Some application issues…

The remaining topics of this course is to introduce some key ideas on how to design control systems for some practical problems such as flight control and hard disk drive servo systems.

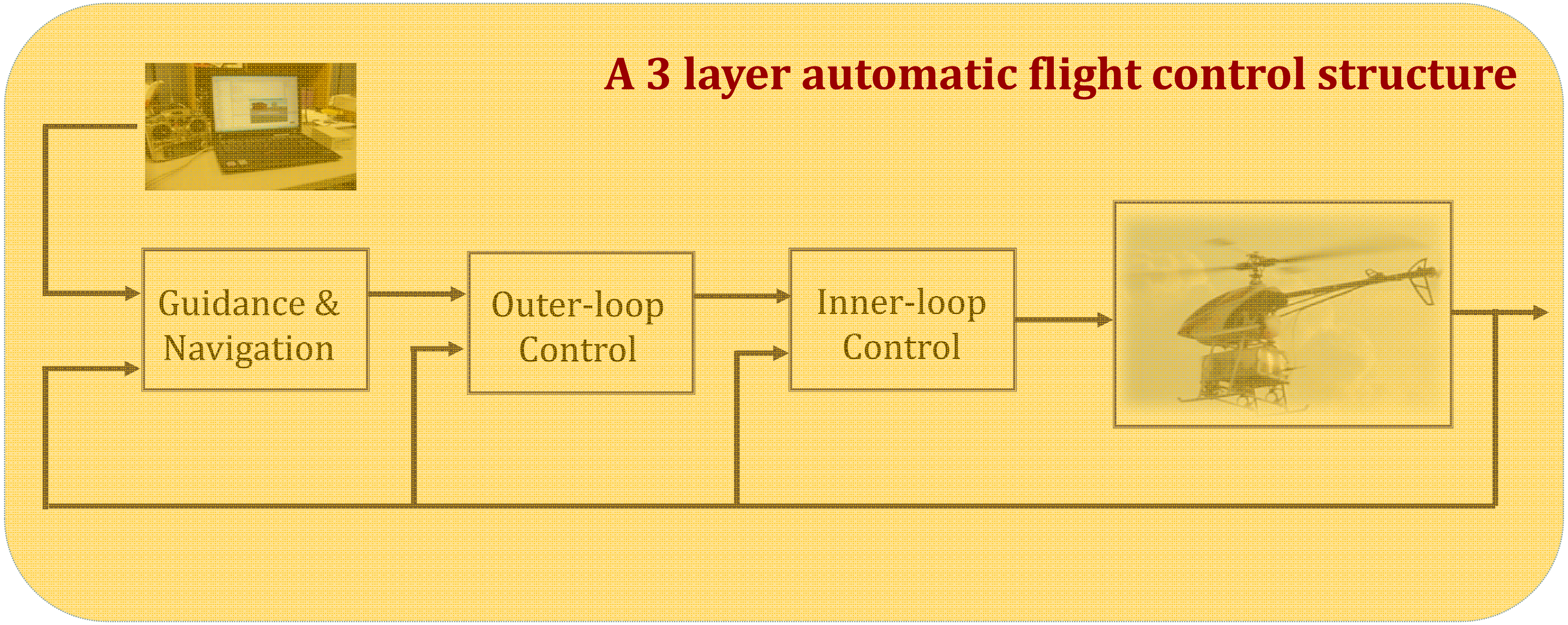
Interested readers should refer to the following monograph for more detailed materials…

Cai, Chen, Lee ◊ *Unmanned Rotorcraft Systems* ◊ Springer, New York, 2011

Chen, Lee, Peng, Venkataramanan ◊ *Hard Disk Drive Servo Systems* (2nd ed.) ◊ Springer, New York, 2006

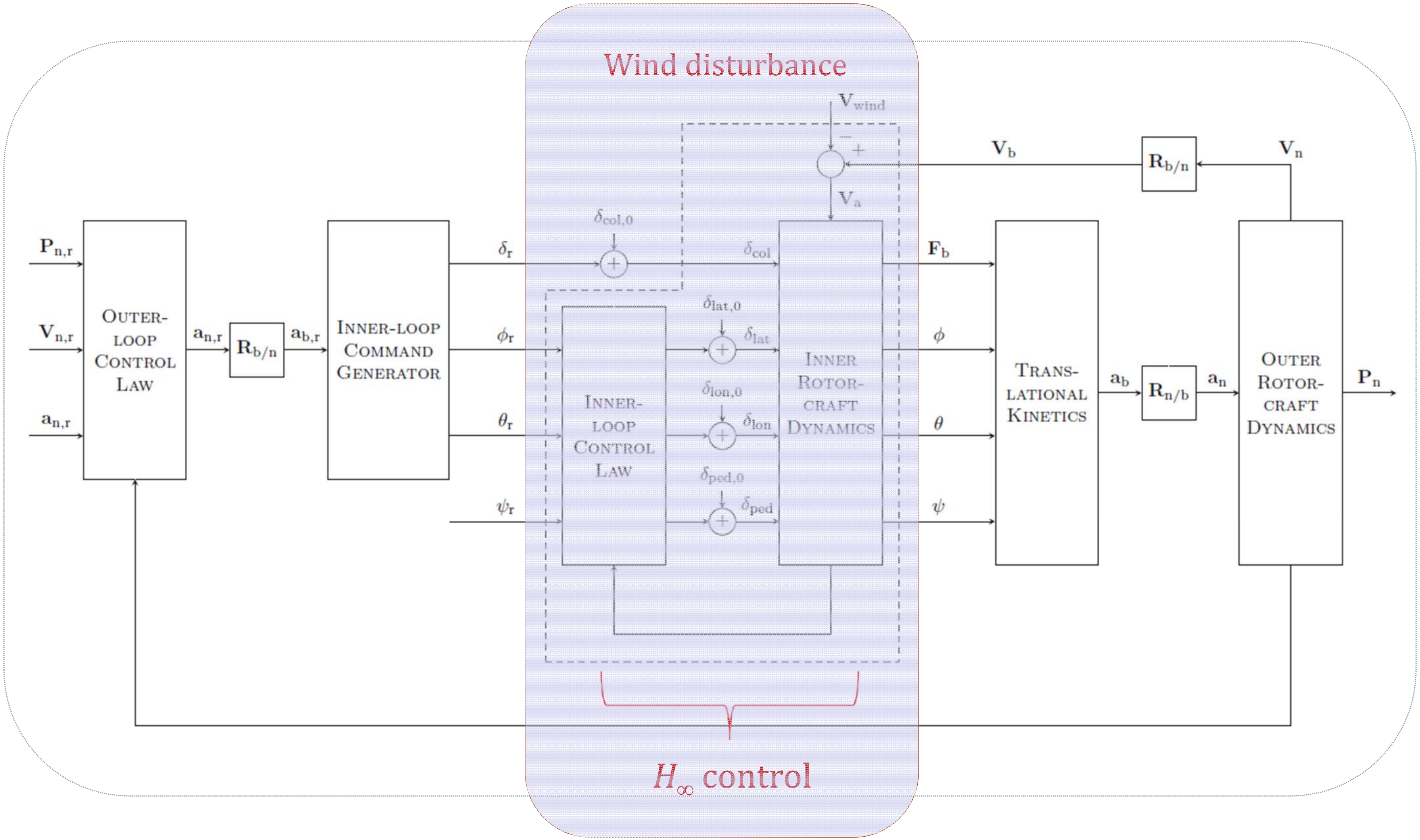
Flight Control Systems

Overall flight control structure

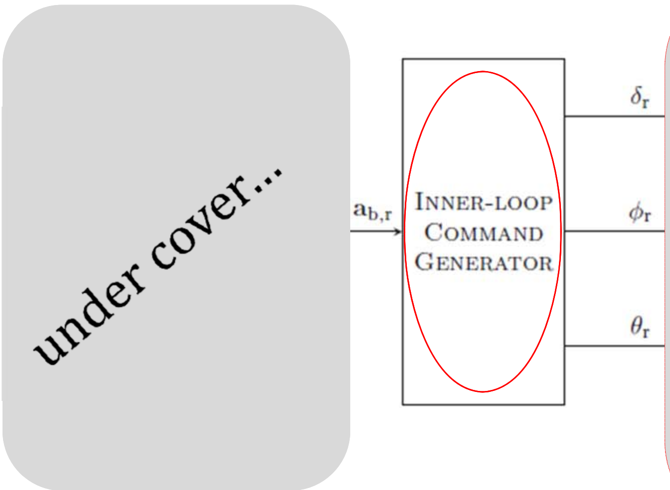


- Inner loop (H_∞) control is to guarantee the stability of the aircraft attitude
- Outer loop (RPT) control is to control the aircraft position and velocity
- Guidance and navigation are to generate references of flight missions for outer-loop

Detailed structure of the flight control system



Inner-loop command generator



The inner-loop command generator is given as

$$\begin{pmatrix} \delta_r \\ \phi_r \\ \theta_r \end{pmatrix} = \mathbf{G}\,\mathbf{a}_{b,r}$$

where **G** is a constant matrix for normal applications…

A nonlinear dynamics inversion might be necessary for high performance maneuvers.

Outer-loop control structure

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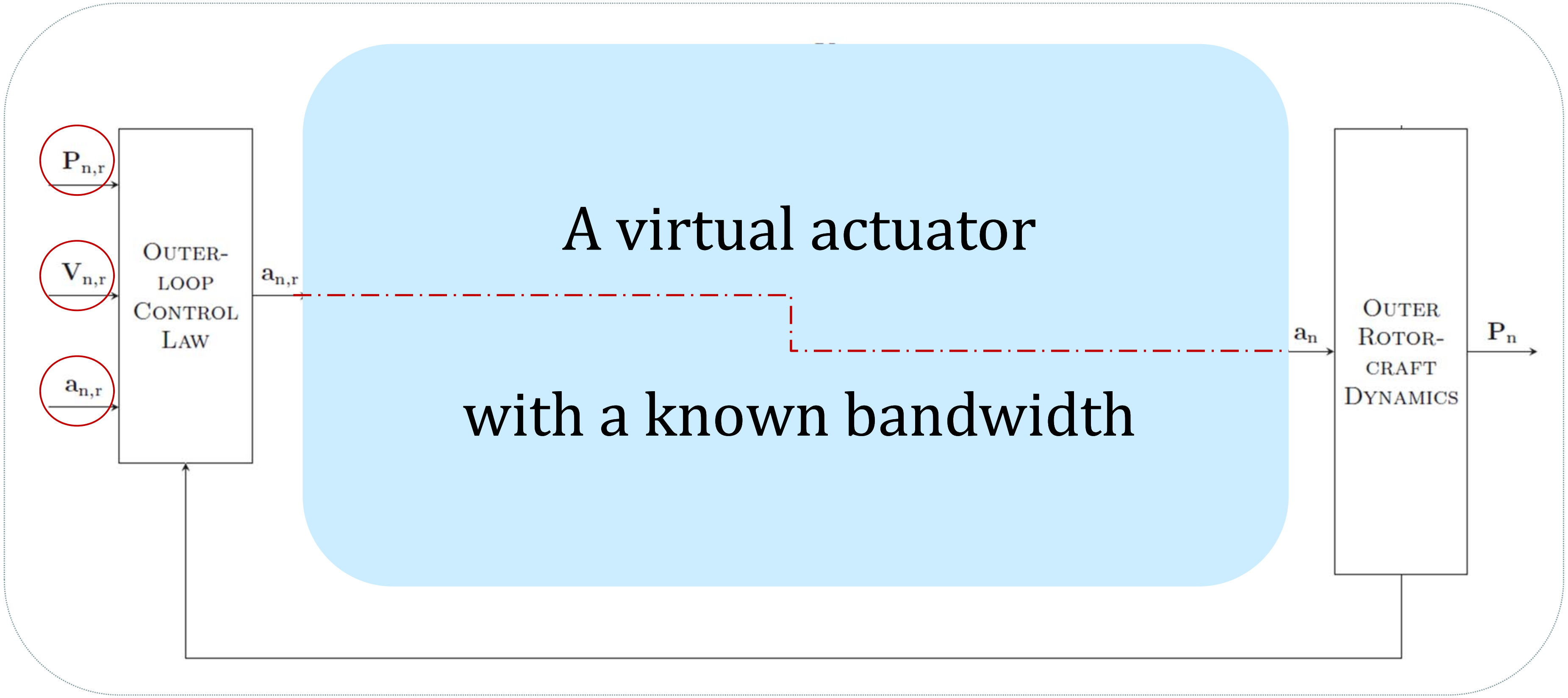
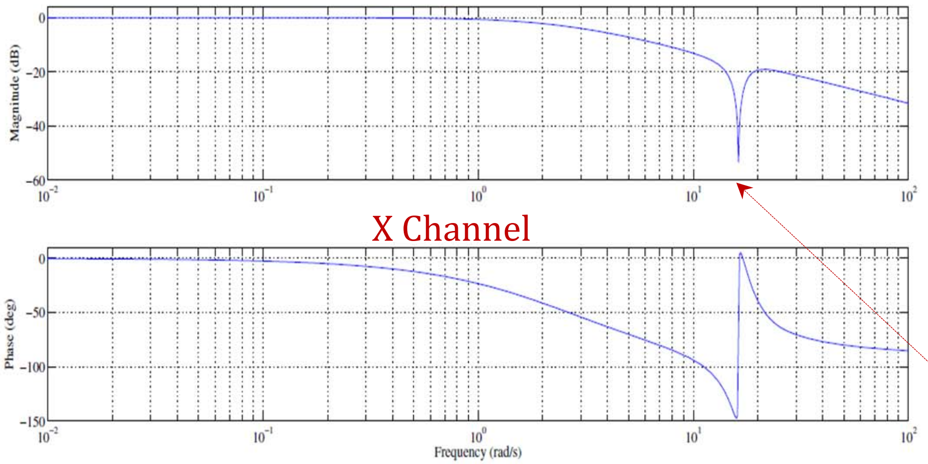
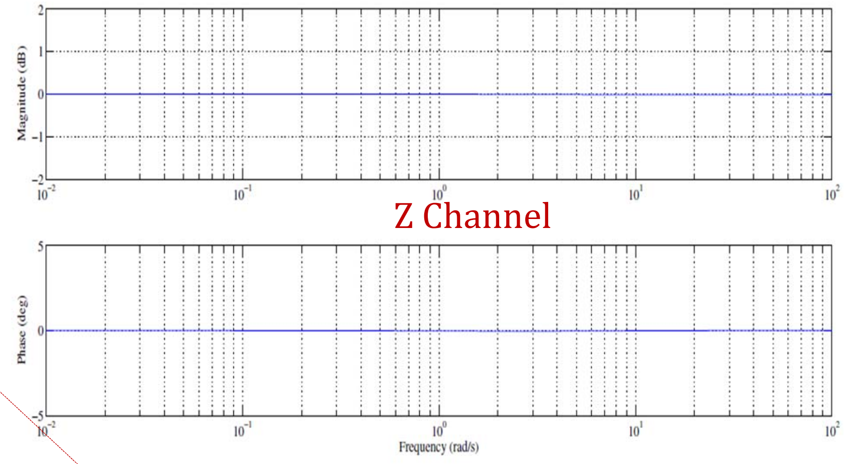


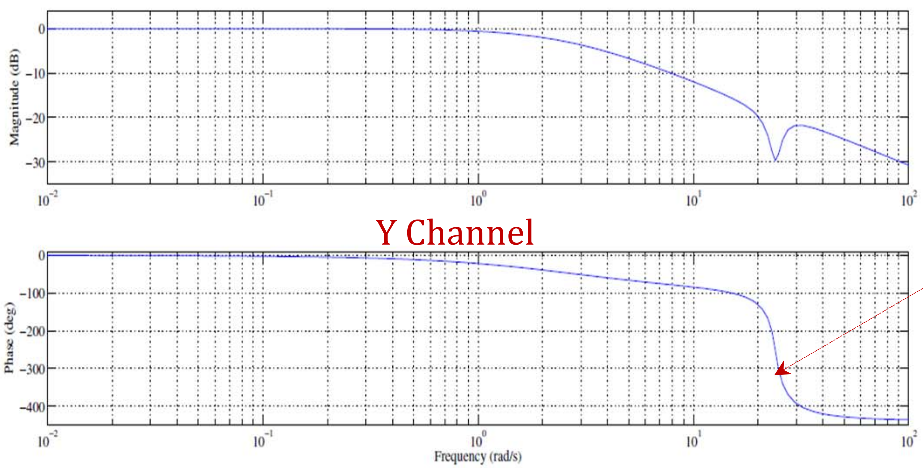
RPT control, which is capable of tracking position, velocity and acceleration references!

Frequency response of the virtual actuator... HeLion & SheLion





Frequency response of the virtual actuator...

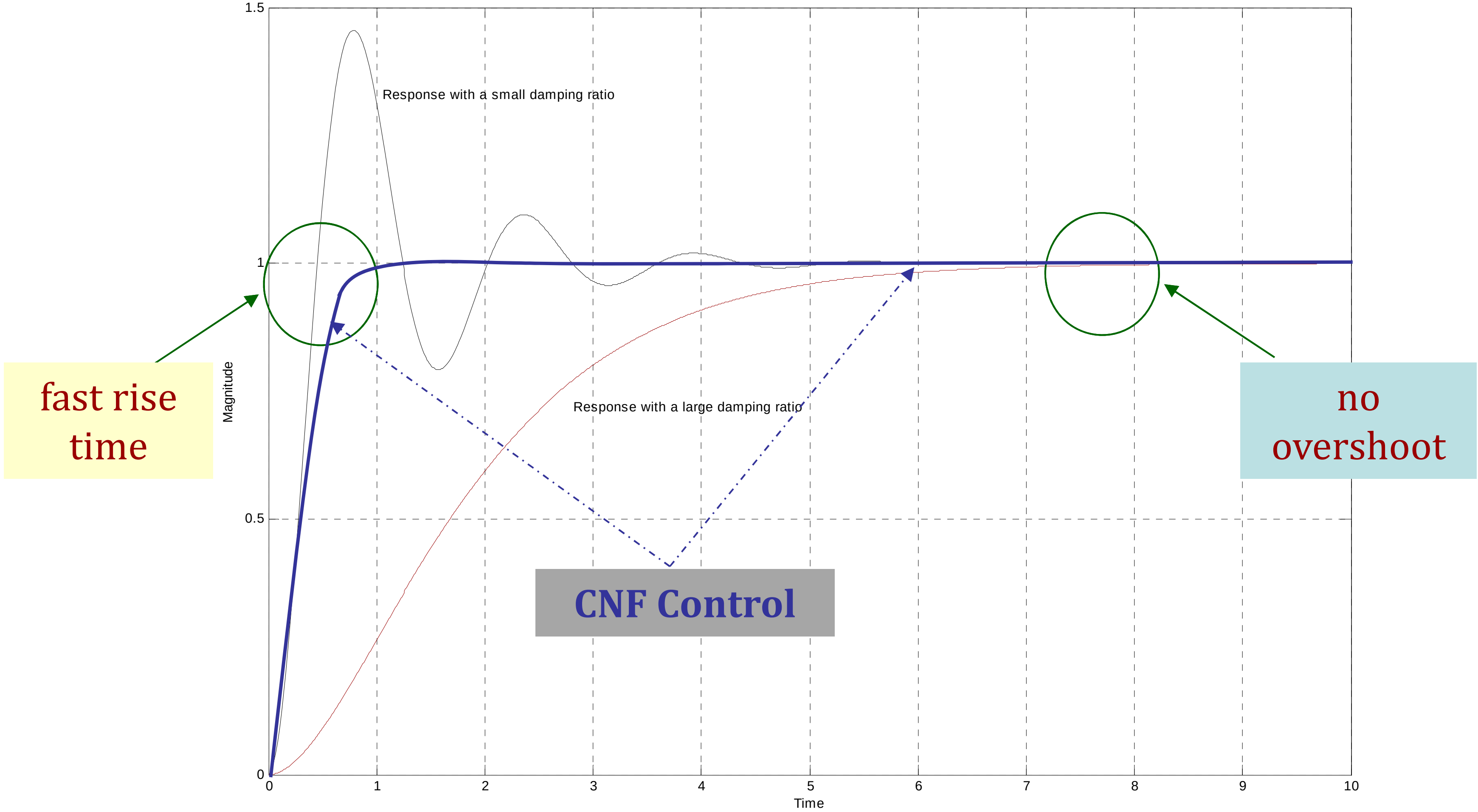


Unstable Zeros!

From practical point of view, it is safe to ignore them so long as the outer-loop bandwidth is within 1 rad/sec...

Composite Nonlinear Feedback Control

Key idea…


fast rise time
no overshoot
CNF Control
Response with a small damping ratio
Response with a large damping ratio
Magnitude
Time
1.5
1
0.5
0
0
1
2
3
4
5
6
7
8
9
10

CNF Control

CNF control consists of a linear law and a nonlinear feedback law with any switching element. Unlike the PTOS and MSC control laws, both the linear and nonlinear controllers in CNF are in operation all the time. The linear part is designed to yield a closed-loop system with a small damping ratio for a quick response. On the other hand, the nonlinear part is used to increase the damping ratio of the closed-loop system as the system output approaches the target reference to reduce the overshoot caused by the linear part.

As will be seen shortly, CNF control has emerged as an effective tool in design many control systems that require faster dynamical response and small overshoot or undershoot.

CNF Control Formulation

Specifically, we consider a linear system with an amplitude constrained actuator, characterized by

$$\begin{cases} \dot{x} = A\,x + B\,\mathrm{sat}(u) + Ew, \quad x(0) = x_0 \\ y = C_1\,x \\ h = C_2\,x \end{cases} \tag{5.89}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}$, $y \in \mathbb{R}^p$, $h \in \mathbb{R}$ and $w \in \mathbb{R}$ are, respectively, the state, control input, measurement output, controlled output and disturbance input of the system. A, B, C_1, C_2 and E are appropriate dimensional constant matrices. The function, sat: $\mathbb{R} \to \mathbb{R}$, represents the actuator saturation defined as

$$\mathrm{sat}(u) = \mathrm{sgn}(u) \min\{\, u_{\max},\ |u|\,\} \tag{5.90}$$

with $u_{\max}$ being the saturation level of the input. The assumptions on the given system are made:

1. (A, B) is stabilizable,
2. (A, C_1) is detectable,
3. (A, B, C_2) is invertible and has no invariant zero at $s = 0$,
4. w is bounded unknown constant disturbance, and
5. h is part of y, *i.e.* h is also measurable.

Note that all these assumptions are fairly standard for tracking control. We aim to design an enhanced CNF control law for the system with disturbances such that the resulting controlled output would track a target reference (set point), say r, as fast and as smooth as possible without having steady-state error.

Preliminary Setup

We first follow the usual practice to augment an integrator into the system by defining

$$\dot{x}_{\rm i} := \kappa_{\rm i} e := \kappa_{\rm i}(h - r) = \kappa_{\rm i} C_2 x - \kappa_{\rm i} r \tag{5.91}$$

which is implementable as h is assumed to be measurable, and $\kappa_{\rm i}$ is a positive scalar to be selected to yield an appropriate integration action. The augmented system is then given as follows,

$$\begin{cases} \dot{\bar{x}} = \bar{A}\,\bar{x} + \bar{B}\,\mathrm{sat}(u) + \bar{B}_{\rm r}\,r + \bar{E}\,w \\ \bar{y} = \bar{C}_1\,\bar{x} \\ h = \bar{C}_2\,\bar{x} \end{cases} \tag{5.92}$$

where

$$\bar{x} = \begin{pmatrix} x_{\rm i} \\ x \end{pmatrix}, \quad \bar{x}_0 = \begin{pmatrix} 0 \\ x_0 \end{pmatrix}, \quad \bar{y} = \begin{pmatrix} x_{\rm i} \\ y \end{pmatrix} \tag{5.93}$$

$$\bar{A} = \begin{bmatrix} 0 & \kappa_{\rm i} C_2 \\ 0 & A \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} 0 \\ B \end{bmatrix}, \quad \bar{B}_{\rm r} = \begin{bmatrix} -\kappa_{\rm i} \\ 0 \end{bmatrix} \tag{5.94}$$

and

$$\bar{E} = \begin{bmatrix} 0 \\ E \end{bmatrix}, \quad \bar{C}_1 = \begin{bmatrix} 1 & 0 \\ 0 & C_1 \end{bmatrix}, \quad \bar{C}_2 = [\,0 \quad C_2\,] \tag{5.95}$$

Step-by-Step Design Procedure for State Feedback Case

STEP 5.C.W.S.1: design a linear feedback control law,

$$u_L = F\bar{x} + G\,r \tag{5.96}$$

where F is chosen such that 1) $\bar{A} + \bar{B}F$ is an asymptotically stable matrix, and 2) the closed-loop system $\bar{C}_2(sI - \bar{A} - \bar{B}F)^{-1}\bar{B}$ has certain desired properties. Let us partition $F = [\,F_i \quad F_x\,]$ in conformity with x_i and x. The general guideline in the design of such an F is to place the closed-loop pole of $\bar{A} + \bar{B}F$ corresponding to the integration mode, x_i, to be sufficiently closer to the imaginary axis compared to the rest eigenvalues, which implies that F_i is a relatively small scalar. The remaining closed-loop poles of $\bar{A} + \bar{B}F$ are placed to have a dominating pair with a small damping ratio, which in turn would yield a fast rise time in the closed-loop system response. Finally, G is chosen as

$$G = -[C_2(A + BF_x)^{-1}B]^{-1} \tag{5.97}$$

which is well defined as (A, B, C_2) is assumed to have no invariant zeros at $s = 0$ and $A+BF_x$ is nonsingular whenever $\bar{A}+\bar{B}F$ is stable and F_i is relatively small.

STEP 5.C.W.S.2: given a positive-definite matrix $W \in \mathbb{R}^{(n+1)\times(n+1)}$, we solve the following Lyapunov equation:

$$(\bar{A} + \bar{B}F)'P + P(\bar{A} + \bar{B}F) = -W \tag{5.98}$$

for $P > 0$. Such a solution is always existent as $\bar{A}+\bar{B}F$ is asymptotically stable. The nonlinear feedback portion of the enhanced CNF control law, $u_{\rm N}$, is given by

$$u_{\rm N} = \rho(e)\bar{B}'P(\bar{x} - \bar{x}_{\rm e}) \tag{5.99}$$

where $\rho(e)$, with $e = h - r$ being the tracking error, is a smooth and nonpositive function of $|e|$, and tends to a constant as $t \to \infty$. It is used to gradually change the system closed-loop damping ratio to yield a better tracking performance. The choices of the design parameters, $\rho(e)$ and W, will be discussed later. Next, we define

$$G_{\rm e} := \begin{bmatrix} 0 \\ -(A + BF_{\rm x})^{-1}BG \end{bmatrix} \quad \text{and} \quad \bar{x}_{\rm e} := G_{\rm e}\, r \tag{5.100}$$

STEP 5.C.W.S.3: the linear feedback control law and nonlinear feedback portion derived in the previous steps are now combined to form an enhanced CNF control law,

$$u = u_{\rm L} + u_{\rm N} = F\bar{x} + G\,r + \rho(e)\bar{B}'P(\bar{x} - \bar{x}_{\rm e}) \tag{5.101}$$

Main Result

Theorem 5.8. *Consider the given system in Equation 5.89 with $y = x$ and the disturbance w being bounded by a non-negative scalar τ_w, i.e. $|w| \le \tau_w$. Let*

$$\gamma := 2\tau_w \lambda_{\max}(PW^{-1}) \left(\bar{E}' P \bar{E}\right)^{1/2} \tag{5.102}$$

Then, for any $\rho(e)$, which is a smooth and nonpositive function of $|e|$ and tends to a constant as $t \to \infty$, the enhanced CNF control law in Equation 5.101 internally stabilizes the given plant and drives the system controlled output h to track the step reference of amplitude r from an initial state $\bar{x}_0$ asymptotically without steady-state error, provided that the following conditions are satisfied:

1. *There exist positive scalars $\delta \in (0, 1)$ and $c_\delta > \gamma^2$ such that*

$$\forall \bar{x} \in \boldsymbol{X}(F, c_\delta) := \{\, \bar{x} \;:\; \bar{x}' P \bar{x} \le c_\delta \,\} \quad \Rightarrow \quad |F\bar{x}| \le (1 - \delta) u_{\max} \tag{5.103}$$

2. *The initial condition, $\bar{x}_0$, satisfies*

$$\bar{x}_0 - \bar{x}_{\mathrm{e}} \in \boldsymbol{X}(F, c_\delta) \tag{5.104}$$

3. *The level of the target reference, r, satisfies*

$$|H\, r| \le \delta u_{\max} \tag{5.105}$$

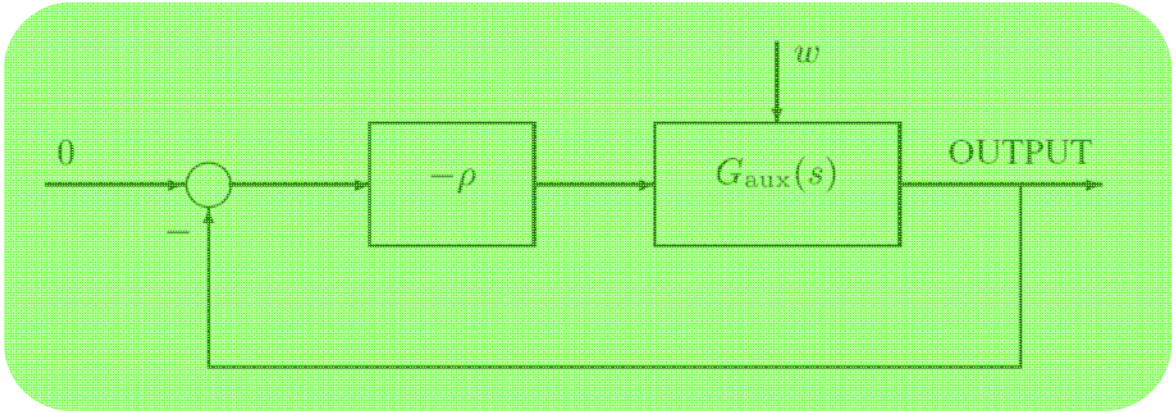
where $H := FG_{\mathrm{e}} + G$.

Selection of Nonlinear Feedback Parameters

The closed-loop system comprising the augmented plant and the CNF control law can be expressed as

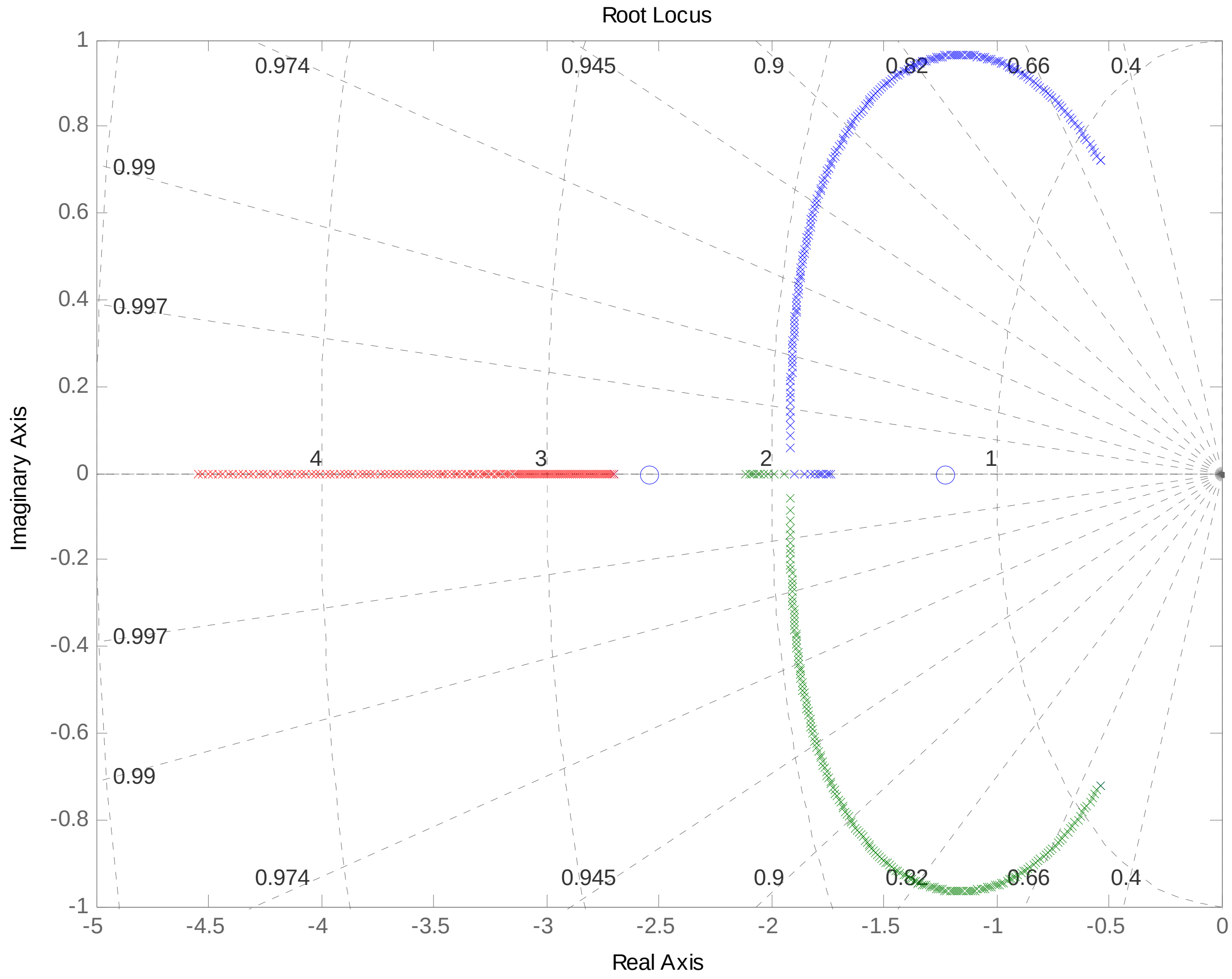
$$\dot{\tilde{x}} = (\bar{A} + \bar{B}F + \rho\bar{B}\bar{B}'P)\tilde{x} + \bar{E}w$$

It is clear that the closed-loop poles can be changed by the nonlinear function ρ. Such a mechanism can be interpreted using the classical root locus theory



where $G_{\rm aux}(s) := C_{\rm aux}(sI - A_{\rm aux})^{-1}B_{\rm aux} := \bar{B}'P(sI - \bar{A} - \bar{B}F)^{-1}\bar{B}$, whose zeros can be properly pre-selected by choosing an appropriate another design parameter W (this is related to the so-called zero placement problem).

Illustration of closed-loop system poles vs the nonlinear function…



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The following is one of the many choices of the nonlinear function:

$$\rho(e) = -\beta \left| \mathrm{e}^{-\alpha|e|} - \mathrm{e}^{-\alpha|h(0)-r|} \right|$$

where α and β are appropriate positive scalars that can be chosen to yield a desired performance, *i.e.* fast settling time and small overshoot. This function $\rho(e)$ changes from 0 to $\rho_0 = -\beta \left|1 - \mathrm{e}^{-\alpha|h(0)-r|}\right|$ as the tracking error approaches zero. At the initial stage, when the controlled output, h, is far away from the final set point, $\rho(e)$ is small and the effect of the nonlinear part on the overall system is very limited. When the controlled output, h, approaches the set point, $\rho(e) \approx \rho_0$, and the nonlinear control law becomes effective. In general, the parameter ρ_0 is chosen such that the poles of $\bar{A} + \bar{B}F + \rho_0 \bar{B}\bar{B}'P$ are in the desired locations, *e.g.*, the dominated poles have a large damping ratio, which would reduce the overshoot of the output response. Note that the choice of $\rho(e)$ is nonunique.

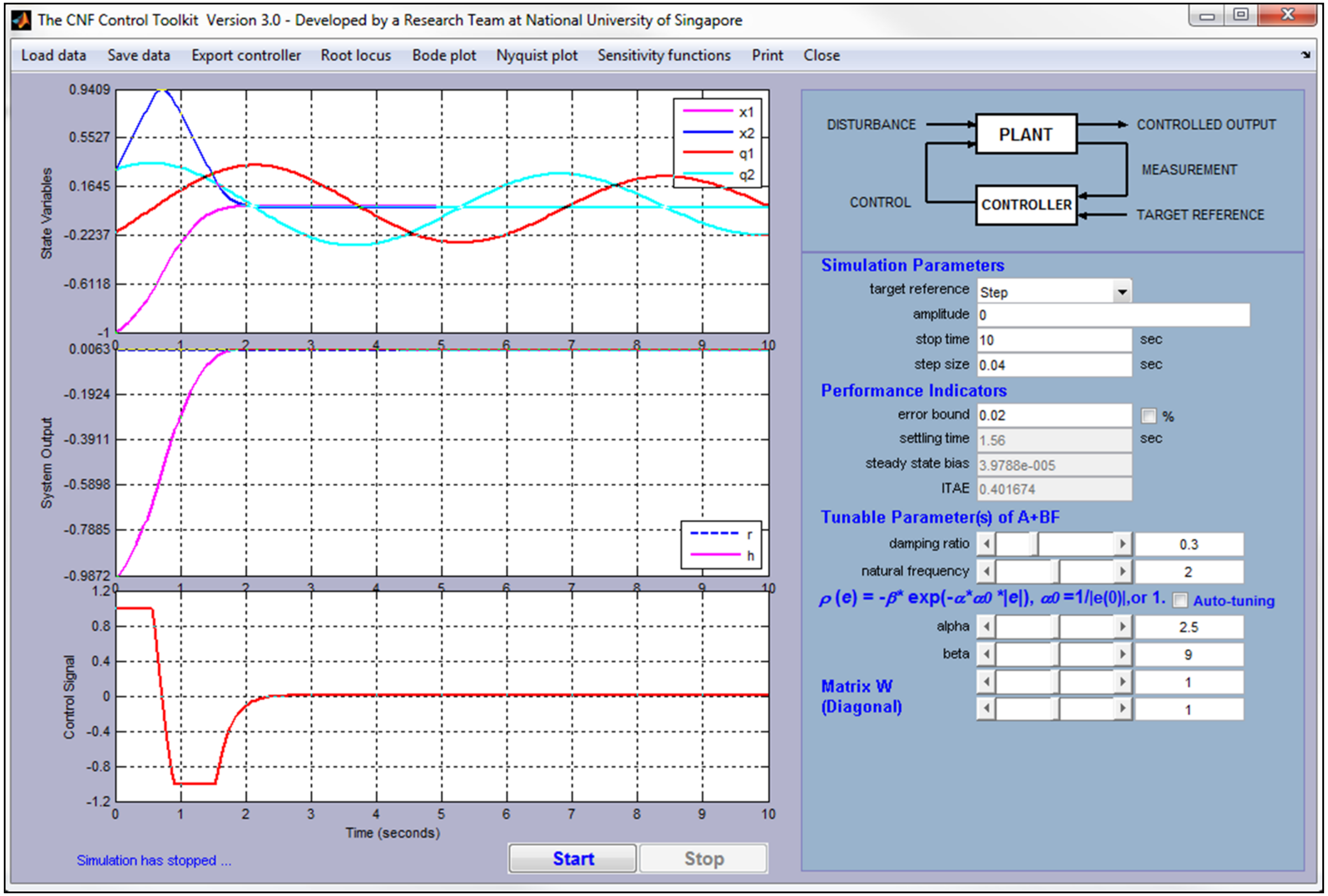
Zongli Lin

Kemao Peng

Guoyang Cheng

Weiyao Lan

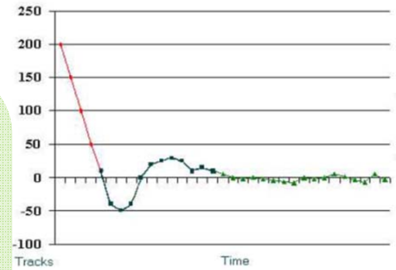
CNF Control Toolkit – The Main Panel

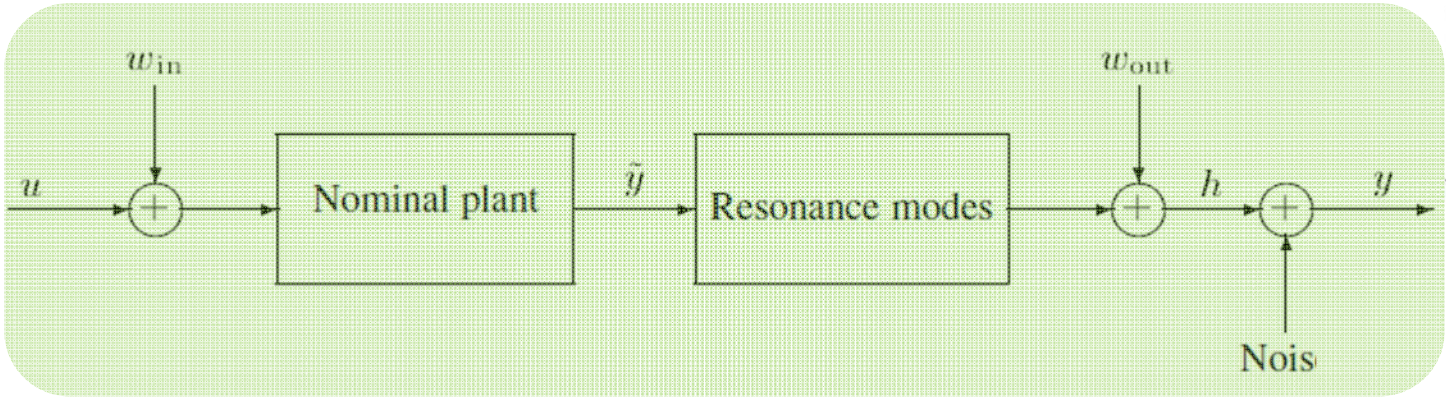


A Benchmark Problem

Before ending this book, we post in this chapter a typical HDD servo control design problem. The problem has been tackled in the previous chapters using several design methods, such as PID, RPT, CNF, PTOS and MSC control. We feel that it can serve as an interesting and excellent benchmark example for testing other linear and nonlinear control techniques.

We recall that the complete dynamics model of a Maxtor (Model 51536U3) hard drive VCM actuator can be depicted as in Figure 11.1:





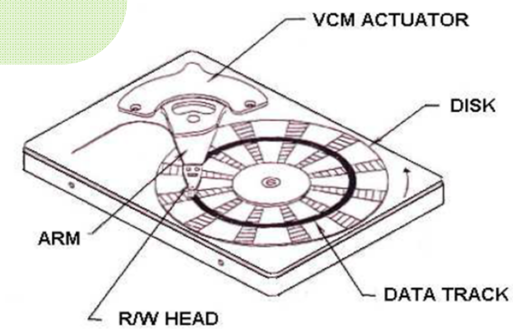


Figure 11.1. Block diagram of the dynamical model of the hard drive VCM actu

The nominal plant of the HDD VCM actuator is characterized by the following second-order system:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 6.4013 \times 10^7 \end{bmatrix} \Big(\mathrm{sat}(u) + w_{\mathrm{in}} \Big) \tag{11.1}$$

and

$$\tilde{y} = [1 \quad 0]\, x \tag{11.2}$$

where the control input u is limited within ± 3 V and w_{in} is an unknown input disturbance with $|w_{\mathrm{in}}| \leq 3$ mV. For simplicity and for simulation purpose, we assume that the unknown disturbance $w_{\mathrm{in}} = -3$ mV. The measurement output available for control, *i.e.* y (in μm), is the measured displacement of the VCM R/W head and is given by

$$y = \left[\prod_{i=1}^{4} G_{\mathrm{r},i}(s) \right] \tilde{y} + w_{\mathrm{out}} + \text{Noise} \tag{11.3}$$

where the transfer functions of the resonance modes are given by

$$\left.\begin{aligned} G_{r,1}(s) &= \frac{0.912s^2 + 457.4s + 1.433(1+\delta)\times 10^8}{s^2 + 359.2s + 1.433(1+\delta)\times 10^8} \\ G_{r,2}(s) &= \frac{0.7586s^2 + 962.2s + 2.491(1+\delta)\times 10^8}{s^2 + 789.1s + 2.491(1+\delta)\times 10^8} \\ G_{r,3}(s) &= \frac{9.917(1+\delta)\times 10^8}{s^2 + 1575s + 9.917(1+\delta)\times 10^8} \\ G_{r,4}(s) &= \frac{2.731(1+\delta)\times 10^9}{s^2 + 2613s + 2.731(1+\delta)\times 10^9} \end{aligned}\right\} \tag{11.4}$$

with $-20\% \leq \delta \leq 20\%$ represents the variation of the resonance modes of the actual actuators whose resonant dynamics change from time to time and also from disk to disk in a batch of million drives. Note that many new hard drives in the market nowadays might have resonance modes at much higher frequencies (such as those for the IBM microdrives studied in Chapter 9). But, structurewise, they are almost the same. The output disturbance (in μm), which is mainly the repeatable runouts, is given by

$$w_{\text{out}} = 0.1\sin(110\pi t) + 0.05\sin(220\pi t) + 0.02\sin(440\pi t) + 0.01\sin(880\pi t) \tag{11.5}$$

and the measurement noise is assumed to be a zero-mean Gaussian white noise with a variance $\sigma_n^2 = 9\times 10^{-6}$ (μm)^2.

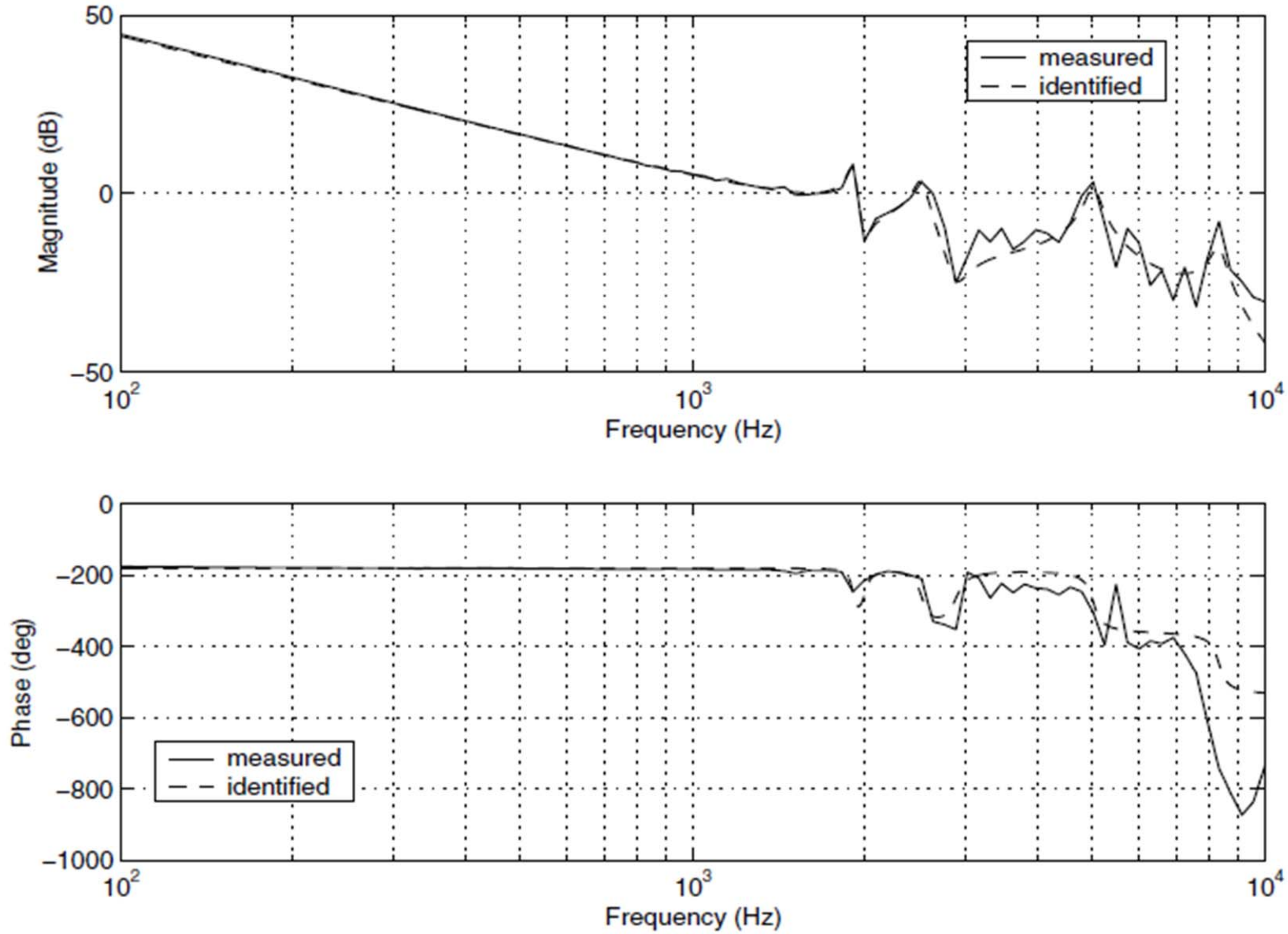


Figure 6.1. Frequency responses of the actual and identified VCM actuator models

The problem is to design a controller such that when it is applied to the VCM actuator system, the resulting closed-loop system is asymptotically stable and the actual displacement of the actuator, *i.e.* h, tracks a reference $r = 1$ μm. The overall design has to meet the following specifications:

1. the overshoot of the actual actuator output is less than 5%;
2. the mean of the steady-state error is zero;
3. the gain margin and phase margin of the overall design are, respectively ,greater than 6 dB and $30°$; and
4. the maximum peaks of the sensitivity and complementary sensitivity functions are less than 6 dB.

The results of Chapter 6 show that the 5% settling times of our design using the CNF control technique are, respectively, 0.80 ms in simulation and 0.85 ms in actual hardware implementation. We note that the simulation result can be further improved if we do not consider actual hardware constraints in our design. For example, the CNF control law given below meets all design specifications and achieves a 5% settling time of 0.68 ms. It is obtained by using the toolkit of [55] under the option of the pole-placement method with a damping ratio of 0.1 and a natural frequency of 2800 rad/sec together with a diagonal matrix $W = \text{diag}\{1.5,\ 0.01,\ 2 \times 10^{-10}\}$.

The CNF Control Law together with an appropriate notch filter:

$$\begin{pmatrix} \dot{x}_{\rm i} \\ \dot{x}_{\rm v} \end{pmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -4000 \end{bmatrix} \begin{pmatrix} x_{\rm i} \\ x_{\rm v} \end{pmatrix} - \begin{bmatrix} -10 \\ 1.6 \times 10^7 \end{bmatrix} y + \begin{bmatrix} 0 \\ 6.4013 \times 10^7 \end{bmatrix} \mathrm{sat}(\tilde{u}) - \begin{pmatrix} 10r \\ 0 \end{pmatrix}$$

$$\tilde{u} = \rho(e) \begin{bmatrix} 0.61237 & 0.049449 & 8.4622 \times 10^{-5} \end{bmatrix} \begin{pmatrix} x_{\rm i} \\ y - r \\ x_{\rm v} + 4000y \end{pmatrix} - \begin{bmatrix} 1.2248 & 0.12335 & 1.0310 \times 10^{-5} \end{bmatrix} \begin{pmatrix} x_{\rm i} \\ y - r \\ x_{\rm v} + 4000y \end{pmatrix}$$

where

$$\rho(e) = -2.65 \left| \mathrm{e}^{-0.7|e|} - \mathrm{e}^{-0.7} \right|$$

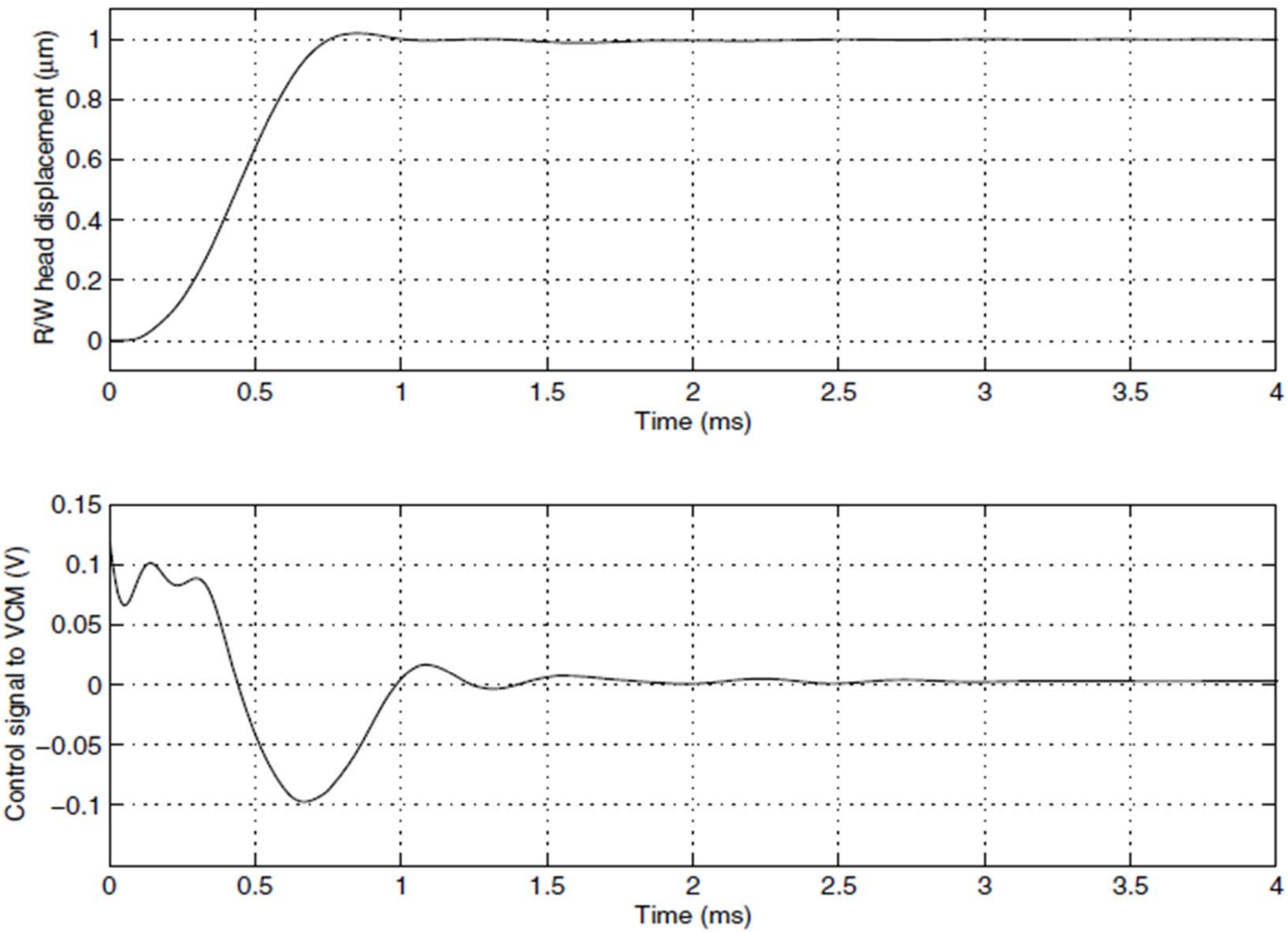
and

$$u = G_{\rm notch}(s) \cdot \tilde{u}$$

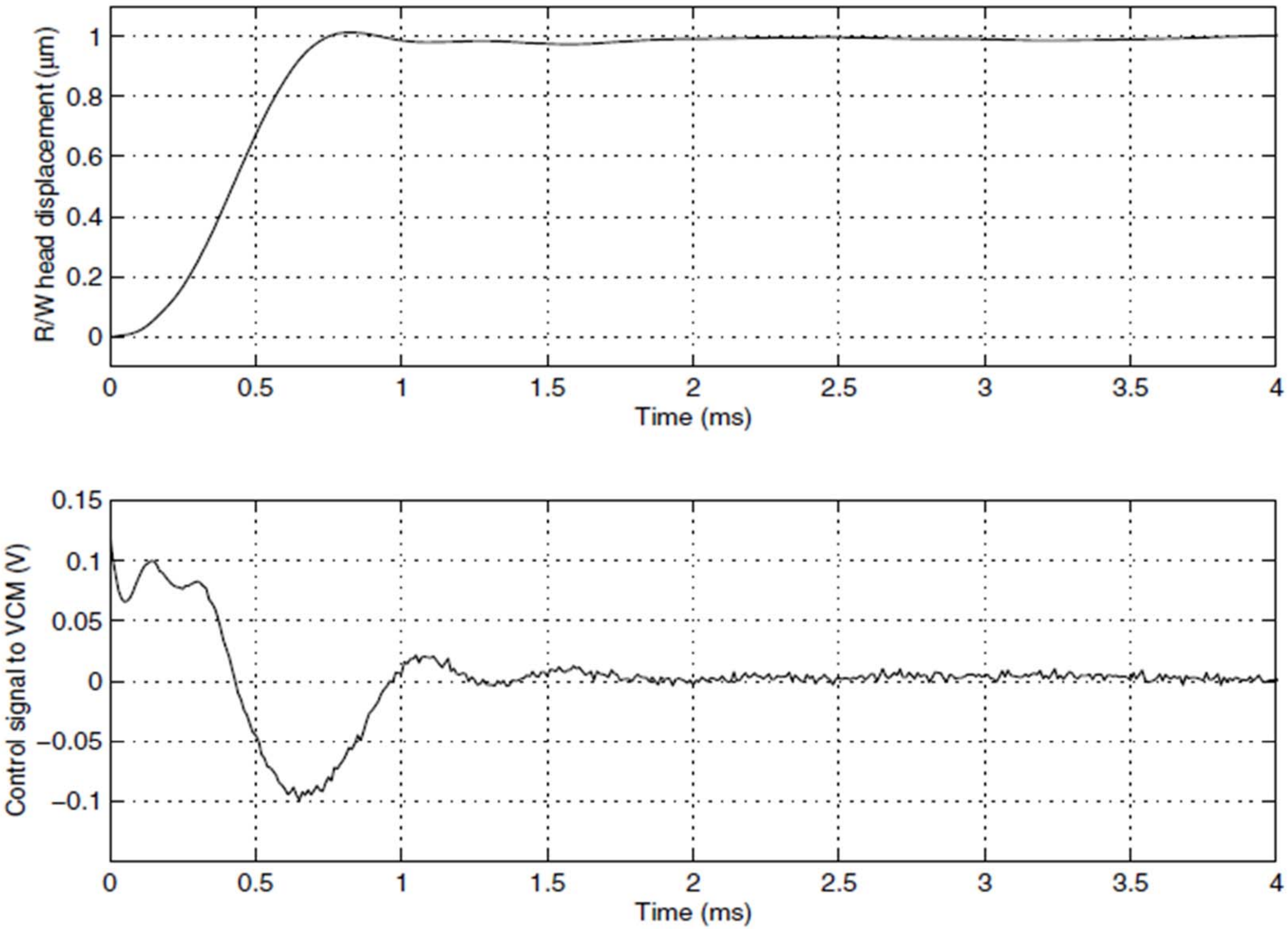
$$G_{\rm notch}(s) = \left(\frac{s^2 + 238.8s + 1.425 \times 10^8}{s^2 + 2388s + 1.425 \times 10^8} \right) \times \left(\frac{s^2 + 314.2s + 2.467 \times 10^8}{s^2 + 3142s + 2.467 \times 10^8} \right) \times \left(\frac{s^2 + 628.3s + 9.87 \times 10^8}{s^2 + 12570s + 9.87 \times 10^8} \right)$$

The simulation results obtained with $\delta = 0$ given in Figures 11.2 to 11.4 show that all the design specifications have been achieved. In particular, the resulting 5% settling time is 0.68 ms, the gain margin is 7.85 dB and the phase margin is 44.7°, and finally, the maximum values of the sensitivity and complementary sensitivity functions are less than 5 dB. The overall control system can still produce a satisfactory result and satisfy all the design specifications by varying the resonance modes with the value of δ changing from -20% to 20%.

Nonetheless, we invite interested readers to challenge our design. Noting that for the track-following case, *i.e.* when $r = 1$ μm, the control signal is far below its saturation level. Because of the bandwidth constraint of the overall system, it is not possible (and not necessary) to utilize the full scale of the control input to the actuator in the track-following stage. However, in the track-seeking case or equivalently by setting a larger target reference, say $r = 500$ μm, the very problem can serve as a good testbed for control techniques developed for systems with actuator saturation. Interested readers are referred to Chapter 7 for more information on track seeking of HDD servo systems.



(a) h and u for the system without output disturbance and noise

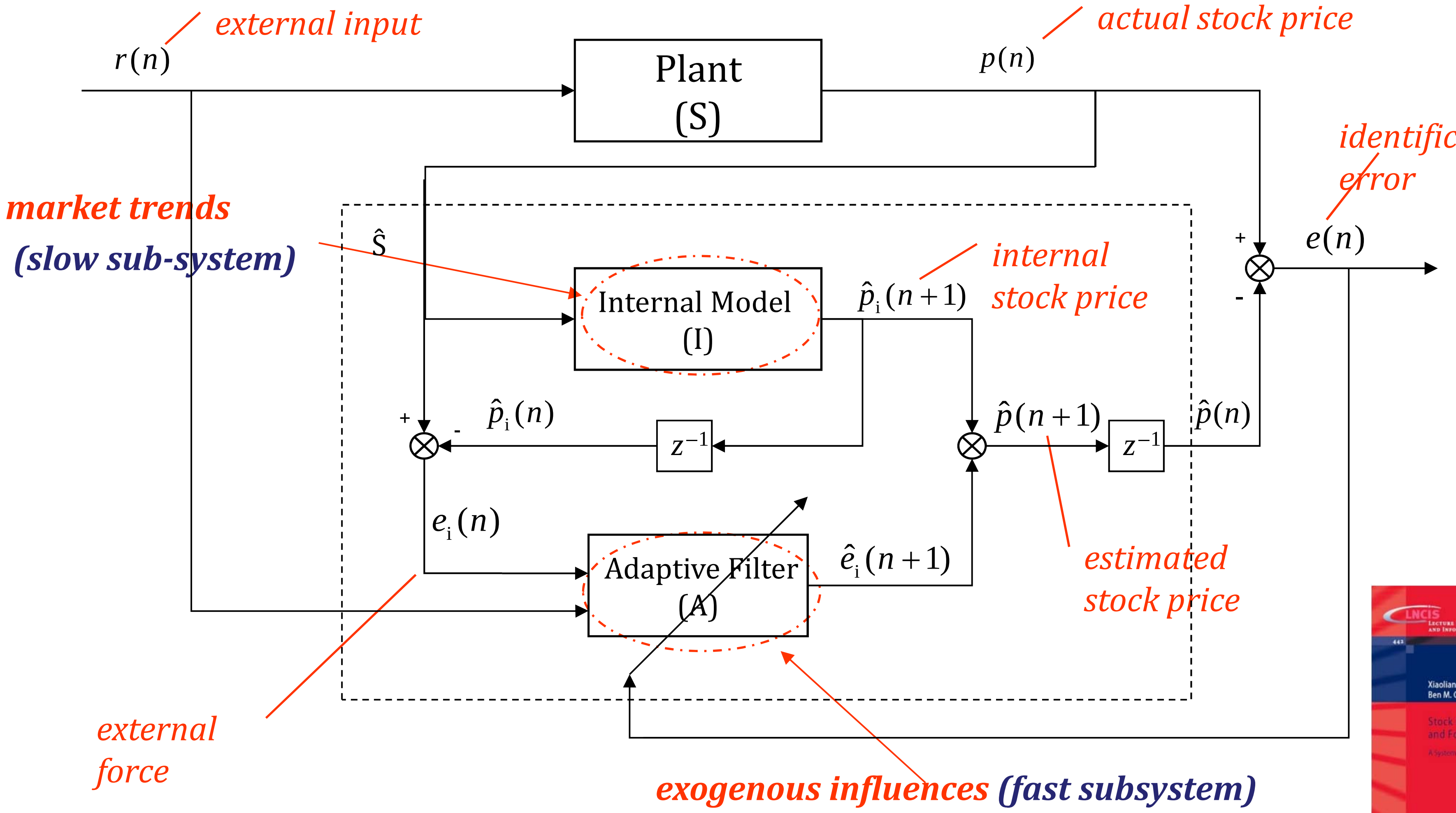


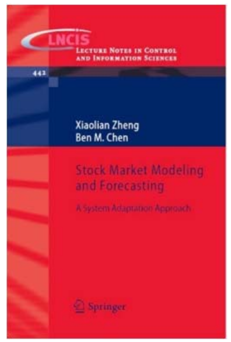
(b) h and u for the system with output disturbance and noise

Figure 11.2. Output responses and control signals of the CNF control system

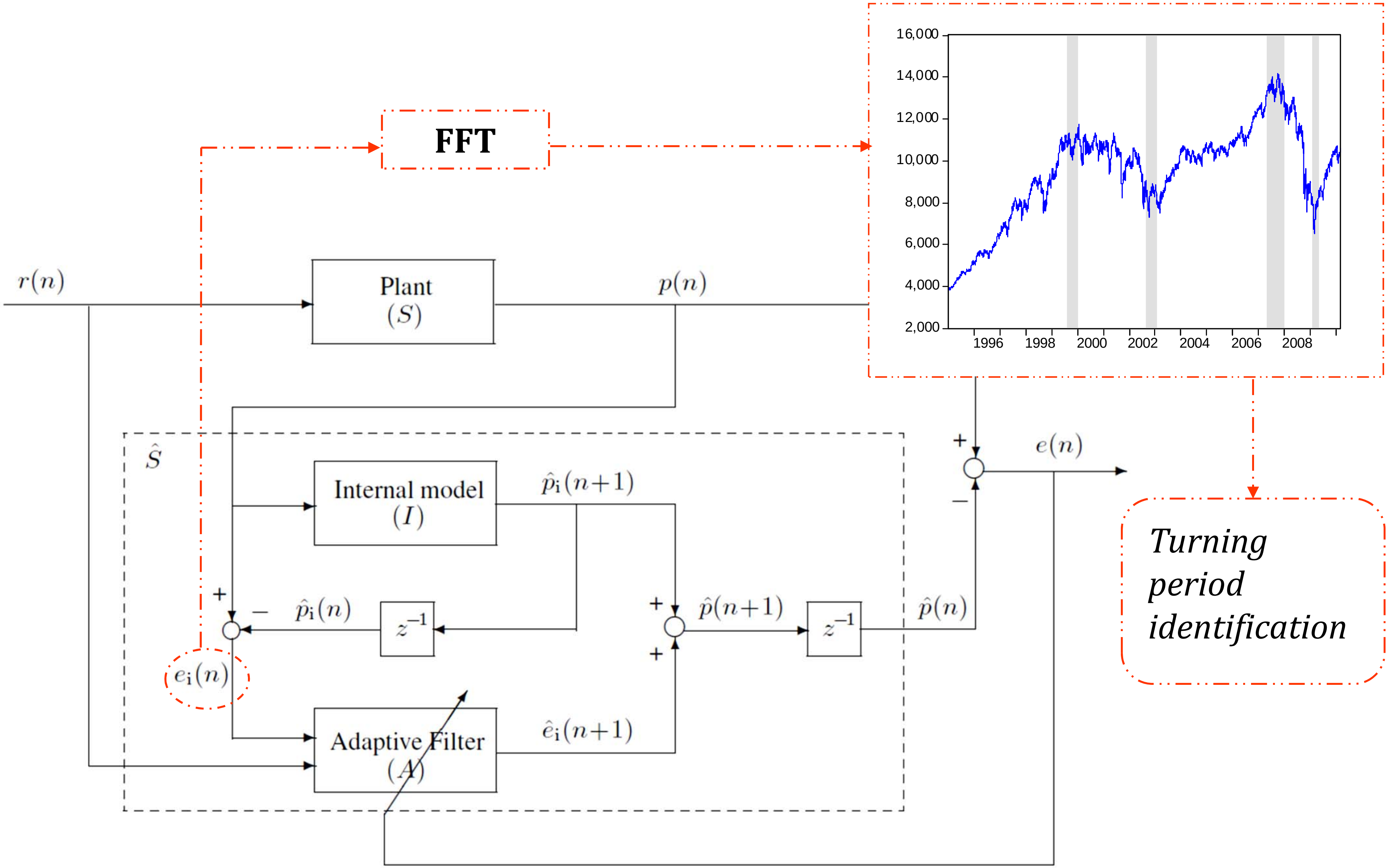
Final Note: Systems Adaptation Framework for Stock Modeling

What is the market input?





Turning period forecasting



That's all, folks!

Bugs Bunny
1940–

Thank You!

Life is a bunch of integrated chains! *B.M.C.*

生命无非就是一堆积分串！

有的自始至终，有的有始无终，有的无始有终，有的无始无终