Linear Systems and Control

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... this is not a regular course at NUS. it is designed just for my own graduate students ...

What is a system?

A system is a set of integrated chains of things.

What is control?

Control is to regulate a system to desired performance.

This course is aimed to answer the following questions:

- Why is the commonly used PID a bad controller?
- What control performance can one expect from a given system?
- Why are system nonminimum phase zeros bad for control?
- What else are bad to be controlled?
- When an airplane passes through turbulences, why can it maintain its position while its body is shaking badly?
- When and how can disturbances, uncertainties and nonlinearities be attenuated through proper control system design?
- What is the best way to design a control system?
    - to design a good control law? or
    - to design a good system?
- How to design a good system through sensor and actuator selection?
- Why is PID not bad at all after all?
- How to improve control performance?

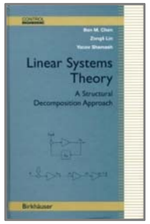
Course Outline

- Introduction
- Background Materials
- Review of Basic Linear Systems Theory
- Decompositions of Unforced and/or Unsensed Systems
- Decompositions of Proper Systems
- Structural Assignment via Sensor/Actuator Selection
- Time-Scale and Eigenstructure Assignment
- H_2 and H_∞ Control
- Disturbance Decoupling Control Problems
- RPT Control
- Flight Control Systems Design
- CNF Control

Systems

Control

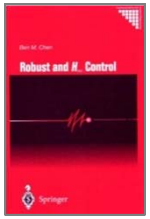
References



B. M. Chen, Z. Lin, Y. Shamash

Linear Systems Theory: A Structural Decomposition Approach

Birkhäuser, Boston, 2004



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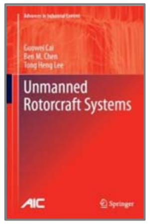
Robust and H_∞ Control

Springer, New York, 2000



G. Cai, B. M. Chen, T. H. Lee

Unmanned Rotorcraft Systems

Springer, New York, 2011

B. M. Chen, T. H. Lee, K. Peng, V. Venkataramanan

Hard Disk Drive Servo Systems (2nd Edition)

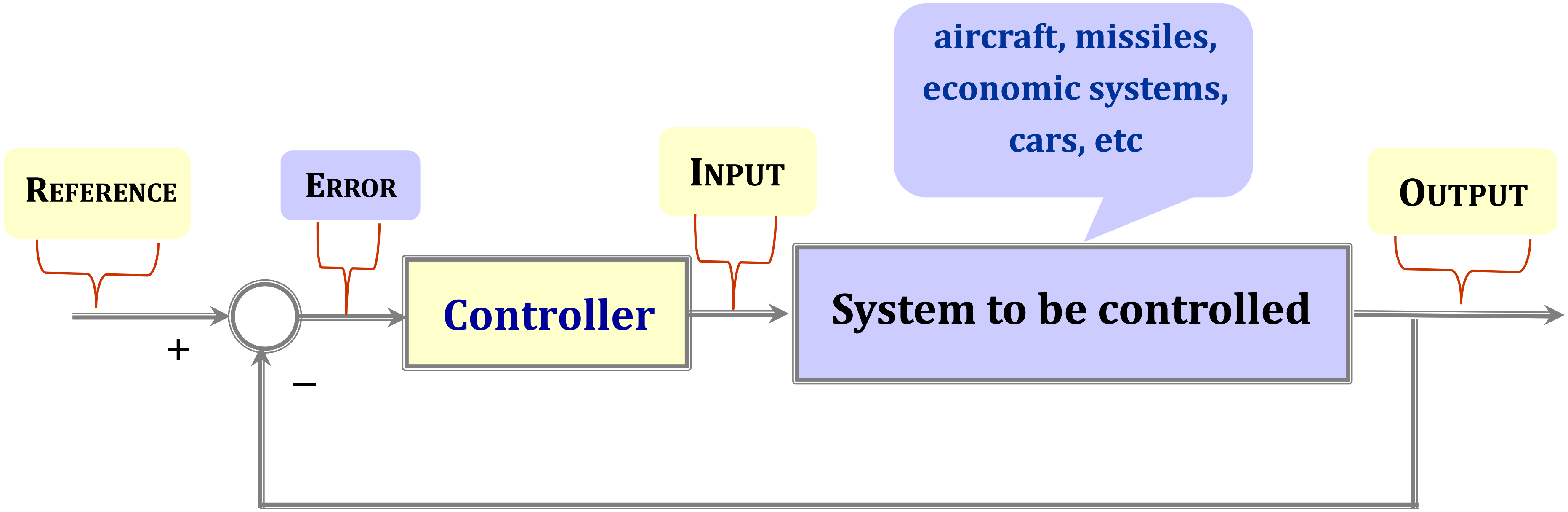
Springer, New York, 2006

All these books are available and downloadable online from SpringerLink at the NUS Library.

Only the chapter on CNF control will be covered.

Introduction

Classical Control System Structure



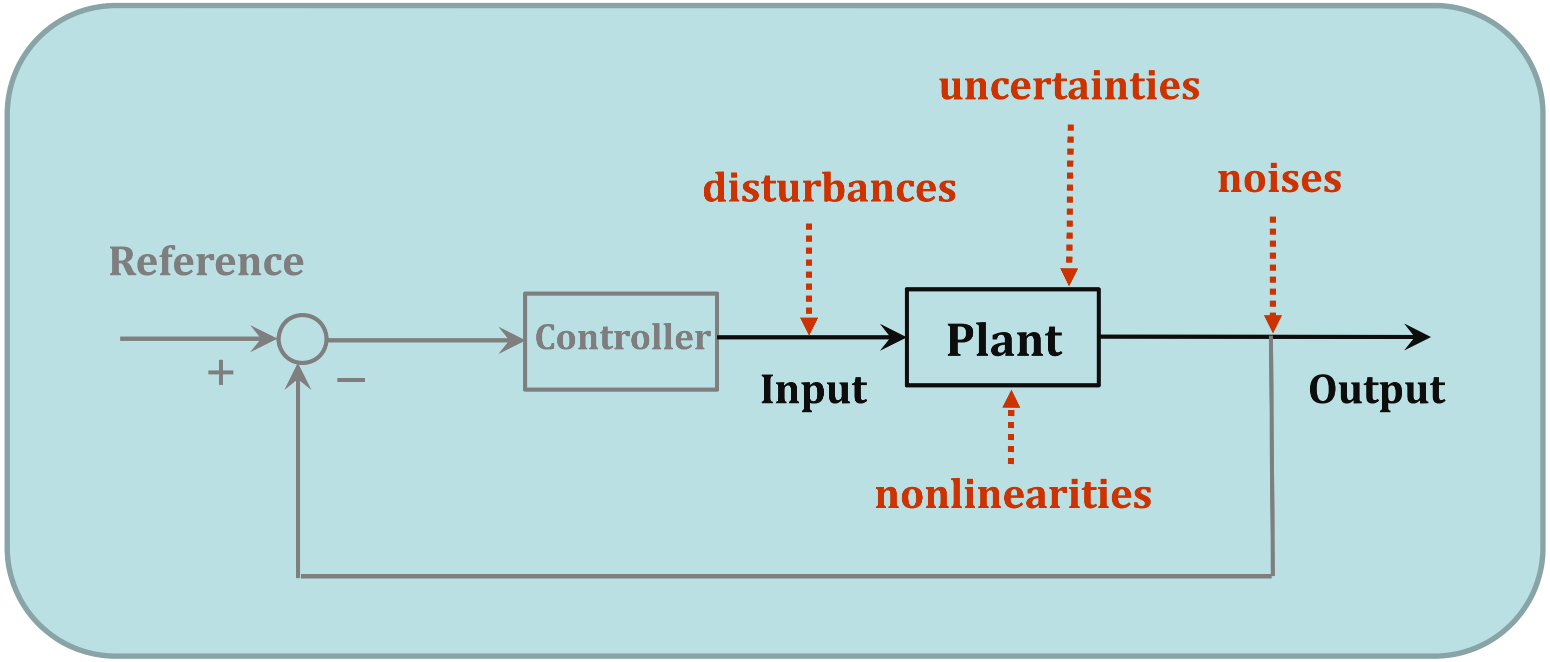
Objective: To make the system **OUTPUT** and the desired **REFERENCE** as close as possible, i.e., to make the **ERROR** as small as possible.

Issues: (1) How to describe the system to be controlled? **(Systems)**

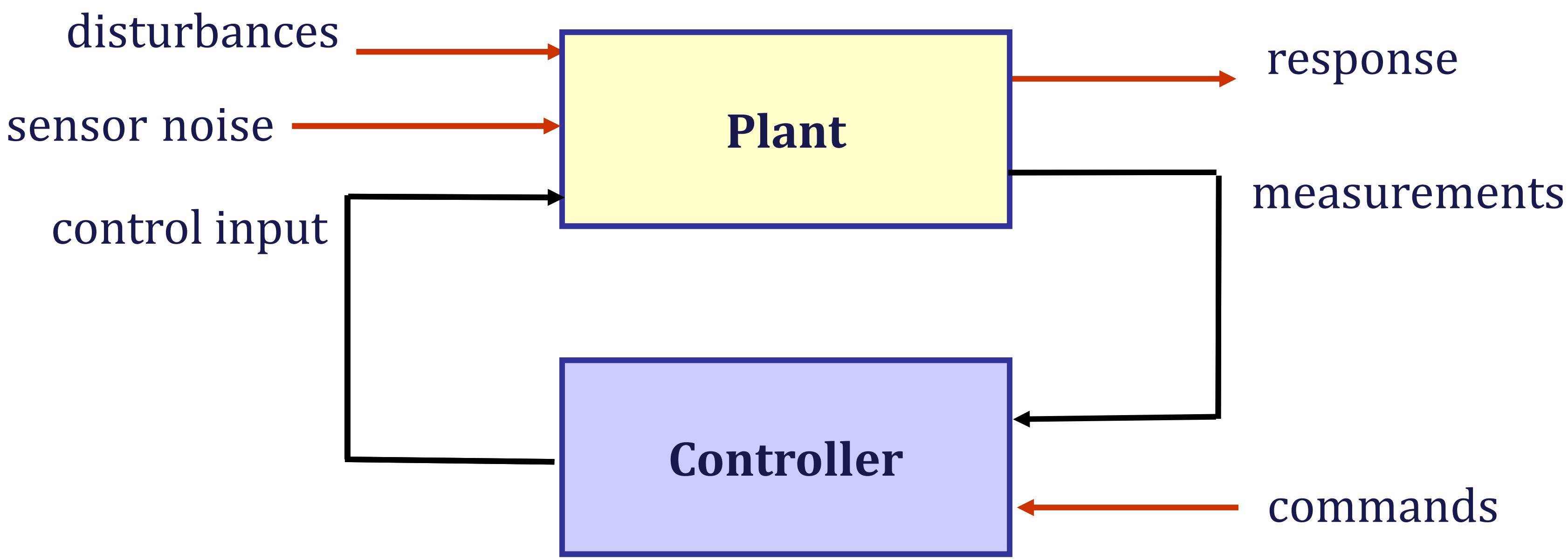
(2) How to design the controller? **(Control)**

Model Uncertainties, Nonlinearities and Disturbances

There are many other factors of life have to be carefully considered when dealing with real-life problems. These factors include:



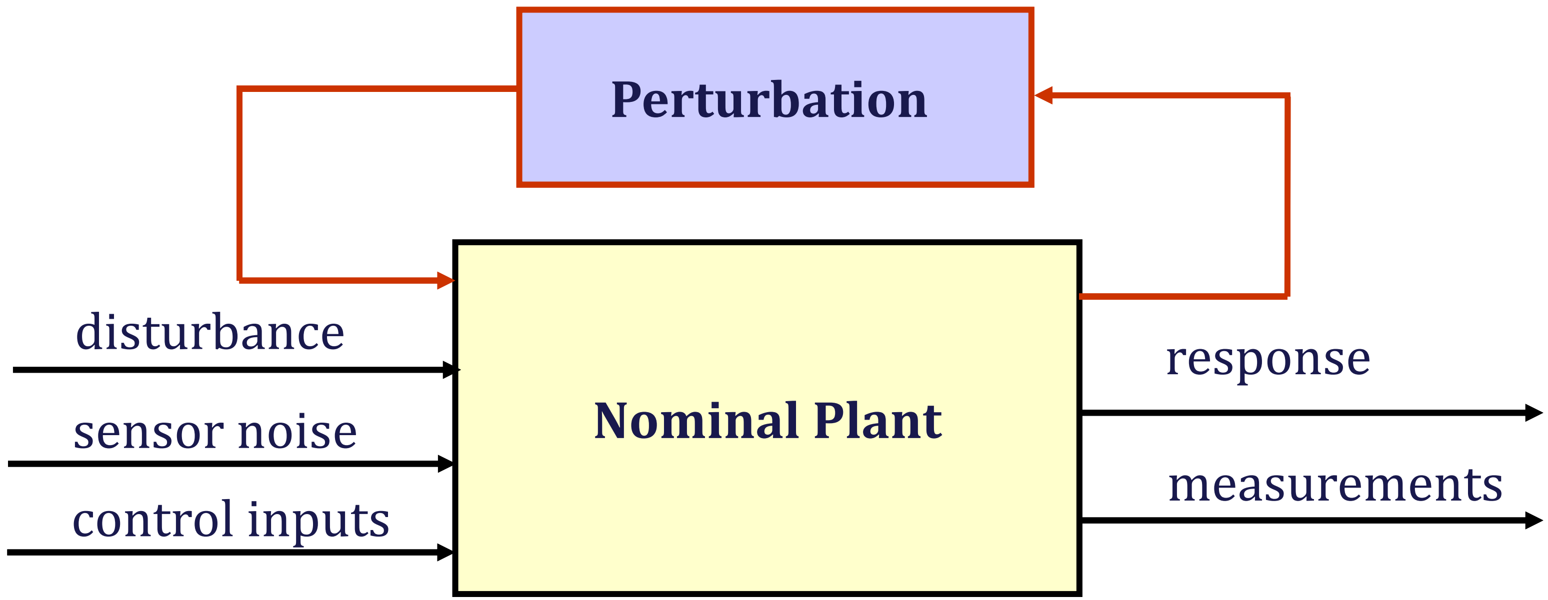
Modern Control System Structure



Controller Objective: To provide desired responses in face of

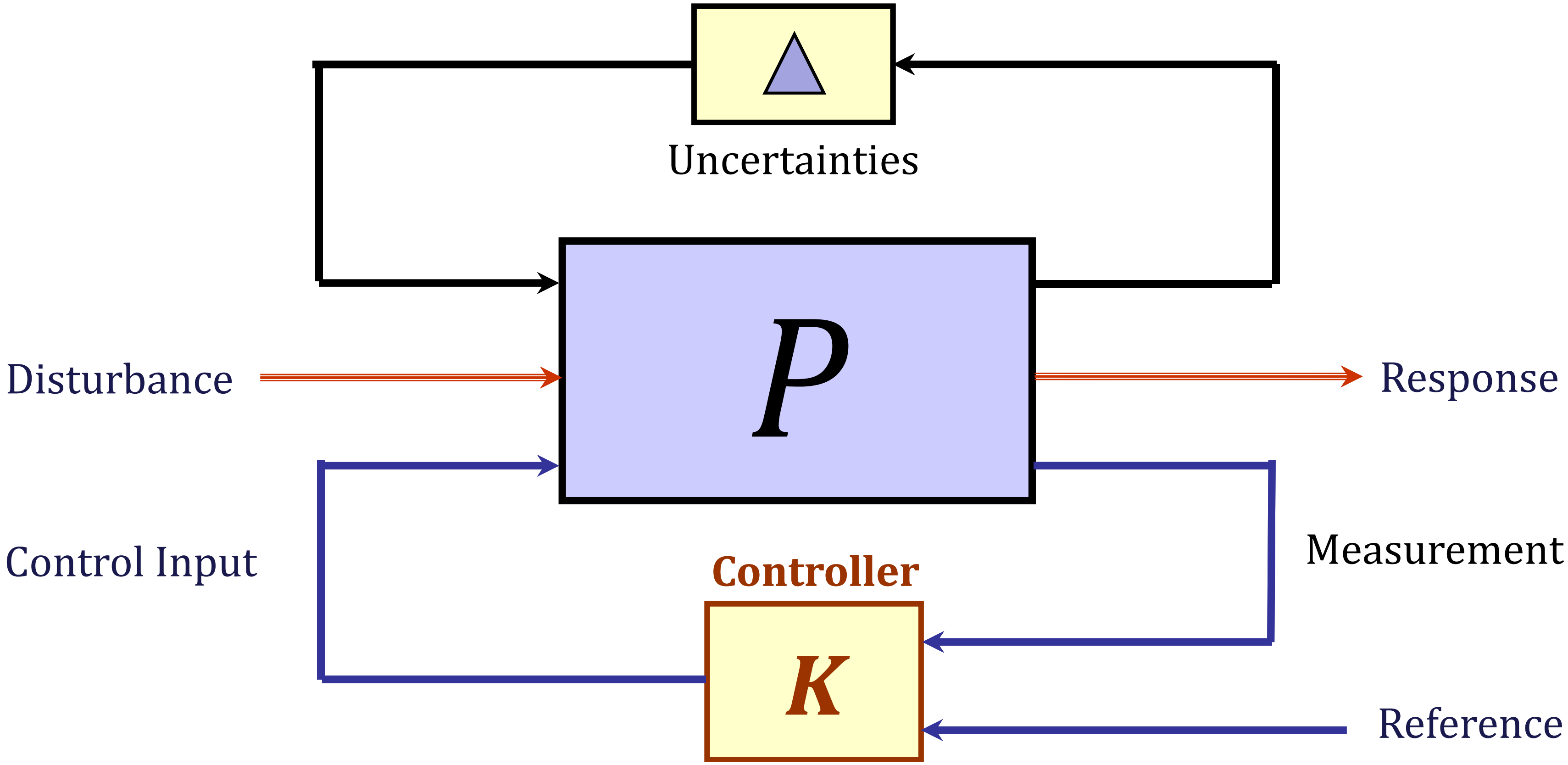
- Uncertain plant dynamics + External inputs $\left\{ \begin{array}{l} \text{disturbances} \\ \text{sensor noise} \\ \text{control input} \end{array} \right.$

Representation of Uncertain Plant Dynamics



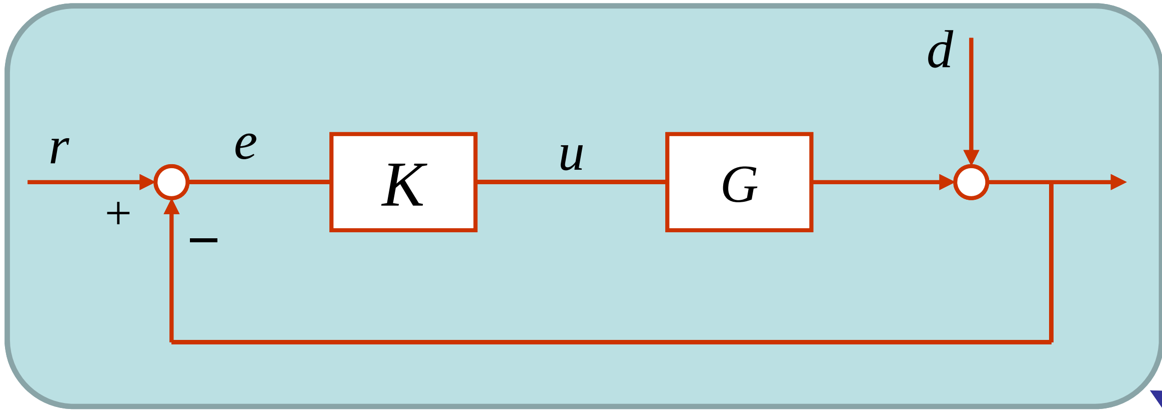
- ✓ Nominal Plant is an FDLTI System
- ✓ Perturbation is Member of Set of Possible Perturbations

A General Control System Structure...

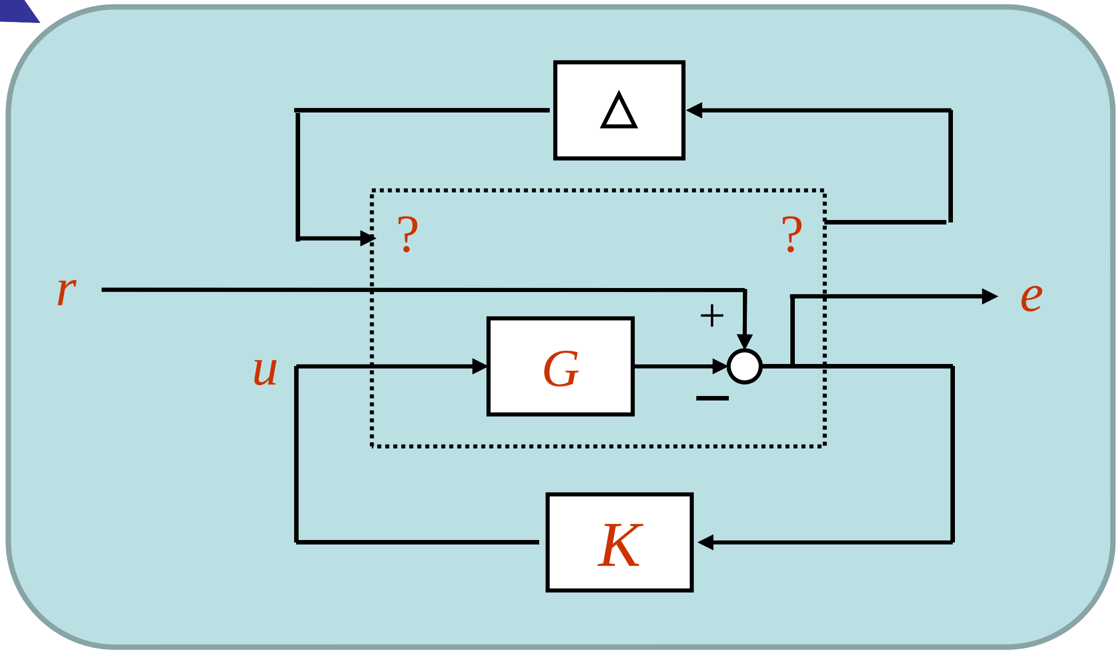


It is aimed to design an appropriate control law such that the resulting overall closed-loop system is stable in face of disturbance and uncertainties while maintaining good response performance (settling time, overshoot, etc…).

Classical vs Modern Control Structures



PID control is a special case of the modern control structure…



What is the fundamental difference between the classical and modern control system structures?

Mathematical Background

2.2 Vector Spaces and Subspaces

We assume that the reader is familiar with the basic definitions of scalar fields and vector spaces.

Let $\mathcal{X}$ be a *vector space* over a certain scalar field $\mathbb{K}$. A subset of $\mathcal{X}$, say $\mathcal{S}$, is said to be a *subspace* of $\mathcal{X}$ if $\mathcal{S}$ itself is a vector space over $\mathbb{K}$. The *dimension* of a subspace $\mathcal{S}$, denoted by $\dim \mathcal{S}$, is defined as the maximal possible number of linearly independent vectors in $\mathcal{S}$.

We say that vectors $s_1, s_2, \ldots, s_k \in \mathcal{S}$, $k = \dim \mathcal{S}$, form a *basis* for $\mathcal{S}$ if they are *linearly independent*, *i.e.*, $\sum_{i=1}^{k} \alpha_i s_i = 0$ holds only if $\alpha_i = 0$. Two subspaces $\mathcal{V}$ and $\mathcal{W}$ are said to be independent if $\mathcal{V} \cap \mathcal{W} = \{0\}$.

Throughout the book, we will focus our attention on two common vector spaces, *i.e.*, $\mathbb{R}^n$ (with a scalar field $\mathbb{K}=\mathbb{R}$) and $\mathbb{C}^n$ (with a scalar field $\mathbb{K}=\mathbb{C}$), and their subspaces. Thus, the *inner product* of two vectors, say x and y, in either $\mathbb{R}^n$ or $\mathbb{C}^n$, is given by

$$\langle x, y \rangle = x^{\mathrm{H}} y = x_1^* y_1 + x_2^* y_2 + \cdots + x_n^* y_n, \tag{2.2.1}$$

where $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_n$ are respectively the entries of x and y, x^{H} is the conjugate transpose of x, and x_i^* is the complex conjugate of x_i. Vectors x and y are said to be *orthogonal* if $\langle x, y \rangle = 0$.

Definition 2.2.1 (Sums of subspaces). *Let $\mathcal{V}$ and $\mathcal{W}$ be the subspaces of a vector space $\mathcal{X}$. Then, the* sum of subspaces *$\mathcal{V}$ and $\mathcal{W}$ is defined as*

$$\mathcal{S} = \mathcal{V} + \mathcal{W} := \{v + w \mid v \in \mathcal{V},\ w \in \mathcal{W}\}. \tag{2.2.2}$$

If $\mathcal{V}$ and $\mathcal{W}$ are independent, then $\mathcal{S}$ is also called the direct sum *of $\mathcal{V}$ and $\mathcal{W}$ and is denoted by $\mathcal{S} = \mathcal{V} \oplus \mathcal{W}$. Obviously, in both cases, $\mathcal{S}$ is a subspace of $\mathcal{X}$.*

Definition 2.2.2 (Orthogonal complement subspace). *Let $\mathcal{V}$ be a subspace of a vector space $\mathcal{X}$. Then, the* orthogonal complement *of $\mathcal{V}$ is defined as*

$$\mathcal{V}^\perp := \{x \in \mathcal{X} \mid \langle x, v\rangle = 0,\ \forall v \in \mathcal{V}\}. \tag{2.2.3}$$

Again, $\mathcal{V}^\perp$ is a subspace of $\mathcal{X}$.

Definition 2.2.3 (Quotient space and codimension). *Let $\mathcal{V}$ be a subspace of a vector space $\mathcal{X}$. The coset of an element $x \in \mathcal{X}$ with respect to $\mathcal{V}$, denoted by $x + \mathcal{V}$, is defined as*

$$x + \mathcal{V} := \{w \mid w = x + v,\ v \in \mathcal{V}\}. \tag{2.2.4}$$

Under the algebraic operations defined by

$$(w + \mathcal{V}) + (x + \mathcal{V}) = (w + x) + \mathcal{V} \tag{2.2.5}$$

and

$$\alpha(w + \mathcal{V}) = \alpha w + \mathcal{V}, \tag{2.2.6}$$

it is straightforward to verify that all the cosets constitute the elements of a vector space. The resulting space is called the quotient space *or* factor space *of $\mathcal{X}$ by $\mathcal{V}$ (or modulo $\mathcal{V}$) and is denoted by $\mathcal{X}/\mathcal{V}$. Its dimension is called the* codimension of *$\mathcal{V}$ and is denoted by* codim $\mathcal{V}$,

$$\operatorname{codim} \mathcal{V} = \dim \mathcal{X}/\mathcal{V} = \dim \mathcal{X} - \dim \mathcal{V}. \tag{2.2.7}$$

Note that $\mathcal{X}/\mathcal{V}$ is not a subspace of $\mathcal{X}$ unless $\mathcal{V} = \{0\}$.

Definition 2.2.4 (Kernel and image of a matrix). *Given* $A \in \mathbb{C}^{m\times n}$ *(or* $\mathbb{R}^{m\times n}$*), a linear map from* $\mathcal{X}=\mathbb{C}^n$ *(or* $\mathbb{R}^n$*) to* $\mathcal{Y}=\mathbb{C}^m$ *(or* $\mathbb{R}^m$*), the* kernel *or* null space *of* A *is defined as*

$$\ker(A) := \{x \in \mathcal{X} \mid Ax = 0\}, \tag{2.2.8}$$

and the image *or* range space *of* A *is defined as*

$$\operatorname{im}(A) = A\mathcal{X} := \{Ax \mid x \in \mathcal{X}\}. \tag{2.2.9}$$

Obviously, $\ker(A)$ *is a subspace of* $\mathcal{X}$*, and* $\operatorname{im}(A)$ *is a subspace of* $\mathcal{Y}$*.*

Definition 2.2.5 (Inverse image of a subspace). *Given* $A \in \mathbb{C}^{m\times n}$ *(or* $\mathbb{R}^{m\times n}$*), a linear map from* $\mathcal{X}=\mathbb{C}^n$ *(or* $\mathbb{R}^n$*) to* $\mathcal{Y}=\mathbb{C}^m$ *(or* $\mathbb{R}^m$*), and* $\mathcal{V}$*, a subspace of* $\mathcal{Y}$*, the* inverse image *of* $\mathcal{V}$ *associated with the linear map is defined as*

$$A^{-1}\{\mathcal{V}\} := \{x \in \mathcal{X} \mid Ax \in \mathcal{V}\}, \tag{2.2.10}$$

which clearly is a subspace of $\mathcal{X}$*.*

Definition 2.2.6 (Invariant subspace). *Given* $A \in \mathbb{C}^{n\times n}$ *(or* $\mathbb{R}^{n\times n}$*), a linear map from* $\mathcal{X}=\mathbb{C}^n$ *(or* $\mathbb{R}^n$*) to* $\mathcal{X}$*, a subspace* $\mathcal{V}$ *of* $\mathcal{X}$ *is said to be* A-invariant *if*

$$A\mathcal{V} \subset \mathcal{V}. \tag{2.2.11}$$

Such a $\mathcal{V}$ *is also called an* invariant subspace *of* A*.*

2.4 Norms

Norms measure the length or size of a vector or a matrix. Norms are also defined for signals and rational transfer functions.

Given a linear space $\mathcal{X}$ over a scalar field $\mathbb{K}$, any real-valued scalar function of $x \in \mathcal{X}$ (usually denoted by $\|x\|$) is said to be a *norm* on $\mathcal{X}$ if it satisfies the following properties:

1. $\|x\| > 0$ if $x \neq 0$ and $\|x\| = 0$ if $x = 0$;
2. $\|\alpha x\| = |\alpha| \cdot \|x\|, \forall \alpha \in \mathbb{K}, \forall x \in \mathcal{X}$; and
3. $\|x + z\| \leq \|x\| + \|z\|, \forall x, z \in \mathcal{X}$.

2.4.1 Norms of Vectors

The following p-norms are the most commonly used norms on the vector space $\mathbb{C}^n$:

$$\|x\|_p := \left(\sum_{i=1}^{n} |x_i|^p \right)^{1/p}, \quad 1 \leq p < \infty, \tag{2.4.1}$$

and

$$\|x\|_\infty := \max_i |x_i|, \tag{2.4.2}$$

where $x_1, x_2, \ldots, x_n$ are the elements of $x \in \mathbb{C}^n$. In particular, $\|x\|_2$ is also called the *Euclidean norm* of x and is denoted by $|x|$ for simplicity.

2.4.2 Norms of Matrices

Given a matrix $A = [a_{ij}] \in \mathbb{C}^{m \times n}$, its *Frobenius norm* is defined as

$$\|A\|_{\mathrm{F}} := \left(\sum_{i=1}^{m} \sum_{j=1}^{n} |a_{ij}|^2 \right)^{1/2} = \left(\sum_{i=1}^{\min\{m,n\}} \sigma_i(A) \right)^{1/2}. \tag{2.4.3}$$

The *p-norm* of A is a norm induced from the vector p-norm, *i.e.*,

$$\|A\|_p := \sup_{x \neq 0} \frac{\|Ax\|_p}{\|x\|_p} = \sup_{\|x\|_p = 1} \|Ax\|_p. \tag{2.4.4}$$

In particular, for $p = 1, 2, \infty$, we have

$$\|A\|_1 = \max_j \sum_{i=1}^{m} |a_{ij}|, \tag{2.4.5}$$

$$\|A\|_2 = \sqrt{\lambda_{\max}(A^{\mathrm{H}} A)} = \sigma_{\max}(A), \tag{2.4.6}$$

which is also called the *spectral norm* of A, and

$$\|A\|_\infty = \max_i \sum_{j=1}^{n} |a_{ij}|. \tag{2.4.7}$$

It can be shown that

$$\|A\| \geq \rho(A), \tag{2.4.8}$$

where $\|A\|$ is any norm of A and $\rho(A)$ is the spectral radius of A. Also note that all these matrix norms are invariant under unitary transformations.

2.4.3 Norms of Continuous-time Signals

For any $p \in [1, \infty)$, let L_p^m denote the linear space formed by all measurable signals $g : \mathbb{R}_+ \to \mathbb{R}^m$ such that

$$\int_0^\infty |g(t)|^p dt < \infty.$$

For any $g \in L_p^m$, $p \in [1, \infty)$, its L_p*-norm* is defined as

$$\|g\|_p := \left(\int_0^\infty |g(t)|^p dt \right)^{1/p}, \quad 1 \le p < \infty. \tag{2.4.9}$$

Let L_∞^m denote the linear space formed by all signals $g : \mathbb{R}_+ \to \mathbb{R}^m$ such that

$$|g(t)| < \infty, \quad \forall t \in \mathbb{R}_+.$$

The L_∞-norm of a $g \in L_\infty^m$ is defined as

$$\|g\|_\infty := \sup_{t \ge 0} |g(t)|. \tag{2.4.10}$$

The following *Hölder inequality* of signal norms is useful,

$$\|fg\|_1 \le \|f\|_p \cdot \|g\|_q, \tag{2.4.11}$$

where $1 < p < \infty$ and $1/p + 1/q = 1$. It can also be shown that if $g(t) \in L_1 \cap L_\infty$, then $g(t) \in L_2$.

2.4.5 Norms of Continuous-time Systems

Given a stable and proper continuous-time system with a transfer matrix $G(s)$, its H_2*-norm* is defined as

$$\|G\|_2 := \left(\frac{1}{2\pi} \text{trace} \left[\int_{-\infty}^{\infty} G(j\omega) G(j\omega)^{\mathrm{H}} d\omega \right] \right)^{1/2}, \tag{2.4.14}$$

and its H_∞-norm is defined as

$$\|G\|_\infty := \sup_{\omega \in [0,\infty)} \sigma_{\max}[G(j\omega)] = \sup_{\|w\|_2 = 1} \frac{\|h\|_2}{\|w\|_2}, \tag{2.4.15}$$

where $w(t)$ and $h(t)$ are respectively the input and output of $G(s)$.

Let (A, B, C, D) be a state space realization of the stable transfer matrix, $G(s)$, *i.e.*, $G(s) = C(sI - A)^{-1}B + D$. It is straightforward to verify that $\|G\|_2 < \infty$ if and only if $D = 0$. In the case of $D = 0$, $\|G\|_2$ can be exactly computed by solving either one of the following Lyapunov equations:

$$A'P + PA = -C'C, \quad AQ + QA' = -BB', \tag{2.4.16}$$

for unique solution $P > 0$ or $Q > 0$. More specifically,

$$\|G\|_2 = \sqrt{\text{trace}\,(B'PB)} = \sqrt{\text{trace}\,(CQC')}. \tag{2.4.17}$$