Q.1. Consider the circuit given in Figure Q.1 below, in which $u(t)$ is a voltage source.

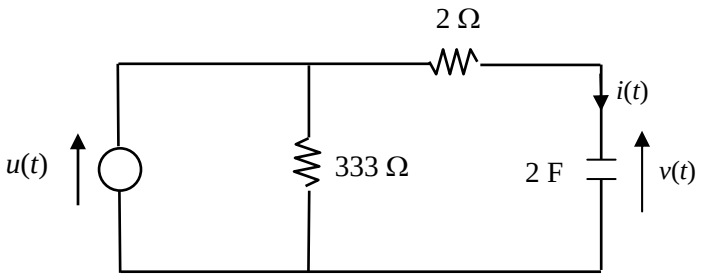

Figure Q.1

(a) Derive a time-domain model for the circuit in terms of the capacitor voltage, $v(t)$.

(10 marks)

Solution: It is simple to observe from the above circuit that

$$i(t) = 2\frac{dv(t)}{dt}$$

and the by applying KVL to the outer loop, we obtain

$$4\frac{dv(t)}{dt} + v(t) = u(t)$$

(b) Is the model in Part (a) linear and time-invariant? Why?

(4 marks)

Solution: Yes. All the components in the circuit are linear and time-invariant.

(c) Assume that the capacitor is initially discharged and the voltage source $u(t) = 2$ V. Determine an explicit expression for the capacitor voltage $v(t)$.

(7 marks)

Solution: From the ODE or the circuit, we can easily observe that the steady state solution

$$v_{ss} = 2$$

and the transient response can be obtained by first find the roots if its characteristic polynomial, i.e.,

$$4z + 1 = 0 \quad \Rightarrow \quad z_1 = -\frac{1}{4} = -0.25 \quad \Rightarrow \quad v_{tr}(t) = k\, e^{-0.25t}$$

The complete solution is then given by

$$v(t) = v_{ss} + v_{tr}(t) = 2 + k\, e^{-0.25t}$$

The assumption that the capacitor is initially discharged implies $v(0) = 0$ and thus $k = -2$. Hence

$$v(t) = v_{ss} + v_{tr}(t) = 2 - 2e^{-0.25t}$$

(d) Is the system is stable. Why?

(4 marks)

Solution: Yes. Its characteristic polynomial has a negative root.

Q.2. The transfer function of a television receiver has a frequency (magnitude) response as shown in Figure Q.2 below:

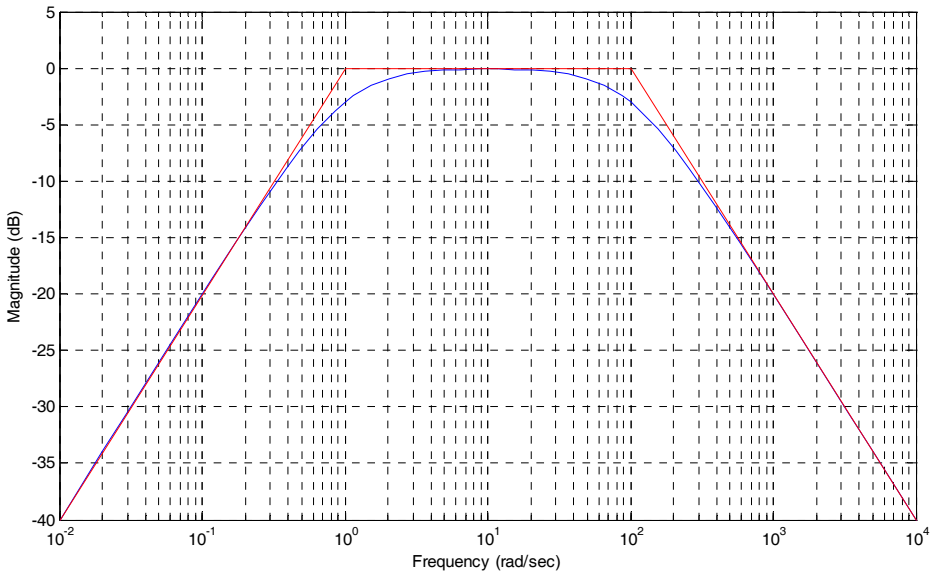

Figure Q.2

(a) Does the system have an integrator? Why?

(4 marks)

Solution: No. The magnitude response does not roll off at low frequency.

(b) Does the system have a differentiator? Why?

(4 marks)

Solution: Yes. The magnitude response does roll up 20 dB per decade at low frequency.

(c) Determine the transfer function of the system?

(9 marks)

Solution: From the asymptotes, we can obtain the transfer function

$$G(s) = \frac{s}{(1+s)(1+s/100)} = \frac{100s}{(s+1)(s+100)}$$

(d) Determine the magnitude of its output signal when its input is $\cos(1000t + 13°)$?

(4 marks)

Solution: From the given magnitude response, its gain = – 20 dB = 0.1 at ω = 1000 rad/s. Thus, the magnitude of the corresponding output signal is 0.1.

(e) What is the DC gain of the system?

(4 marks)

Solution: The DC gain is 0.