Q.1 Using mesh analysis, derive (but DO NOT simplify or solve) the equation for determining the loop currents in the circuit of Fig. 1.

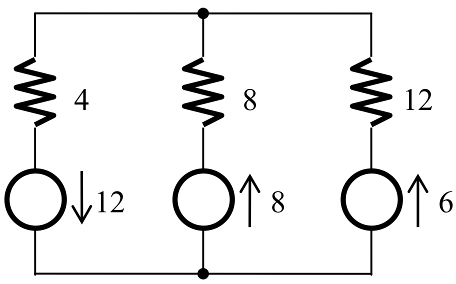


Fig. 1

(20 Marks)

First, define the loop currents i_1 and i_2 to flow clockwise in the loops containing the 12/8 and 8/6 V sources.

Relating loop to branch currents and applying KVL:

$$-12 - 4i_1 - 8(i_1 - i_2) - 8 = 0 \quad \Rightarrow \quad -12i_1 + 8i_2 = 20$$

$$8 - 8(i_2 - i_1) - 12i_2 - 6 = 0 \quad \Rightarrow \quad 8i_1 - 20i_2 = -2$$

In matrix form:

$$\begin{bmatrix} -12 & 8 \\ 8 & -20 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 20 \\ -2 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} -2.1818 \\ -0.7727 \end{bmatrix}$$

Q.2 Determine the voltage across terminals A and B in the circuit of Fig. 2 by using superposition. Hence, determine the Thevenin and Norton equivalent circuits as seen from these two terminals.

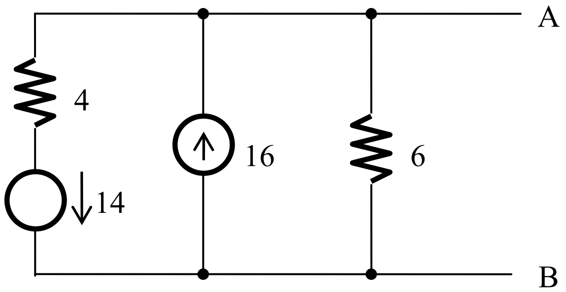


Fig. 2

(20 Marks)

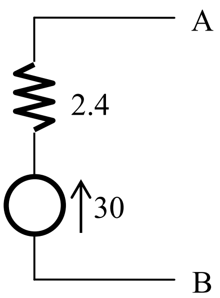
Using superposition, the open circuit voltage across A and B is

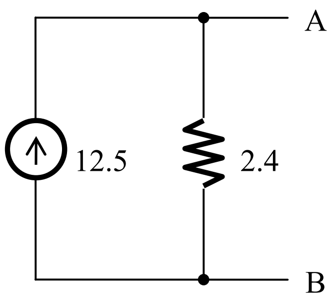
$$v_{AB} = -14\left(\frac{6}{4+6}\right) + 16\left(\frac{4}{4+6}\right)6 = 30$$

The Thevenin resistance is

$$R = \frac{6(4)}{6+4} = 2.4$$

Thus, the equivalent circuits are





Q.3 Determine the rms voltage and current for the unknown device in the circuit of Fig. 3. Write down the voltage phasor, current phasor and the impedance. Is the device a capacitor, inductor or resistor? Determine the corresponding capacitance, inductance or resistance.

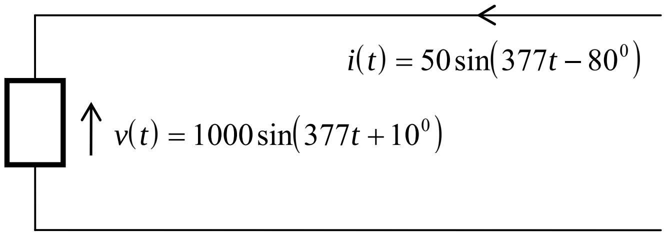


Fig. 3

(20 Marks)

The rms values are

$$v_{rms} = \frac{1000}{\sqrt{2}} \text{ and } i_{rms} = \frac{50}{\sqrt{2}}$$

The phasors and impedance are

$$V = \frac{1000}{\sqrt{2}} e^{j-80^0} \text{ and } I = \frac{50}{\sqrt{2}} e^{-j170^0}$$

$$Z = \frac{V}{I} = 20e^{j90^0} = 20j$$

Since the impedance for an inductor is $j\omega L$, the device is an inductor and

$$\omega L = 20$$

$$L = \frac{20}{\omega} = \frac{20}{377} = 0.0531$$

Q.4 A resistor, capacitor and inductor are connected in series to a supply in the circuit of Fig. 4. Using the impedances shown, determine the total power consumed and the power factor.

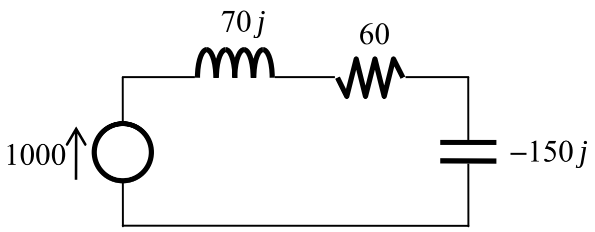


Fig. 4

(20 Marks)

The circuit impedance and current are

$$Z = 70j + 60 - 150j = 60 - 80j$$

$$I = \frac{1000}{Z} = \frac{1000}{60{-}80j} = \frac{1000}{\sqrt{60^2 + 80^2}\, e^{j\tan^{-1}\left(\frac{-80}{60}\right)}} = 10e^{j\tan^{-1}\left(\frac{4}{3}\right)} = 10e^{j0.9273}$$

The power consumed is

$$p = \mathrm{Re}\left[1000^* I\right] = 1000 \cdot 10\cos\left[\tan^{-1}\left(\frac{4}{3}\right)\right] = 10000\left(\frac{3}{5}\right) = 6000$$

The power factor is

$$\cos\left\{\arg\left[\frac{I}{1000}\right]\right\} = \cos\left[\tan^{-1}\left(\frac{4}{3}\right)\right] = 0.6 \text{ leading}$$

Q.5 In the circuit of Fig. 5, the capacitance of the capacitor is C and the frequency of operation is 20 kHz. Express the impedance of the capacitor in terms of C. Hence, find C so that the circuit as seen by the source becomes purely resistive. Under this condition when the circuit is said be in resonance, what can one say about the woltage across the capacitor?

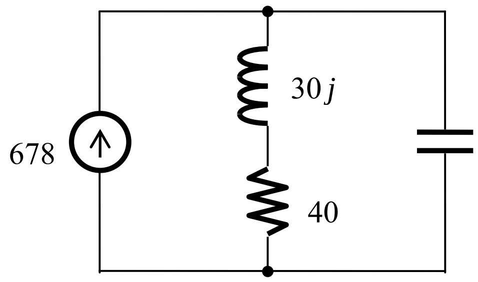


Fig. 5

(20 Marks)

The impedance of the capacitor and the circuit are given by

$$Z_C = \frac{1}{j2\pi(20000)C} = \frac{-j}{40000\pi C}$$

$$\frac{1}{Z} = \frac{1}{Z_C} + \frac{1}{40+30j} = j40000\pi C + \frac{40-30j}{50^2} = \frac{40}{50^2} + j\left(40000\pi C - \frac{30}{50^2}\right)$$

The circuit becomes purely resistive if

$$40000\pi C - \frac{30}{50^2} = 0 \quad \Rightarrow \quad C = \frac{30}{50^2(40000\pi)} = 0.0955\ \mu\text{F}$$

When this happens, $|1/Z|$ will be minimized or $|Z|$ will be maximized. Thus, if the current is fixed, the voltage across the capacitor will be maximized.