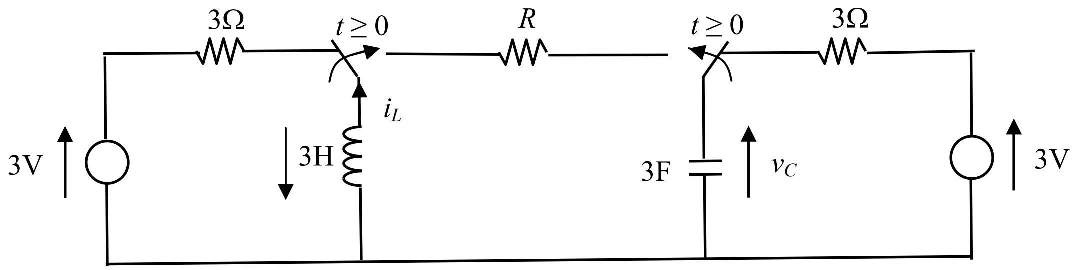


Fig. 4

(a) Determine the capacitor voltage and inductor current for $t < 0$.

(b) Derive the governing differential equation in term of the capacitor voltage for $t \ge 0$.

(c) Determine the value of R such that the circuit has a critically damped response.

(d) For the value of R obtained in Part (c), determine the complete response of the inductor current for all time t.

<u>**Solution:**</u>

(a) Determine the capacitor voltage and inductor current for $t < 0$.

$$i_L = -1, \qquad V_C = 3$$

(b) Derive the governing differential equation in term of the capacitor voltage for $t \ge 0$.

$$9\frac{d^2 v_C}{dt^2} + 3R\frac{dv_C}{dt} + v_C = 0 \qquad or \qquad \frac{d^2 v_C}{dt^2} + \frac{R}{3}\frac{dv_C}{dt} + \frac{1}{9}v_C = 0$$

(c) Determine the value of R such that the circuit has a critically damped response.

$$9z^2 + 3Rz + 1 = 0 \quad \Rightarrow \quad (3R)^2 - 4\times 9 = 0 \quad \Rightarrow \quad R = 2$$

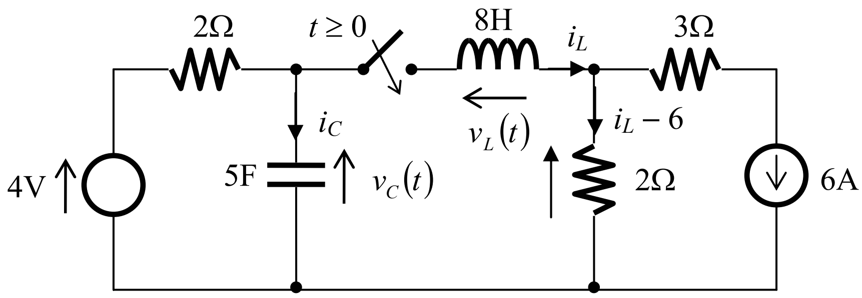
(d) For the value of R obtained in Part (c), determine the complete response of the inductor current for all time t.

$$z_{1,2} = -\frac{1}{3} \quad \Rightarrow \quad v_C(t) = v_{ss} + (k_1 + k_2 t)e^{-\frac{1}{3}t} = 0 + (k_1 + k_2 t)e^{-\frac{1}{3}t} \quad \Rightarrow \quad v_C(0) = 3 = k_1$$

$$\Rightarrow \quad i_L(t) = (3k_2 - k_1 - k_2 t)e^{-\frac{1}{3}t} \quad \Rightarrow \quad i_L(0) = 3k_2 - k_1 = -1 \quad \Rightarrow \quad k_2 = \frac{2}{3}$$

$$= -\left(1 + \frac{2}{3}t\right)e^{-\frac{1}{3}t} \qquad t \ge 0$$

Q.5 In the circuit of Fig. 5, the switch has been in the position shown for a long time and is thrown to the other position for time $t \geq 0$. Determine the inductor current and voltage as well as the capacitor current and voltage just before and just after the switch has been moved to the final position. Determine the steady state current of the voltage source and the steady state voltage of the current source.



Solution: For $t < 0$, it is clear to observe from the circuit that

$$i_L(t) = 0, \quad v_L(t) = 0, \quad v_C(t) = 4, \quad i_C(t) = 0$$

which are the currents and voltages of the inductor and capacitor just before the switch is closed. When the switch is just turned on, the inductor current and capacitor voltage have to be continuous. Thus,

$$i_L(t) = 0, \quad v_C(t) = 4$$

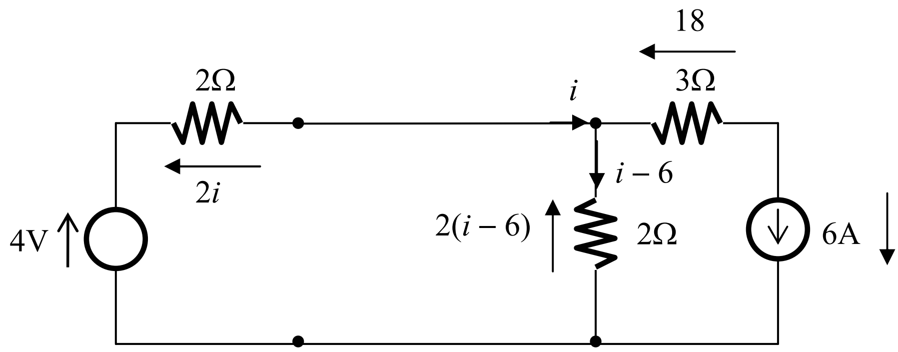
and the current following through the 2Ω resistor on the right is –6 A and thus its voltage is –12V. Applying KVL to the center loop, we obtain

$$v_L(0^+) = 16$$

When the switch is just closed, the current passing through the 2Ω resistor on the left is 0 as both the voltage source and capacitor have a voltage equal to 4V. Applying KCL to the node connecting the resistor, capacitor and inductor, it is obvious that

$$i_C(0^+) = 0$$

For the steady state after the switch being closed for a long time, the capacitor is again open-circuited and the inductor is short-circuited. The circuit is equivalent to



Applying KVL to the left loop, we have

$$2i + 2(i - 6) = 4 \quad \Rightarrow \quad i = 4$$

The voltage of the current source = 18 – 2 (i – 6) = 22 V.