National University of Singapore
Department of Electrical & Computer Engineering
EE2010E Systems & Control

EXPERIMENT C1

COMPUTER-AIDED ANALYSIS OF SYSTEM CHARACTERISTICS

✪ *Please ensure that you read this manual carefully and familiarise yourself with MATLAB before attending the laboratory session.*
✪ *Questions marked with the symbol ✍ must be answered in your log book.*

1. Objective

The aim of this experiment is to use MATLAB computational package to perform computer-aided system analysis and design.

2. Introduction

Computer-aided packages for control system analysis and design are commonly available nowadays, and they have proved to be very useful. In this experiment, the student will use MATLAB to study a system for controlling the position of a car on a level surface. MATLAB is essentially a matrix manipulation package while the Control System Toolbox is a collection of functions for the modelling, analysis, and design of automatic control systems.

3. Preliminaries

Before attending the lab, you should familiarise yourself with MATLAB by reading "Section 2.10 The Simulation of Systems Using MATLAB" in *Modern Control Systems* by R. Dorf and R. Bishop. The following commands may come in handy during the experiment:

roots	Find roots of a polynomial
tf	Creation of transfer functions
pzmap	Generate the pole-zero map of a LTI system
series	Series connection of two LTI systems
feedback	Feedback connection of two LTI systems
impulse	Generate the impulse response of a LTI system
step	Generate the step response of a LTI system
bode	Produces the Bode plot of a LTI system

4. Experiment

Consider the vehicle on a horizontal surface, as illustrated in Figure 1.

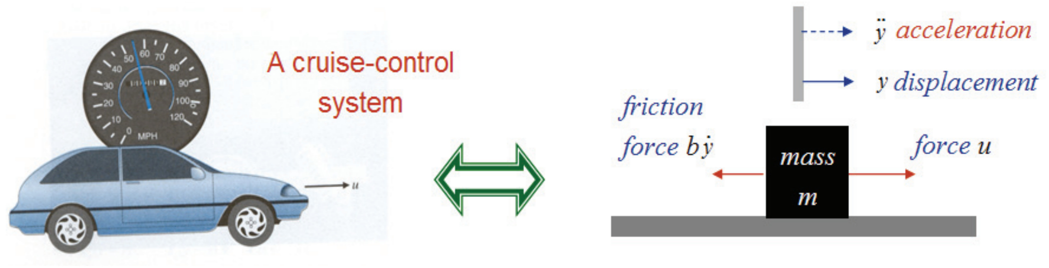

Figure 1: Vehicle on a horizontal surface

Suppose the position of the car at time t, relative to some reference point, is $y(t)$. It follows from Newton's second law of motion ($f = ma$) that

$$m\frac{\mathrm{d}^2y(t)}{\mathrm{d}t^2} = u(t) - f(t) \tag{1}$$

where m is the mass of the car, $u(t)$ is the applied force and $f(t)$ is the frictional forces retarding the motion of the car. For simplicity, it may be assumed that the frictional force is proportional to the car speed i.e.

$$f(t) = b\frac{\mathrm{d}y(t)}{\mathrm{d}t}$$

where b is the coefficient representing frictional losses (Unit of b is $\frac{\mathrm{N}}{m/s}$). Substituting $f(t) = b\dot{x}$ into Equation (1), the second-order linear differential equation that relates the car position, $y(t)$, to the applied force is as follows:

$$m\frac{\mathrm{d}^2y(t)}{\mathrm{d}t^2} + b\frac{\mathrm{d}y(t)}{\mathrm{d}t} = u(t) \tag{2}$$

An autonomous vehicle is constructed by using an infra-red sensor to "sense" the car position, and then using the following expression to "compute" the force that should be applied to the car

$$u(t) = K[y_r(t) - y(t)] \tag{3}$$

where $y_r(t)$ is the desired position of the car at time t and K is the gain of the *proportional controller*. The block diagram of the resulting *position control system* that relates the target position to the actual position of the autonomous vehicle is shown in Figure 2. Such a system may be used to guide an autonomous vehicle, such as the *Pathfinder* rover, to a particular destination.

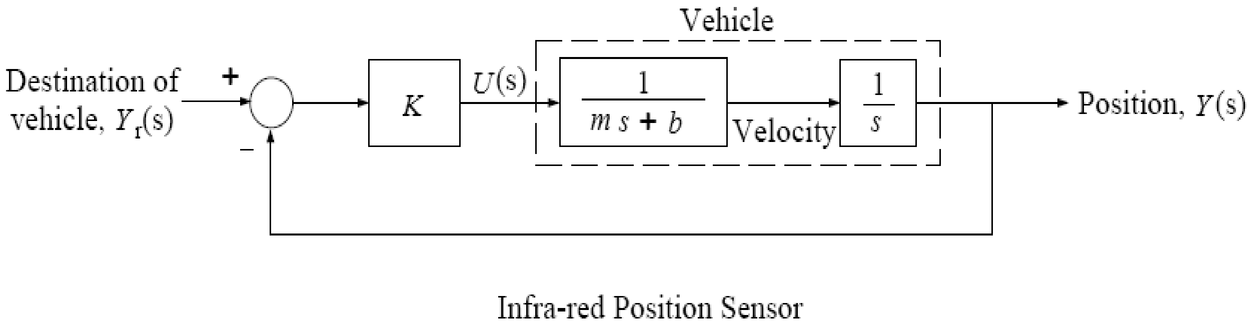

Figure 2: Vehicle position control system

Part 4.1: Analysis of the vehicle position control system

This section aims at analysing the behaviour of the vehicle position control system shown in Figure 2. First, use your matriculation number, U0**ABCD**, to identify the parameters of the position control system:

- Mass of the car, m is $(1 + 0.1D)$
- Friction coefficient $b = 0.6 + (C + D)/50$
- $K = 1$

Open-loop System

- ✍ Derive the open-loop transfer function of the vehicle from the applied force $u(t)$ to the position of the car $y(t)$.
- ✍ Draw the Bode plot of the open-loop transfer function using MATLAB.
- ✍ Compute the unit impulse response of the open-loop transfer function and plot it using MATLAB.
- ✍ Compute the unit step response of the open-loop transfer function and plot it using MATLAB.
- ✍ Is the open-loop system BIBO stable?

We will derive in the next part that the closed-loop system from the reference $y_r(t)$ to the position of the car $y(t)$ has a transfer function of the following form:

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

which has a unit step response

$$y(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(t\omega_n\sqrt{1-\zeta^2} + \phi\right)$$

where the rate of decay of the exponential function, $-\zeta\omega_n$, is the real part of the poles and the frequency of the sinusoid, $\omega_n\sqrt{1-\zeta^2}$, is the imaginary part of the poles. In addition,

➜ t_r is a measure of how quickly the vehicle is able to reach the vicinity of the destination.

➜ t_s is the time taken for the vehicle to arrive and stop at its destination

➜ M_p is the maximum amount by which the vehicle exceeds/overshoots its destination.

Assume that the vehicle is at the reference position when $t = 0$ and it needs to move to a location that is 1 m away from the starting point. Then, $y_r(t)$ may be modelled by a unit step function and the step response provides information about the position of the car at various instances in time.

Closed-loop System

✍ Derive the closed-loop transfer function from the reference $y_r(t)$ to the position of the car $y(t)$?

✍ What are the zeros and poles of the closed-loop system?

✍ What is the damping ratio and the natural frequency of the closed-loop system?

✍ What is the rise time, settling time, peak time and overshoot of the closed-loop system?

✍ Plot the unit step response of the closed-loop system using MATLAB and verify the transient properties computed in the previous question.

✍ Explain why the autonomous vehicle will always stop exactly at the desired final destination that is 1m away from the starting point i.e. $y_{ss} = \lim_{t\to\infty} y(t) = 1$ (*Hint: Using the final value theorem of Laplace transform*).

✍ Draw the Bode plot of the closed-loop system. Use the obtained Bode plot to compute the steady state solution of the system due to the unit step input.

✍ Is the closed-loop system BIBO stable?

Part 4.2: Design of a vehicle position control loop (Optional)

Suppose there is a need to build an autonomous vehicle that fulfils the following criteria:-

i. It should reach a target position within 10 seconds

ii. It can exceed the by a maximum of 8% of the step change

The frictional constant, b, will change only if there is a significant change in the road surface (e.g. in going from a paved to a unpaved surface) so b cannot be independently varied. However, the remaining parameters (mass m and the proportional constant K) can be changed.

❒ Determine the values that m and K should assume in order to meet the given specification.

❒ What are the system poles?

❒ Use MATLAB to verify that the position control system has been correctly designed.

THE END